Limited Liability, Asset Price Bubbles and the Credit Cycle. The Role of Monetary Policy.*

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PRELIMINARY, COMMENTS ARE WELCOME

Abstract

This paper suggests that non-fundamental component in asset prices is one of the drivers of financial and credit cycle. Presented model builds on the financial accelerator literature by including a stock market where limitedly-liable investors trade stocks of productive firms with stochastic productivities. Investors borrow funds from the banking sector and can go bankrupt. Their limited liability induces a moral hazard problem which shifts demand for risk and drives prices of risky assets above fundamental value. Embedding the contracting problem in a New Keynesian general equilibrium framework, the model shows that loose monetary policy induces loose credit conditions and leads to a rise in both fundamental and non-fundamental components of stock prices. Positive shock to non-fundamental component triggers a financial cycle: collateral values rise, lending rate and default rate decreases. These effects reverse after several quarters, inducing a credit crunch. The credit boom lasts only while stock market growth maintains sufficient momentum. However, monetary policy does not reduce volatility of inflation and output gap by reacting to asset prices.

Keywords: credit cycle, limited liability, non-fundamental asset pricing, collateral value, monetary policy

JEL Codes: E32, E44, E52, G10

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1 Introduction

Asset price dynamics are one of the major drivers of credit and financial cycle. As asset prices enter the balance sheets of both financial and non-financial companies, they determine the collateral constraints of borrowers and contribute to how much the banks are willing to lend at a given risk premium. An asset price consists fundamental and non-fundamental components. In this paper, the fundamental value represents the part of the asset price which can be justified by discounted future dividend income under efficient market allocation. The remaining part of the asset price is labeled as non-fundamental and can be interpreted as a bubble. Empirical observations suggest that the non-fundamental component is more volatile than the fundamental, which tends to be rather stable as it follows the flow of dividends. Consequently, the non-fundamental (or bubble) component may be an important driver of the credit cycle. This paper builds a general equilibrium model based on the financial accelerator literature, extending the standard model by an asset (stock) market where assets are endogenously priced above their fundamental values. The assets traded are shares of productive firms, modeled as claims on future returns on capital. These returns (dividends) are stochastic and subject to idiosyncratic productivity shocks. There is an informational asymmetry between lenders (commercial banks) and borrowers (stock market investors), which induces the costly-state-verification problem and gives rise to debt contracts where the leveraged stock market investors are limitedly liable for the outcomes of their investment decisions. The limited liability induces excessive risk-taking by the investors, and leads to overpricing on the market for risky assets. As the overpriced assets enter collateral constraints of borrowers, the value of the non-fundamental (as well as fundamental) component has implications for the real economy: it affects the amount of lending, investment and output. Moreover, the asset price dynamics also affect the rate of loan defaults. Expansive monetary policy boosts both the fundamental and the non-fundamental component. Through the collateral constraints of stock market investors, the higher stock prices induce lower borrowing rates and higher investment. A positive shock to non-fundamental component of the asset price eases the collateral constraint and temporarily decreases lending rate. Although the default rate immediately declines with lower interest rates, it picks up later when the lending rates soar as asset price shock fades out, restricting the collateral constraint. Despite the suggested importance of asset prices for fluctuations in macroeconomic and financial variables, the estimated monetary policy efficiency frontiers show that the central bank achieves lowest combinations of volatilities of inflation and output gap by not reacting to either asset prices or their non-fundamental component.

The paper is organized as follows. Section 2 presents empirical motivation and Section 3 relates the paper to other literature. Section 4 describes the financial intermediation in partial equilibrium setup, describing the interactions between banks, limitedly-liable investors and firms, and shows that this setup leads to inflated prices on the asset market. Section 5 describes an extension, where the overpriced assets have longer maturities and can be used as collateral in the periods before maturity. Section 6 extends the model to general equilibrium by describing the relation to the remaining sectors of the economy, and section 7 presents the

