The Transition to a Mixed Pension System in a Small Open Economy

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Abstract. *The paper investigates the macroeconomic and welfare effects of a gradual transition from a pay-as-you-go (PAYG) pension system to a mixed system comprising a PAYG pillar and a fully-funded (FF) pillar. The analyzing framework consists of an overlapping generations (OLG) model with lifetime uncertainty characterized by perpetual youth households. Agents engage in educational activities at the start of their life, create human capital that is used during the working period to rent it to firms, and, later on in life, retire and are paid a pension benefit. The constructed model allows for a hump-shaped human capital age profile and for a realistic method for computing pension benefits using a pension point scheme. Several pension reforms are simulated in the context of a calibrated version of the model. The findings indicate that, when accompanied by an increase in retirement age, the shift to a mixed pension system is Pareto improving and alleviates the burden of public debt.*

Keywords: overlapping generations, uncertain life, pension reform, hump-shaped human capital profile

JEL Classification: D91, F41, H55, J26

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1 Introduction

The trade-off between welfare of future and current generations is one of the key issues in macroeconomic research. Economists and policy makers have long realized that many decisions have a dynamic and intergenerational nature and that merely focusing on either the immediate effects or – at the other extreme – the long-run effects of different policy options does not reveal potentially important transitory effects. Two types of inter-temporal models that can account for these transitory

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effects stand out in the literature. The mainstream Ramsey-Cass-Koopmans model assumes a representative consumer that lives infinitely. Since individuals currently alive have identical marginal propensities to consume, it is not possible to study the impact of policies that redistribute wealth between workers and retiree, nor it is possible to study the impact of demographic changes, such as the aging of the population. The second strand of these models was developed by Diamond (1965) based on earlier insights of Samuelson. The Diamond model, also known as the overlapping generations (OLG) model, allows turnover in the population such that new individuals are born and old individuals die in each time period. It turns out that the OLG model is well suited to study pension reforms and demographic changes because it allows the modeling of individuals according to their age. Since individuals attempt to maximize their lifetime utility in the OLG model, they are willing to save more at younger ages so as to enjoy a higher level of consumption at older ages if the return on their savings is greater than their subjective discount rate of the future.

Auerbach et al. (1989) simulated the economic dynamics of an ageing population in Japan, Germany, Sweden and US using the Auerbach-Kotlikoff model, a dynamic OLG model with certain life. The household sector comprises 75 overlapping generations. Government expenditure depends on the age composition of the population. The social security system operates on a balanced budget and pay-as-you-go (PAYG) basis. In the context of a closed economy, the simulation results show that, if there is no change in the retirement age, the national saving rate in each of the four OECD countries is projected to drop significantly from the 1985 level due to the ageing of the population. The authors also simulated several policy responses such as rising the retirement age and reducing the social security benefit. By increasing the retirement age gradually by two years, it is found that the social security contribution rate is lower than in the baseline scenario and the national saving rate is higher in the long run. A similar outcome is found if the social security benefit is reduced by 1% per annum between 1990 and 2010. The macroeconomic effects of a pension reform in the context of the ageing population were reinvestigated by Hviding and Mérette (1998) using a version of the Auerbach-Kotlikoff model. They studied the impacts of four basic pension reforms on seven OECD countries, namely Canada, France, Italy, Japan, Sweden, UK and US. They found that the increase in the wage-income tax and the decline in national saving rate and capital returns due to ageing cannot be offset by any of the four simulated policy reforms. It is found that the gradual removal of public pensions is the most effective in the long run, but the increase in the retirement age is the most effective in the medium run. Shimasawa (2004) constructed a dynamic OLG model with endogenous growth to study the population ageing in Japan. The model differs from the Auerbach-Kotlikoff model by allowing the accumulation of human capital through education. Qualitatively, the simulation results are similar to those of the Auerbach-Kotlikoff model. Sadahiro and Shimasawa (2004) developed a two-country dynamic OLG model to study how the differential ageing process across countries affects international capital flow. They conclude that international capital flows from the ageing country to the population-growing country given that other aspects of these two countries are the same. Schneider et al. (2004) employ a simplified Auerbach-Kotlikoff model to estimate a Czech pension system reform proposal based on the introduction of a funded pillar, financed on par with the traditional PAYG pillar. The simulations showed that a switch from the PAYG to a funded system would contribute to faster capital accumulation, higher wage growth, lower taxation, higher economic growth and higher lifetime utility for all generations. The authors propose an unorthodox sequencing of the pension reform ("reverse-sequencing") in which the pre-retirement generations would enter the reformed system first and show that this sequencing maintains the Pareto efficiency condition for all age cohorts, but it gives governments more flexibility in the reform process.

Although Auerbach-Kotlikoff model is widely used in policy analysis it cannot account for life uncertainty. The economic implications of lifetime uncertainty were first studied in the context of a dynamic consumption-saving model by Yaari (1965). He showed that, faced with a positive mortality rate, individual agents will discount future utility more heavily due to the uncertainty of survival. Yaari's insights were embedded in a general equilibrium growth model by Blanchard (1985). The Blanchard-Yaari overlapping generations model have become a mainstream framework for macroeconomic analysis during the last two decades. Typical areas of application of these models are demographic changes and economic growth (de la Croix and Licandro, 1999; Kalemni-Ozcan et al., 2000), or social security and ageing (Bettendorf and Heijdra, 2006). Bettendorf and Heijdra (2006) employed a Blanchard-Yaari OLG model that includes a rudimentary pension system to study two types of pension reform, namely a decrease in the pension benefit and an increase in the retirement age. That found that both reforms have qualitatively the same macroeconomic effects leading to long-run increases in consumption and financial assets but a decrease in the capital stock. However the intergenerational welfare effects associated with the two types of pension reform are different. For the pension reduction, the retired generations loose out as a result of it. An increase in the pension age leaves pensioners unaffected. The oldest of the working-age generations are worse off and future generations are better off as a result of both reforms.

Recently, Heijdra and Romp (2008) develop a dynamic OLG model with more realistic assumptions about the probability of death. Since the model incorporates more extensive age-dependency it gives rise to a non-monotonic welfare effect on existing generations of different shocks. Heijdra and Romp (2009b) extend the model to include endogenous retirement use a calibration of the model to compute the general equilibrium effects of various large demographic shocks and several policy reform measures. Echevarrıa and Iza (2006) study the effect of introducing a PAYG pension system on GDP growth, taking into account the social security impact on education and retirement age incentives. The authors studied the relationship between the size of social security and per capita GDP growth rate and found that such a relationship is mostly negative, except for very low values for the social security contribution rate. Echevarrıa and Iza (2006) make the empirical implausible assumption that the instantaneous utility function is linear and that there is no depreciation of individual human capital while individuals remain on-the-job.

The aim of this paper consists in developing a framework for analyzing macroeconomic and welfare effects of pension reforms, especially of a reform consisting in a transition from a PAYG pension system to a mixed pension system consisting both of a PAYG pillar and fully funded (FF) pilar. We employ an analytical framework consisting in a continuous time OLG model with lifetime uncertainty à la Blanchard (1985). We extend the existing literature in several directions. First, we extend Bettendorf and Heijdra (2006) by incorporating a more realistic PAYG pension system based on pension points. Second, we generalize Heijdra and Romp (2009a) by considering three stages of life, namely a schooling period, a working period, and a retirement period. Third, we extend the analysis of Echevarrıa and Iza (2006) by including a concave, rather than linear, instantaneous utility function, and by modeling a public pension system with more realistic features. Fourth, our model differs from those mentioned above in that we allow for a hump-shaped human capital age profile. Finally, we enrich the Blanchard-Yaari OLG framework by analyzing the effects of various pension reforms consisting in a gradual transition from a PAYG pension system to a mixed PAYG-FF system, including a "reverse-sequencing" pension reform that was investigated by Schneider et al. (2004) in the context of a model with life certainty.

The remainder of this paper is organized as follows. Section 2 describes the underlying general equilibrium model. We present the structure of the economy: the households, the demography, the aggregate production technology, the pension system and derive the steady state. Section 3 provides some numerical results regarding the macroeconomic effects as well as the intergenerational welfare effects of various pension reforms. Section 4 concludes the paper. The proofs of some of the results are provided in the appendix.

