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NEWS AND NOTICES

SEVENTY YEARS OF PROFESSOR VLASTIMIL PTÁK
BIOGRAPHY AND INTERVIEW

On November 8, 1995 an outstanding Czech mathematician working in areas of operator theory and linear algebra, Professor Vlastimil Pták, celebrated his seventieth birthday.

Vlastimil Pták was born on November 8, 1925 in Prague. He studied mathematics and physics at the Charles University in Prague from the year 1945. Already during his studies he worked as a teaching assistant at the Czech Technical University. After obtaining his RNDr. (rerum naturalium doctor) degree he became a research student of Professor M. Katětov. In 1952 he joined the Central Mathematics Institute, later the Mathematics Institute of the Czechoslovak Academy of Sciences where he stayed until his retirement. V. Pták received his Candidate of Science degree (CSc., the equivalent of PhD.) in 1955 (thesis [6], [7]). In 1963 he defended his thesis for the degree of DrSc. (Doctor of Sciences). In 1965 he was appointed a full professor of mathematics at the Charles University.

V. Pták has been the head of the Functional Analysis Department at the Institute since 1960.

The seminar in functional analysis, which he has led for many years, has educated a number of research workers. The annual spring workshops in functional analysis, which he founded and whose organization he has led for twenty two years, represented a source of inspiration for numerous guests. For many years he has delivered a special course in functional analysis at the Charles University. He has also supervised many postgraduate students.

V. Pták is a member of the editorial boards for the Czechoslovak Mathematical Journal and for Linear Algebra and its Applications.

He has held important offices in the organization of Czechoslovak science, for example on committees for scientific degrees and on the board for national projects of basic research.

For his outstanding results in functional analysis, V. Pták was granted the Czechoslovak Federal Prize in 1966.

The author takes the opportunity of wishing Professor Vlastimil Pták good health and favourable conditions for his further creative work and joins a short interview about V. Pták's scientific results.

INTERVIEWER: Could you tell us when you decided to carry out research in functional analysis, and what your decision was based on?

PTÁK: My goal was to dig down to the fundamental principles which underlie mathematical analysis, to identify the common core of the problems of analysis. I came to the conclusion that the key issues to be understood were the continuity of inverse operations and the interchangeability of limiting processes. Functional analysis appeared to be the framework which might allow that understanding.

INTERVIEWER: In spite of your infinite-dimensional interests you never ceased to work in *algebra*.

PTÁK: Questions which combine the methods of analysis and algebra have always attracted me. It is fascinating when a question about infinitary processes is resolved by an argument of finite algebraic character or vice versa. Striking examples are provided by questions about the spectral radius. In this connection I have managed to throw some light on two significant problems. One concerns the notion of critical exponent, which relates the infinitary quantity $\lim \sqrt[n]{|a^n|}$ to its finite sections. The other was the development of an algebraic analogue of the theory of C^* algebras, in which the rôle of the norm is replaced by the purely algebraic function

$$p(x) = r(x^*x)^{\frac{1}{2}}$$

r being the spectral radius.

INTERVIEWER: We'll say more about that later. Could you tell us something about your proof of the *Jordan normal form* theorem?

PTÁK: Even as a student I noticed that methods based on quality, besides being effective in a geometric (or coordinate-free) approach to infinite-dimensional questions, also furnished a natural (and in my opinion, the simplest) way of establishing the Jordan form of a matrix.

INTERVIEWER: Work in *linear algebra* makes up a substantial part of your publications.

PTÁK: Although my main efforts were directed towards the theory of operators in infinite-dimensional spaces, I was always conscious of its close connections with matrix theory. A good example is the theory of the critical exponent in Hilbert space, where a finite problem leads in a natural way to infinite-dimensional matters.

The starting point was the following result [38][1]. If A is a linear operator on the n-dimensional Hilbert space such that $|A| \leqslant 1$ and $r(A) < 1$ then $|A^n| < 1$. My proof was based on the pigeonhole principle. Paper [58] offers a strengthening of this qualitative theorem to a quantitative result; as a technical tool to this end I introduced the infinite companion matrix.

[1] We refer here to the list of V. Pták's publications which occurred in [R4]. At the end of this paper we present the continuation of this list for approximately the last ten years.

