Decentralized Supervisory Control with Communicating Supervisors Based on Top-Down Coordination Control

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Motivation

- Very few constructive results in decentralized supervisory control
- Least restrictive solutions do not exist
- Reason: Lack of structure unlike modular control problems
- Idea: modular over-approximation and use of multi-level coordination control
- Local supervisors are assisted by coordinators: modular control with coordination
- For large number of components central coordinator requires too much coordination on the global level
- Therefore, multilevel coordination control is used

Procedure of Coordination Control

Given

$$G = G_1 \| G_2 \text{ with } G_i = (Q, A_i, \delta_i, q_0^i)$$

and a (global) specification language K.
Coordination control consists in:

- Set $A_k = A_1 \cap A_2$, natural projection: $P_k : A^* \to A_k^*$
- Extend A_k so that $K = P_{1+k}(K) \| P_{2+k}(K) \| P_k(K)$[1]
- Construct a coordinator $G_k = P_k(G_1) \| P_k(G_2)$ over $A_k \supseteq A_1 \cap A_2$
- Compute supervisor S_k for G_k with respect to $P_k(K)$
- Compute supervisors S_{i+k} for $G_i \| [S_k / G_k]$ with respect to $P_{i+k}(K)$
- Construct a nonblocking coordinator for the computed supervisors

[1] $P_{1+k} : (A_1 \cup A_2)^* \to (A_1 \cup A_k)^*$

Conditional decomposability

Definition

A language K is said to be conditionally decomposable with respect to event sets A_1, A_2, A_k if it can be written as

$$K = P_{1+k}(K) \| P_{2+k}(K) \| P_k(K) .$$

There always exists such a A_k!

It is a good candidate for coordinator alphabet

Remarks:

1. For systems with many components central coordination requires too many events in A_k!
2. Can be checked in polynomial time in the number of agents!

Review of Coordination Control

- Consider generators G_1, G_2, G_k over A_1, A_2, A_k, respectively.
- Let $K \subseteq L_m(G_1 \| G_2 \| G_k)$ be a specification language.
- G_k makes G_1 and G_2 conditionally independent
- K and $\overline{K}$ are conditionally decomposable wrt A_1, A_2, A_k.
- Aim: determine supervisors S_1, S_2, S_k so that the closed-loop system with the coordinator satisfies

$$L(S_1/[G_1 \| (S_k/G_k)]) \parallel L(S_2/[G_2 \| (S_k/G_k)]) = \overline{K}$$

and

$$L_m(S_1/[G_1 \| (S_k/G_k)]) \parallel L_m(S_2/[G_2 \| (S_k/G_k)]) = K\,.$$

Coordinator

Algorithm (Construction of a coordinator)

G_1 and G_2 over A_1 and A_2, respectively. Construct the event set A_k and the coordinator G_k as follows:

1. Set $A_k = A_1 \cap A_2$
2. Extend A_k so that K and $\overline{K}$ are conditionally decomposable
3. Extend A_k so that P_k is $L(G_i)$-observer
4. Define $G_k = P_k(G_1) \parallel P_k(G_2)$

Existence of the solution

Theorem.(Existential result) There exist supervisors S_1, S_2, S_k such that

$$L(S_1/[G_1\|(S_k/G_k)]) \parallel L(S_2/[G_2\|(S_k/G_k)]) = \overline{K}$$

if and only if K conditionally controllable with respect to generators G_1, G_2, G_k and event sets $A_{1,u}$, $A_{2,u}$, $A_{k,u}$.
Definition(Conditional Controllability.) K is called conditionally controllable for G_1, G_2, G_k and $A_{1,u}$, $A_{2,u}$, $A_{k,u}$ if

1. $P_k(K)$ is controllable with respect to $L(G_k)$ and $A_{k,u}$,
2. $P_{1+k}(K)$ is controllable with respect to $L(G_1)\|\overline{P_k(K)}$ and $A_{1+k,u}$,
3. $P_{2+k}(K)$ is controllable with respect to $L(G_2)\|\overline{P_k(K)}$ and $A_{2+k,u}$.