2 The model

2.1 Households

At each instant, disconnected generations are born and agents face a constant age-independent probability of death, denoted by β. Each generation is represented by an economic agent who, according to the life-cycle model, has specific age-related consumption and saving patterns and who maximizes her utility over her entire lifetime.

2.1.1 Individual households

The utility function in period t of the representative agent born at time v is denoted by $U(v,t)$ and takes the following form:

$$U(v,t) = \int_t^{\infty} \ln(c(v,s)) \cdot e^{-(\rho+\beta)\cdot(s-t)} ds \tag{1}$$

where $u(\cdot)$ is the instantaneous utility function, $c(v,t)$ is the consumption at time t of an agent born at time v, ρ is the rate of time preference, and β the instantaneous probability of death. Intuitively, $e^{-\beta(s-v)}$is the probability that an agent born at time v is still alive at time s. Therefore, as pointed out by Yaari (1965), future utility is discounted not only because of pure time preference but also because of lifetime uncertainty.

The wealth of the agent consists of financial and human wealth (*i.e.* the present value of after-tax non-asset income). The finitely-lived agents accumulate both physical and human capital. We assume that individual agents accumulate human capital by engaging in full-time educational activities at the start of life until the age S, receiving an education subsidy from the government, $g(v,t)$. The education period is exogenous and constant. Our assumption is in line with the endogenous education literature concluding that the optimal schooling period is constant if the instantaneous probability of death is age independent (de la Croix and Licandro, 1999; Heijdra and Romp, 2009a).

The model incorporates exogenous educational subsidies. There is a large literature that compares educational subsidies, pension benefits and the taxes paid over the lifetime in the form of generational accounting exercises. The generation of current retirees made an investment when they were working by paying taxes which were partially used to educate their offspring. In turn, the debt incurred by the young for being educated through this system is repaid through their own social security contributions when they become middle-aged. In a PAYG pension system, these contributions are transferred to the elderly as pensions and intergenerational risk sharing is achieved, that is not possible in a fully funded system. Therefore, a mixed system with a PAYG element in a pension system is generally welfare enhancing because of the possibility of intergenerational risk sharing (Dutta et al., 2000). Boldrin and Montes (2005) show that if borrowing for education is not possible, then a combined public education and pension system that uses lump sum taxes and transfers can replicate the first-best allocation achieved in an economy without taxes where borrowing for education is allowed. Slobodyan and Vinogradov (2006) estimated for the Czech Republic the model in Boldrin and Montes (2005) and their findings indicate that paying for educating the next generation provides a significantly higher return (in the form of pensions) than the interest "paid" on educational loans.

Thereafter, the agent works full time and the available human capital is rented out to competitive producers for a wage income, $w(v,t)$. The wage is proportional to the human capital:

$$w(v,t) = \overline{w} \cdot h(v,t) \tag{2}$$

where $\overline{w}$ is the market-determined rental rate of human capital and $h(v,t)$ is the human capital of the representative agent born at time v.

Labor supply is exogenous and each agent supplies a single unit of labor. During the working period she pays a contribution to the pension system with rate $\overline{\tau}^{SSC}$. After retirement the agent receives a pension benefit, $p(v,t)$, until death. The retirement age, R, is also exogenously given. All through life, the agent pays a lump sum tax $\tau^{LS}(v,t)$.

In the context of a small open economy, the domestic interest rate equals the foreign interest rate, r^f, which is assumed constant and exogenous:

$$r = r^f \tag{3}$$

The budget identity of the representative agent born at time v is given by:

$$\begin{aligned}\dot{a}(v,t) = [r+\beta]\cdot a(v,t) + g(v,t)\cdot \mathbf{1}_{(v,v+S)}(t) + [1-\overline{\tau}^{SSC}]w(v,t)\cdot \mathbf{1}_{(v+S,v+R)}(t) \\ + p(v,t)\cdot \mathbf{1}_{(v+R,\infty)}(t) - \tau^{LS}(v,t) - c(v,t)\end{aligned} \tag{4}$$

where $a(v,t)$ is the financial wealth and $\dot{a}(v,t) := \dfrac{da(v,t)}{dt}$.

We follow Yaari (1965) and Blanchard (1985), by assuming the existence of a perfectly competitive life insurance sector which offers actuarially fair annuity contracts to the agents. Yaari (1965) pointed out that the uncertainty attached to life can be hedged by purchasing a life annuity from an insurance company. He showed that it is optimal for the individual, in the absence of a bequest motive, to hold all her financial assets in annuities. Therefore, the return on financial assets is equal to the return on annuities, which is proved to be equal to $r+\beta$.

If there is a large consensus on the production function of consumption and investment goods, there is no real evidence on the choice of the production function of human capital. The human capital production function employed in our model includes a 'shoulders of giants' type externality, as proposed by Azariadis and Drazen (1990). At the end of the schooling period the human capital is given by $A_H h(v)^{\varepsilon_H} S$, were ε_H quantifies this externality and $h(v)$ is the average human capital at birth. It seems reasonable to assume that for a given education period, the human capital that the individual accumulates is higher when the knowledge in the economy as a whole is higher. Special cases are employed by de la Croix and Licandro (1999), Boucekkine et al. (2002), Echevarrıa and Iza (2006) who set $\varepsilon_H = 1$, and by Kalemni-

Ozcan et al. (2000), who set $\varepsilon_H = 0$. Heijdra and Romp (2009a) pointed out that if $\varepsilon_H < 1$ the model exhibits exogenous growth, but if $\varepsilon_H = 1$ the growth is endogenous.

Classical uncertain life OLG models (Blanchard, 1985; Bettendorf and Heijdra, 2006) include a simplification, in that it is assumed that the wage income of individuals declines with age. This is, however, contrary to empirical findings. Our model differs in that we allow for a hump-shaped human capital age profile. This specification allows for a time profile that corresponds fairly closely to empirical observations, showing a rise with age and experience when individuals are relatively young but then eventually declining with age as individuals approach retirement years. Following Bryant and McKibbin (2004), we postulate that the evolution of efficiency of the human capital with the age is given by the function:

$$E(u) = a_1 \cdot e^{-\alpha_1 \cdot (u-S)} + a_2 \cdot e^{-\alpha_2 \cdot (u-S)} + a_3 \cdot e^{-\alpha_3 \cdot (u-S)}, \quad a_1 + a_2 + a_3 = 1 \tag{5}$$

Therefore, the human capital of the representative agent born at age v is given by:

$$h(v,t) = \begin{cases} 0 & , \ t < v + S \\ A_H h(v)^{\varepsilon_H} S \cdot E(t-v), & v + S < t < v + R \\ 0 & , \ t > v + R \end{cases} \tag{6}$$

The mechanism for human capital formation employed in our model is different from the Uzawa (1964) and Lucas (1988) framework, who assume a human capital production technology similar to that of goods (consumption and physical capital). Bils and Klenow (2000) pointed out that specifying the human capital as a function of the education period and of the efficiency profile is an improvement because it ensures that the theoretical specification broadly match the age–earnings profiles observed in actual datasets. After an individual enters the labor force, his or her labor income rises with age and experience, reaches a peak in late middle age, and then declines gradually for the rest of life. Mincer (1974) derived that that the log of the individual's wage is linearly related to that individual's years of schooling, years of experience, and years of experience squared.

In the planning period t, the household born at time v chooses the paths for consumption and for financial assets in order to maximize lifetime utility (1) subject to the flow budget identity (4) and the lifetime solvency condition, taking as given the initial level of financial assets $a(v,t)$. It follows that in the planning period the household allocates a proportion of her total wealth to consumption:

$$c(v,t) = \frac{1}{\Phi(t)}\left[a(v,t) + a^H(v,t)\right] \tag{7}$$

where $a^H(v,t)$ is a measure of the individual's human wealth and $1/\Phi(t)$ is the marginal propensity to consume out of wealth, with $\Phi(t)$ given by $\Phi(t) = \int_t^{\infty} \exp\{-(\rho+\beta)\cdot(s-t)\}ds$, which can be written as $\dot{\Phi}(t) = -1 + (\rho+\beta)\cdot\Phi(t)$, and, therefore, $1/\Phi(t) = \rho + \beta$.