The infinite companion matrix C_∞ (with n rows) [58], [97], [149], [152] is a matrix which has the identity matrix in its first n columns $c_0, \dots, c_{n-1}$ and continues in such a manner that each further column is a fixed linear combination of the preceding n columns:

$$c_{n+k} = \sum_{i=0}^{n-1} a_i c_{i+k}, \quad k = 0, 1, \dots$$

It follows that the n consecutive columns $c_k, \dots, c_{k+n-1}$ form the power C^k of the companion matrix C of the polynomial $\varphi(x) = x^n - \sum a_i x^i$. If $\varphi(0) \neq 0$, one can also adjoin columns with negative indices and extend the same property for negative powers. Explicit (combinatorial) formulae for all entries of C_∞ as polynomials in the coefficients of C are also available.

The infinite companion matrix represents a concrete explicit realization of a compression of the shift operator and establishes a connection with the dilation theory of D. Sarason and Sz. Nagy. Dilations have to be taken in a more general sense since C^∞ represents a projection operator which is not necessarily orthogonal as in the theory of Sz. Nagy. Paper [58], where the infinite companion was introduced, represents an independent and different approach to the dilation theory, based on the fact that the cone of positive matrices is generated by tensors of the form aa^*.

These considerations lead, in a natural manner, to the investigation of lifting intertwining relations. In a joint paper with P. Vrbová [135], we have defined generalized Hankel operators by an intertwining relation of the form $XT_1^* = T_2X$, T_1 and T_2 being two given contractions. It turned out that, in order to obtain a Nehari type theorem (in other words, for a lifting to exist), it is necessary to impose an additional condition, R-boundedness.

Besides the joint work with M. Fiedler, devoted to the analogous finite-dimensional relation $C^T H = HC$, which characterizes Hankel matrices, and to its generalization for polynomials with zeros at infinity, some of my papers use the idea of lifting intertwining relations to study Hankel, Bézout and Loewner matrices [131], in particular operators defined by Lyapunov type relations $X - SXC = W$ which may be used to characterize Bézoutians [123]. This yields explicit factorization formulae for polynomials in C, for the Schur-Cohn matrix etc.

Interviewer: You have written a series of joint papers on linear algebra with M. Fiedler. How did this long collaboration begin? How did your differing scientific orientations influence your work together, and how did you arrive at questions of common interest?

Pták: The intersection of our interests is quite wide, so there was ample scope for collaboration, and in our particular circumstances of the time it was not only desirable but practically forced. When the Mathematical Institute took over the building in Žitná Street Fiedler and I were assigned a shared office which was unsuited to independent individual study. There was only one window, and only one desk had natural light. The prevailing system of "research management" required physical presence throughout the working day. In those circumstances the best solution was for us to work together on topics of common interest, foremost among them the question of the convergence of iterative processes. Subsequently we both wrote independent papers on this topic.

At the time the question of the rate of convergence of iterative processes was a fascinating and topical one. Independently of similar research of R. S. Varga [R2] we discussed in [16] the dependence of the rate of convergence of the Gauss-Seidel iterative method (for a symmetric system) on the choice of splitting. In the following years, we invented (independently of Feingold and Varga) the notion of generalized norms [39]. It turned out that for studying generalized norms, the theory of M-matrices, introduced by A. Ostrowski and studied by Ky Fan, Koteljanskij, Varga and others, was an important tool. This led to the paper [37], which was intended as a collection of useful results. To our surprise, this paper became one of the most cited papers in linear algebra. Even our notations Z, for the class of matrices with nonpositive off-diagonal entries, and P, for the class of matrices with positive principal minors, have been universally accepted. We returned to this topic later in [51], [54] and [65]. In [53], analogies between the class of positive definite matrices and M-matrices were discussed.

Another class closely connected with the class of M-matrices is that of diagonally dominant matrices. In [54], we characterized this class and a new class of weakly diagonally dominant matrices.

In [44] we used the following idea for the numerical inversion of a large matrix of the form $I - A$. Let A be partitioned in an appropriate way so that entries within each block do not differ too much. Let $\tilde{A}$ be a matrix with constant blocks which approximates A. Then the inversion of $I - \tilde{A}$ is performed by inverting a small matrix (with as many rows as is the number of block rows in A). Inversion of $I - A$ is then obtained by an appropriate updating of $(I - \tilde{A})^{-1}$.

In [41] and [48], similar ideas lead to estimates of eigenvalues by the approximation of given irreducible matrices by reducible ones.

