ANALYTICAL AND NUMERICAL APPROACH TO THE AIRFOIL STABILITY COMPUTATION

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1 Introduction

The fluid structure interaction (FSI) represents an important task in many applications. This paper deals with interaction of airfoil and fluid flow and compares two possible ways of computation of the stability boundaries of the system. The airfoil has two degrees of freedom represented by translation $h(t)$ and rotation $\alpha(t)$, see Figure 1. The flow acts on the airfoil with aerodynamic forces depending on the flow velocity. Two methods for calculation of the critical flow velocity, when the airfoil loses aeroleastic stability, are described.

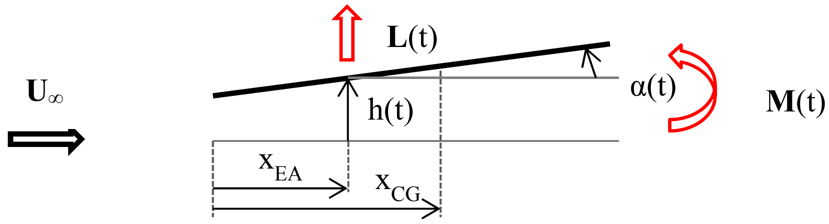

Figure 1: Scheme of the airfoil with center of gravity (CG) and elastic axis (EA) vibrating with two degrees of freedom.

2 Stability boundaries computation

2.1 Analytical approach

Analytical approach is represented by P-K method [1], which is based on the calculation of eigenvalues of the equation of motion for the airfoil and includes the analytical expression of the aerodynamic forces. This expression is based on Theodorsen unsteady aerodynamics. The resulting equation can be written in the form

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{B}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{Q}(\mathrm{k}, \mathbf{U}_\infty), \qquad (1)$$

where $\mathbf{M}, \mathbf{B}, \mathbf{K}$ are the mass, damping a stiffness matrices, $\mathbf{x}(t) = [h \quad \alpha]^T$ is the position vector and $\mathbf{Q}(\mathrm{k}, \mathbf{U}_\infty)$ the matrix of aerodynamic forces,

$$\boldsymbol{k} = \frac{\boldsymbol{\omega b}}{\boldsymbol{U_\infty}} \qquad (2)$$

is the reduced frequency, where ω is the circular frequency, b is the reference length and $\mathbf{U}_\infty$ flow velocity. The eigenvalues of (1) can be written in the form $p_i = g + i\omega$ and the algorithm is based on iterative process, which is described in a flowchart in Figure 2. The matrix of aerodynamic forces can be found in [3].

2.2 Numerical approach

Numerical approach was based on a complete solution of FSI problem using the finite volume method (FVM). The flow field changes the airfoil position and this change influences the flow field. The equations of motion of the airfoil and the equations **x*t*+*B*x*t*+*K*x*t*=Qk,U*∞*,** (**1**) are computed from the total pressure acting on the airfoil and they are expressed with respect to Figure 3 as

$$\boldsymbol{L} = \int d p_i n_y \, ds$$
$$\boldsymbol{M} = \int d p_i \left(n_y (x_i - x_{EO}) + n_x (y_i - y_{EO}) \right) \, ds \qquad (3)$$

where d is span of the airfoil, $\boldsymbol{n} = (n_x \quad n_y)$ is the unit outer normal to the airfoil surface and p_i total pressure in the point $[x_i \; y_i]$.

