

Decentralized Supervisory Control with Communicating Supervisors Based on Top-Down Coordination Control

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Abstract—In this paper we present a new approach to decentralized supervisory control of large automata with communicating supervisors. We first generalize the recently developed top-down architecture of multilevel coordination control with a hierarchical structure of groups of subsystems, their respective coordinators and supervisors. Namely, in the case where the equivalent conditions for achieving a specification language fail to be satisfied, we propose sufficient conditions for a distributed computation of the supremal achievable sublanguage. We then apply the obtained constructive results of multilevel coordination control to decentralized supervisory control with communication, where local supervisors of subsystems within a group communicate with each other via the coordinator of the group. Our approach is illustrated by an example.

I. INTRODUCTION

Decentralized supervisory control has been introduced in [12] aiming at decreasing the computational complexity of supervisory control of large automata without a product structure based on a single (centralized) supervisor. As it turned out that *coobservability* of a specification language, the necessary condition to achieve the specification in decentralized supervisory control, is too strong, two different approaches have been proposed. The first one is based on communication between supervisors to achieve coobservability with respect to observable alphabets enriched by communicated event occurrences, cf. [1] and [10]. The second one consists in proposing new and more general decentralized supervisory control architectures that lead to weaker notions of coobservability. The original notion of coobservability [12] has been called conjunctive and permissive (C & P), while an alternative architecture called disjunctive and antipermissive (D & A) has been proposed in [14] together with their combination. Among weaker concepts of coobservability, one should cite a general architecture combining both architectures [14] or even more general architectures with possible several levels of inferencing leading to conditional versions of coobservability [15].

Coordination control [8] can be seen as a trade-off between a purely local (modular) control synthesis that often fails in achieving sufficiently permissive supervisors and leads to blocking, and a global control synthesis that is too expensive from the computational complexity viewpoint. We have extended coordination control to the multilevel setting with a hierarchical structure of groups of subsystems, their

respective coordinators, and supervisors in [7]. The top-down approach proposed therein along with the corresponding notions of conditional decomposability and conditional controllability enables to compute the supervisors only at the lowest level. Unlike centralized coordination, supervisors of subsystems communicate only within the groups on the same level of hierarchy via the group coordinators located on the next upper level of hierarchy.

In [5], we have proposed a generic approach that consists in applying the results of coordination control of automata with a synchronous-product structure to decentralized control of automata without an explicit product structure. It is based on the over-approximation of the automaton without a product structure by a product of its natural projections to the alphabet of local supervisors. However, a communication structure is not interesting, because all local supervisors communicate with each other via a coordinator.

In this paper we benefit from recent results of multilevel coordination control. First, earlier existential results [7] are extended to a construction procedure that computes the least restrictive solution for the top-down architecture of multilevel coordination control. These results are then applied to the original decentralized control problem, which is solved using an underlying communication scheme, where local supervisors of subsystems within a group communicate with each other via a coordinator of the group. The optimal solution obtained by coordination control leads to a (possibly non-optimal) solution of the original decentralized control problem.

II. EXISTENTIAL RESULTS OF MULTILEVEL COORDINATION CONTROL

Basic notions and notational conventions are first recalled. The free monoid of strings over an alphabet A is denoted by A^* . A language is a subset of A^* . The prefix closure of a language $L \subseteq A^*$ is the set of all prefixes of all its strings and is denoted by $\bar{L} = \{w \in A^* \mid \text{there exists } v \in A^* \text{ such that } wv \in L\}$; L is prefix-closed if $L = \bar{L}$. All languages considered in this paper are assumed to be prefix-closed.

A *generator* is a quadruple $G = (Q, A, f, q_0)$ consisting of a finite set of *states* Q , a finite *alphabet* A , a *partial transition function* $f : Q \times A \rightarrow Q$, and an *initial state* $q_0 \in Q$. The function f can be extended in the standard way to strings, i.e. $f : Q \times A^* \rightarrow Q$. We recall that $L(G) = \{s \in A^* \mid f(q_0, s) \in Q\}$ is called the *generated language* of G .

A *controlled generator* over an alphabet A is a triple (G, A_c, Γ) , where G is a generator over A , $A_c \subseteq A$ is a set of *controllable events*, $A_u = A \setminus A_c$ is the set of *uncontrollable*

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events, and $\Gamma = \{\gamma \subseteq A \mid A_u \subseteq \gamma\}$ is the *set of control patterns*. A *supervisor* for a controlled generator (G, A_c, Γ) is a map $S : L(G) \rightarrow \Gamma$. The *closed-loop system* associated with the controlled generator (G, A_c, Γ) and the supervisor S is defined as the minimal language $L(S/G)$ such that $\varepsilon \in L(S/G)$ and, for any $s \in L(S/G)$ with $sa \in L(G)$ and $a \in S(s)$, sa belongs to $L(S/G)$.

A language $K \subseteq A^*$ is *controllable* with respect to L and A_u if $KA_u \cap L \subseteq K$.

A *projection* $P : A^* \rightarrow B^*$, for $B \subseteq A$, is a homomorphism defined as $P(a) = \varepsilon$, for $a \in A \setminus B$, and $P(a) = a$, for $a \in B$. The *inverse image* of P , denoted by $P^{-1} : B^* \rightarrow A^*$, is defined as $P^{-1}(w) = \{s \in A^* \mid P(s) = w\}$. These definitions can be extended to languages. For alphabets $A_i, A_j, A_\ell \subseteq A$, we use P_ℓ^{i+j} to denote the projection from $(A_i \cup A_j)^*$ to A_ℓ^* . If $A_i \cup A_j = A$, we simply write P_ℓ . Moreover, $A_{i,u} = A_i \cap A_u$ denotes the set of locally uncontrollable events. For a generator G and a projection P , $P(G)$ denotes the minimal generator such that $L(P(G)) = P(L(G))$. The reader is referred to [2] for a construction.