In recent years our collaboration has concentrated especially on the topics of Hankel, Bézout and Loewner matrices. As is well-known, these classes have connections with pairs of polynomials and their relation to interpolation problems. An important property of Hankel, and Bézout matrices, is their intertwining with the associated companion matrix and its transpose. We studied the corresponding relations in [133], [134]. In [137], we investigated the relation between Bézout and Loewner matrices, that is matrices of the form $((c_i - d_j)/(y_i - z_j))$.

I pursued the study of iterative processes and the spectral radius further in my individual work later. One theme here led to the introduction of the critical exponent. Another theme was a general study of iteration and its connection with the

Closed Graph Theorem, which ultimately bore fruit in the discovery of the method of nondiscrete induction. This technique applies also to nonlinear problems.

The Closed Graph Theorem essentially states that, subject to some completeness assumptions, every reasonably defined mapping is continuous. The attempt to give a quantitative strengthening of this theorem led to the "Subtraction Theorem". In the course of my investigations I came to realise that the rôle of the linearity of the mappings in question was inessential. The realisation led me to the notation of a *small function* and thereby to the method of nondiscrete induction, which permitted a sharpening of error estimates for a range of iterative processes.

INTERVIEWER: *Topological vector spaces* were one of your research interests. Among other things, your Ph.D. thesis was on this topic. How did you come to this subject?

PTÁK: At the time I wrote those papers there was a lively interest in non-metrizable linear spaces, stimulated by the needs of distribution theory and the fact that in operator theory one often has to work with non-metrizable topologies. I was fascinated by the beautiful and profound result of Banach to the effect that in complete normed linear spaces the solution of a linear equation is automatically a continuous function of the right hand side. The abstract core of this result is the Open Mapping Theorem. My research was aimed at an understanding of this theorem and of the rôle played by completeness. I hoped to discover in what form and under what assumptions an analogous statement would hold for non-metrizable spaces. The first step is to reformulate the Open Mapping Theorem.

In its classical formulation, "a continuous linear mapping of a Banach space E onto a Banach space F is open", the completeness of E and of F is used in a different manner; in fact, the completeness of F may be replaced by a weaker condition, the Baire property. The starting point was a formulation eliminating the rôle of the image space.

To this end, the classical Open Mapping Theorem may be formulated as follows: Consider a Banach space E and a normed linear space F with closed unit balls U and V respectively. Let T be a continuous linear mapping of E into F. If the closure of TU contains a neighbourhood of zero, let us say βV, then the open mapping theorem says that TU itself contains a neighbourhood of zero, in fact TU contains $\beta' V$ for any $\beta' < \beta$. For brevity, it will be convenient to have a name for mappings with this property: a mapping f will be called *almost open* if, for each x and each neighbourhood U of x the closure of $f(U)$ is a neighbourhood of $f(x)$. The classical Open Mapping Theorem may thus be formulated as follows. *Every almost open bounded linear mapping of a Banach space into a normed space is open.*

A series of my papers was devoted to the problem of characterizing those locally convex topological linear spaces E such that any continuous and almost open linear mapping of E into some locally convex topological vector space is already open. The results are [7]; for normed spaces E this property is equivalent to completeness. In a general case, this property implies completeness but is not equivalent to it. Spaces with this property (called *B-complete or Pt k spaces*) may be characterized by properties of their duals. The space E is B-complete if and only if every subspace M of the dual space E^d such that $M \cap U^0$ is w^*-closed for any neighbourhood of zero U is already w^*-closed. An analogous characterization of completeness (obtained independently of A. Grothendieck) differs from this one in that this implication is postulated for hyperplanes M only. By considering spaces $C(T)$ of continuous functions in the compact open topology and relating topological properties of T and $C(T)$ it was possible to construct T for which $C(T)$ is complete but not B-complete; as well as a complete $C(T)$ with an incomplete quotient. The theory and its further developments are described in detail in a section of the monograph on linear spaces by G. Köthe.

INTERVIEWER: What is the connection between these results and your work on the method of *non-discrete induction* in *nonlinear analysis*?

PTÁK: At a first glance I suppose it is unclear how my work in nonlinear analysis follows logically from a study of the Closed Graph Theorem.

