# Advantages of (non)linear methods in assessing climate interaction structure

## strategies and lessons learned

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DAMES
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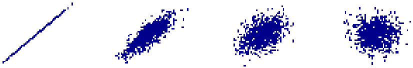

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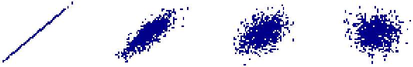

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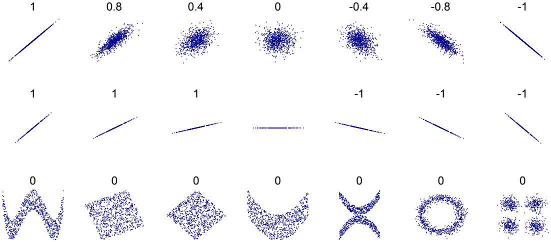

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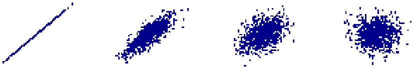

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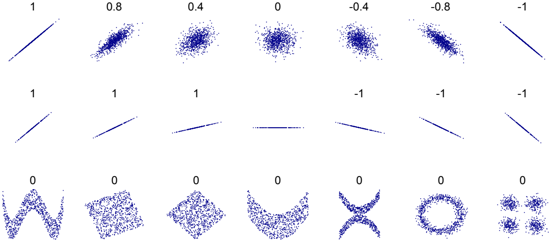


Mutual information:

$$I(X; Y) = \sum_{y \in Y} \sum_{x \in X} p(x, y) \log \left( \frac{p(x, y)}{p(x)\, p(y)} \right)$$

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- BUT ... nonlinear methods also have downsides!
  - implementation
  - interpretation
  - sensitivity and bias

  $\Rightarrow$ **Is linear correlation sufficient?**
- fMRI: [Hlinka et al., 2011, Neuroimage], climate: [Hlinka et al., in prep.]

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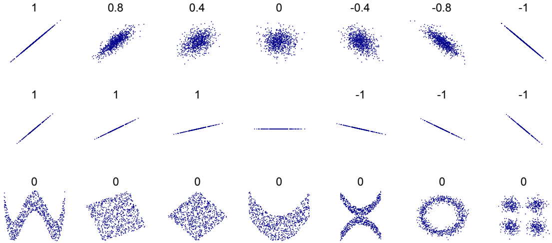
    $I(X,Y) \geq -\frac{1}{2}log(1-\rho^2_{X,Y})$
- $\Rightarrow$ we can quantify the extra dependence (mutual information) that is not captured by linear correlation:
  $I_{extra} = I(X,Y) - I_{Gauss}(\rho_{X,Y})$

# Vizualization



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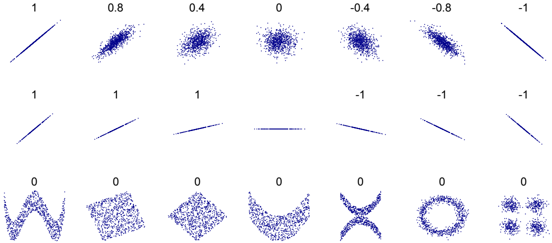


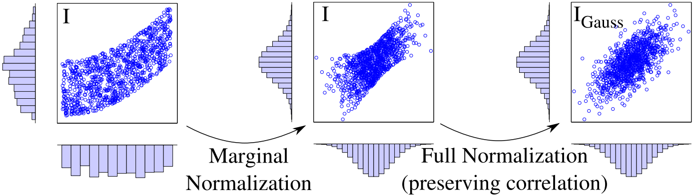

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# Data and methods

Data: NCEP/NCAR reanalysis dataset

- surface air temperatures
- monthly data (years 1948 - 2007; 720 timepoints)
- global grid $73 \times 144$ points ($2.5\,\mathrm{deg} \times 2.5\,\mathrm{deg}$ sampling)
- yearly cycle removed (anomalies)