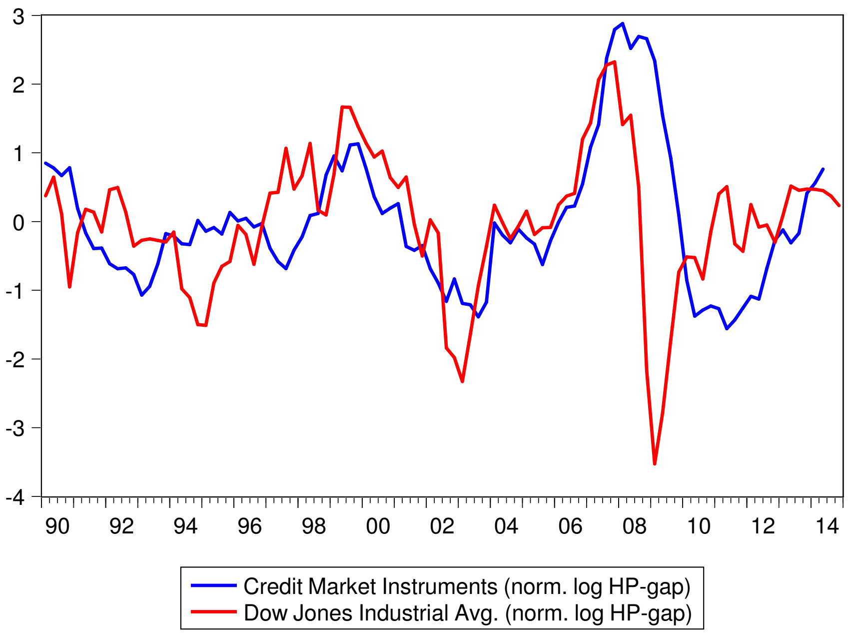

Figure 1: Asset prices leading the credit cycle

responses of model variables to shocks and discusses the implications for efficient monetary policy.

2 Empirical Motivation

This paper claims that business cycle is to a large extent driven by asset price developments, where the non-fundamental component of asset price plays a prominent role. Assuming that assets can be used as collateral, asset price boom leads to an easing of collateral constraints and subsequent increase in lending, investment and real activity. Figure 1 shows the normalized log-deviations (from HP trend) of Dow Jones Industrial Average index and the total value of credit market instruments since 1990. It is apparent that in this period, the Dow Jones index, used as a proxy for asset prices, was a leading indicator for the amount of credit. Credit boom in late 90's was preceded by a steady growth in asset prices. The asset price bust of the dot-com bubble in 2001 was followed by a negative gap in credit, which began to close only after the asset prices rebounded in 2003. In 2007, again it were the asset prices (both stock and real-estate) which preceded the credit crunch associated with the global financial crisis in 2007-2009. Lending began to pick up only after asset prices returned to growth after 2010.

Also the simple Granger causality tests (Table 1) on a quarterly sample of US data (FRED database of St. Louis Fed) ranging from 1949Q1 to 2014Q4 suggest that it is rather asset prices that lead the credit cycle than vice versa (all in percentage changes). Assuming that the fundamental value of an asset price should reflect dividend income, proxy for the fundamental

Pairwise Granger Causality Tests Sample: 1949Q1 2014Q4 Lags: 4			
Null Hypothesis:	Obs	F-Statistic	Prob.
DLOG(DJIA) does not Granger Cause DLOG(CREDIT)	246	4.86702	0.0009
DLOG(CREDIT) does not Granger Cause DLOG(DJIA)		1.72503	0.1451
D(LOG_NONFUND) does not Granger Cause DLOG(CREDIT)	243	2.48795	0.0442
DLOG(CREDIT) does not Granger Cause D(LOG_NONFUND)		1.42583	0.2261
D(LOG_FUND) does not Granger Cause DLOG(CREDIT)	243	1.28622	0.2761
DLOG(CREDIT) does not Granger Cause D(LOG_FUND)		1.80739	0.1281

Table 1: Asset prices Granger cause credit cycle

component was obtained by regressing the log of dividend income (including also dividend income lags and leads of up to 4th order) on the log of asset prices. The residual of this regression is a naïve estimate of the non-fundamental component of asset prices. The Granger causality tests show that this non-fundamental component (a bubble) leads the credit cycle, rather than the fundamental value. As a side note, the amount of lending, in turn, predicts the fundamental returns with marginal significance, possibly because credit-financed investment was followed by higher profits and dividends.