Let G be a generator over an alphabet A . Given a specification $K \subseteq L(G)$, the aim of supervisory control is to find a supervisor S such that $L(S/G) = K$. Such a supervisor exists if and only if K is controllable with respect to $L(G)$ and A_u , see [2].

The synchronous product of languages $L_i \subseteq A_i^*$, for $i = 1, \dots, n$, is defined as $\|_{i=1}^n L_i = \cap_{i=1}^n P_i^{-1}(L_i) \subseteq A^*$, where $A = \cup_{i=1}^n A_i$, and $P_i : A^* \rightarrow A_i^*$ are projections to local alphabets. The corresponding synchronous product of generators G_i (see [2] for definition) satisfies $L(\|_{i=1}^n G_i) = \|_{i=1}^n L(G_i)$.

A projection $Q : A^* \rightarrow B^*$ is an *L-observer* for a language $L \subseteq A^*$ if, for every $t \in Q(L)$ and $s \in L$, $Q(s)$ is a prefix of t implies that there exists $u \in A^*$ such that $su \in L$ and $Q(su) = t$, cf. [13].

We need the following obvious lemma and results below.

Lemma 1: For any language $L \subseteq A^*$ and projections $P_1 : A^* \rightarrow B_1^*$ and $P_2 : A^* \rightarrow B_2^*$ with $B_2 \subseteq B_1 \subseteq A$, it holds that $P_1(L) \| P_2(L) = P_1(L)$. ■

Lemma 2 (Lemma 4.3 in [3]): Let $K \subseteq L$ be controllable with respect to L and A_u . If the natural projection $P : A^* \rightarrow A_o^*$ is an *L-observer* and OCC for L , then $P(K)$ is controllable with respect to $P(L)$ and $A_u \cap A_o$. ■

Lemma 3 (Proposition 4.6 in [3]): For $i = 1, \dots, n$, let $K_i \subseteq L_i$ be controllable with respect to $L_i \subseteq A_i^*$ and $A_{i,u}$, then $\|_{i=1}^n K_i$ is controllable with respect to $\|_{i=1}^n L_i$ and $\cup_{i=1}^n A_{i,u}$. ■

Now we recall the basic notions of coordination control [8]. A language K over $\cup_{i=1}^n A_i$ is *conditionally decomposable with respect to alphabets* $(A_i)_{i=1}^n$ and A_k , where $\cup_{1 \leq i, j \leq n}^{i \neq j} (A_i \cap A_j) \subseteq A_k \subseteq \cup_{i=1}^n A_i$, if

$$K = \|_{i=1}^n P_{i+k}(K),$$

for projections P_{i+k} from $\cup_{j=1}^n A_j$ to $A_i \cup A_k$, for $i = 1, \dots, n$. Alphabet A_k is referred to as a coordinator alphabet and satisfies the *conditional independence* property, that is, A_k includes all shared events: $\cup_{1 \leq i, j \leq n}^{i \neq j} (A_i \cap A_j) \subseteq A_k$. This has the following well known impact.

Lemma 4 ([3]): Let $P_k : A^* \rightarrow A_k^*$ be a projection, and let L_i be a language over A_i , for $i = 1, \dots, n$, and $\cup_{1 \leq i, j \leq n}^{i \neq j} (A_i \cap A_j) \subseteq A_k$. Then $P_k(\|_{i=1}^n L_i) = \|_{i=1}^n P_k(L_i)$. ■

The idea of coordination control is to first construct a supervisor S_k such that the closed-loop system $L(S_k/G_k)$ satisfies the "coordinator part" of the specification given by $P_k(K)$ and then local supervisors S_i for plants $G_i \| (S_k/G_k)$, for $i = 1, \dots, n$, such that the closed-loop system $L(S_i/[G_i \| (S_k/G_k)])$ satisfies the corresponding part of the specification given by $P_{i+k}(K)$. Conditional controllability along with conditional decomposability form an equivalent condition for a language to be achieved by the closed-loop system within our coordination control architecture, see below.

A language $K \subseteq L(G_1 \| G_2 \| \dots \| G_n \| G_k)$ is *conditionally controllable* for generators G_1, G_2, \dots, G_n and a coordinator G_k and uncontrollable alphabets $A_{1,u}, A_{2,u}, \dots, A_{n,u}$, and $A_{k,u}$ if (1) $P_k(K)$ is controllable with respect to $L(G_k)$ and $A_{k,u}$ and (2) $P_{i+k}(K)$ is controllable with respect to $L(G_i) \| P_k(K)$ and $A_{i+k,u} = A_{i+k,u} = (A_i \cup A_k) \cap A_u$, for $i = 1, 2, \dots, n$.

Recall that every conditionally controllable and conditionally decomposable language is controllable [8]. The main existential result of [8] states that for a specification $K \subseteq A^*$ that is conditionally decomposable, there exist supervisors S_1, S_2, \dots, S_n , and S_k such that $\|_{i=1}^n L(S_i/[G_i \| (S_k/G_k)]) = K$ if and only if K is conditionally controllable.

III. CONSTRUCTIVE RESULTS OF TWO-LEVEL COORDINATION CONTROL

In this section we assume that $G = G_1 \| G_2 \| \dots \| G_n$ and that the subsystems are organized into m groups I_j , for $j = 1, 2, \dots, m$. The notation

$$A_{I_r} = \bigcup_{i \in I_r} A_i$$

is used in the paper. Here P_{I_r} denotes the projection $P_{I_r} : A^* \rightarrow A_{I_r}^*$. The notation for a projection to extended group events $P_{I_r+k} : A^* \rightarrow (A_k \cup A_{I_r})^*$ should be self-explanatory. We have introduced the corresponding notion of conditional decomposability in [7].