Human wealth represents the present discounted value of after-tax non-interest income, using the annuity rate of interest for discounting:

$$\begin{aligned} a^H(v,t) = \int_t^{\infty} [&g(v,s)\mathbf{1}_{(v,v+S)}(s) + (1-\bar{\tau}^{SSC})w(v,s)\mathbf{1}_{(v+S,v+R)}(s) \\ &+ p(v,s)\mathbf{1}_{(v+R,\infty)}(s) - \tau^{LS}(v,s)]\cdot e^{-(r+\beta)(s-t)}ds \end{aligned} \tag{8}$$

The dynamics of human wealth is given by:

$$\begin{aligned} \dot{a}^H(v,t) = (r+\beta)a^H(v,t) - [&g(v,s)\mathbf{1}_{(v,v+S)}(s) + (1-\bar{\tau}^{SSC})w(v,s)\mathbf{1}_{(v+S,v+R)}(s) \\ &+ p(v,s)\mathbf{1}_{(v+R,\infty)}(s) - \tau^{LS}(v,s)] \end{aligned} \tag{9}$$

An alternative characterization of the household's optimal dynamic plans makes use of the Euler equation for consumption which can be written as follows:

$$\frac{\dot{c}(v,t)}{c(v,t)} = r - \rho \tag{10}$$

The consumption Euler equation relates the optimal consumption growth rate to the difference between the interest rate and the pure rate of time preference. The instantaneous mortality rate does not appear in this expression because households fully insure against the adverse effects of lifetime uncertainty (Yaari, 1965).

2.1.2 Demography

Blanchard's original theoretical exposition assumed for convenience that the population is stationary (*i.e.* the birth rate equals the death rate) and there is no growth in productivity. As Buiter (1988) and Weil (1989) showed in detail, however, the model can be readily adapted to cover the cases of a growing population and growth in productivity.

In order to allow for non-zero population growth, we employ the analytical framework developed by Buiter (1988). This framework assumes that the probability of death β and the birth rate b are not equal and thus allows for net population change. We denote the population size at time t by $N(t)$. In the absence of international migration, the growth rate of the population, n, is equal to the difference between the birth and death rates:

$$\frac{\dot{N}(t)}{N(t)} = n = b - \beta . \tag{11}$$

The size of a newborn generation at time v is assumed to be proportional to the current population:

$$N(v,v) = bN(v) \tag{12}$$

Since cohorts are assumed to be large, the size of each generation falls exponentially according to:

$$N(v,t) = N(v,v) \cdot e^{-\beta \cdot (t-v)} \tag{13}$$

It follows that the generational population weights, $n(v,t)$, is given by:

$$n(v,t) = \frac{L(v,t)}{L(t)} = be^{-b(t-v)} \tag{14}$$

The population proportion at time t of generation born at time v depends only on the age of that generation. Given this demographic structure, $1-e^{-bS}$ represents the fraction of pupils and students, $e^{-bS} - e^{-bR}$ the fraction of workers and e^{-bR} is the population fraction of pensioners. Therefore, the old-age dependency ratio is thus given by $e^{-bR} \big/ e^{-bS} - e^{-bR} = 1 \big/ e^{b(R-S)} - 1$.

2.1.3 Aggregate household sector

Per capita total population variables are calculated as the integral of the generation-specific values weighted by the corresponding generation weights:

$$x(t) = \int_{-\infty}^{t} n(v,t) \cdot x(v,t) dv \tag{15}$$

where $x(v)$ and $x(v,t)$ are, respectively, per capita total population variable at time t and the level of the corresponding variable for the generation born at time v.

For example, per capita consumption at time t, $c(t)$, is given by:

$$c(t) = \int_{-\infty}^{t} n(v,t) \cdot c(v,t) dv \tag{16}$$

where $n(v,t)$ and $c(v,t)$ are defined in, respectively, (14) and (7) above.

Exact aggregation of consumption is possible in this framework (*i.e.* $c(t) = \frac{1}{\Phi(t)}\left[a(t) + a^H(t)\right]$) because the mortality rate is age independent. The 'Euler equation' for per capita consumption can nevertheless be obtained by differentiating (16) with respect to time:

$$\frac{\dot{c}(t)}{c(t)} = [r - \rho] + \left[b \cdot \frac{c(t,t)}{c(t)} - \beta\right] - n \tag{17}$$

The first term on the right-hand side in equation (17) is the growth rate of individual consumption and the second term is a generational turnover term. Growth in full population consumption is boosted because of the arrival of new agents (who start to consume out of human wealth) and it is slowed down by the death of some individuals in the population. The third term corrects for population growth.

Per capita financial wealth is defined as $a(t) = \int_{-\infty}^{t} n(v,t) \cdot a(v,t)dv$. By differentiating this expression with respect to t and noting that $a(t,t) = 0$ (i.e. newborns do not have financial wealth) we find an equation for financial wealth dynamics:

$$\dot{a}(t) = (r - n)a(t) + g(t) + (1 - \bar{\tau}^{ssc}) \cdot w(t) - \tau^{LS}(t) + p(t) - c(t) \tag{18}$$

where the per capita level of education subsidy is $g(t) = \int_{t-S}^{t} n(v,t) \cdot g(v,t)dv$, of the gross wage is $w(t) = \int_{t-R}^{t-S} n(v,t) \cdot w(v,t)dv$, of the pension benefit is $p(t) = \int_{-\infty}^{t-R} n(v,t) \cdot p(v,t)dv$ and of the lump sum tax is $\tau^{LS}(t) = \int_{-\infty}^{t} n(v,t) \cdot \tau^{LS}(v,t)dv$.

Annuity payments drop out of the expression for per capita asset accumulation because they constitute transfers (via the life insurance companies) from households that die to households that stay alive.

Per capita human wealth is defined as $a^H(t) = \int_{-\infty}^{t} n(v,t) \cdot a^H(v,t)dv$. The dynamics for financial wealth is given by:

$$\begin{aligned} \dot{a}^H(t) = (r - n)a^H(t) + b \cdot a^H(t,t) \\ - \left[g(t) + (1 - \bar{\tau}^{SSC})w(t) + p(t) - \tau^{LS}(t)\right] \end{aligned} \tag{19}$$

Per capita human capital is defined as

$$h(t)=\int_{t-R}^{t-S} n(v,t)\cdot h(v,t)dv=\int_{t-R}^{t-S} n(v,t)A_H h(v)^{\varepsilon_H} S\cdot E(t-v)dv \tag{20}$$

The form of the individual human capital efficiency function defined in (5) permits one to write $h(t)$ as the sum of three components $h_1(t)$, $h_2(t)$ and $h_3(t)$, where each component reflects an exponential term in (5). Specifically, if one defines:

$$h_i(t)=\int_{t-R}^{t-S} n(v,t)A_H h(v)^{\varepsilon_H} S\cdot E_i(t-v)dv \tag{21}$$

where $E_i(u)=a_i\cdot e^{-\alpha_i\cdot(u-S)}$, the dynamics for human capital is given by:

$$\begin{aligned}\dot{h}(t)=n(t-S,t)A_H h(t-S)^{\varepsilon_H} S\cdot E(S)-n(t-R,t)A_H h(t-R)^{\varepsilon_H} S\cdot E(R)\\ -n\cdot h(t)-\beta\cdot h(t)-\alpha_1\cdot h_1(t)-\alpha_2\cdot h_2(t)-\alpha_3\cdot h_3(t)\end{aligned} \tag{22}$$

The intuition behind the delay differential equation (22) is that at a particular time t a new generation (*i.e.* the one born at time $t-S$) with human capital given by $A_H h(t-S)^{\varepsilon_H} S\cdot E(S)$ enters the labor market and the one generation at time $t-R$ with human capital given by $A_H h(t-R)^{\varepsilon_H} S\cdot E(R)$ retires. Also the changes in human capital depend on the death rate and on the relative productivity experiences of existing workers. The specific values of the coefficients of the individual human capital efficiency function play an important role in determining the movements of per capita human capital over time.

2.2 Firms

Firms are perfectly competitive and employ physical and human capital to produce a homogeneous good, $Y(t)$, under constant returns to scale Cobb-Douglas technology:

$$Y(t)=A_Y\cdot K(t)^{\varepsilon_Y}\cdot H(t)^{1-\varepsilon_Y} \tag{23}$$

where $K(t)$ denotes the physical capital stock, $H(t)=N(t)h(t)$ denotes the human capital stock and A_Y represents total factor productivity.

The representative firm rents human and physical capital to maximize its operating profit defined as follows:

$$\Pi(t)=Y(t)-\overline{w}\cdot H(t)-r^K\cdot K(t) \tag{24}$$

where $\overline{w}$ denotes the rental rate of human capital and r^K the rental rate on capital.