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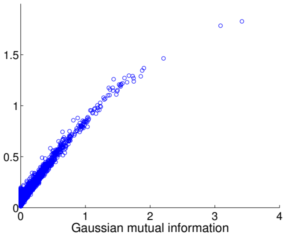
Methods: interaction/dependence quantification

- **nonlinear**: $\hat{I}(X, Y)$mutual information (pdf estimated using equiprobable binning; N=8)
- **linear**: $\hat{\rho}(X, Y)$, $\hat{I}_{Gauss}(X, Y)$, $\tilde{\hat{I}}_{Gauss}(X, Y)$
- **extra-linear**: $\hat{I}_{extra} = \hat{I}(X, Y) - \tilde{\hat{I}}_{Gauss}(X, Y)$

# Results: Existence



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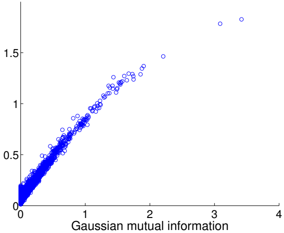
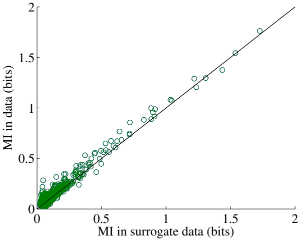



Controling for method bias:



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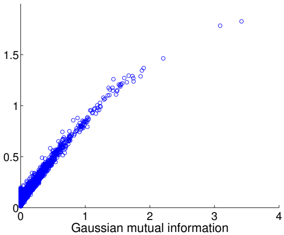


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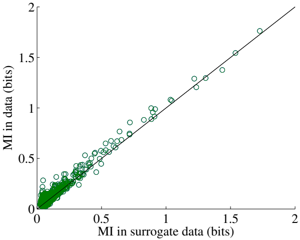


Statistical testing: 15% links above 95th percentile

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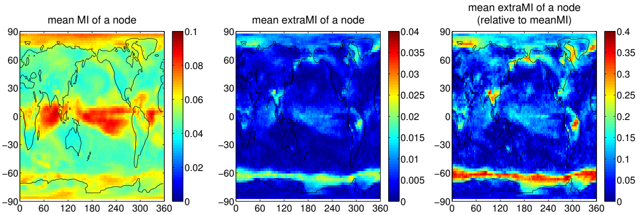

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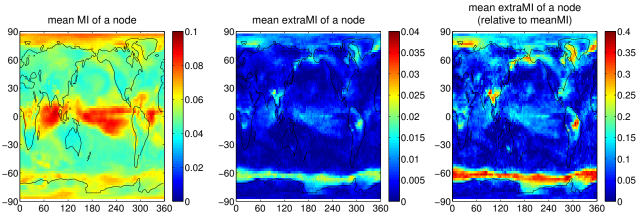
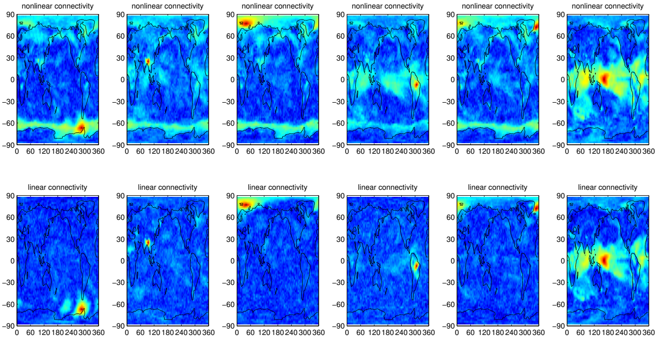
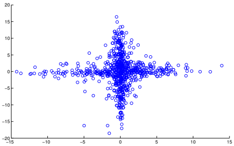
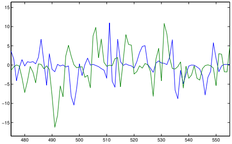