The investigation of the Closed Graph Theorem may be also pursued in another direction: by staying within the realm of metrizable spaces but striving for a quantitative strengthening. The starting point was the so called *Subtraction Theorem*, a quantitative version of the Open Mapping Theorem. In a note [14] published in 1956 the following quantitative improvement of the classical theorem was established. Given two Banach spaces E and F with closed unit balls U and V and a linear mapping $T\colon E \to F$ such that $TU + \alpha V \supset \beta V$ for some numbers $0 < \alpha < \beta$, then

$$TU \supset (\beta - \alpha)V$$

(This "optical illusion" suggests that the neighbourhoods are being subtracted, and it explains the name.) The classical assumption $(TU)^- \supset \beta V$ is thus weakened; elements of βV do not need to be approximated by elements of TU arbitrarily well. It suffices to have a much less precise approximation, the only essential condition being that $\alpha < \beta$.

To obtain a nonlinear analogue it was necessary to find an appropriate generalization of the condition $\alpha < \beta$. In the proof of the Subtraction Theorem it was already obvious that the assumption $\alpha < \beta$ was used in the form $\frac{\alpha}{\beta} < 1$ with the consequence that the geometric series with this quotient converge.

It is not at all obvious how to obtain an analogue of the condition $\frac{\alpha}{\beta} < 1$ in the absence of linearity. To obtain a suitable generalization it is necessary to interpret the quotient of the two numbers α, β as a quotient in the sense of composition of functions of the two linear functions on the positive axis $a(t) = \alpha t$ and $b(t) = \beta t$. The smallness of $\frac{\alpha}{\beta}$ has to be replaced by the requirement that the quotient function is a small function in the sense that the sum of its iterates converges. As matter of speaking, the classical theory has shifted one level up, with the multiplication of numbers being replaced by a composition of functions [115].

A positive function w defined on the positive axis is said to be a small function (or a rate of convergence) if the series

$$t + w(t) + w\big(w(t)\big) + \ldots = s(t)$$

is convergent (in particular, if w is the linear function $w(t) = \alpha t$ then w is small if and only if $\alpha < 1$). The functions w and s are connected by the functional equation $s(t) - t = s\big(w(t)\big)$, the nonlinear analogue of the relation for the geometric series

$$\frac{1}{1-\alpha} - 1 = \frac{\alpha}{1-\alpha}.$$

The nonlinear version of the Subtraction Theorem may now be formulated [80] as follows:

Let (E, d) be a complete metric space and w a rate of convergence. For each $t > 0$ let $Z(t)$ be a subset of E and denote by $Z(0)$ the limit of the family $Z(\cdot)$

$$Z(0) = \bigcap_{s>0} \Big(\bigcup_{r \leqslant s} Z(r) \Big)^{-}.$$

Suppose that

$$Z(t) \subset \bigcup \big(Z\big(w(t)\big), t\big)$$

for each $t > 0$. Then

$$Z(t) \subset \bigcup \big(Z(0), s(t)\big)$$

for each $t > 0$.

These ideas form the basis of the method of *a nondiscrete induction* [80], [142] which reduces the investigation of the convergence of iterative processes to the problem of finding a suitable rate of convergence w so that

$$|x_{n+1} - x_n| \leqslant w(|x_n - x_{n-1}|)$$

for all n. As a consequence this yields estimates for the solution x_*

$$|x_* - x_n| \leqslant s(|x_n - x_{n-1}|).$$

The function w is obtained as the solution of a system of certain functional inequations; if these inequations describe the essential features of the process, the solution

w and the corresponding s yield sharp estimates and optimal convergence criteria. Thus, for instance, the Newton process [79], [81] is described by

$$w(t) = \frac{t^2}{2(t^2 + a)^{1/2}}$$

The advantage of replacing the order of convergence (a mere number) by a function consists in a closer fit of the estimates. They can be sharp throughout the whole process, not only asymptotically.

INTERVIEWER: Some of your theorems depend on combinatorial principles. Is *combinatorics* one of your favourite disciplines?

PTÁK: I have never studied combinatorics systematically. There have been some rewarding instances, however, when I have managed to identify and clarify a combinatorial basis for the problems from other branches of mathematics. This has happened for two circles of problems. One is the question of interchanging repeated limits; its combinatorial core is far from obvious.

INTERVIEWER: We'll come back to that. For an application to matrices you had to relate the *combinatorial structures of a matrix and its powers*.