The first-order conditions characterizing the firm's optimal plans are:

$$\begin{cases} r^K = \dfrac{\partial Y}{\partial K} = A_Y \cdot \varepsilon_Y \cdot \left(\dfrac{h(t)}{k(t)}\right)^{1-\varepsilon_Y} \\ \overline{w} = \dfrac{\partial Y}{\partial H} = A_Y \cdot (1-\varepsilon_y) \cdot \left(\dfrac{h(t)}{k(t)}\right)^{-\varepsilon_Y} \end{cases} \tag{25}$$

For each factor of production, the marginal product is equated to the rental rate. Since the technology features constant returns to scale, pure profits are zero.

On the other hand, the household as a portfolio investor chooses its capital accumulation decision by maximizing the present value of cash flows from the capital stock, defined as:

$$V(t) = \int_t^\infty \left(r^k(s) \cdot K(s) - I(s)\right) \cdot e^{-\int_t^s r(u)du} ds \tag{26}$$

where $I(t)$ denotes gross investment.

The capital accumulation identity is given by:

$$\dot{K}(t) = I(t) - \delta \cdot K(t) \tag{27}$$

where δ is the constant depreciation rate of capital.

The investor chooses paths for gross investment and the capital stock in order to maximize (26) subject to (27) and taking as given the initial capital stock and the path of the rental rate. The first-order condition for this optimization problem is:

$$r^K = r + \delta \tag{28}$$

Since there are no adjustment costs on investment, the value of the firm equals the replacement value of the capital stock, *i.e.* $V(t) = K(t)$. Since the constant interest rate pins down the ratio between human and physical capital, it follows from equation (25) that the rental rate of human capital is time-invariant.

2.3 Government

We abstract from government consumption, although the model can be easily extended to incorporate exogenous spending on goods and services. The government makes transfer payments to households (education subsidies and pensions), raises revenues by taxing households through lump sum taxes and a proportional social security contribution, and pays interest on its outstanding stock of debt.

More specifically, the tax system takes the following form. All through life, the agent born at time v pays a lump sum tax equal to:

$$\tau^{LS}(v,t) = \bar{\tau}^{LS} \cdot h(v)^{\varepsilon_H} \tag{29}$$

where $\bar{\tau}^{LS}$ is exogenous.

During the schooling period, the agent receives from the government a study grant equal to:

$$g(v,t) = \bar{g} \cdot h(v)^{\varepsilon_H} \tag{30}$$

where $\bar{g}$ is exogenous.

During working life, the agent faces a social security contribution on wage earnings equal to $\bar{\tau}^{SSC} w(v,t)$ where $\bar{\tau}^{SSC}$ is exogenous.

After retirement, the agent receives from the government a pension equal to $p(v,t)$. We employ a PAYG pension system based on pension points. More specifically, at time t during working period, the agent born at time v accumulates pension points equal to the fraction between her wage and the rental rate of human capital. These points are averaged over his entire working period, and the number of pension points that he is entitled at the beginning of the retirement period is given by:

$$pp(v) = \frac{A_H h(v)^{\varepsilon_H} S}{R-S} \sum_{i=1}^{3} \frac{a_i}{\alpha_i} \cdot \left[1 - e^{-\alpha_i \cdot (R-S)}\right] \tag{31}$$

Therefore, the pension received by the agent at $t > v + R$ is equal to:

$$p(v,t) = \bar{p} \cdot pp(v) \cdot \bar{w}(t) \tag{32}$$

where $\bar{p}$ represents the value of a pension point and is exogenous.

The dynamics of per capita government debt, $d(t)$, is given by:

$$\dot{d}(t) = (r-n)d(t) + \left[g(t) - \tau^{LS}(t)\right] + \left[p(t) - \bar{\tau}^{SSC} \cdot w(t)\right] \tag{33}$$

Using the government solvency condition the intertemporal budget constraint of the government can be written as:

$$d(t) = \int_t^{\infty} \left\{ \left[\tau^{LS}(s) - g(s)\right] + \left[\bar{\tau}^{SSC} w(s) - p(s)\right] \right\} \cdot e^{-(r-n)(s-t)} ds \tag{34}$$

Therefore, the outstanding debt must be exactly matched by the present value of future primary surpluses.

2.4 Further model components

Financial wealth can be held in the form of claims on domestic capital, domestic government debt, and net foreign assets:

$$a(t) = k(t) + d(t) + f(t) \tag{35}$$

where $f(t)$ is per capita net foreign assets.

The clearing condition in the goods market is given by:

$$y(t) = c(t) + i(t) + nx(t) \tag{36}$$

where $nx(t)$ is per capita net exports.

The dynamics of per capita net foreign assets is given by:

$$\dot{f}(t) = \left(r^f - n\right)f(t) + y(t) - c(t) - i(t) \tag{37}$$

Under the assumptions of the tax system, the human wealth is given by:

$$a^H(v,t) = h(v)^{\varepsilon_H} \cdot a^H(t-v) \tag{38}$$

where $a^H(u)$ is a function only of age and has a form depending on the status of the agent (*i.e.* student, working, retired). For a student (*i.e.* $u < S$) we have that:

$$\begin{aligned} a^H(u) = {} & \frac{\overline{g}}{r+\beta}\left[1-e^{-(r+\beta)\cdot(S-u)}\right] \\ & + (1-\overline{\tau}^{SSC})\overline{w}A_H S \cdot e^{-(r+\beta)\cdot(S-u)} \cdot \sum_{i=1}^{3} \frac{a_i}{r+\beta+\alpha_i} \cdot \left[1-e^{-(r+\beta+\alpha_i)\cdot(R-S)}\right] \\ & + \frac{\overline{p}\cdot\overline{w}}{r+\beta}\frac{A_H S}{R-S}\sum_{i=1}^{3}\frac{a_i}{\alpha_i}\cdot\left[1-e^{-\alpha_i\cdot(R-S)}\right]\cdot e^{-(r+\beta)\cdot(R-u)} - \frac{\overline{\tau}^{LS}}{r+\beta} \end{aligned} \tag{39}$$

For a working agent (i.e. $S < u < R$) we have that:

$$\begin{aligned} a^H(u) = {} & (1-\overline{\tau}^{SSC})\overline{w}A_H S \cdot e^{-(r+\beta)\cdot(S-u)} \cdot \sum_{i=1}^{3} \frac{a_i}{r+\beta+\alpha_i} \cdot \left[e^{-(r+\beta+\alpha_i)\cdot(u-S)} - e^{-(r+\beta+\alpha_i)\cdot(R-S)}\right] \\ & + \frac{\overline{p}\cdot\overline{w}}{r+\beta}\frac{A_H S}{R-S}\sum_{i=1}^{3}\frac{a_i}{\alpha_i}\cdot\left[1-e^{-\alpha_i\cdot(R-S)}\right]\cdot e^{-(r+\beta)\cdot(R-u)} - \frac{\overline{\tau}^{LS}}{r+\beta} \end{aligned} \tag{40}$$

For a retired person (*i.e.* $u > R$) we have that:

$$a^H(u) = \frac{\overline{p}\cdot\overline{w}}{r+\beta}\frac{A_H S}{R-S}\sum_{i=1}^{3}\frac{a_i}{\alpha_i}\cdot\left[1-e^{-\alpha_i\cdot(R-S)}\right] - \frac{\overline{\tau}^{LS}}{r+\beta} \tag{41}$$

2.5 Steady State

In the presence of non-zero population growth, the model gives rise to ongoing economic growth. Therefore, the analysis is performed using per capita variables. As pointed by Heijdra and Romp (2009a) the value of ε_H is critical in determining whether the model exhibits exogenous or endogenous growth. We will restrict our attention to the case of exogenous growth. Hence we assume that the intergenerational knowledge transfer incorporated in the human capital production function (6) is subject to diminishing returns (i.e. $\varepsilon_H < 1$). In this section we analytically characterize the steady-state of the model and study its sensitivity with respect to various parameters and policy variables.

In the steady state all per capita aggregate variables are constant, so their levels grow at the population growth rate. The steady-state values for all variables are designated by means of a hat overstrike.