3 Related Literature

The global financial and European debt crises have pointed out the importance of financial sector in transmitting and amplifying economic shocks. The literature has reacted to this increased interest by building on the general equilibrium models with financial frictions of the late 1990s. Most important contributions involve works by Carlstrom & Fuerst (1997) and the subsequent synthesis of Bernanke *et al.* (1999) who integrate the previous models of financial frictions with New Keynesian rigidities and therefore are able to analyze the role of monetary policy. Another stream of literature builds on Kiyotaki & Moore (1997), which establishes a link between collateral values and business cycle, but does not explicitly model loan defaults. In recent years, enormous work has been done to incorporate other aspects of financial intermediation, such as the role of collateral constraints in housing market Iacoviello & Neri (2010) or the role of unconventional tools of monetary policy (Gertler & Karadi, 2011; Cúrdia & Woodford, 2011; Gertler & Karadi, 2013). The canonical model for the analysis of financial frictions and policy responses during the crisis has been set up by Gertler & Kiyotaki (2010). The role of liquidity constraints for the possibility of bank runs has been analyzed by Gertler & Kiyotaki (2013). The effects of the recently widely adopted tools of macro-prudential regulation has been explored by many, eg. Kashyap *et al.* (2014). Similarly to the present paper, Farhi & Tirole (2012) have concluded that a form of limited liability gives rise to elevated risk preference.

This paper presents a general equilibrium model capable of monetary policy simulations, which captures the characteristics of financial and credit cycle. It is inspired by a broadly held view of how do large financial crises develop and spread. Closest to our view is possibly the idea of Adrian & Shin (2010) and the model of Allen & Gale (2000). While Allen & Gale (2000) provide a rational explanation (albeit only in partial equilibrium setup) for overpricing of risky assets when the pricing is done by limitedly-liable investors, Adrian & Shin (2010) show how changes in the value of assets used as collateral can immediately lead to large swings in the balance sheet sizes (causing a contraction in credit and real activity) when the market is highly leveraged. This paper suggests that asset price and credit booms and subsequent busts could be explained and described using a combination of these two approaches. Because of the incentive structure of financial intermediaries' managers induces excessive risk appetite, there may be an over-investment into risky assets (compared to the efficient market solution), leading to inflated prices. If these overpriced assets are allowed to serve as collateral for further loans, a sufficient growth in asset prices creates a credit boom. The model implies that whenever growth of collateral prices slows down, a credit crunch occurs, with higher lending rates and elevated loan defaults. The model also illustrates how does monetary policy of an inflation-targeting central bank interfere with this mechanism. The simulations suggest that easy monetary policy leads to increase in both the fundamental and the non-fundamental element of asset prices and subsequently increases collateral values, reduces lending rates and boosts economic activity. A shock to asset prices can trigger a financial cycle: shortly after a positive price shock the lending rate falls and rate of defaults decreases as collateral constraint eases. After several quarters, however, the financial cycle reverses: lending rates and the rate of defaults rise as investors' wealth shrinks with the stock market slowdown.

Similarly to Martin & Ventura (2014), the growth of non-fundamental component (or a bubble) is not harmful *per se*, as it allows for more lending, investment and production when collateral is constrained. In a sense, the bubble helps to correct the adverse effects of the financial market imperfection. However, this paper also presents a structural interpretation for how does a bubble emerge, based on an increased risk taking by limitedly-liable investors. This paper also shows that the slow-down in growth of the asset prices is sufficient to reverse the beneficial effect of a bubble. The reversal of the financial cycle causes the real economy variables fall below the benchmark of Bernanke *et al.* (1999).

