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Partitioning of Two-dimensional NURBS Meshes for the Parallel Isogeometric Analysis

Presentation Outline

- **Introduction**
- **Partitioning strategy**
- **Dual graph of isogeometric mesh**
- **Numerical example**
- **Summary**

Introduction – Isogeometric Analysis

- **recently introduced alternative to the FEM**
- **spans the gap between CAD and FEA**
- **employs the same functions for the description of geometry and for the approximation of the solution on that geometry**
  - **eliminates costly FE mesh generation**
  - **geometric preprocessing still required**
- **originally developed for NURBS, later extended to T-splines**
- **outperforms classical FEM in various aspects**
- **still open issues** (e.g. independence of T-spline basis functions)

Introduction – Differences between FEM and IGA

- **dominating computational costs**
  - **FEM - global costs** (solution of overall system of equations)
  - **IGA - local costs** (assembly of governing matrices)
- **support of basis functions**
  - **FEM - local patch of elements** (sparse matrix with narrow band)
  - **IGA - nonlocal patch of knot spans** (denser matrix with wider band)
- **number of unknowns**
  - **FEM - large**
  - **IGA - smaller** (decreasing with degree of basis functions)

Introduction – Parallel Isogeometric Analysis

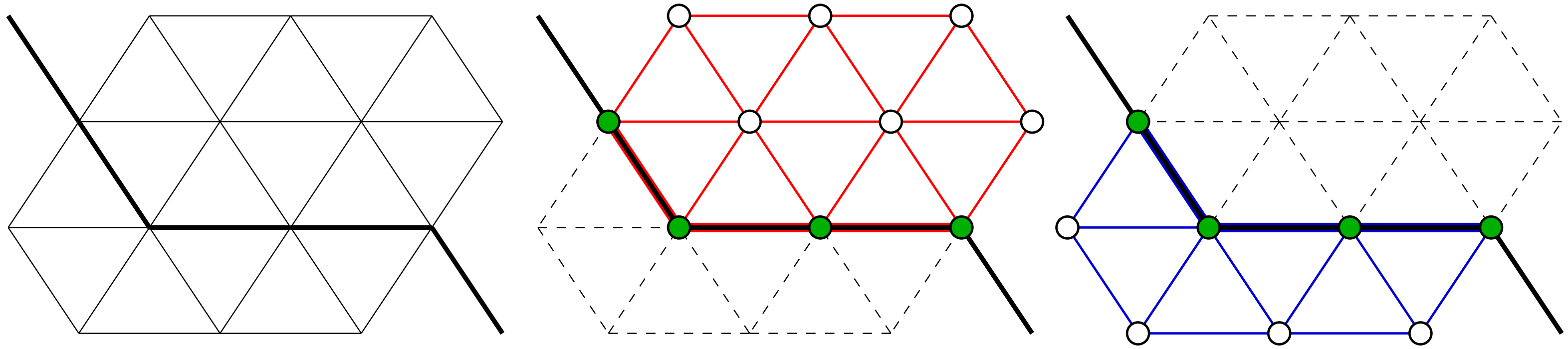
- problem is decomposed in subdomains and processed in parallel on different mutually interconnected CPUs
  - to achieve solution of large scale problems
  - to speedup response of moderate scale problems
- features
  - dominating local computation can be efficiently parallelized
  - not so scalable solution of system of equations is less significant
  - solution of denser system not as efficient
  - increase of relative communication cost
  - optimal decomposition not straightforward

Partitioning Strategy

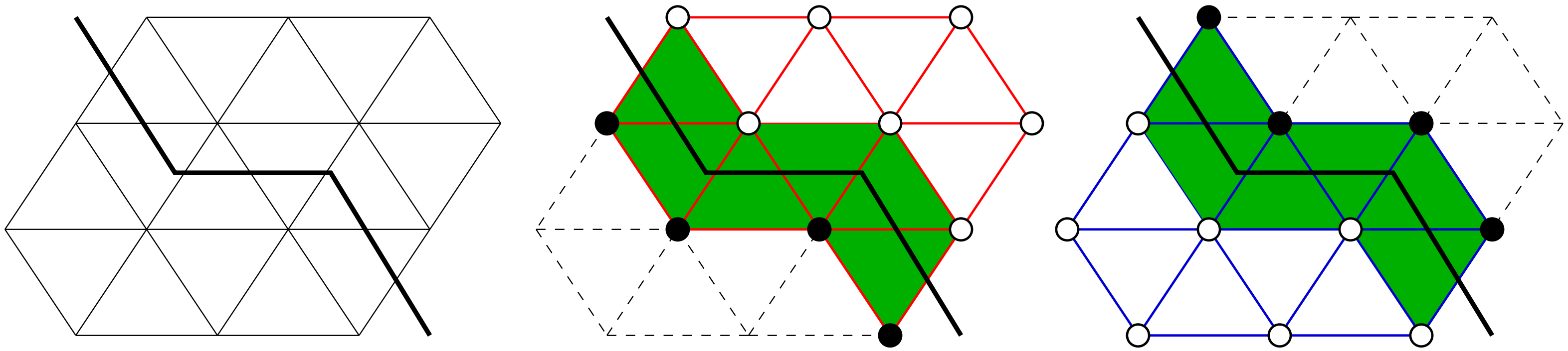
- based on (spatial) domain decomposition concept
  - node-cut partitioning
    - elements uniquely assigned to partitions
    - cut nodes shared between partitions
  - element-cut partitioning
    - nodes uniquely assigned to partitions
    - cut elements shared between partitions
- node-cut generally more efficient than element-cut
  - less inter-processor data transfer
  - duplicated processing of shared elements avoided