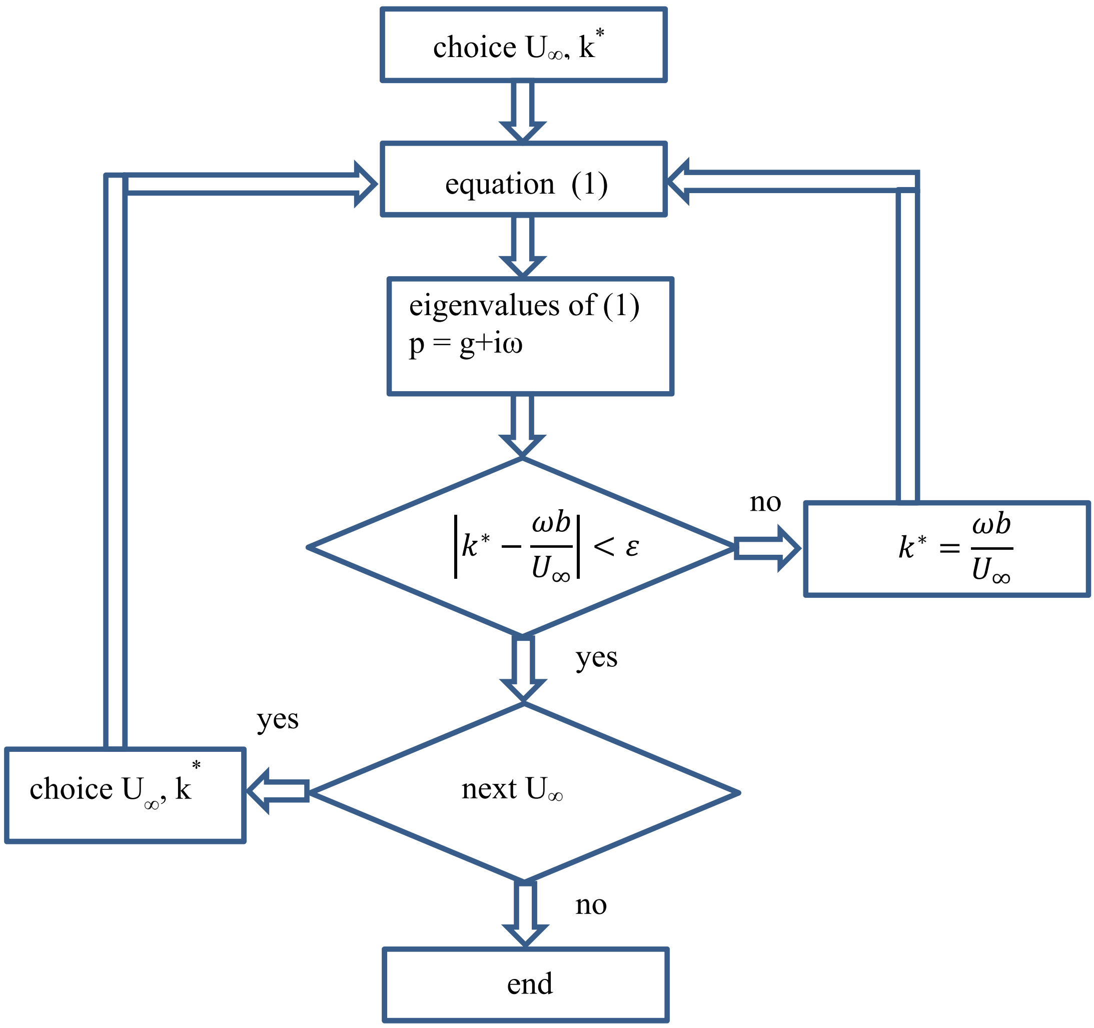

Figure 2: The p-k algorithm is based on iterative procedure

The pressure field is computed from the solution of the unsteady incompressible Navier-Stokes equations describing the fluid flow. The FVM operates with Navier-Stokes equations in a conservative form:

$$\boldsymbol{DW_t} + \boldsymbol{F_x^c} + \boldsymbol{G_y^c} = \frac{1}{Re}\left(\boldsymbol{F_x^v} + \boldsymbol{G_y^v}\right), \qquad (4)$$

where $\boldsymbol{W} = (\rho \quad u \quad v)^T$, $\boldsymbol{F}^c = (u \quad u^2 + p \quad uv)^T$, $\boldsymbol{G}^c = (v \quad uv \quad v^2 + p)^T$, $\boldsymbol{F}^v = (0 \quad u_x \quad v_x)^T$, $\boldsymbol{G}^c = (0 \quad u_y \quad v_y)^T$ and $\boldsymbol{D} = (0 \quad 1 \quad 1)^T$. For more details see [2].