PTÁK: Yes, and an unusual approach to the notion of an oriented graph was helpful here. We define an oriented graph to be an additive mapping $\varphi\colon \exp(N) \to \exp(N)$, that is, one which satisfies, for any $A, B \subset N$,

$$\varphi(A \cup B) = \varphi(A) \cup \varphi(B)$$

We say that φ is irreducible if there is no nontrivial $A \subset N$ such that $\varphi(A) \subset A$. For such graphs one of the following alternative holds:

1) φ^q is a complete graph for sufficiently large q, or

2) φ has a cyclic decomposition.

Sedláček and I subsequently discovered that the length of this cycle is equal to the least common multiple of the lengths of the cycles in φ. A slightly weaker version of this result appeared almost simultaneously in [R2].

We associate with a given matrix M a directed graph in the above sense by defining

$$\varphi(A) = \{j\colon \exists i \in A, m_{ij} \neq 0\}.$$

We call this the graph of M. For a non-negative matrix M the graph of M^k is the k-th iterate of the graph of M. The heart of a series of results about non-negative matrices is precisely the stated combinatorial result about graphs.

INTERVIEWER: How are the *nuclear waste disposal problem*, *Fubini's theorem* and *weak compactness* related?

PTÁK: It is a little obscure how results on the interchange of limits, like Lebesgue's dominated convergence theorem and Fubini's theorem, connect with results on the combinatorial structure of families of finite sets.

The main result makes it possible to reduce theorems on the exchange of two limit operations to a combinatorial lemma on families of finite sets. This approach is based on a combinatorial existence theorem for convex means. It also makes it possible to eliminate the integration theory from results on weak compactness.

I formulated the combinatorial principle in question in 1959 [28]. One way of stating it is in the form of the "nuclear waste disposal problem".

We are given a set S and a certain amount of radioactive waste to be disposed of. The material is to be divided into a finite number of parts to be located at a finite number of suitably chosen points in S. Furthermore, we are given a family W of subsets of S; we may think of these subsets as being protected areas to be treated with special consideration; care should be taken that the total amount of waste contained in each of the protected areas is as low as possible.

In general, the sets of the family W may overlap in general, since there will be many reasons for a subset of S to be included in W; accordingly, a given point $s \in S$ will belong, in general, to more than one set $w \in W$. If we assume that the total mass to be distributed is one, a weight distribution may be viewed as a convex mean. Given a weight distribution λ, denote, for each $w \in W$, by $\lambda(w)$ the total amount of waste contained in the set w. The problem is to find weight distributions for which the maximum load $\lambda(w)$ as w ranges over W is as small as possible. Obviously the infimum

$$\inf_{\lambda} \max_{w} \lambda(w)$$

contains important information about the combinatorial structure of the family W.

Families for which this infimum is zero are particularly important: their structure may be used to prove an extension theorem for separately continuous functions with many applications in analysis. This approach makes it possible to reduce theorems about the exchange of two limit operations to simple assertions about families of finite sets [28].

To state the theorem [42], [47], we need a notation: If T is a completely regular topological space, denote by $C(T)$ the Banach space of all bounded continuous functions on T. Clearly T may be identified with a subset of the dual space $C(T)^d$ taken in its weak star topology; thus T is embedded in a space with an additional linear structure.

Given two completely regular topological spaces S and T and a bounded separately continuous function f on $S \times T$ we may ask under what condition it may be extended to a separately continuous bilinear form in $C(S)^d \times C(T)^d$. The theorem says that such an extension exists if and only if f satisfies the double limit condition of Banach-Grothendieck.

Two points should be stressed here:

(1) the proof is purely combinatorial;

(2) the countable character of the condition, the conclusion being a statement about topologies not metrizable in general.

The extension theorem contains, as an immediate consequence, e.g. the theorem of Eberlein and the Krein theorem on convex hulls of weakly compact sets.

INTERVIEWER: Let's come back to problems on the spectral radius. You have spoken of *Hermitian algebras*.

PTÁK: I succeeded in clarifying the algebraic essence of some basic facts in the theory of C^*-algebras. It turns out that these results hold under the purely algebraic assumption that an algebra be Hermitian, that is, that the spectra of self-adjoint elements are real.

The classical theorem of Gelfand and Najmark (improved by Glimm and Kadison) states that a Banach algebra with involution satisfying the condition $|x^*x| = |x^*|\,|x|$ for every x is a C^* algebra, in other words, is isometrically isomorphic to a closed selfadjoint subalgebra of $B(\mathcal{H})$ for a suitable Hilbert space $\mathcal{H}$.