Partitioning Strategy – FEM Mesh Partitioning

Node cut

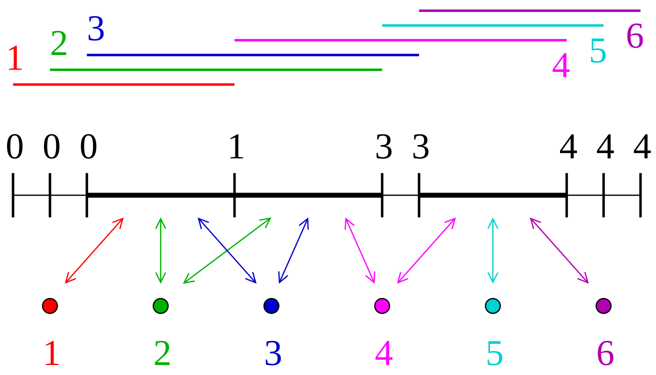


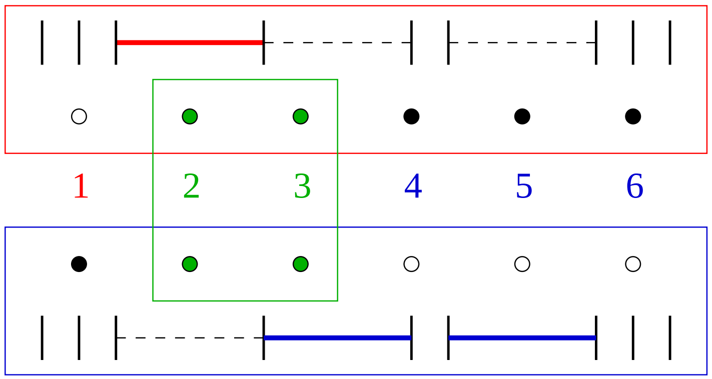
Element cut



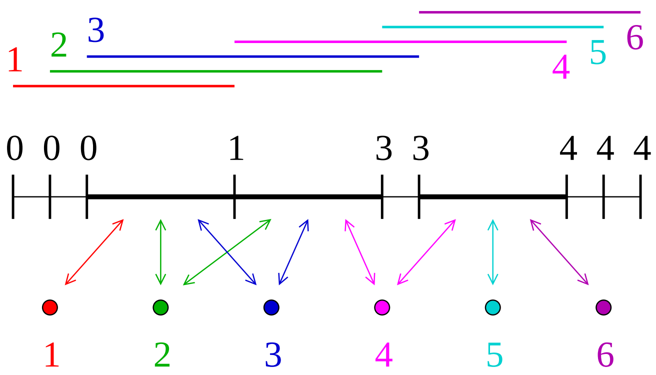
Partitioning Strategy – IGA Mesh Partitioning

