# THE BLOOD FLOW SIMULATIONS USING GENERALIZED NEWTONIAN MODELS

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## Introduction

A numerical investigation of the blood flow is presented in this paper. Human blood is assumed to be shear-thinning non-Newtonian fluid. The viscosity of blood decreases with increasing shear rate (velocity gradient). The shear-thinning viscosity model is used.
In this paper we have investigated the influence of the flow rate on the viscosity magnitude, that is generated by the shear-thinning viscosity model. There were performed a number of simulations with different flow rates to explore the dependencies between the width of the blood vessel and the flow rate of the blood.
We have assumed two-dimensional, steady flow of incompressible fluid. All the regimes of flow are considered to be laminar, the highest Reynolds number is about 300. Numerical method we have used is based on solving Navier-Stokes equations using Finite Volume Method.

## System of Generalized Navier-Stokes equations

$$P\mathsf{W}_t + \mathsf{F}_x + \mathsf{G}_y = \mathsf{R}_x + \mathsf{S}_y \tag{1}$$

where: $\mathsf{W} = (p, u, v)^T$ denotes the vector of unknowns (conservative quantities), $\mathsf{F} = (u, u^2+p, uv)^T$, $\mathsf{G} = (v, uv, v^2 + p)^T$ are the vectors of inviscid (convective) fluxes, $\mathsf{R} = (0, \eta u_x, \eta v_x)^T$, $\mathsf{S} = (0, \eta u_y, \eta v_y)^T$ are the vectors of viscous (diffusive) fluxes, $P = diag(0, 1, 1)$. Discretization of these equations is achieved using Finite Volume Method and MacCormack scheme, details can be found e.g. in [2] or [5].

## Calculation of viscosity

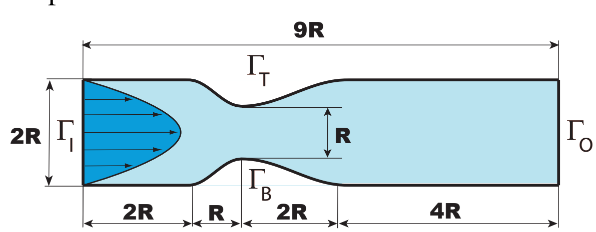
For computing variable viscosity there was used *Modified Cross Model*, where the viscosity decreases from $\eta_0$ to $\eta_\infty$. Such a model is created by fitting an experimental data [4]. Modified Cross Model is given by formula:

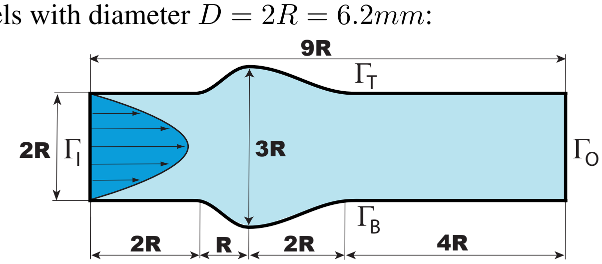
$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)\left[\frac{1}{[1+(\lambda\dot{\gamma})^m]^a}\right], \quad \dot{\gamma} = \frac{1}{2}\sqrt{\mathsf{D}:\mathsf{D}} = \frac{1}{2}\sqrt{\sum_{i,j} d_{i,j}^2}, \quad \mathsf{D} = \frac{1}{2}\left(\frac{\partial \mathbf{u_i}}{\partial x_j} + \frac{\partial \mathbf{u_j}}{\partial x_i}\right) \tag{2}$$

where: $\eta_0 = 0.056$ Pa.s, $\eta_\infty = 0.00345$ Pa.s, $\lambda = 3.736$ s, $m = 2.406$, $a = 0.254$. More information about blood viscosity models can be found e.g. in [4] or [5].