Definition 5 (Two-level conditional decomposability):

A language $K \subseteq A^*$ is called *two-level conditionally decomposable* with respect to alphabets A_1, A_2, \dots, A_n , high-level coordinator alphabet A_k , and low-level coordinator alphabets $A_{k_1}, A_{k_2}, \dots, A_{k_m}$ if

$$K = \|_{r=1}^m P_{I_r+k}(K) \quad \text{and} \quad P_{I_r+k}(K) = \|_{j \in I_r} P_{j+k_r+k}(K)$$

for $r = 1, 2, \dots, m$. □

Remark 6: Unlike the original approach in [7] we propose the following simplification. Instead of using both low-level coordinator alphabets A_{k_j} , $j = 1, 2, \dots, m$, and high-level coordinator alphabet A_k , we will only use low-level coordinator alphabets A_{k_j} , $j = 1, 2, \dots, m$. We recall that A_k contains only events shared between different groups of subsystems, that is, $A_k \supseteq \bigcup_{k, \ell \in \{1, 2, \dots, m\}}^{k \neq \ell} (A_{I_k} \cap A_{I_\ell})$, which is typically a much smaller set than the set of shared events (between two or more subsystems). Thus, for $j = 1, 2, \dots, m$, we define new low-level coordinator alphabets $A_{k_j} := A_k \cup A_{k_j}$ by putting

into the alphabets of group coordinators A_{k_j} also events from the global coordinator set (if this is nonempty). This is possible, because only prefix-closed languages are used, which means that no high-level coordinators for nonblocking are needed. We recall that in the prefix-closed case the coordinators are actually determined by the corresponding alphabets from Definition 5 as a projection of the plant to these alphabets. This simplification is used because this paper is technically involved.

Problem 7 (Two-level coordination control problem):

Let generators G_1, G_2, \dots, G_n be over the alphabets A_1, A_2, \dots, A_n , respectively, and consider the modular system as their synchronous product $G = G_1 \| G_2 \| \dots \| G_n$ along with the two-level hierarchical structure of subsystems organized into groups I_j , for $j = 1, 2, \dots, m$ and $m \leq n$, on the low level. The synchronous products $\|_{i \in I_j} G_i$, for $j = 1, 2, \dots, m$, then represent the m high-level systems. Coordinators G_{k_j} are associated to groups of subsystems $\{G_i \mid i \in I_j\}$, for $j = 1, 2, \dots, m$. The two-level coordination control problem consists in synthesizing supervisors S_i for any low-level systems G_i , for $i = 1, 2, \dots, n$, and higher-level supervisors S_{k_j} supervising the group coordinator G_{k_j} , for $j = 1, 2, \dots, m$, such that the specification is met by the closed-loop system. Then the two-level coordinated and supervised closed-loop system is

$$\|_{j=1}^m \|_{i \in I_j} L(S_i / [G_i \| (S_{k_j} / G_{k_j})]). \quad \triangleleft$$

For a specification K , the coordinator G_{k_j} of the j -th group of subsystems $\{G_i \mid i \in I_j\}$ is computed as follows.

- 1) Set $A_{k_j} = \bigcup_{k, \ell \in I_j}^{k \neq \ell} (A_k \cap A_\ell)$ to be the set of all shared events of systems from the group I_j .
- 2) Extend A_{k_j} so that $P_{I_j+k}(K)$ is conditional decomposable with respect to $(A_i)_{i \in I_j}$ and A_{k_j} , for instance using a method described in [6].
- 3) Let coordinator $G_{k_j} = \|_{i=1}^n P_{k_j}(G_i)$.

With the definition that $A_k \subseteq A_{k_j}$ described in Remark 6, we can simplify $L(G_k) \| L(G_{k_j})$ of [7] to $L(G_{k_j})$. Indeed, by our choice of coordinators $L(G_k) \| L(G_{k_j}) = P_k(L) \| P_{k_j}(L) = P_{k_j}(L) = L(G_{k_j})$, where $L = \|_{i=1}^n$.

Therefore, instead of both low-level coordinators G_{k_j} , for $j = 1, 2, \dots, m$, for subsystems belonging to individual groups $\{G_i \mid i \in I_j\}$ and high-level coordinators G_k that coordinate the different groups, we are using only low-level (group) coordinators G_{k_j} , but over larger alphabets compared to [7].

Since the only known condition ensuring that the projected generator is smaller than the original one is the observer property [13] we might need to further extend alphabets A_{k_j} so that projection P_{k_j} is an $L(G_i)$ -observer, for all $i \in I_j$.

We assume that the specification is prefix-closed, hence the blocking issue is not considered in this paper. Blocking can be handled using coordinators for nonblockingness studied in [8].

The key concept is the following.

Definition 8 ([7]): Consider the setting and notations of Problem 7, and let G_k be a coordinator. A language $K \subseteq L(\|_{i=1}^n G_i)$ is *two-level conditionally controllable* with respect

to generators G_1, G_2, \dots, G_n , local alphabets A_1, A_2, \dots, A_n , low-level coordinator alphabets $A_{k_1}, A_{k_2}, \dots, A_{k_m}$, and uncontrollable alphabet A_u if

- 1) $P_{k_j}(K)$ is controllable with respect to $L(G_{k_j})$ and $A_{k_j,u}$,
- 2) for $j = 1, 2, \dots, m$ and $i \in I_j$, $P_{i+k_j}(K)$ is controllable with respect to $L(G_i) \| P_{k_j}(K)$ and $A_{i+k_j,u}$. \square

Note that we have simplified the original version of two-level conditional controllability from [7] by replacing the composition $L(G_k) \| L(G_{k_j})$ by $L(G_{k_j})$ as discussed in Remark 6. For a future reference we will say that K is two-level conditionally controllable with respect to G_i , for $i \in I_l$, and G_{k_l} , i.e., several items are omitted (but should be clear from the problem formulation).

The following lemma shows how to construct a two-level conditional controllable language as the synchronous composition of conditionally controllable languages for groups.