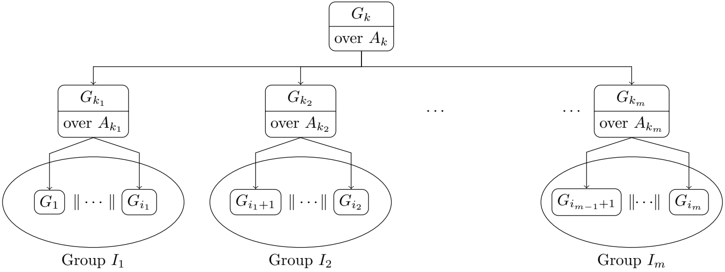
Multilevel control motivation

Centralized coordination suffers from several problems:

- For large n too many events must be included in A_k!
- Too many events need to be communicated among all subsystems
- Coordinator as well as its supervisor are too large
- Our solution: divide subsystems into groups and associate each group with group coordinators that need much less events
- Top level coordination then may have much fewer events as well

Multilevel Hierarchy

Subsystems are organize into groups starting from the lowest level:



$I_j = \{i_{j-1}+1, i_{j-1}+2, \ldots, i_j\}$

$\bigcup_{k,\ell \in \{1,\ldots,m\}}^{k \neq l} (A_{I_k} \cap A_{I_\ell})$ smaller than $\bigcup_{k,\ell \in \{1,\ldots,n\}}^{k \neq l} A_k \cap A_l)$!

2 level Conditional decomposability

Definition (Two-level conditional decomposability)

A language $K = \overline{K} \subseteq A^*$ is called **two-level conditionally decomposable** with respect to alphabets $A_1, \ldots, A_n$, high-level coordinator alphabet A_k, and low-level coordinator alphabets $A_{k_1}, \ldots A_{k_m}$ if

$$K = \|_{r=1}^{m} P_{I_r+k}(K) \qquad \text{and} \qquad P_{I_r+k}(K) = \|_{j\in I_r} P_{j+k_r+k}(K)$$

for $r = 1, \ldots, m$. ◁

Example: 2 level CD vs. CD

Let $K_{12} \subseteq \{a_1,a_2\}^*$, $K_{34} \subseteq \{a_3,a_4\}^*$, and $K = K_{12} \| K_{34}$ (below).

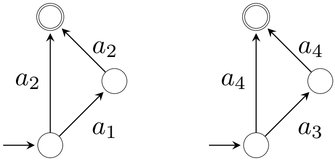


Figure: Generators of languages K_{12} and K_{34}, respectively

Hence, $K = P_{1+2}(K) \| P_{3+4}(K)$, i.e. $A_k = \emptyset$.
For $A_{k_1} = \{a_1\}$ and $A_{k_2} = \{a_4\}$:
$K_{12} = P_{1+k_1}(K_{12}) \| P_{2+k_1}(K_{12})$ and $K_{34} = P_{3+k_2}(K_{34}) \| P_{4+k_2}(K_{34})$.
To make K conditionally decomposable wrt $\{a_i\}$, $i = 1, \ldots, 4$ and $A_{k'}$,
either a_1 or a_2, and either a_3 or a_4 should be in $|A_{k'}| \geq 2$,
whereas $|A_{k_1}| = |A_{k_2}| = 1$ enough for 2-level CD!

Special case of multilevel control problem

Example

Let $G = G_1 \| \dots \| G_4$ with $I_1 = \{1,2\}$ and $I_2 = \{3,4\}$.
G_{k_1} is coordinator of G_1 and G_2
G_{k_2} is coordinator of G_3 and G_4
G_k is coordinator of G_{k_1} and G_{k_2}.

2 level CD:
$K = P_{1+2+k}(K) \| P_{3+4+k}(K)$, $P_{1+2+k}(K) = P_{1+k_1+k}(K) \| P_{2+k_1+k}(K)$,
and $P_{3+4+k}(K) = P_{3+k_2+k}(K) \| P_{4+k_2+k}(K)$.