# Form/origin





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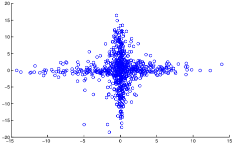


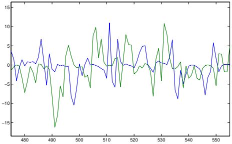


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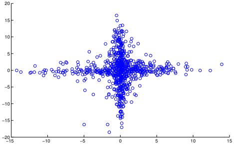

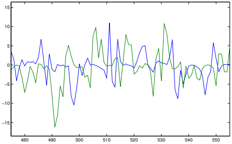

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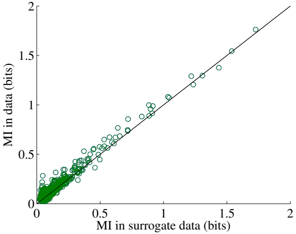

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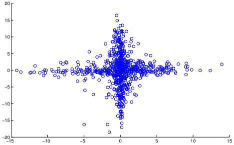
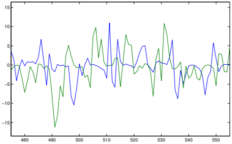

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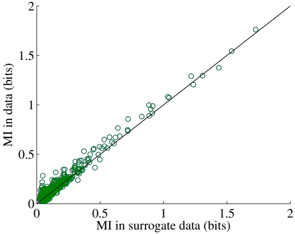
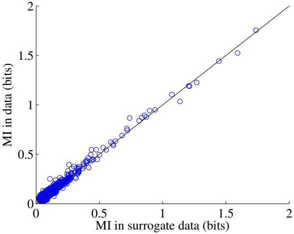
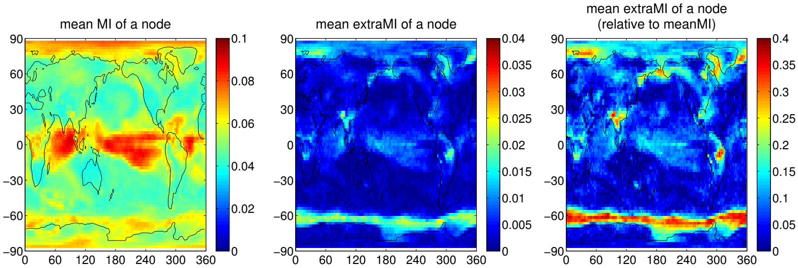


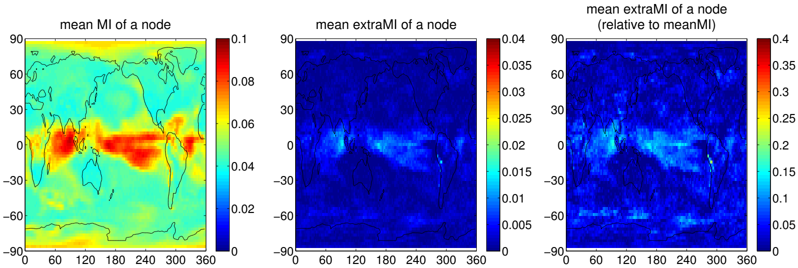



Statistical testing against surrogates: 8% links above 95th percentile

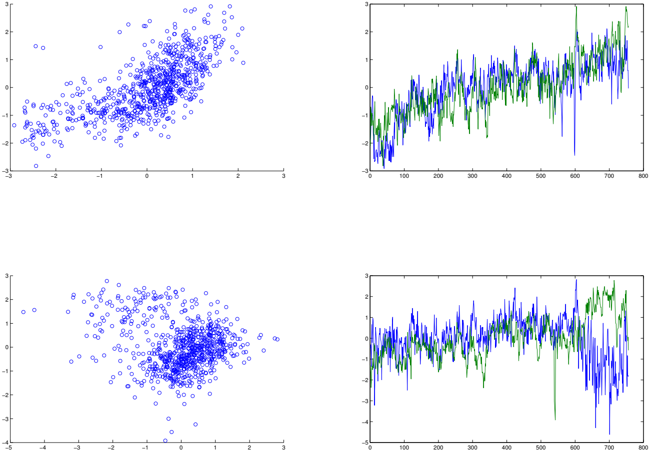
# Temperature anomalies:



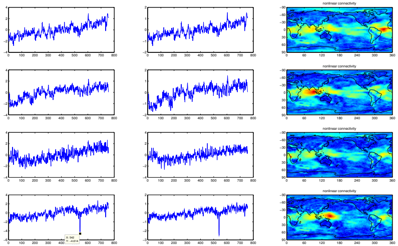

# After additional normalization of variance:

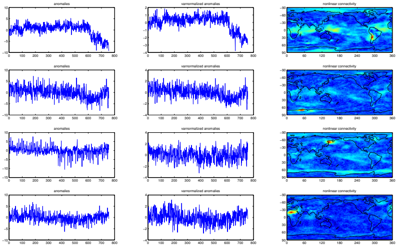


# What about remaining ‘non-linearities’?



# More examples



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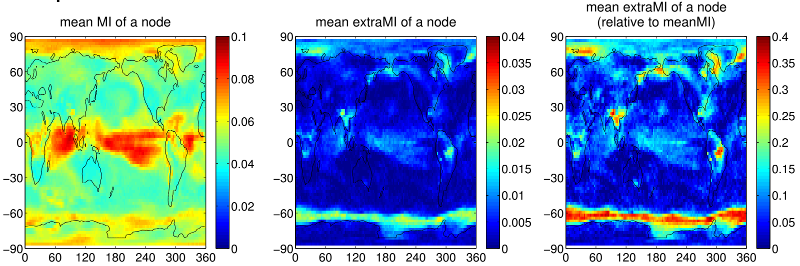


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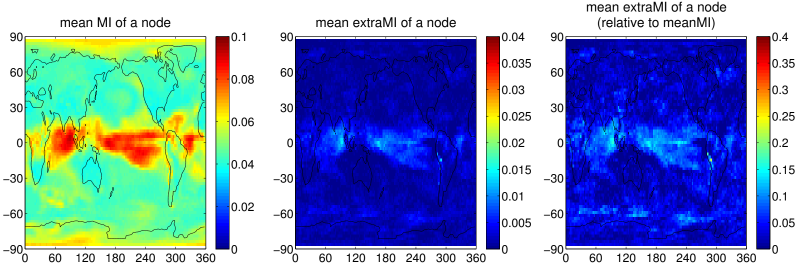


Statistical testing: 6% links above 95th percentile

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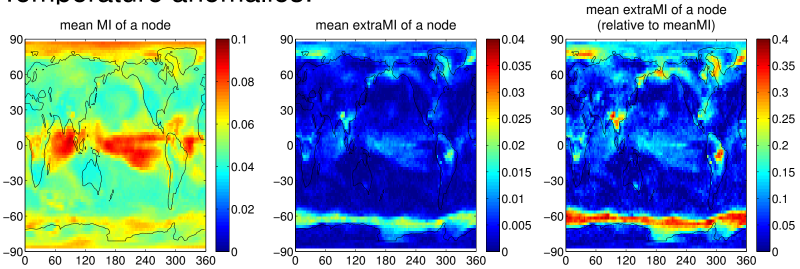
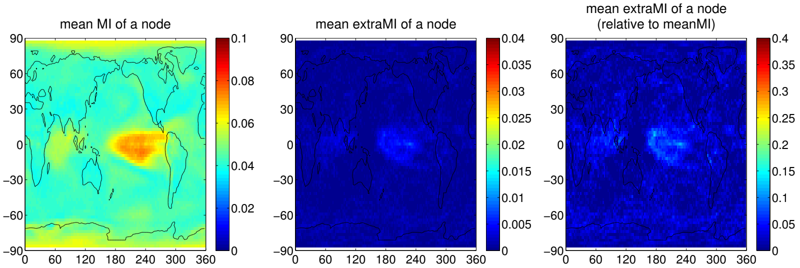