Lemma 9: For all $l = 1, 2, \dots, m$, let the languages $M_l \subseteq A_{I_l}^*$ be conditionally controllable with respect to G_i , for $i \in I_l$, and G_{k_l} , and conditionally decomposable with respect to alphabets A_i , for $i \in I_l$, and A_{k_l} , and $A_{k_\ell} \supseteq A_k \supseteq \bigcup_{k \neq \ell} (A_k \cap A_\ell)$.

If for all $l = 1, 2, \dots, m$, $P_{k_l}^{k_l}$ is a L_{k_l} -observer and OCC for $P_{k_l}(M_l)$, then $\|_{l=1}^m M_l$ is two-level conditionally controllable with respect G_i , for $i \in I_l$, and G_{k_l} , for $l = 1, 2, \dots, m$. \blacksquare

The main existential result of multilevel coordination control is now recalled from [7].

Theorem 10: Consider the setting of Problem 7 (in particular K is two-level conditionally decomposable with respect to local alphabets A_1, A_2, \dots, A_n , high-level coordinator alphabet A_k , and low-level coordinator alphabets $A_{k_1}, A_{k_2}, \dots, A_{k_m}$). There exist supervisors S_i , for $i \in I_j$, for low-level systems within any group of low-level systems $\{G_i \mid i \in I_j\}$, for $j = 1, 2, \dots, m$, and supervisors S_{k_j} , for $j = 1, 2, \dots, m$, for low-level coordinators such that

$$\|_{j=1}^m \|_{i \in I_j} L(S_i / G_i \| (S_{k_j} / G_{k_j})) = K \quad (1)$$

if and only if K is two-level conditionally controllable as defined in Definition 8. \blacksquare

In the last section we have recalled two-level coordination control framework with the main existential result. A natural question is what to do if the specification fails to satisfy the necessary and sufficient conditions for being achievable. We recall from [8] that in the case specification K fails to be conditionally controllable, the supremal conditionally controllable sublanguage always exists and can be computed in a distributive way. First, we show that two-level conditional controllability is closed under language unions as well.

Theorem 11: Two-level conditional controllability is closed under language unions, that is, supremal two-level conditional controllable languages always exist. \blacksquare

By Theorem 11, the supremal two-level conditional controllable sublanguage of a specification K with respect to $A_{k_1}, A_{k_2}, \dots, A_{k_m}$, and A_u , denoted by $\text{sup}2\text{cC}(K, L, A_{i+k_j})$, always exists. Below we propose a procedure to compute the supremal two-level conditional controllable sublanguage $\text{sup}2\text{cC}(K, L, A_{i+k_j})$.

Similarly as in the centralized coordination we introduce the following notation. For all $j = 1, 2, \dots, m$ and $i \in I_j$,

$$\begin{aligned} \sup C_{k_j} &= \sup C(P_{k_j}(K), L(G_{k_j}), A_{k_j, u}) \\ \sup C_{i+k_j} &= \sup C(P_{i+k_j}(K), L(G_i) \parallel \sup C_{k_j}, A_{i+k_j, u}) \end{aligned} \quad (2)$$

where $\sup C(K, L, A_u)$ denotes the supremal controllable sublanguage of K with respect to L and A_u , see [2].

Similarly as in the centralized coordination, the following inclusion always holds true.

Lemma 12: For all $j = 1, 2, \dots, m$ and for all $i \in I_j$, we have that $P_{k_j}^{i+k_j}(\sup C_{i+k_j}) \subseteq \sup C_{k_j}$. ■

Transitivity of controllability is needed later.

Lemma 13 ([8]): Let $K \subseteq L \subseteq M$ be languages over A such that K is controllable with respect to L and A_u , and L is controllable with respect to M and A_u . Then K is controllable with respect to M and A_u . ■

The main constructive result follows.

Theorem 14: Consider Problem 7 and languages defined in (2). If $\cap_{i \in I_j} P_{k_j}^{i+k_j}(\sup C_{i+k_j})$ is controllable with respect to $L(G_{k_j})$ and $A_{k_j, u}$, and if for all $j = 1, 2, \dots, m$, $P_k^{k_j}$ is an L_{k_j} -observer and OCC for L_{k_j} , then $\sup 2cC(K, L, A_{i+k_j}) = \parallel_{j=1}^m \parallel_{i \in I_j} \sup C_{i+k_j}$. ■

Note that controllability of $\cap_{i \in I_j} P_{k_j}^{i+k_j}(\sup C_{i+k_j})$ with respect to $L(G_{k_j})$ and $A_{k_j, u}$ for all $j = 1, 2, \dots, m$, is not a suitable condition for verification. Clearly, for our prefix-closed languages, controllability of $P_{k_j}^{i+k_j}(\sup C_{i+k_j})$ with respect to $L(G_{k_j})$ and $A_{k_j, u}$, for $j = 1, 2, \dots, m$ and $i \in I_j$, implies it. Moreover, two stronger checkable conditions are provided below.

It is easy to see that the equality in Lemma 12 implies the sufficient condition of Theorem 14. Indeed, if for all $j = 1, 2, \dots, m$ and for all $i \in I_j$, we have $P_{k_j}^{i+k_j}(\sup C_{i+k_j}) \subseteq \sup C_{k_j}$, then in particular $P_{k_j}^{i+k_j}(\sup C_{i+k_j})$ is controllable with respect to $L(G_{k_j})$ and $A_{k_j, u}$. Hence, for all $j = 1, 2, \dots, m$, $\cap_{i \in I_j} P_{k_j}^{i+k_j}(\sup C_{i+k_j})$ is controllable with respect to $L(G_{k_j})$ and $A_{k_j, u}$, which proves the following result.