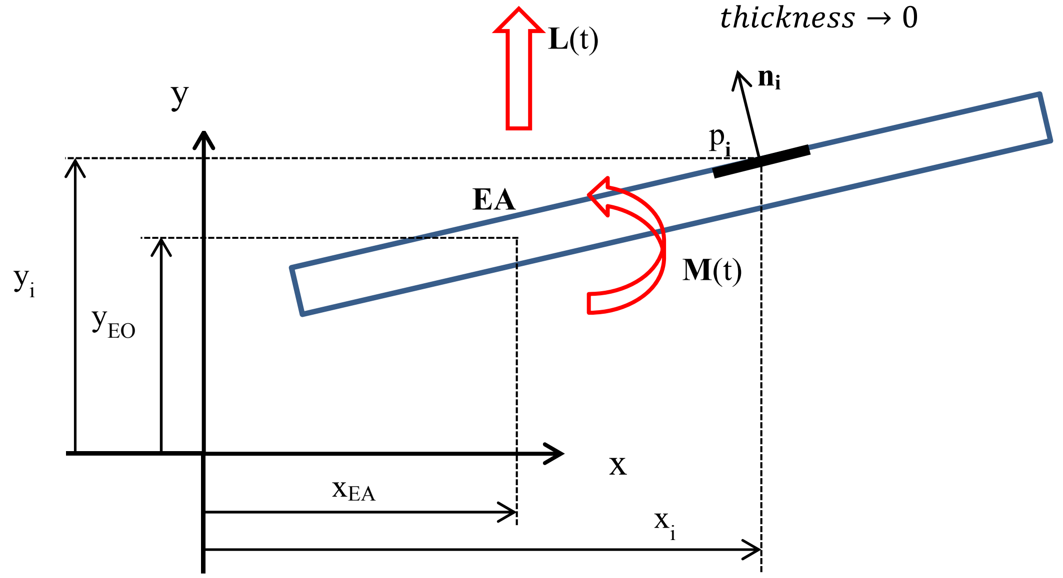

Figure 3: Scheme for calculation of the forces acting on the airfoil.

3 Results

For the numerical experiment, the airfoil with the following structural properties: $m = 0{,}086622\ kg$, $S_\alpha = -779{,}673.10^{-6}\ kg.m$, $I_\alpha = 487{,}291.10\ kg.m^2$, $k_h = 105{,}109\ N.m^{-1}$, $k_\alpha = 3{,}695582\ N.rad^{-1}$ has been chosen. The span was $d = 0{,}079\ m$, the length of the chord $c = 0{,}3\ m$ and the elastic axis was located at 40% of the chord, see [2].

The results based on analytical approach are presented in Figure 2. Considering the graphs it can be concluded: the divergence instability occurs at the flow velocity $u_{div} = 30\ m.s^{-1}$ and the flutter starts at the velocity $u_{flut} = 36\ m.s^{-1}$. The flutter frequency is $f_{flut} = 7{,}5\ Hz$.

The results computed using the FVM are summarized in Table 1. The stability of the system was evaluated from the time response of the airfoil computed for the initial conditions $h = 0.02\ m$, $\alpha = -6\ deg$. The interaction has been computed only for few values of the flow velocity due to high computer time consumption. The graphs of aeroelastic responses are shown in Figure 5-8 for several flow velocities from $U_\infty = 0\ m.s^{-1}$ up to the flow velocity $U_\infty = 40\ m.s^{-1}$. Table 1 shows that the airfoil remained stable for the flow velocities less than and equal to $u_\infty = 29 m.s^{-1}$.

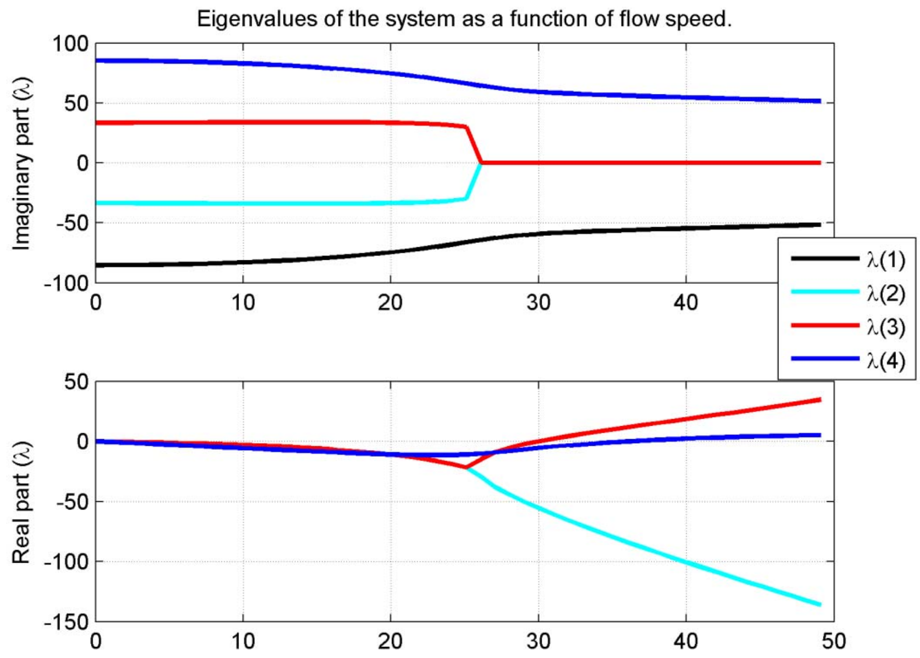

Figure 4: Real (up) and imaginary part (bottom) of eigenvalues of the system as a function of a flow speed.