## Test geometries: axisymmertric stenosis & aneurysm

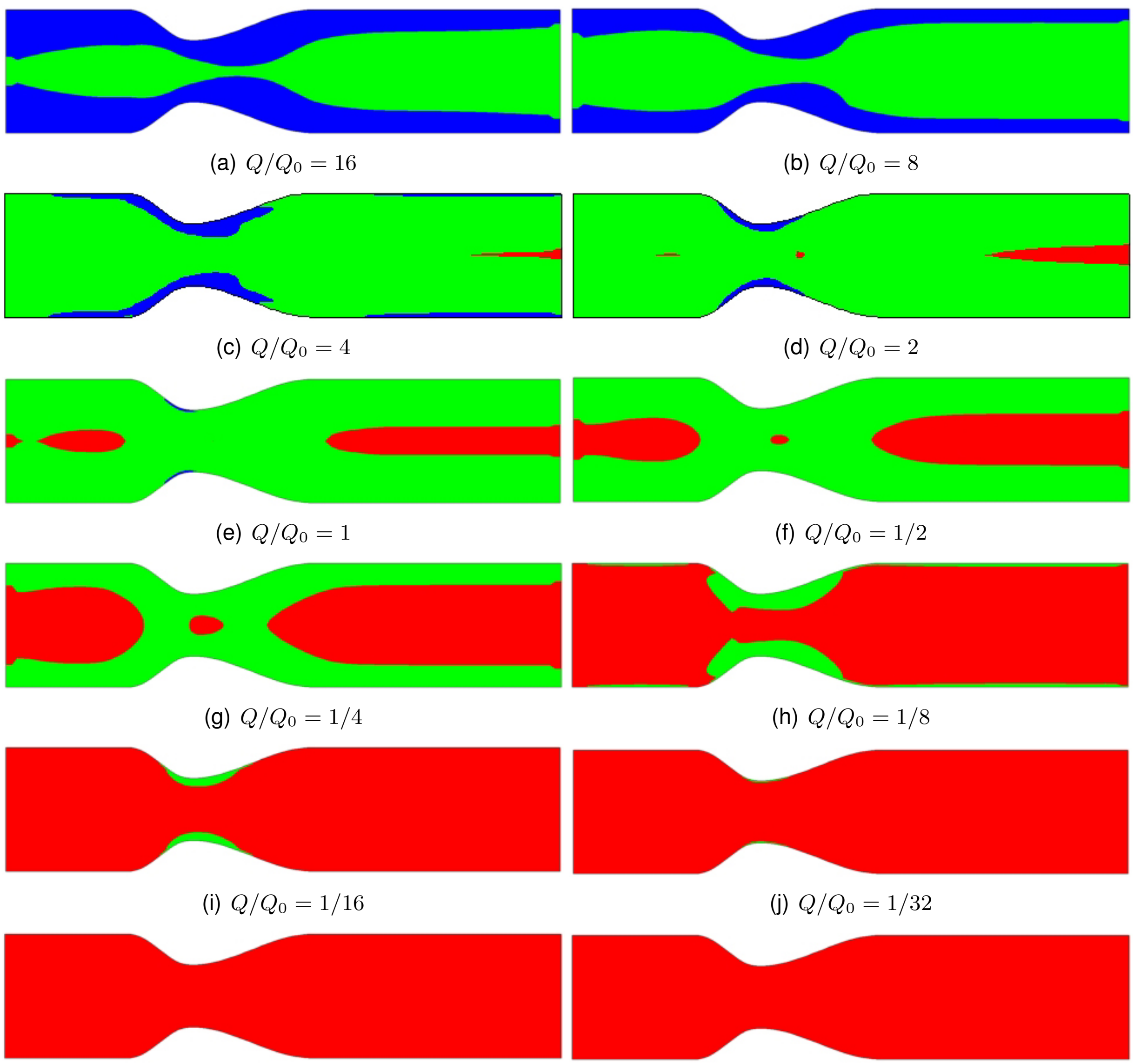
Both computational domains are cosine narrowed/widened channels to imitate vessel stenosis/aneurism. Computational domains are two-dimensional channels with diameter $D = 2R = 6.2mm$:



**Results**

There were performed twelve computations for decreasing flow rates: $Q/Q_0 = \frac{16}{1}, \frac{8}{1}, \frac{4}{1}, \frac{2}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}$, where $Q_0 = 2cm^2/s$. For transparent interpretation of the viscosity magnitude, lets introduce relative viscosity: $\bar{\eta} = (\eta(\dot{\gamma}) - \eta_\infty)/\eta_\infty$ as a measure of viscosity changes. We can find three viscosity regions with different characteristics. For $0 \leq \bar{\eta} < 1$ (blue area), the viscosity is close to $\eta_\infty$ and doesn't change much. This is Low-viscosity region, showing "pseudo-Newtonian" behavior. For $1 \leq \bar{\eta} < 10$ (green area), one can see Highly non-Newtonian region, where the viscosity strongly differs with changing shear rate. For $10 \leq \bar{\eta}$ (red area), there is High-viscosity region, where the viscosity is high due to the low shear rate (small velocity changes). In the following set of figures, one can see relative viscosity magnitude for some selected flow rates:



(a) $Q/Q_0 = 16$ (b) $Q/Q_0 = 8$

(c) $Q/Q_0 = 4$ (d) $Q/Q_0 = 2$

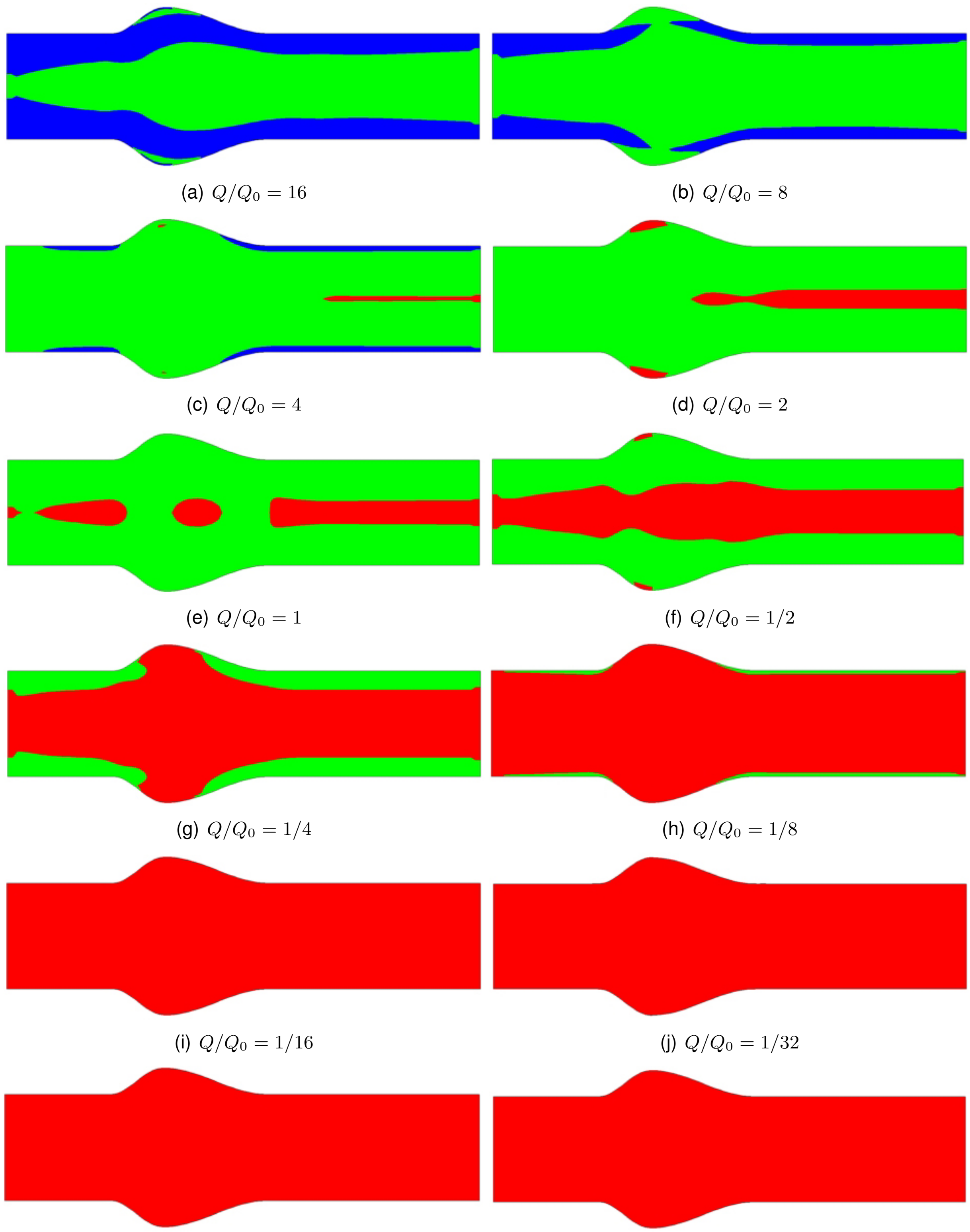
(e) $Q/Q_0 = 1$ (f) $Q/Q_0 = 1/2$

(g) $Q/Q_0 = 1/4$ (h) $Q/Q_0 = 1/8$

(i) $Q/Q_0 = 1/16$ (j) $Q/Q_0 = 1/32$

(k) $Q/Q_0 = 1/64$ (l) $Q/Q_0 = 1/128$

Figure 1: Relative viscosity magnitude, $\bar{\eta} = (\eta(\dot{\gamma}) - \eta_\infty)/\eta_\infty$, stenosis geometry



(a) $Q/Q_0 = 16$ (b) $Q/Q_0 = 8$

(c) $Q/Q_0 = 4$ (d) $Q/Q_0 = 2$

(e) $Q/Q_0 = 1$ (f) $Q/Q_0 = 1/2$

(g) $Q/Q_0 = 1/4$ (h) $Q/Q_0 = 1/8$

(i) $Q/Q_0 = 1/16$ (j) $Q/Q_0 = 1/32$

(k) $Q/Q_0 = 1/64$ (l) $Q/Q_0 = 1/128$

Figure 2: Relative viscosity magnitude, $\bar{\eta} = (\eta(\dot{\gamma}) - \eta_\infty)/\eta_\infty$, aneurism geometry

**Conclusion**

One can see that the *viscosity magnitude strongly depends on the flow rate* (having constant diameter of the channel). For higher flow rates the viscosity is low due to the high shear-rate. Decreasing flow rate the viscosity is increasing due to the low shear-rate.

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