# After additional detrending:

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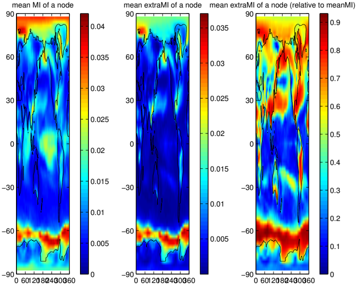

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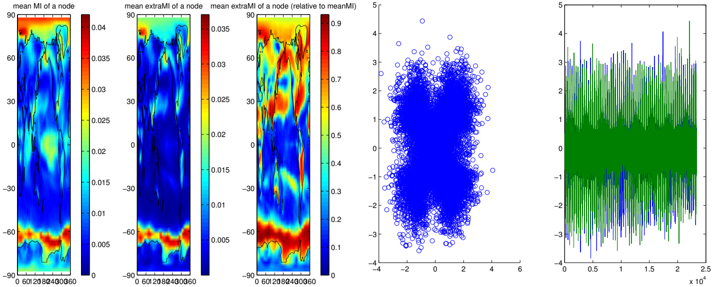

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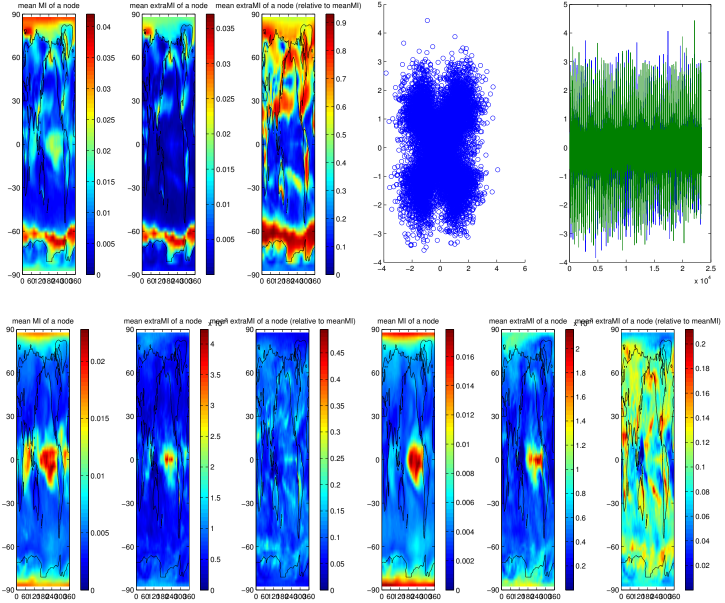
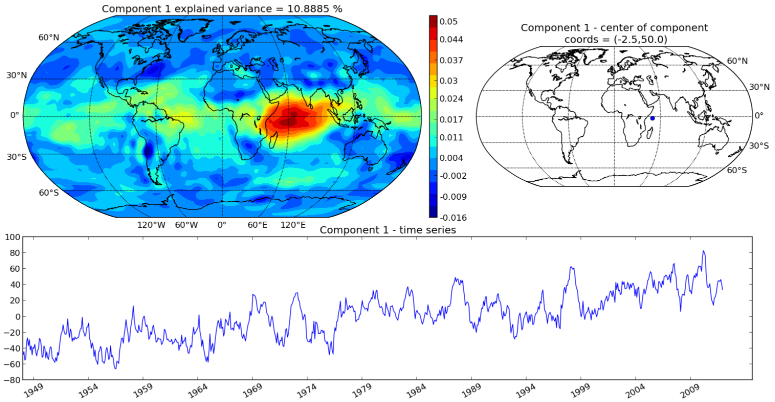


# Working with components



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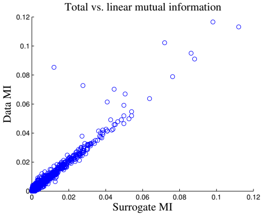