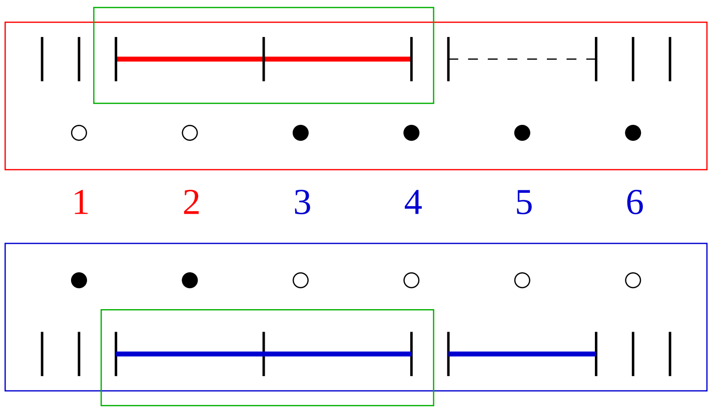
Node cut





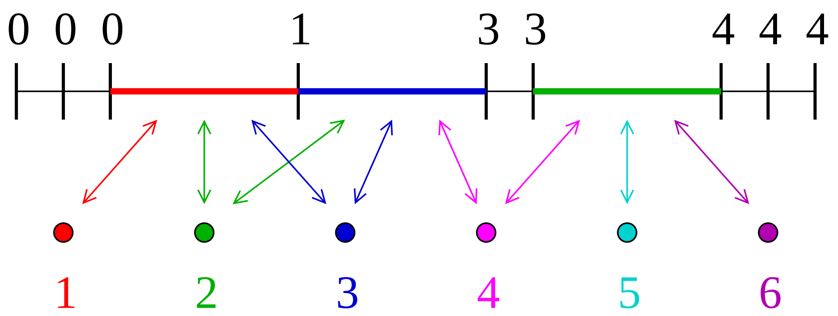
Element cut

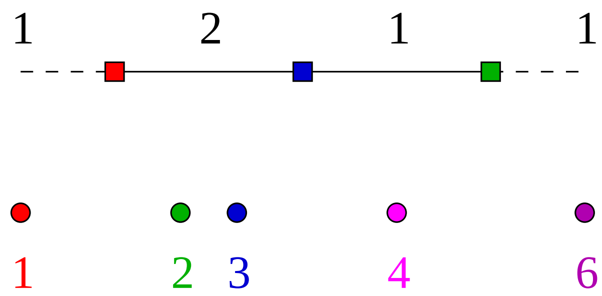




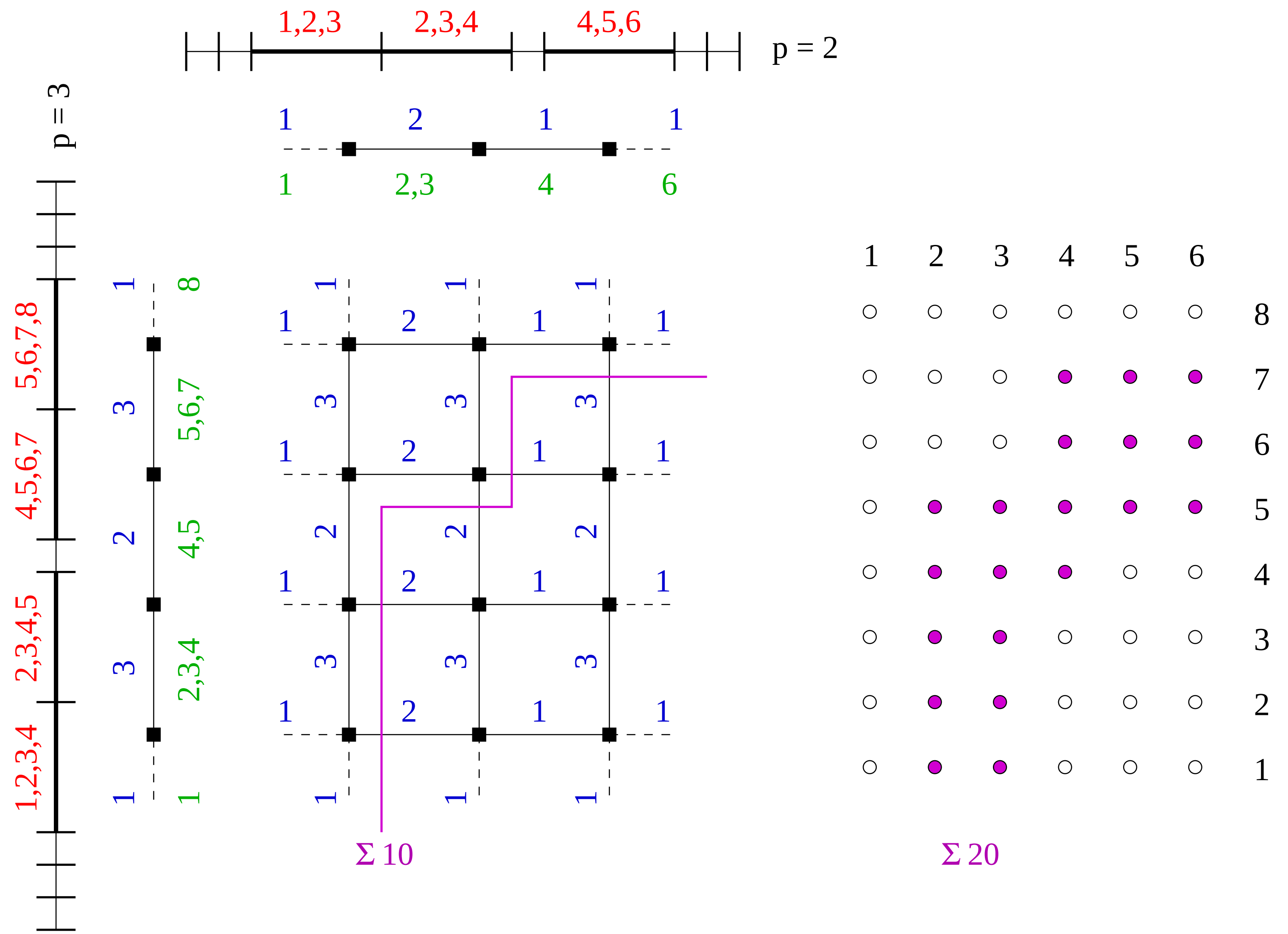
Dual Graph of Isogeometric Mesh

- **graph vertices are formed by non-zero knot spans**
- **graph edges represent the connectivity between non-zero knot spans**
- **weights of graph vertices correspond to computational load**
  (on associated non-zero knot spans)
- **weights of graph edges correspond the number of shared nodes**
  (due to cutting that edge) $w_e = w_{t_i} = max(1, p+1-k_{t_i})$