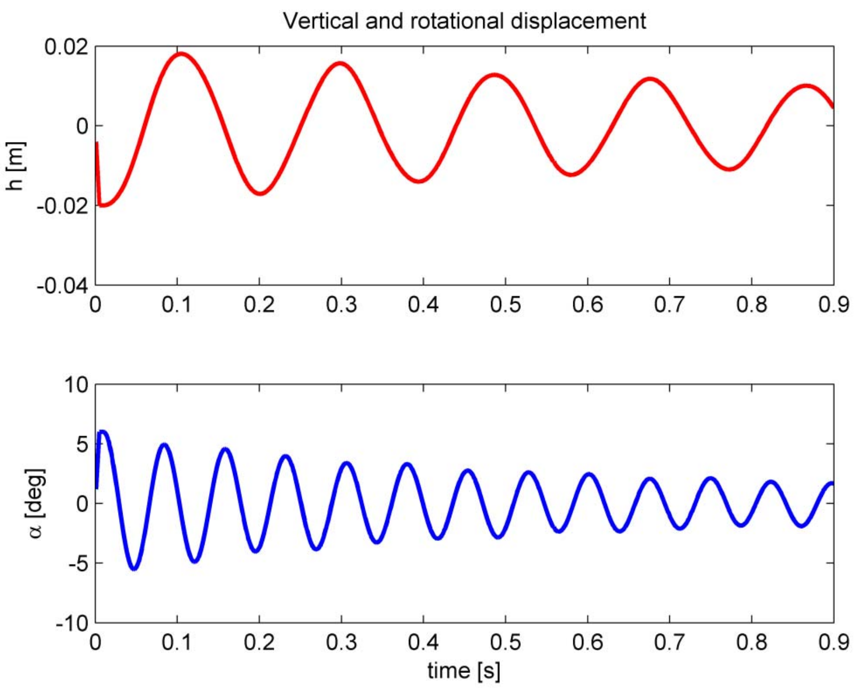

Figure 5: Vertical displacement and angle of rotation in dependence of time for U_{∞}=0 m.s^{-1}

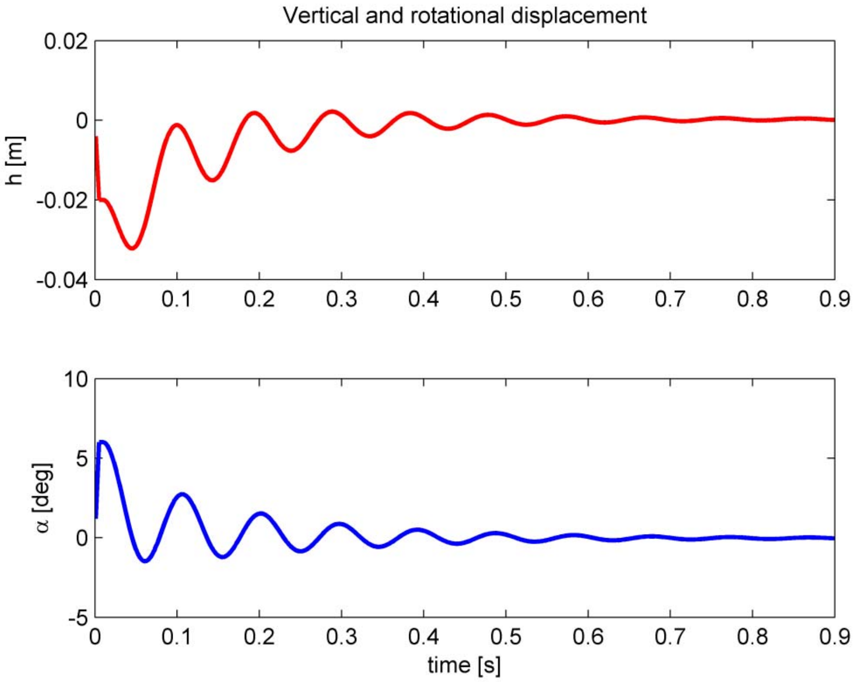

Figure 6: Vertical displacement and angle of rotation in dependence of time for U_∞ = 29 m.s^{-1}