Multilevel coordination control :

$$S_1/[G_1 \parallel (S_{k_1}/G_k \| G_{k_1})] \parallel S_2/[G_2 \parallel (S_{k_1}/G_k \| G_{k_1})] \parallel$$
$$S_3/[G_3 \parallel (S_{k_2}/G_k \| G_{k_2})] \parallel S_4/[G_4 \parallel (S_{k_2}/G_k \| G_{k_2})].$$

◁

Problem of multilevel control

Problem formulation. Consider $G = G_1 \| \ldots \| G_n$ along with the two-level hierarchical structure of subsystems $I_j = \{i_{j-1}+1, i_{j-1}+2, \ldots, i_j\}$, $j = 1, \ldots, m \le n$, on the low level.

Determine supervisors S_i, $i \in I_j$, for groups $\{G_i \mid i \in I_j\}$, $j = 1, \ldots, m$, and supervisors for low-level coordinators combined with the high-level coordinator S_{k_j}, $j = 1, \ldots, m$, such that

$$\|_{j=1}^{m} \|_{i \in I_j} L(S_i / [G_i \parallel (S_{k_j} / G_k \| G_{k_j})]) = K \qquad \triangleleft$$

Main Existential result

Definition (Two level conditional controllability)

$K \subseteq L(\|_{i=1}^{n} G_i \parallel G_k)$ is **two-level conditionally controllable** wrt $G_1, \ldots, G_n$, $A_1, \ldots, A_n$, high-level coordinator alphabet A_k, low-level coordinator alphabets $A_{k_1}, \ldots A_{k_m}$, and A_u if

1. $P_{k_j+k}(K)$ is controllable with respect to $L(G_{k_j} \| G_k)$ and $A_{k_j+k,u}$,
2. for $j = 1, \ldots, m$ and $i \in I_j$, $P_{i+k+k_j}(K)$ is controllable with respect to $L(G_i) \parallel P_{k_j+k}(K)$ and $A_{i+k_j+k,u}$. ◁

Theorem

There exist a set of multilevel supervisors such that such

$$\|_{j=1}^{m} \|_{i \in I_j} L(S_i/G_i \parallel (S_{k_j}/G_k \| G_{k_j})) = K \qquad (1)$$

if and only if K is two-level conditionally controllable.

Computation of supremal cC sublanguages

Theorem

If $\cap_{i\in I_j} P^{i+k_j}_{k_j}(\sup \mathrm{C}_{i+k_j})$ *is controllable with respect to* $L(G_{k_j})$ *and* $A_{k_j,u}$*, and if for all* $j = 1,2,\ldots,m$, $P^{k_j}_k$ *is an* L_{k_j}*-observer and OCC for* L_{k_j}*, then*

$\sup 2\mathrm{cC}(K,L,A_{i+k_j}) = \|_{j=1}^{m} \|_{i\in I_j} \sup \mathrm{C}_{i+k_j}$, *where*

$$\sup \mathrm{C}_{k_j} = \sup \mathrm{C}(P_{k_j}(K), L(G_{k_j}), A_{k_j,u})$$

$$\sup \mathrm{C}_{i+k_j} = \sup \mathrm{C}(P_{i+k_j}(K), L(G_i) \| \sup \mathrm{C}_{k_j}, A_{i+k_j,u})$$

Lemma

For all $l = 1,2,\ldots,m$*, let* $M_l \subseteq A^*_{I_l}$ *be conditionally controllable wrt* G_i*, for* $i \in I_l$*, and* G_{k_l}*, and conditionally decomposable wrt alphabets* A_i*, for* $i \in I_l$*, and* A_{k_l}*, and* $A_{k_\ell} \supseteq A_k \supseteq \bigcup_{k\neq\ell}(A_{I_k} \cap A_{I_\ell})$*.*
If for all l*,* $P^{k_l}_k$ *is a* L_{k_l}*-observer and OCC for* $P_{k_l}(M_l)$*, then* $\|_{l=1}^{m} M_l$ *is two-level conditionally controllable wrt* G_i*, for* $i \in I_l$*, and* G_{k_l}*, for* $l = 1,2,\ldots,m$*.*

Sufficient Conditions

Corollary

If for all $j = 1, 2, \ldots, m$, $P_k^{k_j}$ is an L_{k_j}-observer and OCC for L_{k_j}, and for all $i \in I_j$, $P_{k_j}^{i+k_j}(\sup \mathrm{C}_{i+k_j}) = \sup \mathrm{C}_{k_j}$, then $\sup 2\mathrm{cC}(K, L, A_{i+k_j}) = \|_{j=1}^{m} \|_{i \in I_j} \sup \mathrm{C}_{i+k_j}$.