4 Financial Intermediation Structure and Asset Pricing

Inspired by model of asset bubbles presented by Allen & Gale (2000) and building on the model of financial accelerator in a New Keynesian framework of Bernanke *et al.* (1999), this paper constructs a partial equilibrium model which generates incentives-based overpricing in asset prices. The limited liability of investors (i.e. investors do not suffer the full cost in case of default) induces them to prefer risky assets and price them above their fundamental value. The fundamental value is defined as the price at which the investors would invest their own resources, which is also consistent with the pricing based on the present value of future dividend income. In later sections this contracting problem is embedded in a general equilibrium framework of Bernanke *et al.* (1999) and show that a non-fundamental overpricing of risky assets emerges within this widely-used model framework. This allows to conduct monetary policy experiments to study the impact of monetary policy shocks on the size of the non-fundamental component of asset prices. The size of the non-fundamental element of asset prices has real implications: it affects credit availability and subsequently the amount of lending, investment and economic activity.

Note that the similar incentive structure applies to a setting where an investment fund is managed by a limited-liable fund manager whose salary is dependent on the fund performance. When the return to managed portfolio is sufficiently high, manager's payoff increases with the returns. When the returns become negative, the manager can be fired, but does not directly bear the cost of portfolio loss. The limited liability leads to increased demand for risky assets. The same structure could describe corporate management rewarded in company stock options. In good states of the world, managers can execute their options, in bad states of the world they do not bear the losses. This incentive structure leads to more risky projects being undertaken in comparison to a first-best setting (which we will call fundamental) where managers would decide about the investment of funds owned by themselves. This paper illustrates that the spreading of these managerial incentive schemes in recent decades are at the heart of the increased risk-taking and excess volatility in the financial sector with adverse impact on the macroeconomic volatility.

4.1 Contract timing and payoffs

This model is an extension of Bernanke *et al.* (1999), and extends it by incorporating a stock market where the shares of productive firms are traded by limited-liable investors. Figure 2 depicts the agents in the contracting problem, the flow of funds and the respective interest rates. Risk-neutral investors can invest in risky shares S_{t+1} of productive firms, but have only limited own wealth N_{t+1}. These shares of productive firms have endogenously determined price P_{t+1} and yield a stochastic return equal to the return on firm's installed capital ωR^K_{t+1}, where ω is an idiosyncratic productivity element, which is i.i.d. with $E[\omega] = 1$. R^K_{t+1} is an aggregate capital productivity. The realization of idiosyncratic return ω is not known at the time of investment decision. It is revealed to the investor ex-post, but it cannot be contracted on, which gives rise to the costly-state-verification problem. The investor needs to borrow $B_{t+1} = P_t S_{t+1} - N_{t+1}$. First we assume that an ownership of shares entitles the holder to firm capital returns in one period, but later we will relax this assumption to allow for multi-period asset holdings which enter collateral constraints and lead to more pronounced effects of asset prices on other financial and macroeconomic variables.

The risk-neutral financial intermediaries (banks) are willing to lend B_{t+1} for a contractual rate Z_{t+1} as soon as their expected payoff from the contract exceeds the opportunity cost

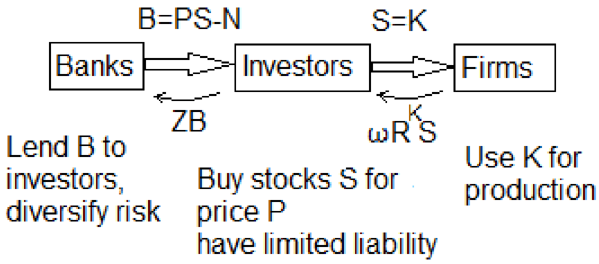

Figure 2: Agents in the financial intermediation, fund flows and interest rates

of investing in a risk-free asset with certain return R_{t+1} (the banking sector is competitive). Lending to investors is generally risky, as investors can default on the loan whenever the realized return from the portfolio is low enough, such that he is not able to repay the borrowing at the contractual rate $Z_{t+1}B_{t+1}$. Let us also assume that the limitedly liable investors are the only agents in the economy who are capable of investing in risky assets. In general, investor can also invest positive amounts in the risk-free asset. However, he will invest zero amount in risk-free asset, as his external financing costs (the contractual rate) will be generally higher than the risk-free return ($Z_{t+1} > R_{t+1}$) because the contractual rate Z_{t+1} would need to compensate the bank for any default risk. The riskiness of investment in not ex-ante observable by the bank, which can only ex-post monitor the returns by paying an agency costs (a standard costly-state/verification problem).