It turns out that a surprising number of the basic properties of C^* algebras may be deduced from a much weaker assumption of a purely algebraic character.

Consider a Banach algebra with an involution (an additive mapping $x \to x^*$ so that $x^{**} = x$, $(\alpha x)^* = \alpha^* x^*$, $(xy)^* = y^* x^*$). No assumptions on the relation between the involution and the topology of the algebra are made, in particular, continuity of the involution is not assumed. The purely algebraic assumption of hermiticity makes it possible to show — by purely algebraic methods — that the most important properties of C^* algebras remain valid even for this wider class of algebras.

A Banach algebra with an involution is said to be hermitian if selfadjoint elements have real spectra. The theory is based on the investigation of the function $x \to r(x^*x)^{1/2}$; the basic result is the following equivalence [68], [71]. A Banach algebra with involution is hermitian if and only if $r(x) \leqslant r(x^*x)^{1/2}$ for every x. This condition represents an algebraic analogue of the C^* condition $|x^*|\,|x| = |x^*x|$. (Indeed, the condition $r(x) \leqslant r(x^*x)^{1/2}$ may be rewritten in the form $r(x^*)r(x) \leqslant r(x^*x)$ and the condition $|x^*|\,|x| = |x^*x|$ is equivalent to the inequality $|x^*|\,|x| \leqslant |x^*x|$). In a similar manner we find that the function $p(x) = r(x^*x)^{1/2}$ is a natural algebraic analogue of the norm in a C^*-algebra and that many metric properties of C^*-algebras remain valid for hermitian algebras if the norm is replaced by p. One of the most important ones: a linear form f such that $f(I) = I$ is a state if and only if it is dominated by p. Now p can be shown to be subadditive if and only if the algebra is Hermitian. It follows that, in a Hermitian algebra, the Hahn-Banach theorem may be applied to obtain sufficiently many states to construct a representation on Hilbert space.

INTERVIEWER: The spectral radius plays an essential rôle in the concept of the critical exponent.

Pták: The motivation of this theory lies in an attempt to mathematically formulate some ideas concerning convergence of iterative procedures. Although we do possess theoretical criteria of convergence for many iterative processes, in numerical practice it is often simpler to ignore the criteria and start the computation; with some luck we observe that after a few steps the differences between the consecutive steps become smaller and smaller and the process converges.

If, on the other hand, the sequence of the first n steps of the process shows no sign of convergence, we are confronted with the following problem: should we continue, in the hope of obtaining an improvement later on or does the behaviour of the first n steps justify the conclusion that the process diverges (and should, consequently, be abandoned)? To make this decision, it is necessary to have some estimates for the number of steps in the process sufficient to distinguish between convergence and divergence. To express it somewhat loosely, we are looking for a number q with the following property: if the process does not start converging before the q-th step then it is divergent. This leads, in a natural manner, to the notion of a critical exponent.

In the particular case of a process $x_{r+1} = Ax_r + y$ in linear algebra, a number of results have been obtained. The precise definition [38] is as follows:

The critical exponent of a Banach space E is the smallest integer q for which the following implication holds: if $T \in B(E)$ and $|T| \leqslant 1$ then $r(T) < 1$ if and only if $|T^q| < 1$.

This definition appears first in the paper [38] which contains a proof that the critical exponent of a Hilbert space equals its dimension. An earlier result in a joint paper with J. Mařík may now be interpreted as the statement that the critical exponent of n-dimensional ℓ_∞ space equals $n^2 - n + 1$.

The definition of the critical exponent admits a quantitative reformulation: given a Banach space with critical exponent q and a number $0 < r < 1$, compute

$$\sup\{|T^q|;\ |T| \leqslant 1,\ r(T) \leqslant r\}.$$

For n-dimensional Hilbert space it is possible to identify the operator T for which the maximum is attained [58]. It is a Toeplitz operator that corresponds to a Möbius function which depends on r. There are deep connections with the theory of complex functions and operator valued Möbius functions. Also, lower estimates for the spectral radius in terms of $|T|$ and $|T^n|$ may be deduced from the theory [104].

Interviewed by Zdeněk Vavřín. The author is thankful to Professor N.I. Young for translation of parts of the interview.

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