Department of Impact and Waves in Solids

On the contact pressure oscillations of an isogeometric contact-impact algorithm

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Introduction

The main difficulty in contact analysis is non-smoothness. It arises from inequality constraints as well as geometric discontinuities inducted by the spatial discretization. Contact analysis based on traditional finite elements utilizes element facets to describe contact surfaces. The facets are C^0 continuous so that the surface normals can experience jump across facet boundaries leading to artificial oscillations in contact force and pressure.
A remedy to this geometric discontinuity could provide isogeometric analysis (IGA). The fundamental idea is to accurately describe a physical domain by proper representation (e.g. NURBS) and then to utilize the same basis for analysis. This is in contrast with the classical finite element method where the basis is given in advance by the element type. Consequently the physical domain could be approximated inaccurately. A more detailed description can be found in [1].
Isogeometric NURBS-based contact analysis has some additional advantages:

- preserving geometric continuity,
- facilitating patch-wise contact search,
- supporting a variationally consistent formulation,
- and having a uniform data structure for the contact surface and the underlying volumes.

B-splines

Let $\boldsymbol{\Xi}^i$, $i = 1, \ldots, d_p$ be the open non-uniform knot vector associated with i^{th} parametric dimension of a patch

$$\boldsymbol{\Xi}^i = \left\{ \underbrace{\xi^i_1, \ldots, \xi^i_{p_i+1}}_{p_i+1 \text{ equal terms}}, \xi^i_{p_i+2}, \ldots, \xi^i_{n_i}, \underbrace{\xi^i_{n_i+1}, \ldots, \xi^i_{n_i+p_i+1}}_{p_i+1 \text{ equal terms}} \right\}, \qquad i = 1, \ldots d_p$$

The B-spline basis functions, $N_{j,p}(\xi)$, are defined by Cox-de Boor recursion formula. For $p = 0$ it is defined as

$$N_{j,0}(\xi) = \begin{cases} 1 & \xi \in [\xi_j, \xi_{j+1})\,, j = 1 \ldots n \\ 0 & \text{otherwise} \end{cases}$$

and for $p > 0$

$$N_{j,p}(\xi) = \frac{\xi - \xi_j}{\xi_{j+p} - \xi_j} N_{j,p-1}(\xi) + \frac{\xi_{j+1+p} - \xi}{\xi_{j+1+p} - \xi_{j+1}} N_{j+1,p-1}(\xi)$$

NURBS

A p^{th} degree NURBS basis function is defined by

$$R^p_j(\xi) = \frac{N_{j,p}(\xi) w_j}{\sum_{\hat{j}=1}^{n} N_{\hat{j},p}(\xi) w_{\hat{j}}}$$

where w_j is referred to as the j^{th} weight. Multivariate NURBS objects can be constructed simply by tensor product of these univariate basis functions. With NURBS basis functions at hand one can introduce surface discretization as

$$\mathbf{x}(\xi^1, \xi^2) = \sum_{j_1=1}^{n_1} \sum_{j_2=1}^{n_2} R^{p_1,p_2}_{j_1,j_2}(\xi^1, \xi^2) \mathbf{P}_{j_1,j_2}$$

where $\mathbf{P}_{j_1,j_2} \in \Re^{d_s}$ is the control net, i.e., the array of coordinates of control points. Adopting the isogeometric concept, an analogous interpolation is used for the unknown displacement field and its variation.

Bibliography

[1] Cottrell JA, Hughes TJR, Bazilevs Y. *Isogeometric Analysis: Toward Integration of CAD and FEA*. John Wiley & Sons, 2009.

[2] Gabriel D, Plešek J, Ulbin M. Symmetry preserving algorithm for large displacement frictionless contact by the pre-discetization penalty method. *International Journal for Numerical Methods in Engineering* December 2004; **61**(15):2615–2638, doi:10.1002/nme.1173.

[3] Hughes TJR. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Dover Publication, 2000.

Explicit dynamic contact algorithm

An algorithm, originally proposed in [2], was adapted to the isogeometric analysis and expanded to explicit dynamics. The main idea is as follow. The contact boundary value problem is formulated in the weak sense

$$\delta \boldsymbol{\Pi}_{\text{int,ext}}(\mathbf{u}, \delta\mathbf{u}) + \delta \boldsymbol{\Pi}_{\text{c}}(\mathbf{u}, \delta\mathbf{u}) = 0$$
$$g_{\text{N}}(\mathbf{u}) \geq 0$$

where $\delta\boldsymbol{\Pi}_{\text{c}}$ was proposed in the form

$$\delta \boldsymbol{\Pi}_{\text{c}}(\mathbf{u}, \delta\mathbf{u}) = -\int_{\Gamma_{\text{c1}}} \varepsilon_{\text{N}} g_{\text{N}} \delta\mathbf{u} \,\mathrm{d}\Gamma - \int_{\Gamma_{\text{c2}}} \varepsilon_{\text{N}} g_{\text{N}} \delta\mathbf{u} \,\mathrm{d}\Gamma$$