Whenever the realized return on the risky asset is below certain threshold, where the investor is unable to pay the loan back, he declares bankruptcy. The threshold for default is the break-even idiosyncratic return $\bar{\omega}$ on the risky asset, defined by the following constraint:

$$Z_{t+1}B_{t+1} = \bar{\omega}R^K_{t+1}S_{t+1} \tag{1}$$

The default-threshold idiosyncratic return $\bar{\omega}$ is such that the borrower will just be able to repay the borrowing B_{t+1} times the contractual rate Z_{t+1}. In case of default, the borrower (bank) pays a fraction μ as auditing costs to collect whatever remained from the project. The participation constraint for the bank can be written as

$$(1 - F(\bar{\omega}))Z_{t+1}B_{t+1} + (1-\mu)\int_0^{\bar{\omega}} \omega R^K_{t+1}S_{t+1}dF(\omega) \geq R_{t+1}B_{t+1} \tag{2}$$

Where the bank's expected payoff from lending to the investors must be higher than the opportunity cost of investing in risk-free bonds. To avoid facing idiosyncratic risk, the banks diversify their loan portfolio among many ex-ante identical investors, charging a flat rate Z_{t+1}. The timing of the financial intermediation contract is the following.

1. Banks lend B_{t+1} to investors for a flat rate Z_{t+1}, which compensates the bank for the ex-ante symmetric risk of investor's default.

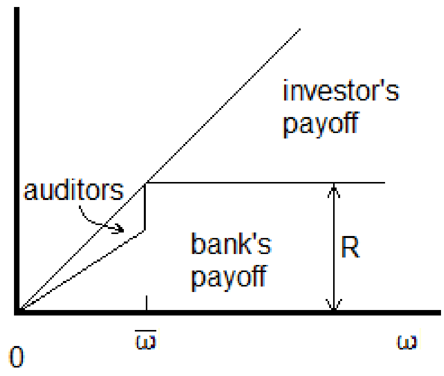

Figure 3: Contract payoffs distribution between bank, investor and auditing costs

2. The stock market opens, where investors may sell and buy risky assets S_{t+1} for an endogenously determined price P_t.

3. The idiosyncratic risk ω is realized, the assets S_{t+1} yield ωR^K_{t+1} to investors.

4. Investor either repays $Z_{t+1}B_{t+1}$ to the bank or defaults. In case of default, the bank pays auditing costs μ and collects the residual value of the investment.

This is a variant of the standard costly-state-verification problem of Townsend (1979), where the described risky debt contract (including the true reporting of default) is Pareto-optimal, as demonstrated by Gale & Hellwig (1985). The contract payoffs are depicted in Figure 3.

In the case of default, the investor's payoff is zero. In case of success, the investor is the residual claimant after satisfying the participation constraint of the bank.

4.2 Investors problem and demand pricing of the risky asset

The investor chooses the amount of risky investment S_{t+1} and the default threshold $\bar{\omega}$ to maximize

$$\max_{S_{t+1},\bar{\omega}} \left[\int_{\bar{\omega}}^{\infty} \omega R^K_{t+1} S_{t+1} dF(\omega) - (1 - F(\bar{\omega})) Z_{t+1} B_{t+1} \right] \tag{3}$$

subject to the bank's participation constraint (2). In other words, the investor takes into account only the "optimistic" part of the return distribution where he has positive profit, and he does not internalize the full cost of losses. Because of the limited liability, investor's payoff is zero in the case of default. This causes the investor's subjective return distribution to be more optimistic than the true fundamental return distribution. We will show that limited liability increases investor's appetite for investing in the risky asset, and raises the stock market prices.[1]

[1]Several conceptual points are worth noting at this point. First, we can think about the idiosyncratic ω realizations as shocks to distinctive sectors of the economy. We conjecture that investors prefer not to diversify their asset holdings among sectors and fully face the idiosyncratic risk. The idiosyncratic risk is preferred by the investors because the limited liability makes the non-diversified risky investment more attractive. Appendix B shows in more detail that limitedly liable investors do not prefer to diversify their asset holdings.