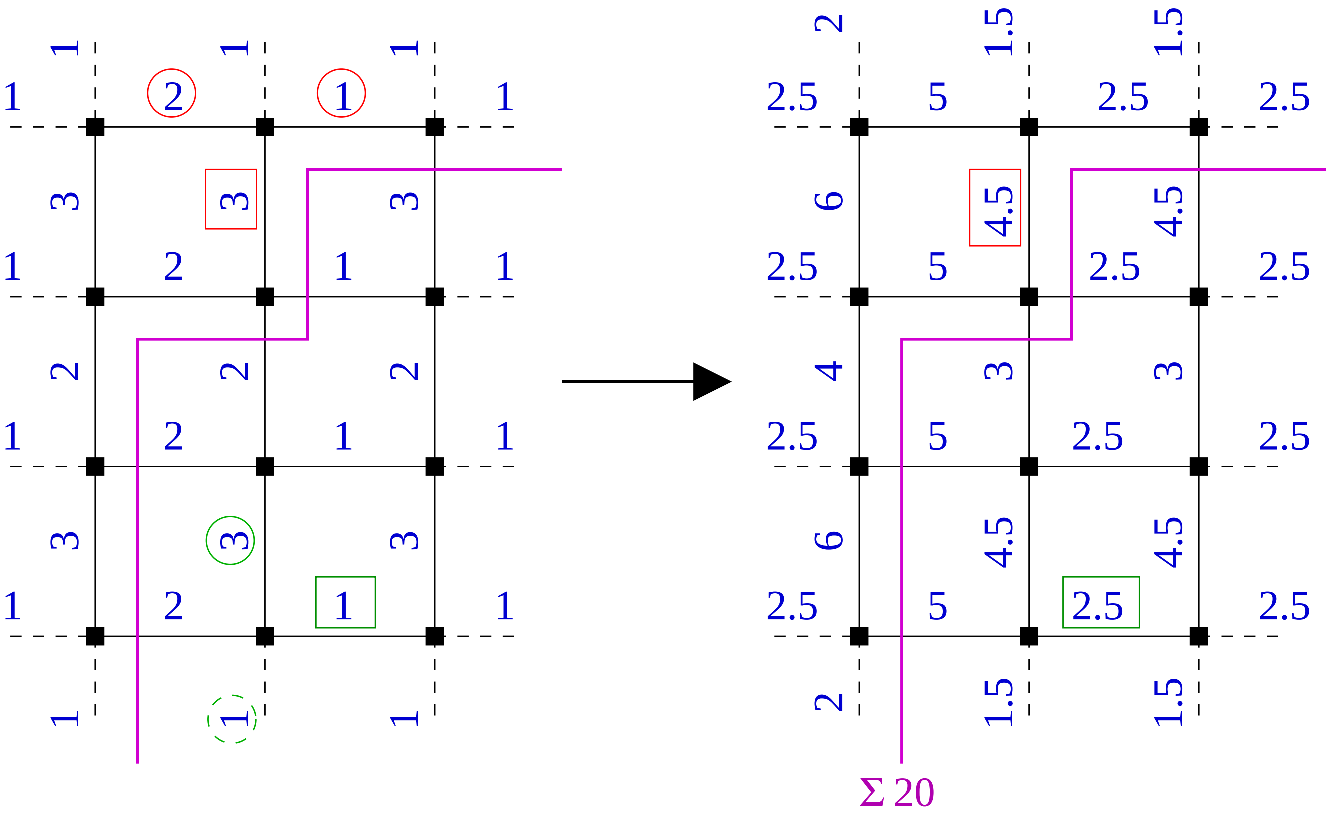


Dual Graph of Isogeometric Mesh – NURBS Surface



Dual Graph of Isogeometric Mesh – NURBS Surface

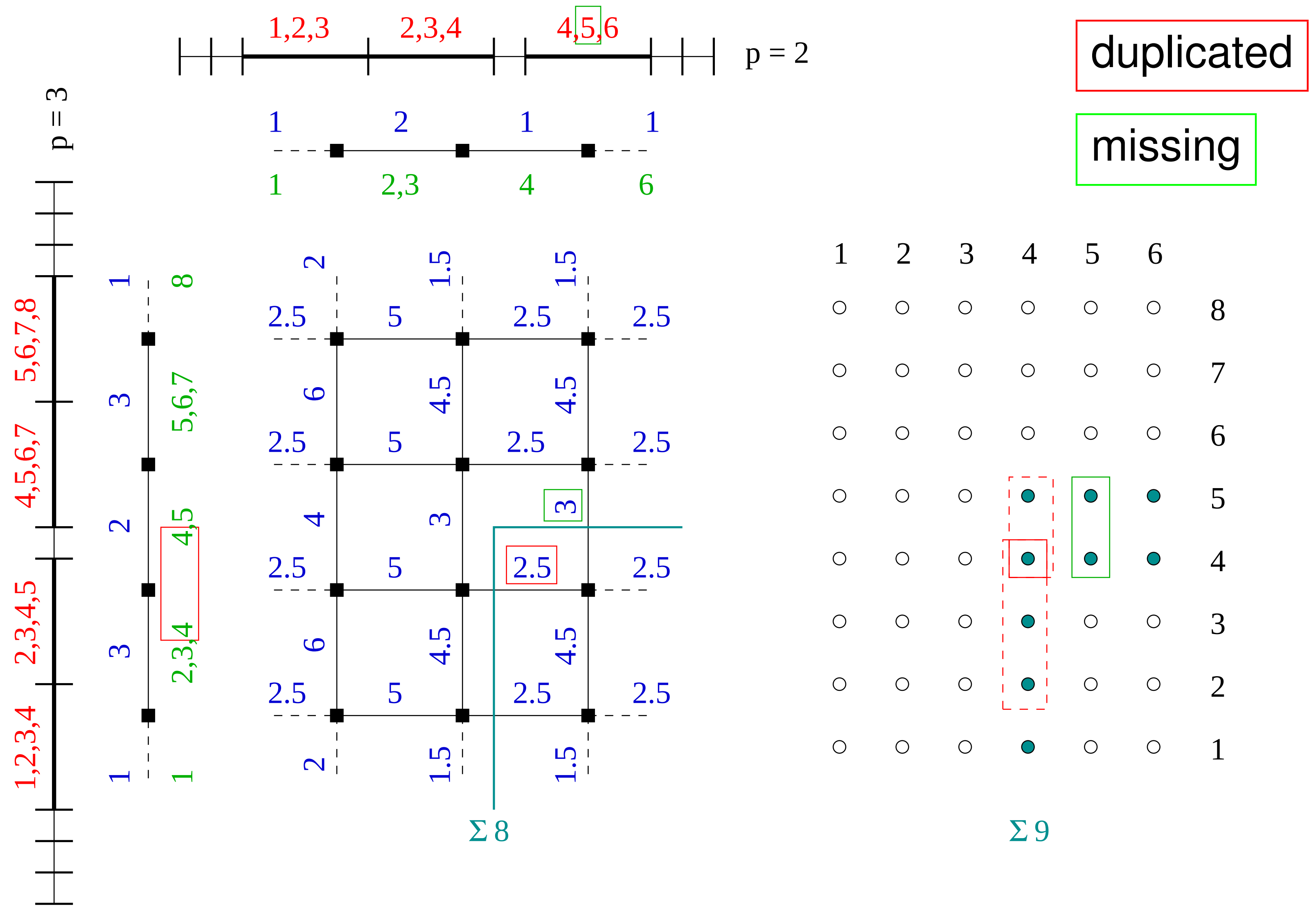
1D weights ⟶ **2D weights**



$$3 \times \left(\frac{2}{2} + \frac{1}{2}\right) = 4.5$$