The steady-state per capita human capital stock, $\hat{h}$, is easily obtained from equation (20):

$$\hat{h}^{1-\varepsilon_H} = A_H S \cdot b e^{-b \cdot S} \sum_{i=1}^{3} \frac{a_i}{b+\alpha_i} \cdot \left[1 - e^{-(b+\alpha_i)\cdot(R-S)}\right] \tag{42}$$

Due to the design of the tax system in the model, steady-state per capita values of some government related variables depend on $\hat{h}$. More specifically, one obtains the following steady state values:

$$\begin{aligned} &\hat{\tau}^{LS} = \bar{\tau}^{LS} \hat{h}^{\varepsilon_H}; \hat{g} = \bar{g} \cdot \hat{h}^{\varepsilon_H} \cdot \left(1 - e^{-bS}\right) \\ &p\hat{p} = \frac{A_H \hat{h}^{\varepsilon_H} S}{R-S} \sum_{i=1}^{3} \frac{a_i}{\alpha_i} \cdot \left[1 - e^{-\alpha_i \cdot (R-S)}\right]; \hat{p} = \bar{p} \cdot p\hat{p} \cdot \bar{w} \cdot e^{-b \cdot R} \end{aligned} \tag{43}$$

The steady state value of per capita government debt is:

$$\hat{d} = \frac{\left[\hat{\tau}^{LS} - \hat{g}\right] + \left[\hat{\tau}^{SSC} \cdot \hat{w} - \hat{p}\right]}{(\hat{r} - n)} \tag{44}$$

In the steady-state equilibrium, all variables applying to individuals can be written solely in terms of their age. We find that:

$$\hat{c}(u) = (\rho + \beta) \cdot \hat{a}^H(0) \cdot e^{\sigma \cdot (\hat{r} - \rho) \cdot u} \tag{45}$$

$$\hat{a}^H(u) = \hat{h}^{\varepsilon_H} a^H(u) \tag{46}$$

$$\hat{a}(u) = \frac{1}{\rho + \beta} \hat{c}(u) - \hat{a}^H(u) \tag{47}$$

Human wealth at birth key 'initial condition' since it is an important determinant for the age profiles for consumption and financial assets. In our open economy framework, the value $\hat{a}^H(0)$ can be easily computed from equation (46).

3 Visualizing pension reform effects

In this section we calibrate the model and analyze eight pension reform scenarios. The first set of two scenarios does not involve a shift to a mixed system. The next set of four scenarios corresponds to a shift between a PAYG system to a mixed PAYG-FF system. The last set of two scenarios correspond to a "reverse-sequencing" (Schneider et al., 2004) shift between a PAYG system to a mixed PAYG-FF system. Barr and Diamond (2009) point out that it is an analytical mistake to focus exclusively on the steady states before and after the reform, ignoring the steps that are necessary to get to that steady state. A move to a fully-funded system or a mixed system generally has major fiscal costs. This kind of reform might impose an added burden on present workers, who have to pay not only their own contributions but also some of the taxes that finance current pensions. Thus, it is mistaken to present the gain to pensioners in later generations as a Pareto improvement, if it comes at the expense of present cohorts. Therefore, we analyze the eight scenarios focusing not only on the steady states, but also on transitional dynamics of key variables and on intergenerational welfare effects.

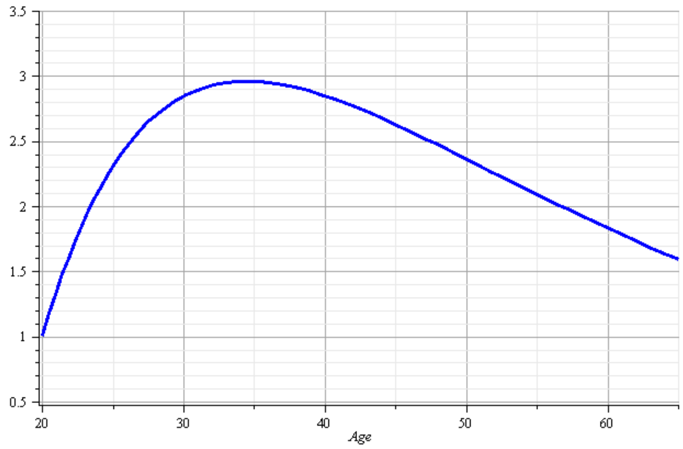
3.1 Calibration

The model is calibrated to capture some features of the Romanian economy. We set the foreign interest rate at $r_f=0.05$, the rate of time preference $\rho=0.04$, the annual depreciation rate of capital $\delta=0.07$, the human capital externality parameter $\varepsilon_H=0.3$, and the capital share parameter in the production function $\varepsilon_Y=0.3$. The scaling parameters are set to unity ($A_Y=A_H=1$). Similar values of the parameters are commonly employed in the pension literature (Echevarrıa and Iza, 2006; Heijdra and Romp, 2009a, 2009b) or in studies analyzing the Romanian macroeconomic environment (Altar et. al, 2008a, 2008b; Caraiani, 2009).

The mortality rate is set to $\beta=0.01375$, corresponding to a life expectancy of 72.7 years. The annual birth rate was chosen $b=0.01775$, and, therefore, the model exhibits an annual population growth rate $n=0.004$. The duration of full-time educational activities is equal to $S=20$ years and the retirement age is set to $R=65$ years. The resulting dependency ratio, $\hat{dr}$, is about 82%, a quite realistic value. For example, in 2009, Romania public pension system relied on only 4,587 million employees to contribute, while the number of pensioners went up to 5,689 million. However, some of these pensioners have not contributed to the public pension system. If only the contributing pensioners are accounted for, the effective dependency ratio in Romania has been in the last few years slightly below 1.

The parameters of the human capital efficiency function are set to $a_1=1$, $a_2=6$, $a_3=-6$, $\alpha_1=0.05$, $\alpha_2=0.03$, $\alpha_3=0.1$, allowing for a hump-shaped profile with a maximum around 35 years (Figure 1).

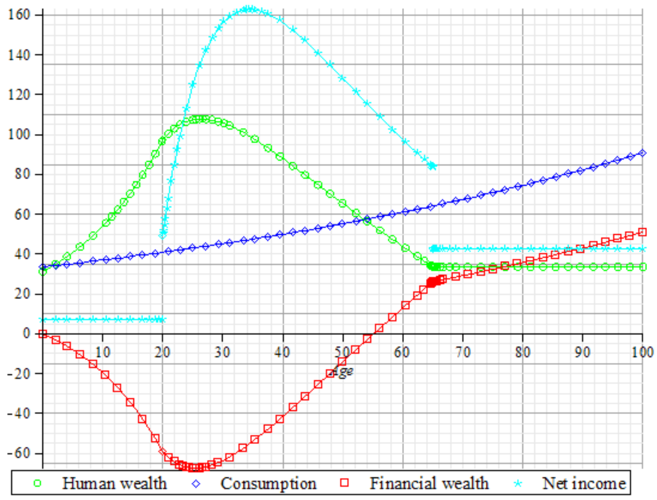
Figure 1. *Human capital efficiency age profile*



Source: own computations

The Social Security Contribution rate is set to $\overline{\tau}^{SSC}=0.2$. The value is comparable to the implicit Social Security Contribution rate in Romania that is close to 22%, although the statutory rate is much larger. The pension point is set to $\overline{p}=0.3$ that corresponds to a ratio between the average pension benefit and the average gross wage of 29.65%. The parameter quantifying the education subsidy is set to $\overline{g}=4.5$ corresponding to an aggregate education subsidy of about 5% of GDP. The lump sum parameter is set to $\overline{\tau}^{LS}=2.5$. Under these circumstances, the public system is sustainable in the long run and the steady state debt to GDP ratio, $\hat{d}/\hat{y}$, is 27%. The output shares of consumption, investment, and net exports are, respectively 80.10%, 18.5%, and 1.39%. The steady state values for the benchmark case are summarized in column 1 of Table 1.

Figure 2. *Steady state profiles for individuals*



Source: own computations

Figure 2 depicts the age profile for the steady state individual consumption, human wealth and financial wealth. The consumption is increasing throughout life, a pattern specific to a "perpetual youth" economy. The human wealth has a hump-shaped profile due to the realistic pattern of human capital efficiency.

3.2 The analyzing apparatus

In this subsection we describe the apparatus for analyzing the transitional and long-run effects on the macroeconomic variables (*i.e.* impulse-response functions) as well as the intergenerational welfare effects of various policy measures related to pension reform.