Second, let us also assume continuum of sectors of measure 1, so that the probability of default represents the fraction of defaulting investors. There are infinitely many firms in each sector, so that firms do not have any

The FOC of the investor's problem with respect to S_{t+1} (using that $B_{t+1} = P_t S_{t+1} - N_{t+1}$) equates the investors' marginal profit of risky investment to zero:

$$\int_{\bar{\omega}}^{\infty} \omega R_{t+1}^K dF(\omega) - (1 - F(\bar{\omega}))Z_{t+1}P_t = 0 \tag{4}$$

This equality will be achieved because of the competitive investors' sector and the convex costs of creating investment opportunities (see below). We assume that there are many ex-ante identical investors and they take the universally charged rate Z as exogenous. In other words, the contractual rate Z is not conditioned on the individual characteristics of investors, such as the individual amount of invested shares. The investors' wealth is equalized across investors' households. From the FOC (4), we can express the price of the risky asset as it can be observed on the stock market:

$$P_t = \frac{1}{Z_{t+1}} \frac{\int_{\bar{\omega}}^{\infty} \omega R_{t+1}^K dF(\omega)}{(1 - F(\bar{\omega}))} \tag{5}$$

We claim that this price of risky shares observed on the stock market is overpriced in comparison to the fundamental value as a result of limited liability of investors. Following Allen & Gale (2000), we define the fundamental value as the price which would the investor pay if he invested his own funds with no borrowing. Such a "fundamental" investor would solve

$$\max_{S_{t+1}} \left[\int_0^{\infty} \left(\omega R_{t+1}^K S_{t+1} + R_{t+1}(N_{t+1} - P_t^F S_{t+1}) \right) dF(\omega) \right] \tag{6}$$

Where the investors own wealth N_{t+1} is sufficient to cover all desired spending on the risky asset and the rest $(N - PK)$ is left for investing in the risk-free asset. The FOC of this problem defines the fundamental price:

$$P_t^F = \frac{1}{R_{t+1}} (E[\omega] R_{t+1}^K) = \frac{R_{t+1}^K}{R_{t+1}} \tag{7}$$

The fundamental price is not more than a present value of one-period-ahead claim on returns from installed capital. Comparing the fundamental price with the price with limited liability, we show that

$$P_t \geq P_t^F \tag{8}$$

The full proof can be found in Appendix A.

We have shown that under certain reasonable assumptions (most notably, the investors enjoy limited liability) the prices which are observed on the stock market are endogenously in-

bargaining power vis-à-vis investors, who become residual claimants. To ensure ex ante symmetry among the wealth of investors at the beginning of each period, we assume that investors gather in investors' households, and each investors' household sends an investor to each sector. The investors in each sector operate individually (importantly, they cannot repay debt using returns from other sectors), coming "home" together and pooling their wealth among household members only after the uncertainty is realized and after returns are paid. Also the below described "creative investors" hand their profits over to the household pool.

Finally, to prevent arbitrage conducted by banks or households that would drive price down to fundamental, the model assumes that trading on the stock market is restricted to the limitedly liable investors, who possess a unique skill of controlling and operating the firms which they own.

flated compared to their fundamental values. The non-fundamental component is the difference between the observed price of the risky asset and its fundamental value.

4.3 Supply of the risky asset

To complete the model of the stock market, we assume that in each household there are investors who "create" the investment opportunities (and sell them to other investors on the stock market). The profit from creating investment opportunity, and the creative investors' objective function is

$$\max_{S_{t+1}} P_t S_{t+1} - c(S_{t+1})) \tag{9}$$

where $c(S_{t+1})$ is an increasing convex cost function which links the costs of creating an investment opportunity to number of created investment assets. To ensure interior equilibrium, assume that $c'(.) > 0$, $c''(.) > 0$, and further that at the steady state $c(S) = 1$. The amount of created stocks S_t is then sold at the stock market at a price P_t, which the competitive creative investors take as exogenous and which potentially exceeds its average costs and creates economic profit. The creative investors are evenly distributed among investors' households, so that any profit from trading on the stock market stays in the investors' sector and is equalized across households to prevent heterogeneous paths of wealth. The first order condition of creative investors' profit maximization problem is

$$P_t = c'(S_{t+1}) \tag{10}$$

This equation describes the supply side of the market for the risky asset.