$$1 \times \left(\frac{1}{1} + \frac{3}{2}\right) = 2.5$$

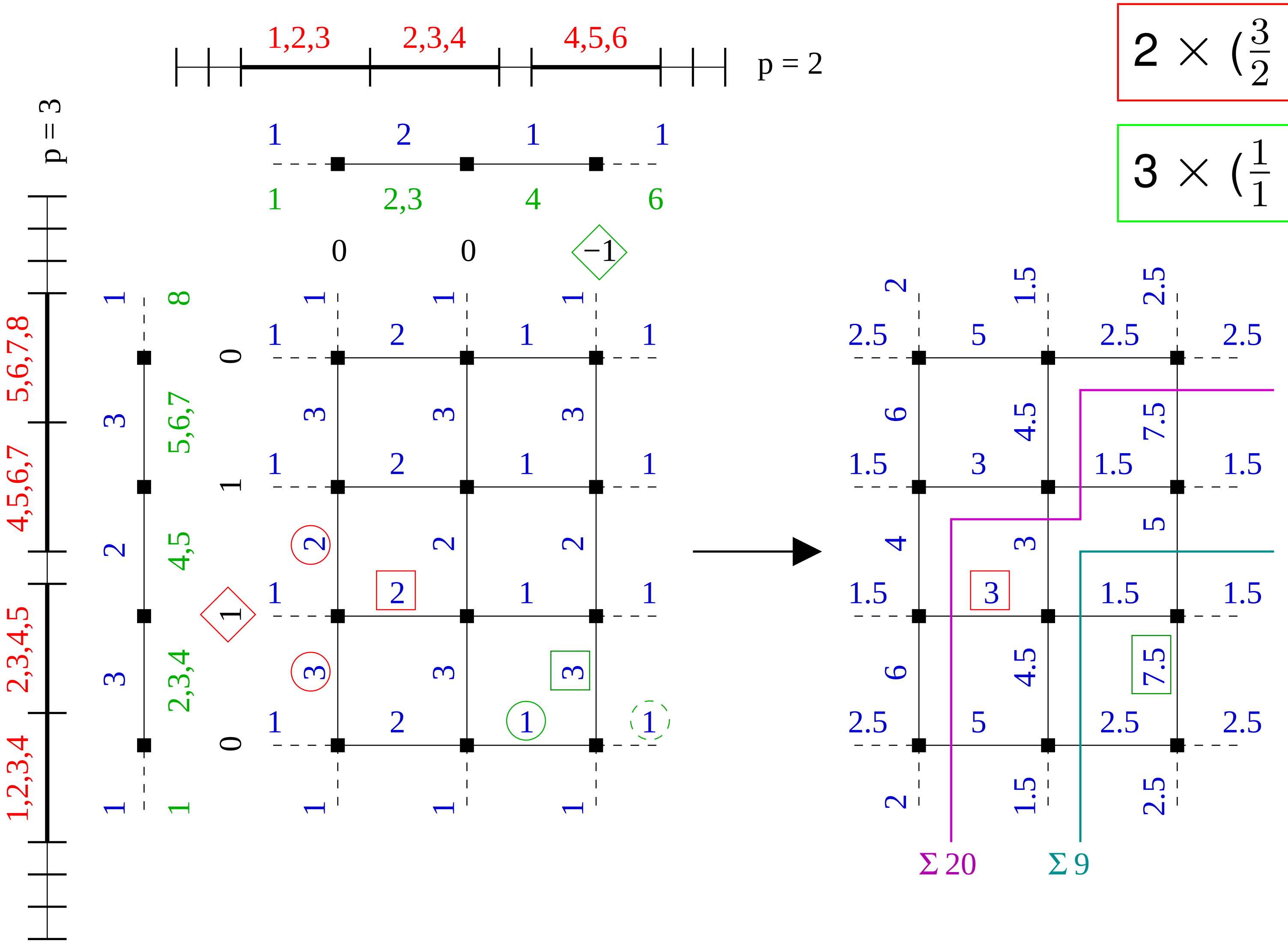
Dual Graph of Isogeometric Mesh – NURBS Surface



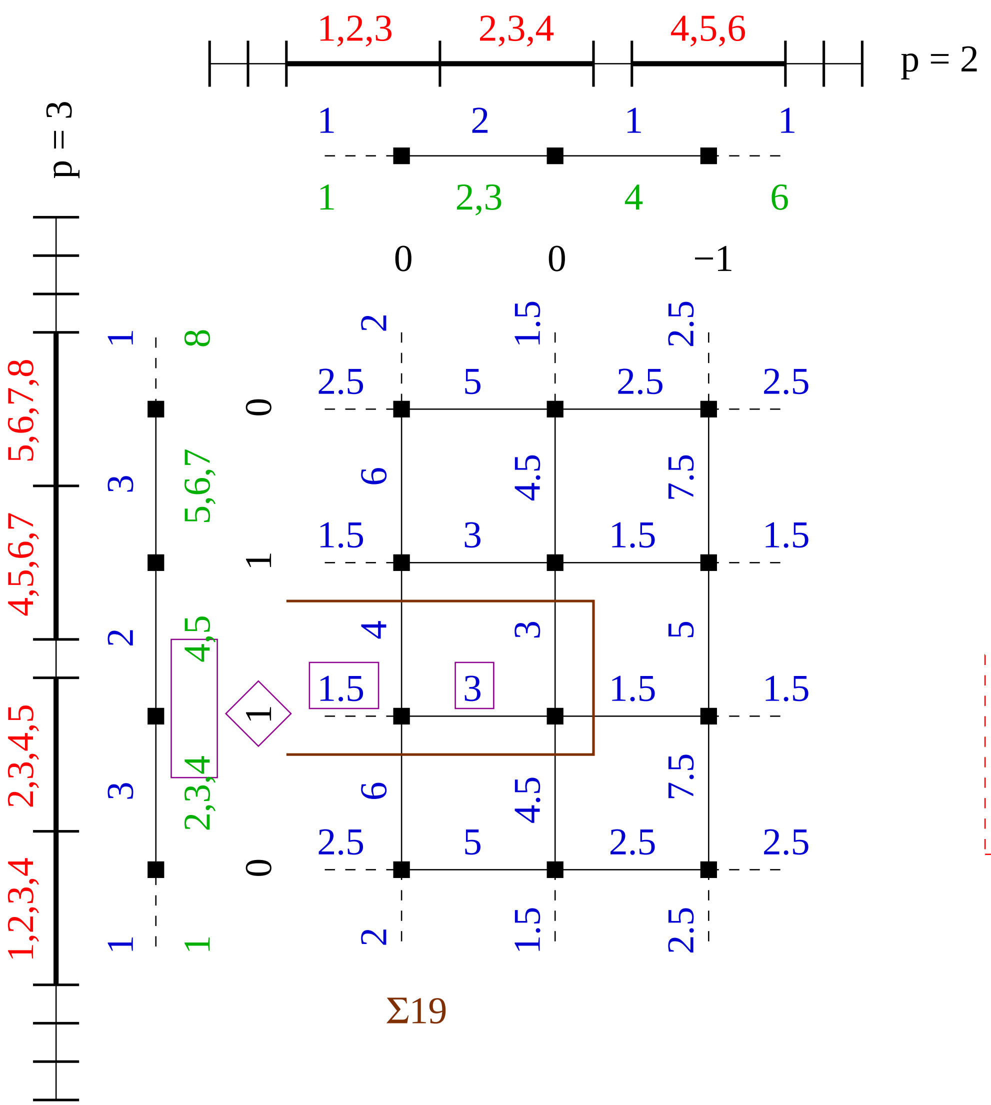
Dual Graph of Isogeometric Mesh – NURBS Surface