Corollary 15: Consider the setting of Problem 7 and the languages defined in (2). If for all $j = 1, 2, \dots, m$, $P_k^{k_j}$ is an L_{k_j} -observer and OCC for L_{k_j} , and for all $i \in I_j$, $P_{k_j}^{i+k_j}(\sup C_{i+k_j}) = \sup C_{k_j}$, then $\sup 2cC(K, L, A_{i+k_j}) = \parallel_{j=1}^m \parallel_{i \in I_j} \sup C_{i+k_j}$. ■

There is yet another sufficient condition that guarantees the controllability requirement in Theorem 14. Namely, local control consistency and observer properties (that are checkable by well-known methods).

Lemma 16: For all $j = 1, 2, \dots, m$ and $i \in I_j$, let $P_{k_j}^{i+k_j}$ be an $(P_i^{i+k_j})^{-1}L(G_i)$ -observer and OCC for $(P_i^{i+k_j})^{-1}L(G_i)$. Then, for all $j = 1, 2, \dots, m$, $\cap_{i \in I_j} P_{k_j}^{i+k_j}(\sup C_{i+k_j})$ is controllable with respect to $L(G_{k_j})$ and $A_{k_j, u}$. ■

We point out that even without the above conditions, $\parallel_{j=1}^m \parallel_{i \in I_j} \sup C_{i+k_j}$ is controllable with respect to L , but we cannot guarantee maximal permissiveness with respect to the two-level coordination control architecture.

IV. DECENTRALIZED SUPERVISORY CONTROL WITH COMMUNICATION

In this section, constructive results of the top-down coordination are applied to decentralized supervisory control with communicating supervisors. To avoid any confusion we use systematically Σ to denote alphabets in decentralized control, while notation A is reserved for alphabets in coordination control. Decentralized supervisory control is now briefly recalled.

A. Decentralized Supervisory Control Problem

Decentralized supervisory control differs from modular or coordination control, because the global system is a large automaton without a product structure. In decentralized supervisory control sensing and actuating capabilities are distributed among local supervisors $(S_i)_{i=1}^n$ such that each S_i observes a subset $\Sigma_{o,i} \subseteq \Sigma$ and based on its observation it can disable its controllable events $\Sigma_{c,i}$. Projections to locally observable events are denoted by $P_i : \Sigma^* \rightarrow \Sigma_{o,i}^*$. We use the notation $\Sigma_c = \cup_{i=1}^n \Sigma_{c,i}$, $\Sigma_o = \cup_{i=1}^n \Sigma_{o,i}$, $\Sigma_u = \Sigma \setminus \Sigma_c$, and $\Sigma_{uo} = \Sigma \setminus \Sigma_o$.

Formally, a local supervisor S_i for a generator G is defined as a mapping $S_i : P_i(L(G)) \rightarrow \Gamma$, where $\Gamma_i = \{\gamma \subseteq \Sigma \mid \gamma \supseteq (\Sigma \setminus \Sigma_{c,i})\}$ is the set of local control patterns, and $S_i(s)$ represents the set of locally enabled events when S_i observes a string $s \in \Sigma_{o,i}^*$. Then the permissive local supervisor law is $S_i(s) = (\Sigma \setminus \Sigma_{c,i}) \cup \{a \in \Sigma_{c,i} \mid \exists s' \in K \text{ with } P_i(s') = P_i(s) \text{ and } s'a \in K\}$. The global control law S is given by conjunction of local ones: for $w \in \Sigma_{o,i}^*$, $S(w) = \cap_{i=1}^n S_i(P_i(w))$. This is why this control architecture is nowadays referred to as conjunctive and permissive.

Definition 17: $K \subseteq L$ is *C&P coobservable* with respect to $L = L(G)$ and $(\Sigma_{o,i})_{i=1}^n$ if for all $s \in K$, $a \in \Sigma_c$, and $sa \in L \setminus K$, there exists $i \in \{1, 2, \dots, n\}$ such that $a \in \Sigma_{c,i}$ and $(P_i^{-1}(P_i(s))\{a\} \cap K = \emptyset)$. □

C&P coobservability can be interpreted in the following way: if we exit from the specification by an event a , then there must exist at least one local supervisor that can control this event ($a \in \Sigma_{c,i}$) and can disable a unambiguously, i.e., all lookalike (for S_i) strings exit the specification as well. Recall that C&P coobservability can be decided in polynomial time [11] in the number of states of the specification and system (but in exponential time in the number of local supervisors).

Since the counterpart of C&P coobservability, called D&A coobservability, is not studied in this paper, C&P coobservability will be referred to as coobservability.

It has been proved in [12] that controllability and coobservability are the necessary and sufficient conditions to achieve a specification as the resulting closed-loop language. For languages that fail to satisfy these conditions it is important to compute a controllable and coobservable sublanguage.

We first recall that decomposability is strongly related to coobservability. A language K is *decomposable* with respect to alphabets $(\Sigma_i)_{i=1}^n$ and L if $K = \parallel_{i=1}^n P_i(K) \cap L$. In the special case $L = \Sigma^*$ decomposability is called *separability* [4], i.e.,

K is *separable* with respect to alphabets $(\Sigma_i)_{i=1}^n$ if $K = \prod_{i=1}^n P_i(K)$.

Now we are ready to recall that (under the alphabet condition that will be shown non-restrictive in the following subsection) separability implies coobservability.

Theorem 18 ([5]): Assume that $\Sigma_{o,i} \cap \Sigma_c \subseteq \Sigma_{c,i}$, for $i = 1, 2, \dots, n$. If K is separable with respect to $(\Sigma_{o,i})_{i=1}^n$, then $K \cap L$ is coobservable with respect to $(\Sigma_{o,i})_{i=1}^n$ and L . ■

B. Construction of controllable and coobservable sublanguages based on two-level coordination control

Now we show how the new constructive results of the top-down approach to multilevel coordination control from Section III can be used to compute sublanguages that are by construction controllable and coobservable.