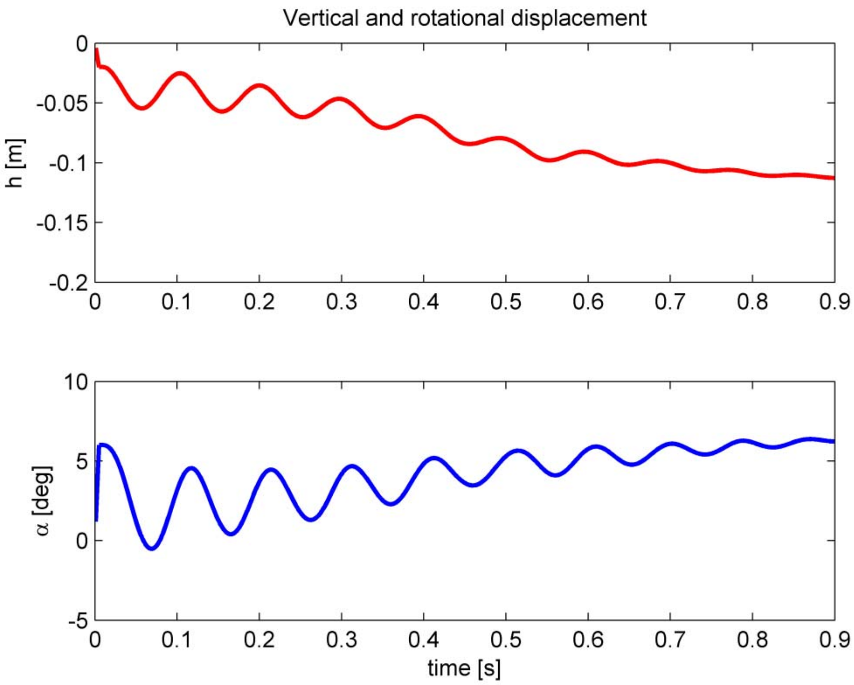

Figure 7: Vertical displacement and angle of rotation in dependence of time for U∞ = 35m.s-1

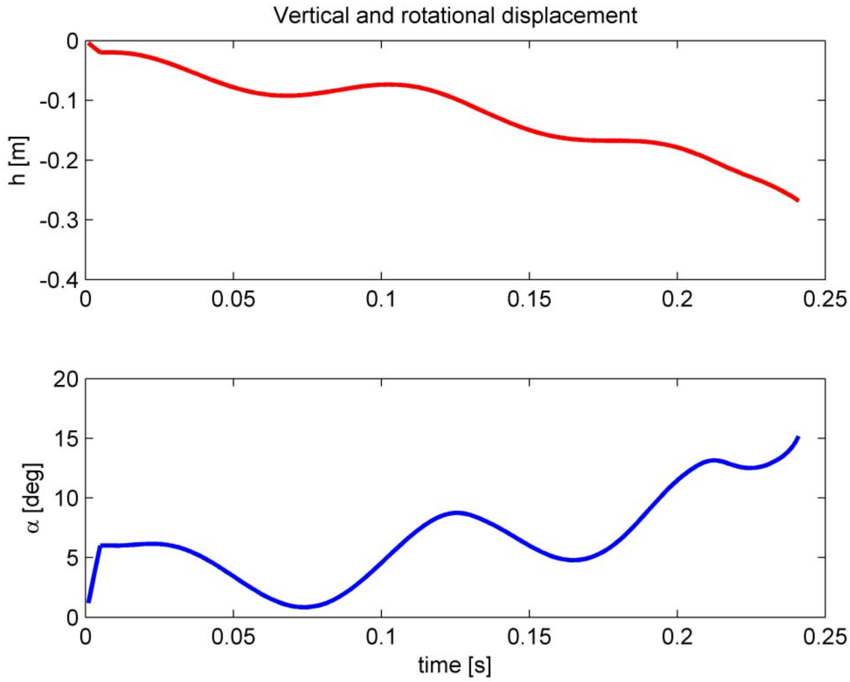

Figure 8: Vertical displacement and angle of rotation in dependence of time for U∞ = 40 m.s-1

Flow velocity [m.s^{-1}]	System	Type of instability
0	stable	-
15	stable	-
28	stable	-
29	stable	-
35	unstable	divergence
40	unstable	divergence

Table 1: Evaluated type of stability as a function of flow velocity computed using the FVM method

4 Conclusion

Both numerical and analytical algorithms have some advantages and disadvantages. The analytical approach has a great benefit in the speed of calculation, which was for the presented range of flow speed less than 0,25 s, nevertheless the algorithm is appropriate just for certain problems. The numerical approach based on FVM method is suitable for solution of a more general fluid-structure interaction problems, the disadvantage is in a high complexity of the sophisticated solution, software requirements and high computer time consumption.

The results of both methods are quite similar in the studied case. The main reason for this conformity is probably the low critical flow velocity. There are some applications, where the low critical speed is expected and the analytical approach can be useful, more complicated tasks have to use the numerical approach.

Acknowledgement
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References

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