It is assumed that the economy is initially in steady state equilibrium and that at time $t = 0$ a modification of the policy variables occurs. Following this shock, the non-predetermined variables (human wealth and consumption) of existing generations (i.e. $v < 0$) react immediately. On the other hand, the predetermined variables (such as financial wealth) stay constant. More specifically, for $v < 0$, we have that:

$$\begin{aligned} &a^H(v,0) \neq \hat{a}^H(-v); a^H(0) \neq \hat{a}^H \\ &a(v,0) = \hat{a}(-v); a(0) = \hat{a} \\ &c(v,0) = (\rho+\beta)\left[a(v,0) + a^H(v,0)\right] \neq \hat{c}(-v); c(0) \neq \hat{c} \end{aligned} \tag{48}$$

The value of $a^H(v,0)$ have to be computed separately for every policy modification. The values in (48) form the initial conditions for the dynamic system that will track the transitional effects.

For example, for pre-shock generations (i.e. $v<0$) the consumption at $t>0$ is given by:

$$c(v,t)=(\rho+\beta)\cdot\left[\hat{a}(-v)+a^H(v,0)\right]\cdot e^{(r-\rho)\cdot t} \tag{49}$$

On the other hand, the consumption at $t>0$ for post-shock generations (i.e. $v>0$) is given by:

$$c(v,t)=(\rho+\beta)\cdot a^H(v,v)\cdot e^{(r-\rho)\cdot(t-v)} \tag{50}$$

Determining the consumption dynamics after the shock allows us to compute the change in welfare for different generations. For existing agents (i.e. $v<0$), the change in welfare is evaluated from the perspective of the period $t=0$:

$$\Delta U(v,0)=\int_0^\infty\left[\ln c(v,s)-\ln\hat{c}(v,s)\right]\cdot e^{-(\rho+\beta)\cdot s}ds\,. \tag{51}$$

For future agents (*i.e.* $v>0$), the change in welfare is evaluated from the perspective of their birth date:

$$\Delta U(v,v)=\int_v^\infty\left[\ln c(v,s)-\ln\hat{c}(v,s)\right]\cdot e^{-(\rho+\beta)\cdot(s-v)}ds\,. \tag{52}$$

3.3 Alternative pension reform scenarios

3.3.1 "no-shift" pension reforms

The first set of two scenarios does not involve a shift to a mixed system, but only the modification of the retirement age. The first alternative corresponds to an increase of 1 year in the retirement age. The policy variable modified to insure that the economy is on the new saddle path is the Social Security Contribution rate, which is reduced to $\overline{\tau}^{SSC}=0.1934$. The second scenario consists also in an increase of 1 year in the retirement age, but the pension point is adjusted to $\overline{p}=0.3083$.

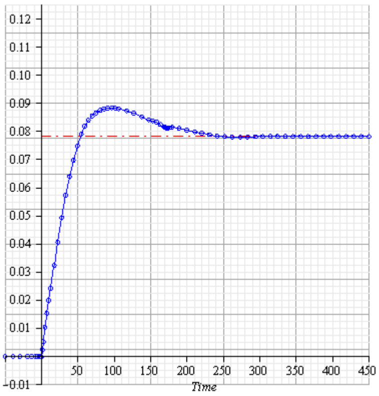
Table 1. *Benchmark steady state and long run effects*

	Benchmark	"no shift" reform		"orthodox" shift from PAYG to a mixed PAYG-FF system reform				"reverse-sequencing" shift reform	
		Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8
R	65	**66**	**66**	65	65	65	**66**	65	**65.125**
$\overline{p}$									
old	0.3	0.3	**0.3083**	0.3	0.3	**0.2925**	**0.3053**	0.3	0.3
new	--	--	--	**0.2925**	**0.2167**	**0.2925**	**0.3053**	**0.2965**	0.3
$\overline{\tau}^{LS}$	2.5	2.5	2.5	**2.5413**	2.5	**2.5130**	2.5	2.5	2.5
$\overline{\tau}^{SSC}$									
old	0.2	**0.1934**	0.2	0.2	0.2	0.2	0.2	0.2	0.2
new	--	--	--	**0.1950**	**0.1950**	**0.1950**	**0.1950**	**0.1950**	**0.1950**
$d\hat{r}$	0.8281	0.7920	0.7920	0.8281	0.8281	0.8281	0.7920	0.8281	0.8145
$\hat{c}/\hat{y}$	0.8010	0.8002	0.7946	0.8023	0.8010	0.8043	0.7990	0.8010	0.8004
$\hat{i}/\hat{y}$	0.1850	0.1850	0.1850	0.1850	0.1850	0.1850	0.1850	0.1850	0.1850
$n\hat{x}/\hat{y}$	0.0140	0.0148	0.0204	0.0127	0.0140	0.0107	0.0160	0.0140	0.0146
$\hat{a}/\hat{y}$	2.4662	2.4642	2.3449	2.5425	3.4147	2.5634	2.4000	2.5001	2.4522
$\hat{d}/\hat{y}$	0.2696	0.2868	0.2874	0.3186	1.2181	0.2961	0.2469	0.3034	0.2699
$\hat{f}/\hat{y}$	-0.3034	-0.3226	-0.4424	-0.2761	-0.3034	-0.2328	-0.3469	-0.3034	-0.3177
$\Delta\%d$	--	0.0782	0.0802	0.1814	3.5170	0.0982	-0.0720	0.1253	0.0027
$\Delta\%c$	--	0.0124	0.0054	0.0016	0.0000	0.0041	0.0110	0.0000	0.0009
$\Delta\%y$	--	0.0135	0.0135	0.0000	0.0000	0.0000	0.0135	0.0000	0.0017

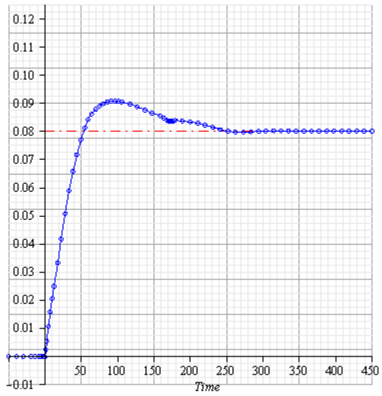
Source: own computations

The steady state profiles of the two scenarios are comparable, with an increase of public debt of approximately 8% with respect to the benchmark. Also, the transitional path of public debt is similar in the two scenarios. However, the welfare effects are different. In the first scenario, rather obviously, all retired generations are unaffected by the pension reform. Post-reform cohorts are better off due to increase in the human wealth associated with a lower SSC contribution rate. The studying generations and working age cohorts have a higher utility than in the benchmark case, with the highest increase in the case of generations just entering the labor market. In the second scenario all generations are better off, including the retired ones since the increase of the pension point benefit them also. However, the highest gain corresponds to the generations near retirement. Future generations obtain a lower increase in welfare that in the first scenario, since there is no reduction in contributions and the increase in the pension point benefit them only when they retire. Therefore, the reforms in the first two scenarios are Pareto improving, but there is increase of public debt of approximately 8% in the steady state, with a maximum of 9% during transition to the new steady state.

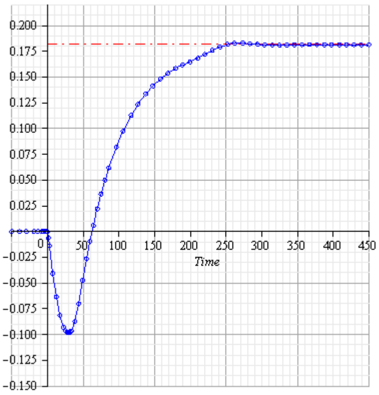
Figure 3. *Change in public debt relative to benchmark*