The idea is that owing to our results of coordination control, the supremal conditionally controllable sublanguage (that is in particular controllable) can be computed as a synchronous product of languages over alphabets enriched by communicated events from group coordinators. Hence the resulting language is by construction not only controllable, but in view of Theorem 18 also coobservable. Moreover, due to the multilevel coordination, coordinator events are not communicated among all subsystems, which was our first approach presented in [5] based on the centralized coordination. Roughly speaking, coordinator events from the centralized coordination are distributed to group coordinator events, and only these are to be communicated among the subsystems belonging to the group.

Recall at this point that alphabets in decentralized control are denoted by Σ , while alphabets in coordination control are denoted by A .

It follows from Theorem 14 and its consequences that the supremal two-level conditionally-controllable sublanguage is decomposable with respect to alphabets $(A_{i+k_j})_{j=1,2,\dots,m, i \in I_j}$.

Note that conditional decomposability with respect to alphabets $(\Sigma_{o,i})_{i=1}^n$ and Σ_k such that $\cup_{i \neq j} (\Sigma_i \cap \Sigma_j) \subseteq \Sigma_k$, i.e.,

$$K = \prod_{i=1}^n P_{i+k}(K),$$

is nothing else but separability of K with respect to $(\Sigma_{o,i} \cup \Sigma_k)_{i=1}^n$. An important feature of separability with respect to the alphabets of this form with intersection between all pairs of alphabets being equal to Σ_k is that it can be checked in polynomial time in the number of local agents [6].

We recall that there always exists Σ_k that makes language K conditionally decomposable with respect to $(\Sigma_i)_{i=1}^n$ and Σ_k , cf. [6]. Moreover, K is conditionally decomposable if and only if there exist $M_i \subseteq \Sigma_{i+k}^*$ such that $K = \prod_{i=1}^n M_i$.

Consider now the setting of decentralized control with subsets of events observable $(\Sigma_{o,i})_{i=1}^n$ and controllable $(\Sigma_{c,i})_{i=1}^n$ by agent (supervisor) i and a specification $K \subseteq L = L(G)$.

We plunge the decentralized control problem into the coordination control problem by setting

$$A_i = \Sigma_{o,i} \quad \text{and} \quad A_{c,i} = \Sigma_{o,i} \cap \Sigma_{c,i}.$$

For simplicity, the same notation for projection is kept, that is, $P_i : A^* \rightarrow A_i^*$. The plant language G will be overapproximated by a two-level modular plant $\prod_{i=1}^n P_i(G)$, that

is, by the parallel composition of projections to events observable by local control agents. We will organize the local supervisors into a two-level hierarchy. Similarly as in multilevel coordination control we will group the agents based on their interactions given by intersection of their alphabets, here observations $A_i = \Sigma_{o,i}$. The idea is to place agents with maximal shared observations to the same groups at the lowest level of the multilevel structure. Then, in an ideal situation, there will be no shared observations between different groups of agents, because all shared observations are realized within the low-level groups. This can be formalized by associating a square matrix with the number of shared events observed by both and try to find after permutation a block matrix structure such that the maximum of shared events is situated in the diagonal blocks, while off-diagonal blocks contain very small numbers (ideally zero matrices). Again, we denote m low-level groups of agents by I_j , where $j = 1, 2, \dots, m$, for $m \leq n$.

According to the two-level top-down architecture, we have to find an extension A_k (a high-level coordinator alphabet) of these "high-level shared observations" $A_{sh} = \bigcup_{k,\ell \in \{1,\dots,m\}}^{k \neq \ell} (A_{I_k} \cap A_{I_\ell})$ such that $A_k \supseteq A_{sh}$ and $K = \prod_{r=1}^m P_{r+k}(K)$. Similarly, we have to extend the shared low-level observations in groups, i.e., $A_{sh,r} = \bigcup_{k,\ell \in I_r}^{k \neq \ell} (A_k \cap A_\ell)$, for $r = 1, 2, \dots, m$, to low-level coordinator alphabets $A_{k,j} \supseteq A_{sh,r}$ such that $P_{r+k}(K) = \prod_{j \in I_r} P_{j+k_r+k}(K)$, i.e., two-level conditional decomposability holds true.

It is a common assumption in both modular and coordination supervisory control that shared events have the same controllability status in all components, i.e., $A_i \cap A_{c,j} \subseteq A_{c,i}$, for $i, j = 1, 2, \dots, n$. Since $A_i = \Sigma_{o,i}$, it is clear that this assumption is equivalent to $\Sigma_{o,i} \cap \Sigma_{c,j} \subseteq \Sigma_{c,i}$, for $i, j = 1, 2, \dots, n$. Hence, by Theorem 18, separability implies coobservability and the condition stated therein is not restrictive. More precisely we have the following.

Lemma 19: If $\Sigma_{o,i} \cap \Sigma_{c,j} \subseteq \Sigma_{c,i}$, for $i, j = 1, 2, \dots, n$, then $(A_i)_{i=1}^n$ and $(A_{c,i})_{i=1}^n$ defined above satisfy $A_i \cap A_{c,j} \subseteq A_{c,i}$, for all $i, j = 1, 2, \dots, n$. ■

Once A_k and A_{k_j} , for $j = 1, 2, \dots, m$, are found such that two-level conditional decomposability holds, we compute languages $L_{k_j} = L(G_{k_j})$, $\sup C_{k_j}$, and $\sup C_{i+k_j}$ as in the previous section, cf. formula (2), where $L(G_i)$ are replaced by $P_i(\prod_{i=1}^n L(G_i))$. Namely, for all $j = 1, 2, \dots, m$, $L_{k_j} = \prod_{i=1}^n P_{k_j}(P_i(L))$, $\sup C_{k_j} = \sup C(P_{k_j}(K), L_{k_j}, A_{k_j,u})$, and for all $i \in I_j$, $\sup C_{i+k_j} = \sup C(P_{i+k_j}(K), P_i(L) \parallel \sup C_{k_j}, A_{i+k_j,u})$.