$$2 \times \left(\frac{3}{2} + \frac{2}{2} - 1\right) = 3.0$$

$$3 \times \left(\frac{1}{1} + \frac{1}{2} + 1\right) = 7.5$$

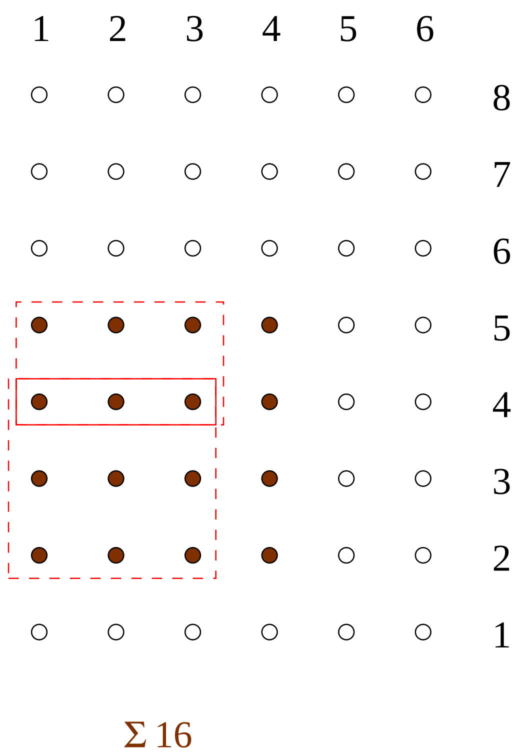


Dual Graph of Isogeometric Mesh – NURBS Surface

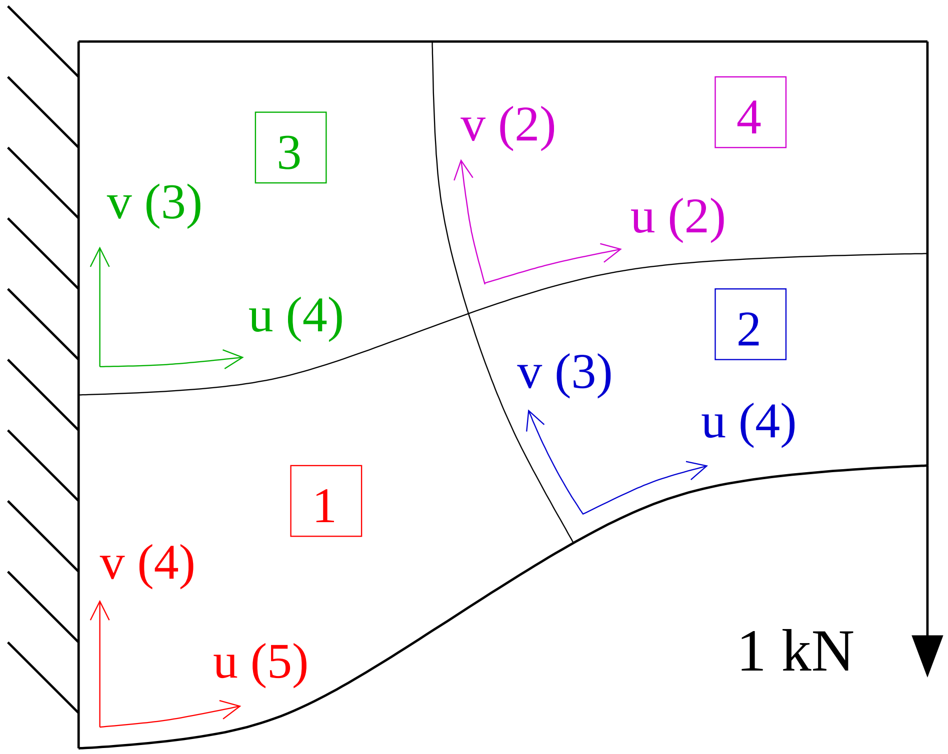


duplicated

$$\boxed{1} \times (\boxed{1.5} + \boxed{3})_{1D} = 3$$

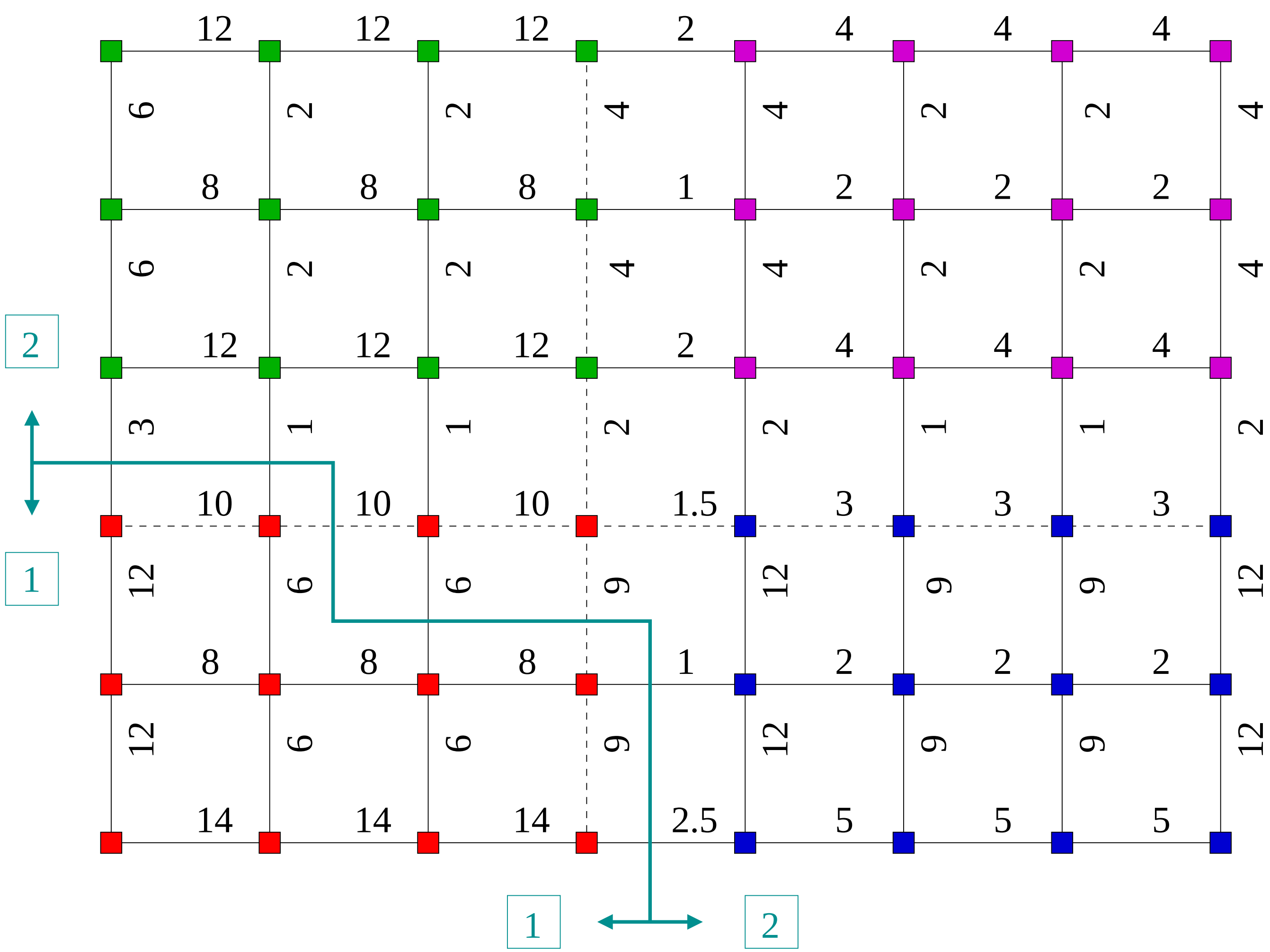


Academic Example – Short Cantilever



- **plane stress regime**