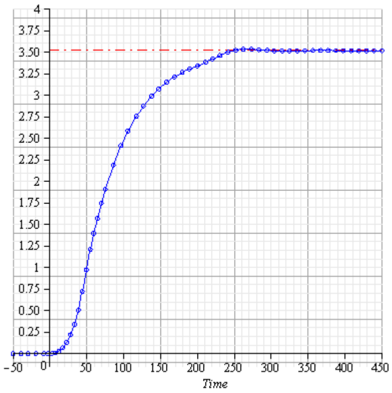
a. Scenario 1



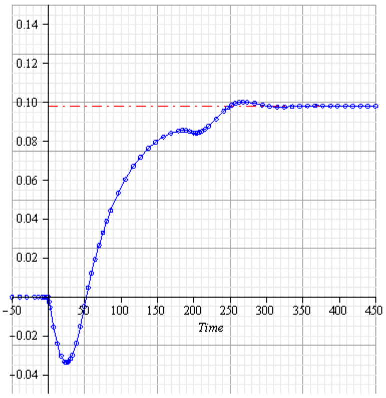
b. Scenario 2



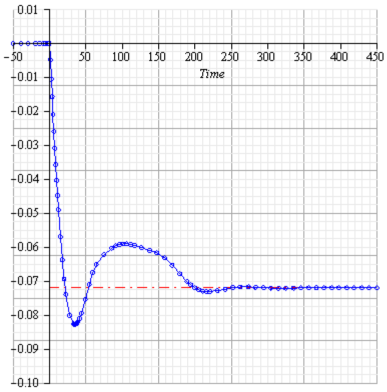
c. Scenario 3



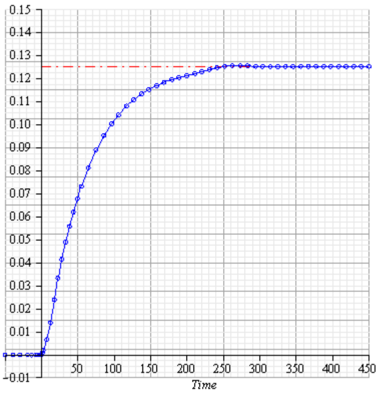
d. Scenario 4



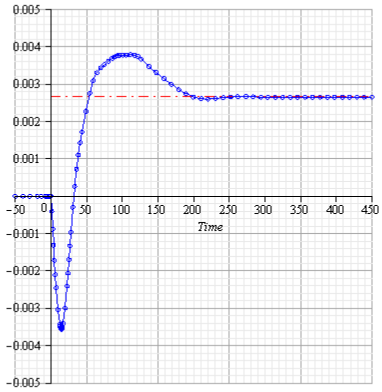
e. Scenario 5



f. Scenario 6



g. Scenario 7



h. Scenario 8

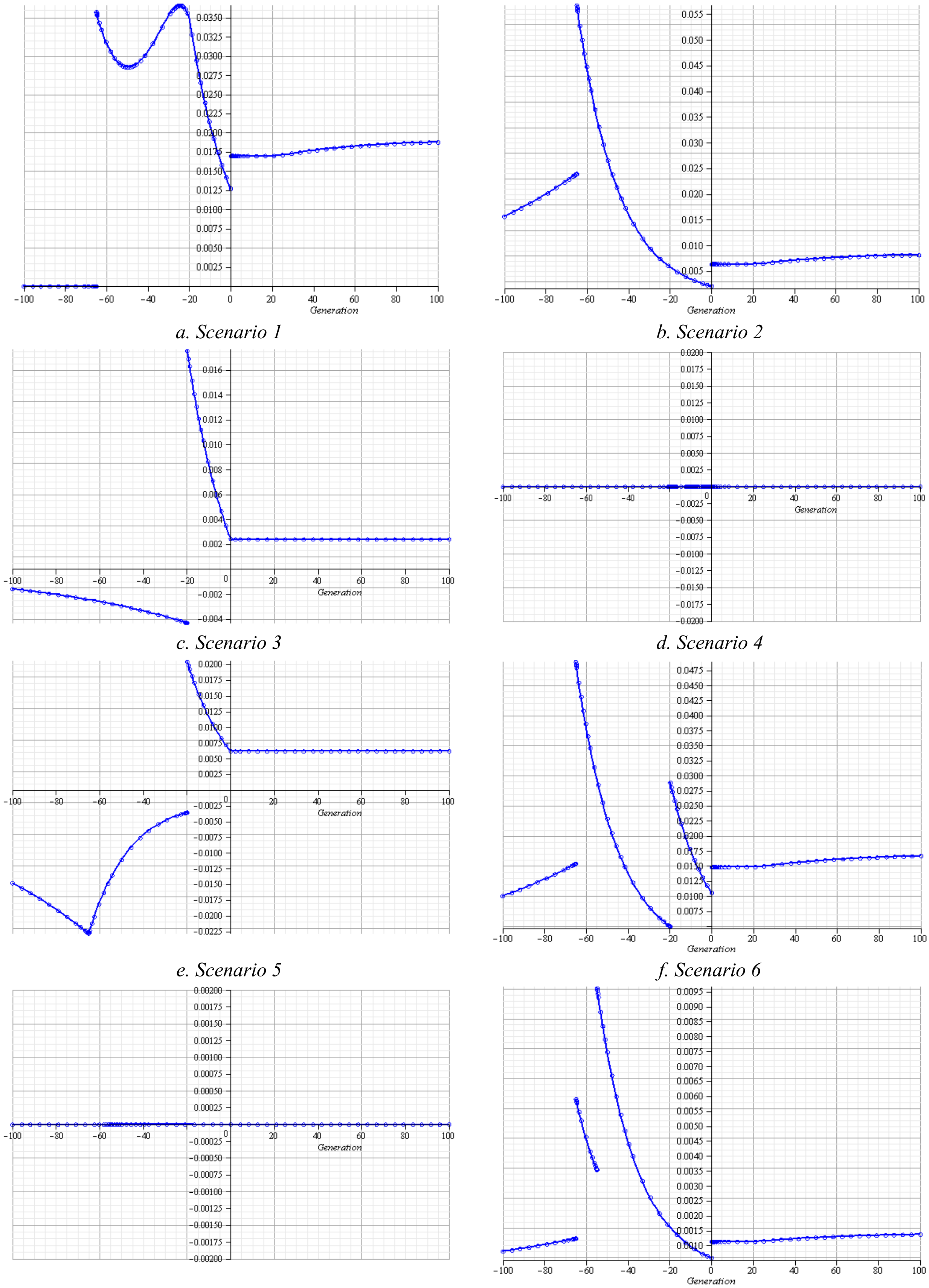
Source: own computations

3.3.2 "orthodox" transition pension reforms

The next set of four scenarios corresponds to a shift between a PAYG system to a mixed PAYG-FF system. The new cohorts entering the workforce at age *S=20* are expected to pay a lower Social Security Contribution rate $\overline{\tau}_{new}^{SSC} = 0.195$ that the workers already enrolled in the public pension system ($\overline{\tau}_{old}^{SSC} = 0.2$). In the same time, in the third scenario, the cohorts in the reformed system will receive a lower pension benefit from the public pension system corresponding to a pension point of $\overline{p}_{new} = 0.2925$, while the other pensioners will be paid a pension point of $\overline{p}_{old} = 0.3$. However, the public system is unsustainable and in order to assure that the economy is on the new saddle path an increase in the lump sum tax is also necessary ($\overline{\tau}^{LS} = 2.5413$). The next three scenarios are variants of the third one. In the fourth scenario the lump sum is held constant and the economy is set on the new saddle path using a sufficient downward adjustment of the pension point of the pensioners of the reformed system ($\overline{p}_{new} = 0.2167$). In the fifth scenario the pension point of all the retirees is reduced to $\overline{p}_{new} = \overline{p}_{old} = 0.2925$ and the lump sum tax is adjusted to $\overline{\tau}^{LS} = 2.5130$. In the sixth scenario, an increase of 1 year in the retirement age takes also place and the pension point is adjusted to put the economy on the new sustainable path ($\overline{p}_{new} = \overline{p}_{old} = 0.3053$).

In the third scenario the transition to the mixed system is financed by a higher lump sum which has a negative effect on human wealth for all cohorts. However, this effect is balanced, for the student generations and post-reform generations, by the lower SSC rate. Therefore, the reform is not Pareto improving since the working age generations and retired cohorts have a lower utility relative to the benchmark. For the first part of the transition period, the public debt decreases due to the higher income from lump sum taxes. After the post-reform workers begin to retire, the public debt begins to increase, reaching a steady state 18% higher than the initial case. The conclusions are similar for the reform in the fifth scenario.

Figure 4. *Change in welfare relative to benchmark*



a. Scenario 1

b. Scenario 2

c. Scenario 3

d. Scenario 4

e. Scenario 5

f. Scenario 6

g. Scenario 7

h. Scenario 8

Source: own computations

The reform in the fourth scenario is welfare neutral since the reduction of SSC rate for post-reformed contributors is fully balanced by the downward adjustment of their value of the pension point. However, since the income from SSC contribution decreases but the pre-reformed contributors are paid the full pension point, there is a staggering increase in public debt to a steady state 350% higher than the benchmark.