Theorem 20: Let $K \subseteq L$ be languages, and let K be two-level conditionally decomposable with respect to $(A_i)_{i=1}^n$, $A_{k,j}$, and A_k . Then $\prod_{j=1}^m \prod_{i \in I_j} \sup C_{i+k_j}$ is a sublanguage of K controllable with respect to L and A_u , and coobservable with respect to L and $(A_{i+k_j})_{j=1,\dots,m, i \in I_j}$.

If $\prod_{j=1}^m \prod_{i \in I_j} P_{k_j}^{i+k_j}(\sup C_{i+k_j})$ is controllable with respect to $L(G_{k_j})$ and $A_{k_j,u}$, and for all $j = 1, 2, \dots, m$, $P_k^{k_j}$ is an L_{k_j} -observer and OCC for L_{k_j} , then $\prod_{j=1}^m \prod_{i \in I_j} \sup C_{i+k_j} = \sup 2cC(K, L, A_{i+k_j})$ is the largest controllable and coobservable language we can obtain using the two-level coordination. ■

Note that according to the above results there are stronger sufficient conditions that are more suitable for verification, namely that for $j = 1, 2, \dots, m$ and $i \in I_j$, either $P_{k_j}^{i+k_j}(\sup C_{i+k_j}) = \sup C_{k_j}$, or $P_{k_j}^{i+k_j}$ are $(P_i^{i+k_j})^{-1}P_i(L)$ -observers and OCC for $(P_i^{i+k_j})^{-1}P_i(L)$. The first condition has the advantage that if it holds, our results can be extended to the non-prefix-closed case. The second condition has the advantage that there are known algorithms to extend the local alphabets such that corresponding projections become OCC and satisfy the observer property. Finally, let us point out that without any assumption (except two-level conditional decomposability) M might be a too small language, because without the above additional conditions we cannot guarantee the optimality with respect to the coordination control ($\sup 2cC(K, L, A_{i+k_j})$) and the optimality is lost twice (it is potentially lost due to over-approximation of L by $\|_{i=1}^n P_i(L)$ unless L is decomposable).

C. Example

Let the languages K and L be given by generators on Figs. 1 and 2, respectively. The alphabets of local agents are $\Sigma_{o,1} = \{a, b, u_1, u\}$, $\Sigma_{o,2} = \{a, b, u_2, u\}$, $\Sigma_{o,3} = \{v, b, v_1, b_1\}$, $\Sigma_{o,4} = \{v, b, v_2, b_2\}$, $\Sigma_{c,1} = \Sigma_{c,2} = \{a, b\}$, $\Sigma_{c,3} = \{v, v_1, b_1\}$, $\Sigma_{c,4} = \{v, v_2, b_2\}$, $\Sigma_u = \{b, u, u_1, u_2\}$, and $\Sigma_c = \Sigma \setminus \Sigma_u$.

Note that K is not controllable with respect to L , because e.g. $v_2v_1b \in K\Sigma_u \cap L$, but $v_2v_1b \notin K$. On the other hand, K is coobservable with respect to L and $\Sigma_{o,i}$, for $i = 1, 2, 3, 4$. However, $N = \sup C(K, L, \Sigma_u)$ is not coobservable with respect to L and $\Sigma_{o,i}$, for $i = 1, 2, 3, 4$, anymore. This is because both $v_1v_2 \in L$ and $v_2v_1 \in L$, $v_1v_2 \in N$, $v_2 \in N$, while $v_2v_1 \notin N$.

It is clear that at least one of the agents 3 and 4 has to observe both v_1 and v_2 in order to issue a correct control decision. This means that it is not clear how to compute a sublanguage that is at the same time controllable and coobservable. Also notice that if b were controllable, K would be controllable, but still not coobservable. We now use our two-level coordination control approach.

First, we project the plant to the alphabets of observable events, cf. Fig. 3. The inclusion $L \subseteq \|_{i=1}^n P_i(L)$ is strict. We set $A_i = \Sigma_{o,i}$ for $i = 1, 2, 3, 4$. Now we have to extend $A_{sh} = (A_{I_1} \cap A_{I_2}) = (A_1 \cup A_2) \cap (A_3 \cup A_4) = \{b\}$ such that for $A_k \supseteq A_{sh}$, and $K = P_{1+2+k}(K) \| P_{3+4+k}(K)$. It turns out that no extension of A_{sh} is needed, i.e., $A_k = \{b\}$.

We now need to find low-level coordination alphabets A_{k_1} and A_{k_2} such that conditions on the low-level for two-level conditionally decomposability hold true. Since $P_{1+2+k}(K) = P_1(K) \| P_2(K)$, there is no need to extend $A_{sh,1} = A_1 \cap A_2 = \{a, u, b\}$ and we take $A_{k_1} = A_{sh,1}$. Therefore, $P_{1+2+k}(K) = P_{1+k_1}(K) \| P_{2+k_1}(K)$, with $A_{k_1} = \{a, u, b\}$. In particular, coordinator L_{k_1} is not needed. Secondly, $P_{3+4+k}(K)$ is not decomposable with respect to A_{3+k} and A_{4+k} . Hence, we have to extend A_k by A_{k_2} to make the equation $P_{3+4+k}(K) = P_{3+k_2}(K) \| P_{4+k_2}(K)$ hold true. We choose $A_{k_2} = \{v_1, b_2, v, b\}$ and it can be checked that $P_{3+4+k}(K)$ is conditionally decomposable with respect to alphabets A_3 , A_4 , and A_{k_2} . The