Moreover: Let $\forall j = 1, 2, \ldots, m$ and $i \in I_j$ $P_{k_j}^{i+k_j}$ be an $(P_i^{i+k_j})^{-1}L(G_i)$-observer and OCC for $(P_i^{i+k_j})^{-1}L(G_i)$. Then, for all $j = 1, 2, \ldots, m$, $\cap_{i \in I_j} P_{k_j}^{i+k_j}(\sup \mathrm{C}_{i+k_j})$ is controllable with respect to $L(G_{k_j})$ and $A_{k_j,u}$.

Corollary

Let $\forall j = 1, 2, \ldots, m$, $P_k^{k_j}$ is an L_{k_j}-observer and OCC for L_{k_j}, and $P_{k_j}^{i+k_j}$ be an $(P_i^{i+k_j})^{-1}L(G_i)$-observer and OCC for $(P_i^{i+k_j})^{-1}L(G_i)$. Then

$\sup 2\mathrm{cC}(K, L, A_{i+k_j}) = \|_{j=1}^{m} \|_{i \in I_j} \sup \mathrm{C}_{i+k_j}$.

Decentralized control

Idea: plunge decentralized control problem into coordination control problem by

$$A_i = \Sigma_{o,i} \quad \text{and} \quad A_{c,i} = \Sigma_{o,i} \cap \Sigma_{c,i}\,.$$

Note that conditional decomposability is just separability of K with respect to $(\Sigma_{o,i} \cup \Sigma_k)_{i=1}^n$.

Theorem

Let $\Sigma_{o,i} \cap \Sigma_c \subseteq \Sigma_{c,i}$, for $i = 1,2,\ldots,n$. If $K = \|_{i=1}^n P_i(K)$ (separable) wrt $(\Sigma_{o,i})_{i=1}^n$, then
$K \cap L$ is coobservable wrt $(\Sigma_{o,i})_{i=1}^n$ and L.

Due to $\Sigma_{o,i} \cap \Sigma_{c,j} \subseteq \Sigma_{c,i}$, for $i,j = 1,2,\ldots,n$, $(A_i)_{i=1}^n$ and $(A_{c,i})_{i=1}^n$ defined above satisfy $A_i \cap A_{c,j} \subseteq A_{c,i}$, for all $i,j = 1,2,\ldots,n$.
Hence, separability implies coobservability

Main result of Decentralized control

Theorem

- *Let* $K \subseteq L$ *and* K *be two-level CD wrt* $(A_i)_{i=1}^n$, $A_{k,j}$, *and* A_k. *Then* $\|_{j=1}^m \|_{i \in I_j} \sup \mathrm{C}_{i+k_j} \subseteq K$ *is controllable wrt* L *and* A_u, *and coobservable wrt* L *and* $(A_{i+k_j})_{j=1,\ldots,m,\, i \in I_j}$.
- *If* $\|_{j=1}^m \cap_{i \in I_j} P_{k_j}^{i+k_j}(\sup \mathrm{C}_{i+k_j})$ *is controllable wrt* $L(G_{k_j})$ *and* $A_{k_j,u}$, *and for all* $j = 1, 2, \ldots, m$, $P_k^{k_j}$ *is an* L_{k_j}*-observer and OCC for* L_{k_j}, *then* $\|_{j=1}^m \|_{i \in I_j} \sup \mathrm{C}_{i+k_j} = \sup 2\mathrm{cC}(K, L, A_{i+k_j})$ *is the largest controllable and coobservable language we can obtain using the two-level coordination.*