The reform in the sixth scenario is Pareto improving, since there is no modification of the lump sum tax and the increase in the pension point benefits both post-reform and pre-reform contributors. Moreover, increased SSC contributions, due to higher retirement age, imply that the public debt is decreasing during the transition period to a steady state 7% lower.

3.3.3 "reverse-sequencing" transition pension reforms

The last set of two scenarios correspond to a "reverse-sequencing" (Schneider et al., 2004) shift between a PAYG system to a mixed PAYG-FF system. In this kind of reform, the preretirement generations enter the reformed system in the last 10 years of their working period. Therefore, in the first 55 years an employee pays the full Social Security Contribution rate $\overline{\tau}_{old}^{SSC} = 0.2$, and afterwards a reduced rate $\overline{\tau}_{new}^{SSC} = 0.195$. In the seventh scenario the pension point of the retiree in the reformed system is adjusted so that the economy is on the new saddle path ($\overline{p}_{new} = 0.2965$). In the final scenario, the retirement age is increased to 65.125 years and the economy is on the saddle path if the pension point is approximately at the initial level of 0.3 ($\overline{p}_{new} = \overline{p}_{old} = 0.3004$).

The reform in the seventh scenario is welfare neutral since the government does not modify the lump sum tax and the reduction of SSC rate for contributors in the new system is balanced by a lower value of the pension point. However, in comparison with scenario four, there is a more manageable increase in public debt to a steady state only 10% higher than the benchmark. The reform in the eighth scenario is Pareto improving but also gives government more flexibility, since there is an increase of less than 1% of steady state public debt and in the first part of the transition period there is a decrease in public debt.

4 Concluding remarks

The paper has investigated the macroeconomic and welfare effects of a gradual transition from a PAYG pension system to a mixed system comprising a PAYG pillar and a FF pillar. The analyzing framework consists of an OLG model with finite life but perpetual youthful households. Agents engage in educational activities at the start of their life, create human capital that is used during the working period to rent it to firms, and, later on in life, retire and are paid a pension benefit. The constructed model allows for a hump-shaped human capital age profile and for a realistic method for computing pension benefits using a pension point scheme. Due to the form of the individual human capital efficiency function, the dynamics for human capital is given by a delay differential equation.

Several pension reforms have been simulated in the context of a calibrated version of the model. The findings indicate that, when accompanied by an increase in retirement age, the shift to a mixed pension system is Pareto improving and alleviates the burden of public debt. A "reverse-sequencing" shift reform is also Pareto improving and only a marginal increase in retirement age is necessary to keep the steady state public debt approximately constant. Moreover, at the beginning of the transition period there is a decrease in public debt, and, therefore, this kind of reform gives more flexibility to the government.

The paper can be extended in several directions, some which we intend to pursue in the near future. First, it would probably be useful to incorporate labor-leisure choice in the framework in order to endogenize the household labor supply. Second, the model can be extended to allow individual agents to choose their optimal retirement age, taking into account the profiles of wages, taxes, and the public pension system. Third, the arguably unrealistic but analytically convenient lump-sum tax that the agents are paying can be replaced with a simple proportional labor income tax. Finally, a more realistic description of the mortality process can be embedded in the framework. Especially in the context of pension reform, these extensions could significantly affect the conclusions of the present paper.

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Appendix

1. Households

The Hamiltonian of the household optimization problem is:

$$\begin{aligned} H &= \ln(c(v,s))\cdot e^{-(\rho+\beta)\cdot(s-t)} + \mu(v,s)\cdot\{(r+\beta)\cdot a(v,s) + g(v,s)\cdot \mathbf{1}_{(v,v+S)}(s) \\ &\quad +[(1-\overline{\tau}^{ssc})\cdot w(v,s)]\cdot \mathbf{1}_{(v+S,v+R)}(s) + p(v,s)\cdot \mathbf{1}_{(v+R,\infty)}(t) - \tau^{LS}(v,s) - c(v,t)\} \end{aligned} \tag{A.1}$$

The first order conditions are:

$$\frac{\partial H}{\partial c} = c(v,s)^{-1}\cdot e^{-(\rho+\beta)\cdot(s-t)} - \mu(v,s) = 0 \tag{A.2}$$

$$\dot{\mu}(v,s) = -\frac{\partial H}{\partial a} = -[r+\beta]\cdot \mu(v,s) \tag{A.3}$$

Using (A.2) and (A.3) one obtains successively that:

$$\dot{\mu}(v,s)\cdot c(v,s) + \mu(v,s)\cdot \dot{c}(v,s) = -(\rho+\beta)\cdot e^{-(\rho+\beta)\cdot(s-t)}$$

$$\frac{\dot{\mu}(v,s)\cdot c(v,s)}{\mu(v,s)\cdot c(v,s)} + \frac{1}{\sigma}\cdot\frac{\dot{c}(v,s)}{c(v,s)} = -(\rho+\beta)$$

$$-[r+\beta] + \frac{\dot{c}(v,s)}{c(v,s)} = -(\rho+\beta)$$

or alternatively the Euler consumption equation:

$$\frac{\dot{c}(v,s)}{c(v,s)} = [r-\rho] \tag{A.4}$$

To obtain the consumption in the planning period we define:

$$R(t,s) = \exp\left[-\left(r+\beta\right)\left(s-t\right)\right] \tag{A.5}$$

and notice that:

$$\begin{aligned} &\frac{d}{ds}(a(v,s)\cdot R(t,s)) = [\dot{a}(v,s) - (r+\beta)\cdot a(v,s)]\cdot R(t,s) \\ &= [g(v,t)\cdot \mathbf{1}_{(v,v+S)}(t) + [1-\overline{\tau}^{SSC}]w(v,t)\cdot \mathbf{1}_{(v+S,v+R)}(t) \\ &\qquad\qquad + p(v,t)\cdot \mathbf{1}_{(v+R,\infty)}(t) - \tau^{LS}(v,t) - c(v,t)]\cdot R(t,s) \end{aligned}$$

Therefore:

$$\lim_{s->\infty} R(t,s)\cdot a(v,s) - a(v,t) = a^{H}(v,t) - \int_{t}^{\infty} c(v,s)\cdot R(t,s)ds$$

Using the transversality condition

$$\lim_{s->\infty} R(t,s)\cdot a(v,s) = 0 \tag{A.6}$$

one obtains that:

$$\int_t^{\infty} c(v,s)\cdot R(t,s)ds = a(v,t) + a^H(v,t) \tag{A.7}$$

Solving the Euler consumption equation (A.4) one obtains that:

$$c(v,s) = c(v,t)\cdot e^{-(\rho+\beta)\cdot(s-t)}\cdot\frac{1}{R(t,s)} \tag{A.8}$$

Using (A.8) in (A.7) we have that:

$$c(v,t)\cdot\Phi(t) = a(v,t) + a^H(v,t) \tag{A.9}$$

where:

$$\Phi(t) = \int_t^{\infty} e^{-(\rho+\beta)\cdot(s-t)}ds = \frac{1}{\rho+\beta} \tag{A.10}$$

2. Firms

The Hamiltonian of the portfolio investor optimization problem with objective function given in (26) is:

$$\mathcal{H} = \left[r^k(s)\cdot K(s) - I(s)\right]\cdot e^{-r(s-t)} + \mu(s)\cdot\left[I(s) - \delta\cdot K(s)\right] \tag{A.11}$$

The first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial I} = 0 = \mu(s) - e^{-r(s-t)} \tag{A.12}$$

$$\dot{\mu}(s) = -\frac{\partial \mathcal{H}}{\partial K} = r^K(s)\cdot\mu(s) - \delta\cdot\mu(s) \tag{A.13}$$

Combining (A.12) and (A.13) one obtains that:

$$r^K(s) = r + \delta \tag{A.14}$$

It follows from (26) that:

$$V(t) = \int_t^{\infty}\left(r\cdot K(s) - \dot{K}(s)\right)\cdot e^{-r(s-t)}ds = -\int_t^{\infty}\left(K(s)e^{-r(s-t)}\right)' ds$$

Using the transversality condition

$$\lim_{s->\infty} K(s)e^{-r(s-t)} = 0 \tag{A.15}$$

one obtains that:

$$V(t) = K(t) \tag{A.16}$$