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Spline boxes for shape optimization

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In shape optimization the aim is to minimize an objective functional which depends on the state variables and on the shape of domain Ω. The major constraint of the optimization problem (OP) is presented by a PDE; for example, a classical problem in mechanics is to find an optimal shape of an elastic body situated in Ω such that its compliance with respect to a given system of loads is minimized.

For solving numerically the OP, the shape of Ω can be parameterized in a number of ways. In any case, all shapes must be regular enough, so some smoothness constraints (also related to the FEM discretization) are to be prescribed. Splines have many advantages in this respect: they are smooth enough, although they allow for large variability of shapes which they describe.

Usually the shape is parameterized using spline patches which describe the surface $\partial\Omega$ and have been created using a convenient software package. The control points of the patches are treated as the optimization (design) variables, however, smoothness must be protected during optimization. Moreover, in order to handle the "interior nodes" of the finite element mesh, a mapping has to be constructed which relates the nodal positions with values of the design parameters. Typically, this can be done upon solving another PDE with a number of right-hand sides.

The *free-from deformation* method forms an alternative approach how to parameterize the shape, but also the whole domain, when necessary. It provides for free the manipulation with "interior mesh nodes" without any additional computations.

The Spline-Box S^I represents a 3D body $\mathcal{B}^I$. We consider a union of boxes: $\mathcal{B} = \sum_{I=1}^{NSB} \mathcal{B}^I$, such that the domain Ω, which is to be "designed", is a subset, i.e. $\Omega \subset \mathcal{B}$. The spline parameterization t is defined in the "initial" configuration $\mathcal{B}_0$. By $\{b\}$, $b = (b_r^{ijk})$, $r = 1, 2, 3$ we denote the control polyhedron and define map $S^I : t \rightarrow x$,

$$x = S^I(\{b\}, \{N\}, t) \equiv \left[\sum_{i=1}^{\bar{i}} \sum_{j=1}^{\bar{j}} \sum_{k=1}^{\bar{k}} b^{ijk} N^i(t_1) N^j(t_2) N^k(t_3) \right]_I \quad , \quad t = (t_r) \in \mathcal{B}_0^I \; ,$$

where $N^i(t_r)$, $t \in]a_r, b_r[$ are the B-spline basis functions. For a defined parameterization $t = x \in \mathcal{B}_0^I$, the Greville abscissae $\{g_r^{ijk}\}^I$ can be computed, so that the following identity property holds: $\mathcal{B}_0^I = S^I(\{g\}, \{N\}, \mathcal{B}_0^I)$.

Domain parameterization and design variables. In general, it is possible to couple spline-boxes with different representations of the common face, see Fig. 1.

This may be important when more variability of shape is required only in some subdomains of $\Omega \subset \mathcal{B}$. In Fig. 1, two attached spline boxes (labelled A and B) have different number of segments, therefore, the C^0-continuity constraint in the matrix form is $\boldsymbol{b}^B - \mathbf{C}\boldsymbol{b}^A = 0$. Such coupling conditions and other constraints on the control points $\{b\}$ lead to a system of linear equalities $B\{b\} = 0$, where $\mathrm{rank}(B) < \dim(\{b\})$. The design variables are introduced as multipliers (weights) of the kernel of matrix B: let $\{d\}^\alpha \in \mathrm{Ker}B$ for $\alpha = 1, \ldots, \bar{\alpha} \leq \mathrm{card}(\mathrm{Ker}B)$. We consider γ_α, $\alpha = 1, \ldots, \bar{\alpha}$ as the *design variables*, such that $\{b\} = \{g\} + \sum_{\alpha=1}^{\bar{\alpha}} \gamma^\alpha \{d\}^\alpha$, where $\{g\}$ describes the fixed initial configuration of spline-boxes.

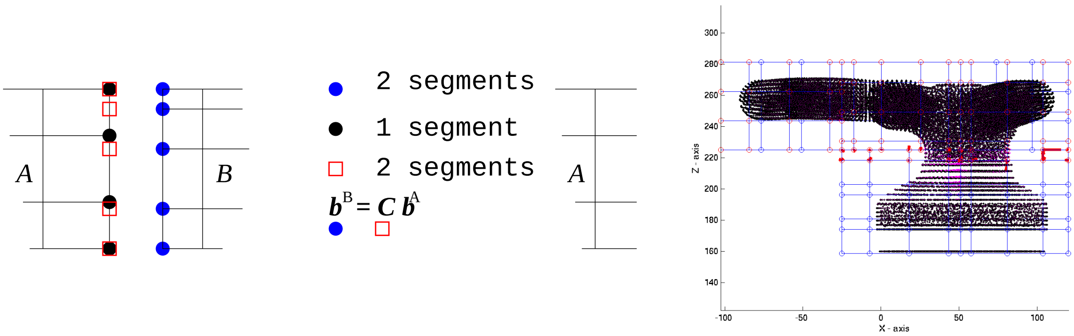

Fig. 1. *Left: Different segmentation of two sticked splines. Right: Complex structure embedded in the spline-boxes.*

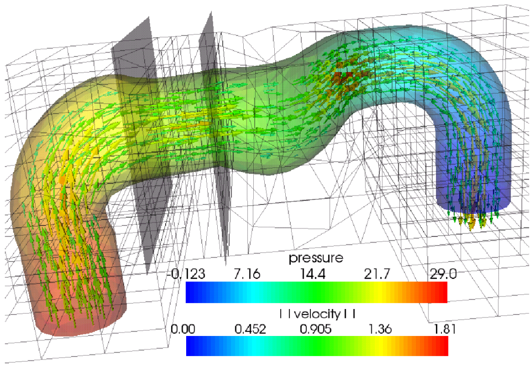

Fig. 2. *Spline-box parameterization of a channel geometry for optimization of flow. [2]*

Remarks. The spline-box parameterization of the shape allows for easy describing very complex geometries. It has been implemented [1] and tested in problems of incompressible flow optimization, where a whole channel was embedded into union of spline-boxes, see Fig. 2.

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[1] Rohan, E.: *SPBOX, User's guide*, NTC, Západočeská univerzita, Plzeň 2007.

[2] Rohan, E., Cimrman, R.: *Shape sensitivity analysis for flow optimization in closed channels.* In Proc. of the conf. Engineering Mechanics 2006, Svratka.