Example: Plant

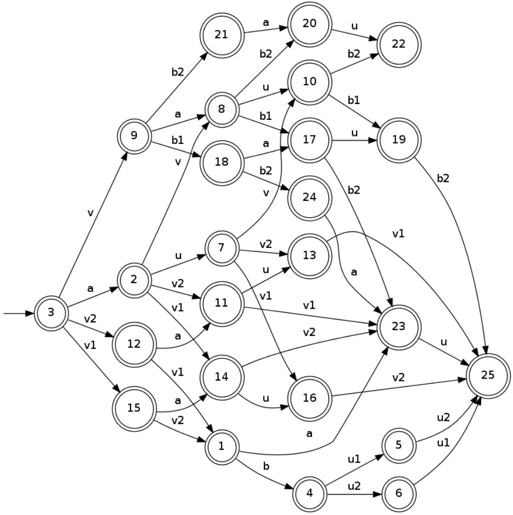


Figure: Plant L

Example: Specification

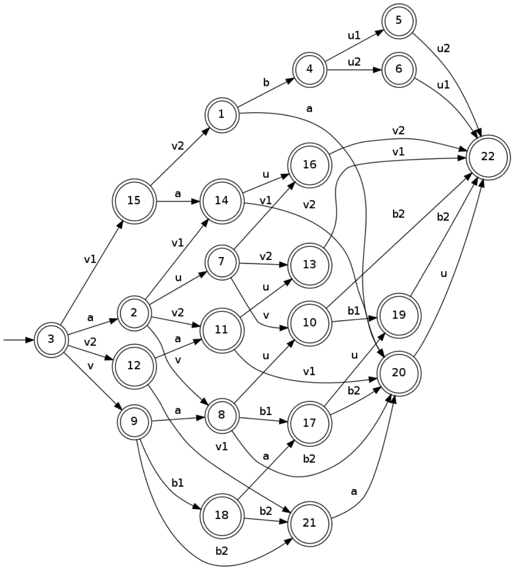


Figure: Specification K

Example: continued

K is not controllable wrt L, e.g.
$v_2 v_1 b \in K\Sigma_u \cap L$, but $v_2 v_1 b \notin K$.
K is coobservable wrt L and $\Sigma_{o,i}$, for $i = 1, 2, 3, 4$.
$N = \sup \mathrm{C}(K, L, \Sigma_u)$ is not coobservable wrt L and $\Sigma_{o,i}$,
e.g. $v_1 v_2 \in L$ and $v_2 v_1 \in L$, $v_1 v_2 \in N$, $v_2 \in N$, while $v_2 v_1 \notin N$.
Intuition: either agents 3 and 4 has to observe both v_1 and v_2 to issue a correct control decision. .

Example continued

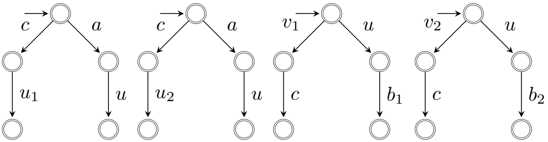


Figure: Projections $P_1(L), \ldots, P_4(L)$

$\sup \mathrm{C}_{1+k_1} \| \sup \mathrm{C}_{2+k_1} \| \sup \mathrm{C}_{3+k_2} \| \sup \mathrm{C}_{4+k_2} = \sup 2\mathrm{cC}(K, L, A_{i+k_j})$ is controllable with respect to L and A_u and coobservable with respect to L and $A_{1+k_1,o}$, $A_{2+k_1,o}$, $A_{3+k_2,o}$, $A_{4+k_2,o}$.

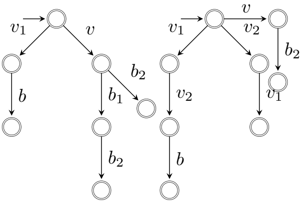


Figure: $P_{3+k_2}(K)$ and $P_{4+k+k_2}(K)$

Conclusions and Perspectives

- Multilevel coordination control for modular systems proposed
- It is based on top-down approach, bottom-up approach also exists
- Extension to constructive results (supremal conditional controllable languages)
- Communications based on multi-level coordination approach are applied to decentralized supervisory control control
- Extension to partial observations