4.4 Value of the investment and wealth accumulation

Using the substitution for $Z_{t+1}B_{t+1}$ from eq. 1, the investors' objective 3 can be rewritten as

$$\max_{S_{t+1},\bar{\omega}} \left[\int_{\bar{\omega}}^{\infty} \omega R_{t+1}^K S_{t+1} dF(\omega) - (1 - F(\bar{\omega}))(\bar{\omega} R_{t+1}^K S_{t+1}) \right] \tag{11}$$

Substituting the last term from the banks' participation constraint 2, we arrive at the following expression for the value of investment:

$$R_t^K S_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R_t^K S_t dF(\omega)}{P_{t-1}S_t - N_t} \right) (P_{t-1}S_t - N_t) \tag{12}$$

As noted above, some of the investors are "creative" and they actively generate investment opportunities, establishing contact with firms. They are able to generate shares of firms, which they will be able to sell for P_t, by paying convex costs $c(S_{t+1})$, which in steady-state equals 1. Their net profit around the steady-state is therefore $S_{t+1}(P_t - 1)$, which they store at the risk-free account for the rest of the period. We assume that these creative investors are uniformly distributed across investors' households, and pool their gain in a representative investors' household wealth. Further imposing the market clearing condition that $S_t = Q_{t-1}K_t$ (i.e. stocks of firms entitle their holders to a share of firms' installed capital), we get the

$$\begin{aligned} V_t &= R_t^K Q_{t-1}K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R_t^K Q_{t-1} K_t dF(\omega)}{P_{t-1}Q_{t-1}K_t - N_t} \right) (P_{t-1}Q_{t-1}K_t - N_t) + R_t Q_{t-1} K_t (P_{t-1} - 1) \\ &= R_t^K Q_{t-1}K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R_t^K Q_{t-1} K_t}{Q_{t-1}K_t - N_t} dF(\omega) Q_{t-1}K_t - N_t \right) (Q_{t-1}K_t - N_t) \end{aligned} \tag{13}$$

which is identical to the wealth accumulation equation of Bernanke *et al.* (1999) (although here it comes from a different structure of financial market). In other words, although we consider a sector of limitedly liable investors in between the firms and banks, and we show that the prices of assets traded on the financial market are inflated. Because of the different incentive structure leading to different decisions about investment, this already does leads to a different allocation of real resources in comparison to Bernanke *et al.* (1999).

However, if the inflated assets are held for more than one period and can be used as collateral for further loans, the overpricing will have a stronger impact on credit availability, lending, investment, and real activity. In the next section we explain how the model economy behaves when the assets (with inflated prices as a result of the above described mechanism) are held for two periods before they mature.

5 Extension: Multi-Period Assets and Collateral Constraint

In this section we consider an extension of the presented model where the agents hold and trade the assets for multiple periods until the assets mature. In that case, the prices of assets held for more than one period will alter the evolution of investors' wealth, which is used as collateral. When prices of these assets would be inflated as in the single-period case described above, the investors' wealth (which serves as collateral) would be inflated as well. Most importantly, the results suggest that if the asset prices maintain sufficient (precisely defined) growth momentum, investors' wealth is higher than in the Bernanke *et al.* (1999) benchmark. If the asset price growth slows down, the wealth decreases below the Bernanke *et al.* (1999) benchmark.