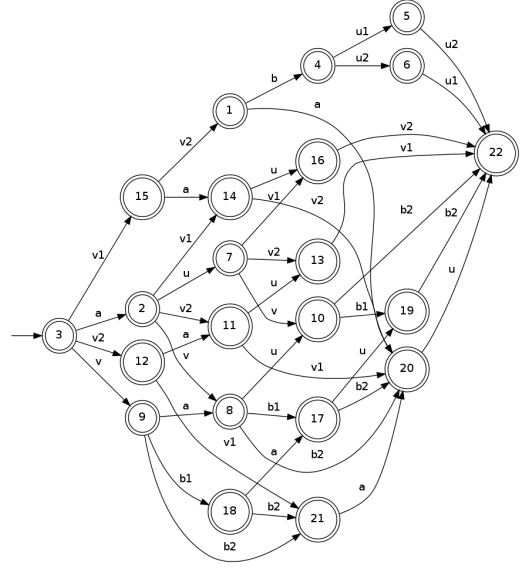


Fig. 1. Specification K

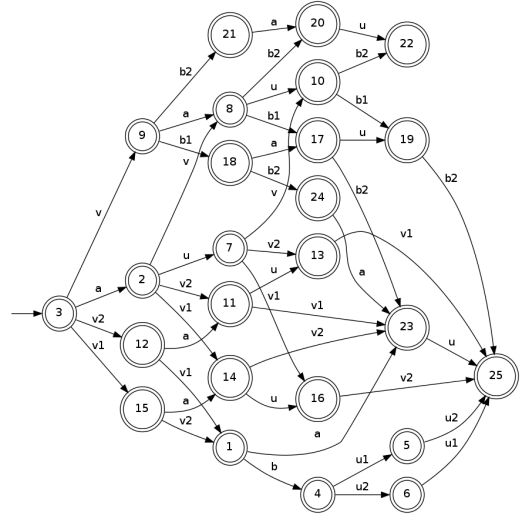


Fig. 2. Plant L

corresponding coordinator for the second group is $L_{k_2} = \|_{i=1}^n P_i(L) = \{\varepsilon, v, vb_2, v_1, v_1b\}$.

Now starts the actual computation of a sublanguage of K that is both controllable with respect to L and A_u and coobservable with respect to L and observable alphabets $A_{1+k_1,o}$, $A_{2+k_1,o}$, $A_{3+k_2,o}$, $A_{4+k_2,o}$. Note that K is not two-level conditionally controllable, hence the constructive procedure is needed. In fact, $P_{4+k_2}(K)$ given in Fig. 4 is not controllable with respect to $P_4(L) \| P_{k_2}(K)$ given in Fig. 5. Therefore, corresponding supervisor S_4 for $P_4(L)$ is not simply given by $P_{4+k_2}(K)$, but has to be computed. Its language is given by $\sup C_{4+k_2} = \sup C(P_{4+k_2}(K), P_4(L) \| \sup C_{k_2}, A_{u,4+k_2})$. First, we compute the supervisor $\sup C_{k_2}$ for coordinator L_{k_2} . We have $\sup C_{k_2} = \sup C(P_{k_2}(K), L_{k_2}, A_{u,k_2}) = P_{k_2}(K) = L_{k_2}$ computed above. Thus, $\sup C_{4+k_2} = \{v_1, v_2, b, v_2, vb_2\}$ is given by Fig. 5. This supervisor simply disables v_1 after v_2 has occurred.

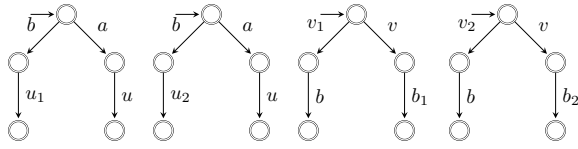


Fig. 3. Projections $P_1(L), \dots, P_4(L)$

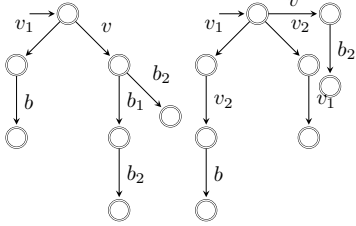


Fig. 4. $P_{3+k_2}(K)$ and $P_{4+k+k_2}(K)$

Similarly, we have to derive the supervisor $\text{sup}C_{3+k_2}$ for $P_3(L)$. We note that $P_{3+k_2}(K)$, given in Fig. 4, is controllable with respect to $P_3(L) \parallel P_{k_2}(K)$. Thus, $\text{sup}C_{3+k_2} = P_{3+k_2}(K)$. Concerning the first group of agents, no control action is required for supervisors $\text{sup}C_{1+k_1}$ and $\text{sup}C_{2+k_1}$, because $P_{1+2}(K) = P_1(L) \parallel P_2(L)$. Moreover, $\text{sup}C_{1+k_1} = P_{1+k_1}(K) = P_1(K) = P_1(L)$ and $\text{sup}C_{2+k_1} = P_{2+k_1}(K) = P_2(K) = P_2(L)$.

Since all conditions in Theorem 14 are satisfied, it holds true that $\text{sup}C_{1+k_1} \parallel \text{sup}C_{2+k_1} \parallel \text{sup}C_{3+k_2} \parallel \text{sup}C_{4+k_2} = \text{sup}2cC(K, L, A_{i+k_j})$. In particular, it is controllable with respect to L and A_u and coobservable with respect to L and $A_{1+k_1, o}, A_{2+k_1, o}, A_{3+k_2, o}, A_{4+k_2, o}$. \triangleleft

V. CONCLUDING REMARKS

We have extended existential results of multilevel coordination control to constructive results with a procedure to compute the supremal sublanguage achievable in the two-level architecture. The constructive results have been applied to decentralized supervisory control with communicating supervisors, where local supervisors communicate with each other via a coordinator of the group.

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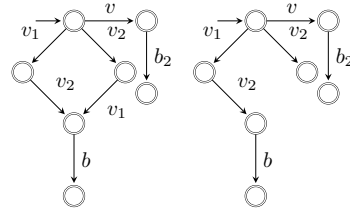


Fig. 5. $L_4 \parallel P_{k_2}(K)$ and $\text{sup}C_{4+k_2}$

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