Note that the contact virtual work is integrated over both contact boundaries Γ_{c1} and Γ_{c2} so that the algorithm preserves symmetry. Consequently, after FE discretization the action-reaction principle is not explicitly fulfilled. However, it should be shown that the equilibrium is recovered during the mesh refinement.
The application of the FEM for spatial discretization and the central difference method (CDM) for temporal integration yields

$$\mathbf{M}\mathbf{U}_{n+1} = \Delta t^2 \left[\mathbf{R} + \mathbf{R}_{\text{c12}}(\mathbf{U}_n) + \mathbf{R}_{\text{c21}}(\mathbf{U}_n) - \mathbf{F}(\mathbf{U}_n)\right] + \mathbf{M}(2\mathbf{U}_n - \mathbf{U}_{n-1})$$

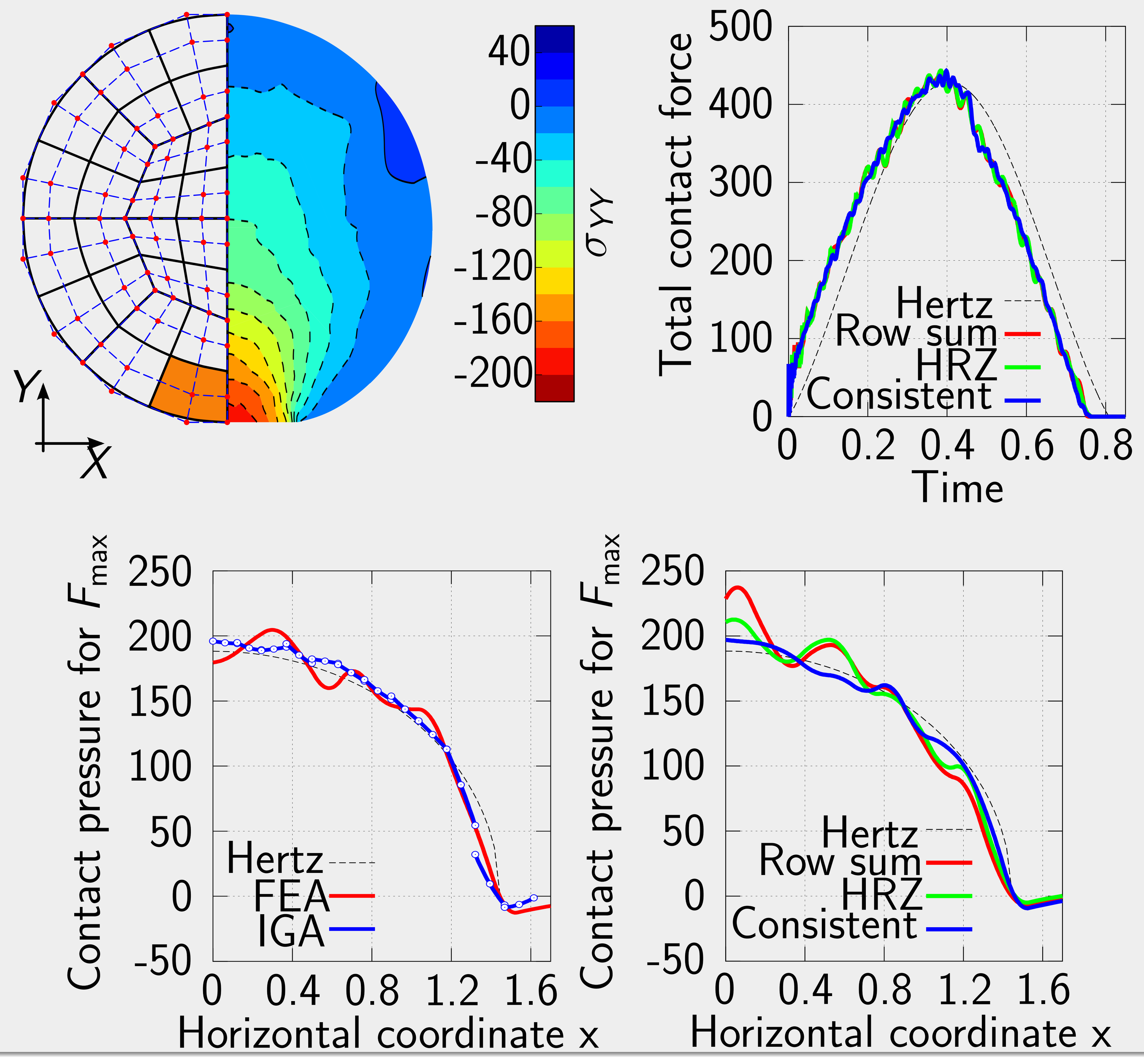
The stability of the integration process requires time step to be smaller or equal to $2/\omega_{\max}$. The consistent element mass matrix rising from the variational formulation has the form

$$\mathbf{M}_e = \int_{\Omega_e} \rho \mathbf{H}^{\text{T}} \mathbf{H} \,\mathrm{d}\Omega$$

The efficient solution of the resulting system of equations requires diagonalization of the mass matrix. The common techniques are the row sum method and HRZ method [3].

Dynamic Hertz problem

The numerical example dealt with frictionless impact of the cylinders. The effect of mass lumping was investigated. The analysis was limited to the second order elements.



Conclusions

This paper addressed the utilization of the NURBS based isogeometric analysis in an explicit contact-impact algorithm. Two main conclusions may be drawn:

- For second order elements and mass matrix lumped by the HRZ method, IGA in comparison with classic FEA leads to a more oscillatory contact force and consequently also contact pressure.
- The oscillations of the contact forces in IGA are minimal for consistent mass matrix.

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