- **linear elasticity**
- **4 NURBS patches**
  - various degree
  - 4 × 3 uniform knot spans
- **interface constraints**
  - top(2) – bottom(4)
  - right(3) – left(4)
  - right(1) – left(2)
  - top(1) – bottom(3)

Academic Example – Partitioning of Dual Graph



relative weights:

- ■ – 21.67
- ■ – 7.23
- ■ – 7.23
- ■ – 1.00

domain weights:

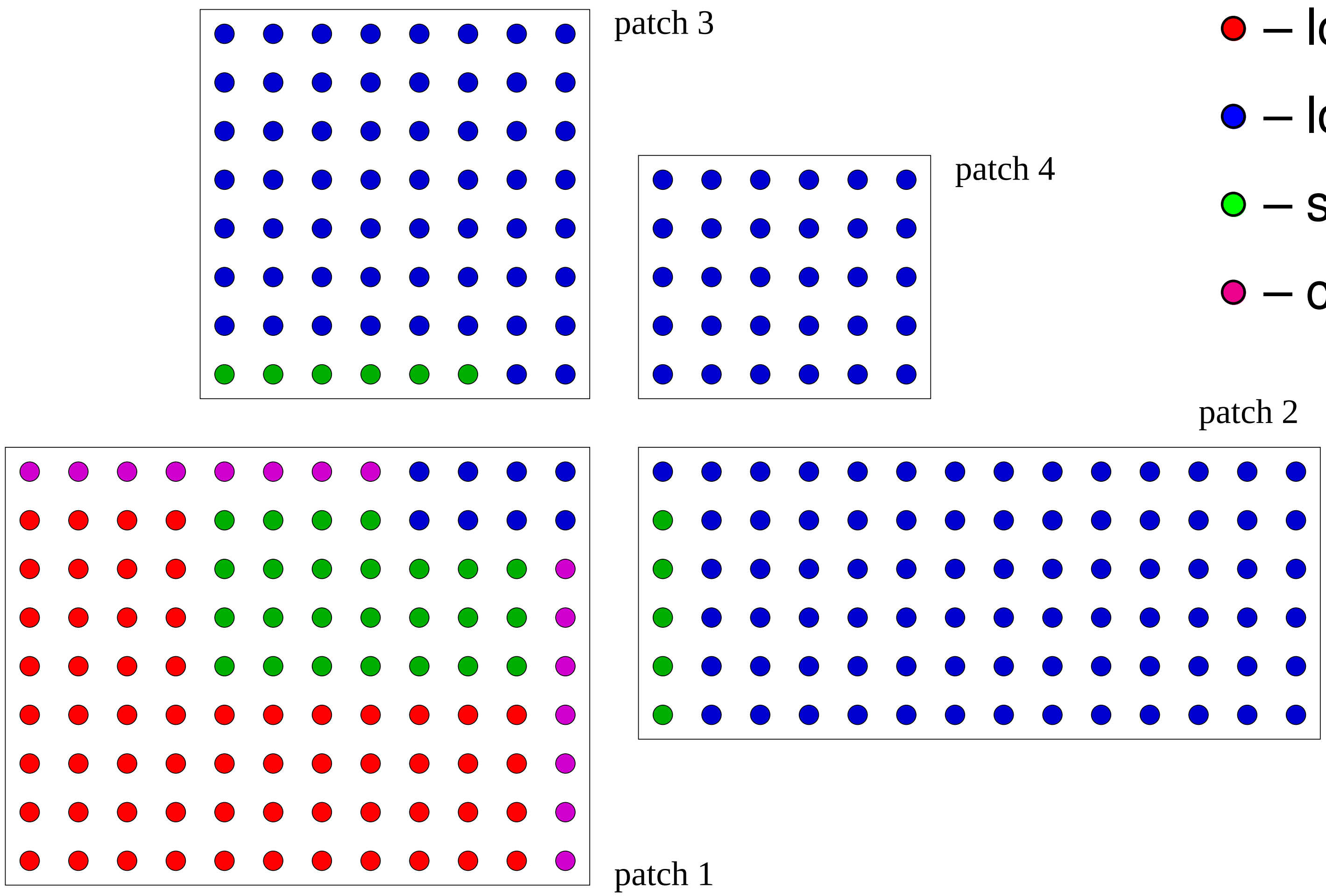
216.7 : 228.8

cut weight:

estimate: **32.5**

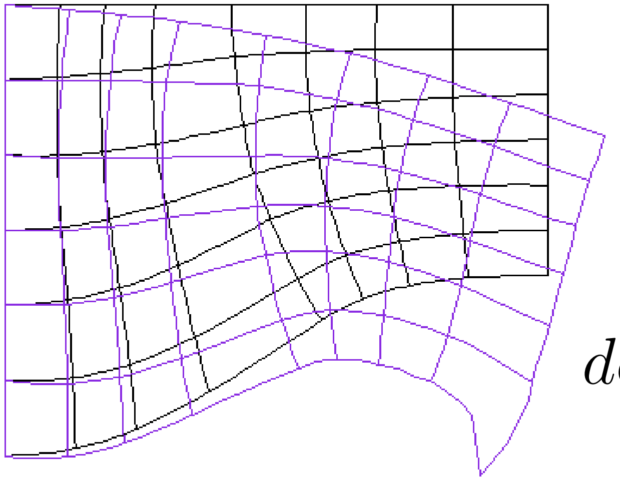
real: **36**

Academic Example – Control Point Classification

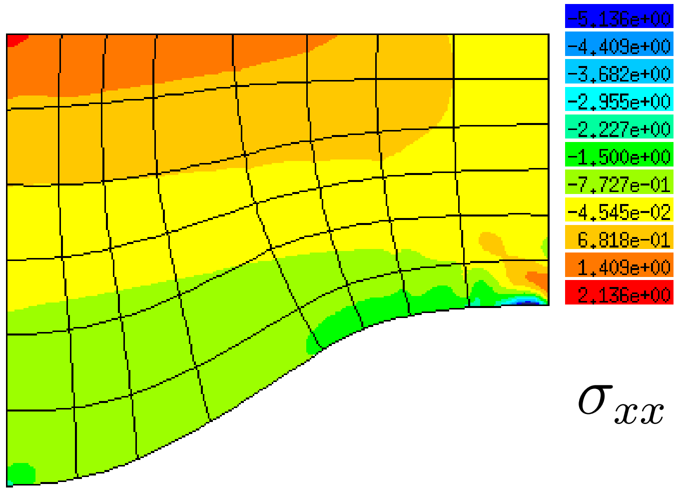


- – local on domain 1
- – local on domain 2
- – shared
- – constrained to shared

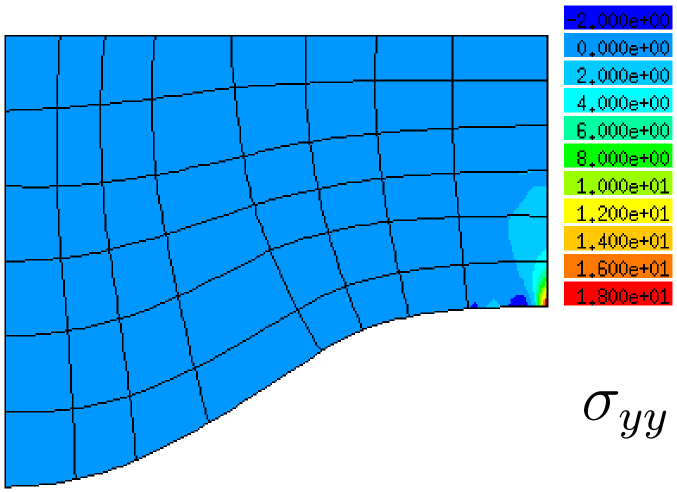
Academic Example – Results



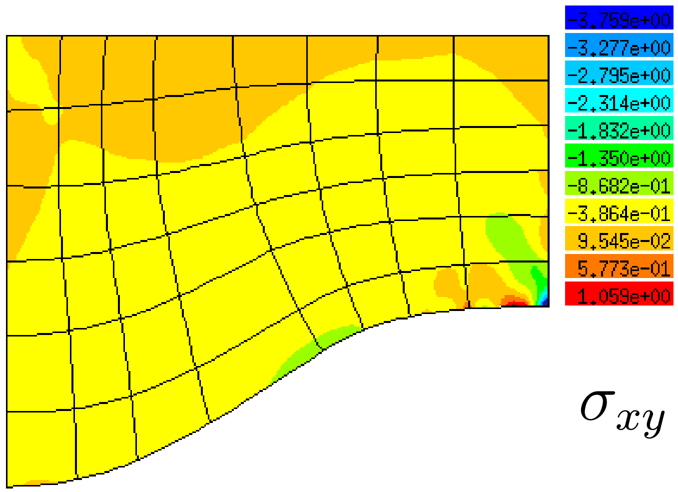
def



σ_{xx}



σ_{yy}



σ_{xy}

Summary

- **novel approach for representation of isogeometric mesh on single NURBS patch by weighted dual graph has been introduced**
  - **allows to use standard graph partitioners to obtain balanced domain decomposition with minimum interface**
- **functionality of the proposed methodology has been successfully demonstrated on simple academic example**
- **standard FE-based paradigms have been adopted**
  - **domain decomposition concept**
  - **communication based on MPI**

Summary – Future work

- improved handling of interface between NURBS patches of the same parameterization
- extension to handling of interface between NURBS patches with different parameterization
- extension to 3D patches

- extension to T-spline patches
- representation of isogeometric mesh by primal graph to allow element-cut partitioning concept