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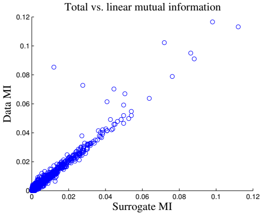


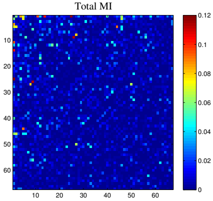


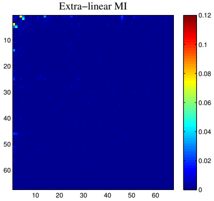

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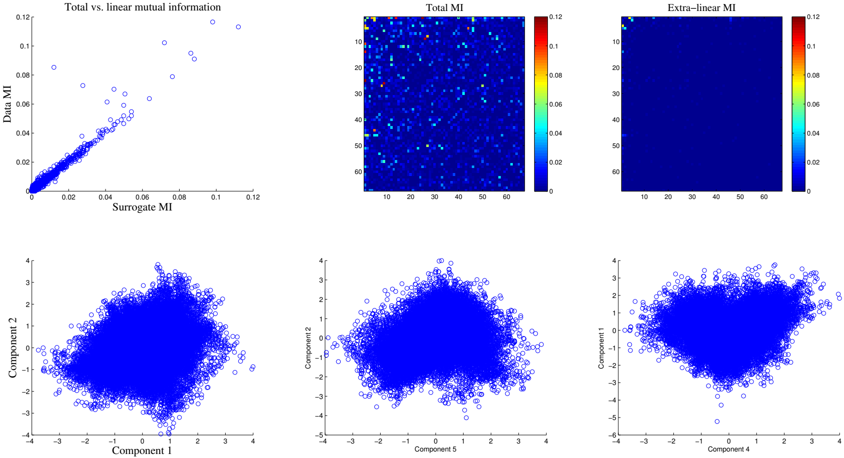
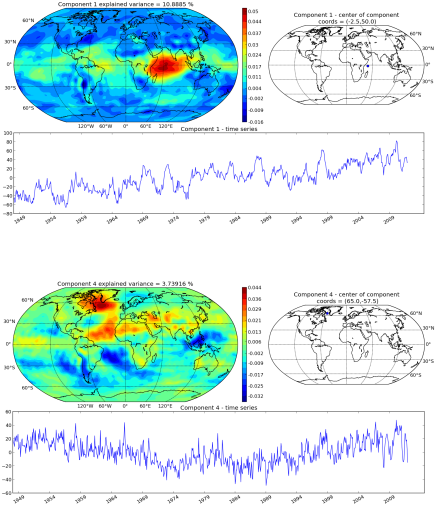
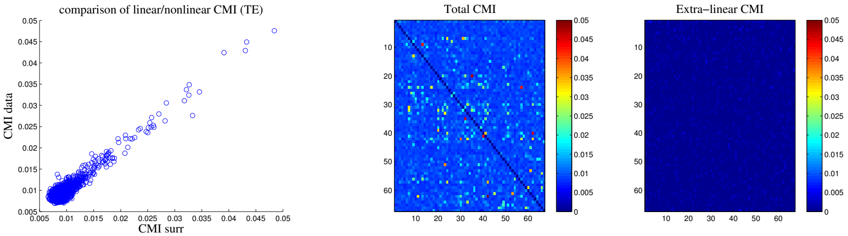


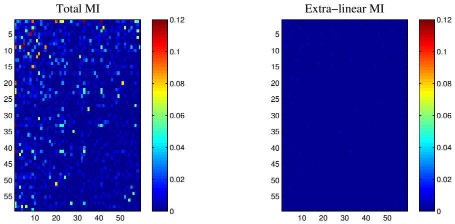
# Example 'nonlinear' link



# Further observations: CMI, SLP




SLP components time series dependence:



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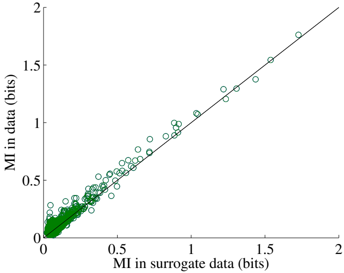

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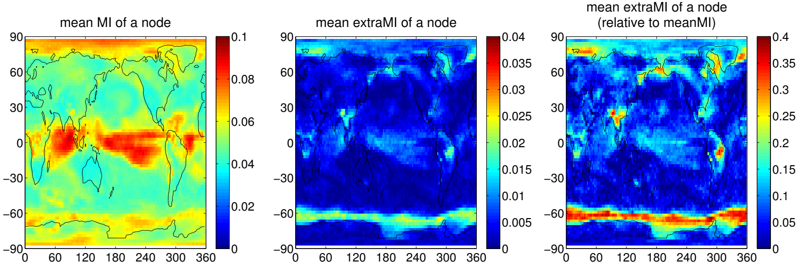

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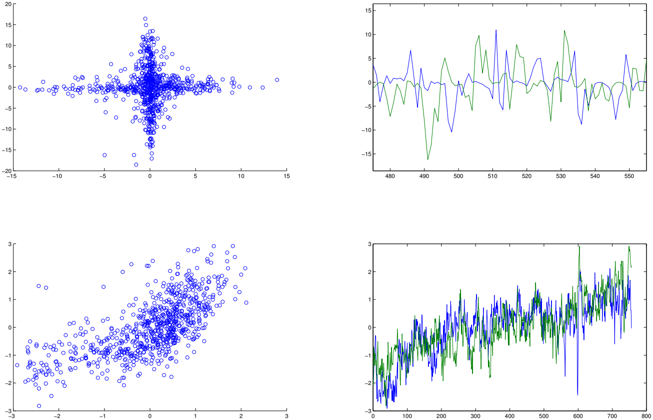
**sources:** strongest non-linearities are non-stationarities



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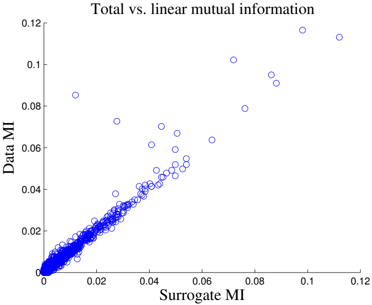
**existence:** deviations from linear dependences (non-linearities) confirmed
**strength:** non-linearities are relatively minor
**localization:** non-linearities are spatially sparse
**sources:** strongest non-linearities are non-stationarities
**data dependence:** sampling, transformations



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Thank you for your attention!

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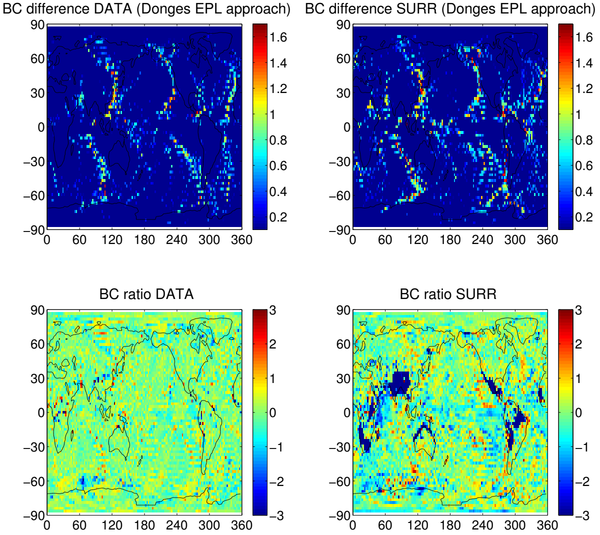
# Relevance for graph topology

## Donges et al., 2009: nonlinearity key for global topology



# Other datasets: ERA