5.1 Financial intermediation contract with two-period assets

Assume there is an asset which is purchased in the first period, held in the second, and in the third period it transforms into a claim for productive capital. As a result, in every period t there two types of assets traded; the old S_{t+1}^{old} (issued in previous period $t-1$, maturing in $t+1$) with price P_t^{old} and the newly issued ones S_t^{new} with price P_t^{new}. The cash-flow constraint, i.e. the relationship between contractual rate Z_{t+1} and threshold idiosyncratic productivity $\bar{\omega}$ (formerly eq. 1) in this case becomes

$$Z_{t+1}B_{t+1} = \bar{\omega} R_{t+1}^K S_{t+1}^{old} + P_{t+1}^{old} S_{t+1}^{new} \tag{14}$$

because additionally to capital returns on maturing assets, the investors will in $t+1$ own previously purchased assets maturing in the following period. The bank participation constraint 2 changes to

$$(1-F(\bar{\omega}))Z_{t+1}B_{t+1} + (1-\mu)\int_0^{\bar{\omega}} \left(\omega R^K_{t+1} S^{old}_{t+1} + P^{old}_{t+1} S^{new}_{t+1}\right) dF(\omega) \geq R_{t+1}B_{t+1} \tag{15}$$

5.2 Investors' objective and asset pricing

The investors' objective (previously eq.3) is to maximize expected profit, which is in the presence of two-period assets defined as

$$\max_{S^{new}_{t+1}, S^{old}_{t+1}, \bar{\omega}} \left[\int_{\bar{\omega}}^{\infty} \left(\omega R^K_{t+1} S^{old}_{t+1} + P^{old}_{t+1} S^{new}_{t+1}\right) dF(\omega) - (1-F(\bar{\omega}))Z_{t+1}B_{t+1}\right] \tag{16}$$

Substituting for $B_{t+1} = P^{new}_t S^{new}_{t+1} + P^{old}_t S^{old}_{t+1} - N_{t+1}$, the first order conditions with respect to S^{new}_{t+1} and S^{old}_{t+1} define the prices

$$P^{old}_t = \frac{1}{Z_{t+1}} \frac{\int_{\bar{\omega}}^{\infty} \omega R^K_{t+1} dF(\omega)}{1-F(\omega)} \tag{17}$$

$$P^{new}_t = \frac{P^{old}_{t+1}}{Z_{t+1}} = \frac{1}{Z_{t+1}Z_{t+2}} \frac{\int_{\bar{\omega}}^{\infty} \omega R^K_{t+2} dF(\omega)}{1-F(\omega)} \tag{18}$$

Which both could be shown to be higher with respect to fundamental values defined as if there would be no information asymmetry (analogously to the case of single-period assets).

5.3 Investors' wealth accumulation under two-period assets

Combining the investors objective 16 with bank participation constraint 15, substituting for $Z_{t+1}B_{t+1}$ using eq.14 and imposing the market-clearing condition that $S^{old}_t = Q_{t-1}K_t$, one can get the evolution of the aggregate value of investors' assets.

$$\begin{aligned} V_t = {} & R^K_t Q_{t-1}K_t + P^{old}_t S^{new}_t \\ & - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \left(\omega R^K_t Q_{t-1}K_t + P^{old}_t S^{new}_t\right) dF(\omega)}{P^{new}_{t-1} S^{new}_t + P^{old}_{t-1} Q_{t-1}K_t - N_t}\right) (P^{new}_{t-1} S^{new}_t + P^{old}_{t-1} Q_{t-1}K_t - N_t) \end{aligned} \tag{19}$$

The value of investment is now different in comparison to the single-period assets and so with the Bernanke *et al.* (1999) benchmark. This is because (overpriced) assets can be used as collateral in the meantime before maturity. However, the purchase of overpriced assets also constitutes extra costs with respect to the Bernanke *et al.* (1999) benchmark. Comparing the wealth accumulation equation with the benchmark, we are able to establish the conditions under the inflated prices of risky assets have expansionary effects on the economy, and when the effect is restrictive.

The question is whether

$$V_t^{INF} - V_t^{BGG} = P_t^{old} S_t^{new}(1 - \mu F(\bar{\omega})) - R_t(P_{t-1}^{old} Q_{t-1} K_t) \lesseqgtr 0 \tag{20}$$

This inequality translates (using that new assets will become old next period, $S_t^{new} = S_{t+1}^{old}$) into a question whether a nominal growth of the stock market maintains sufficient momentum:

$$\frac{P_t^{old} S_{t+1}^{old}}{P_{t-1}^{old} S_t^{old}} \lesseqgtr \frac{R_t}{1 - \mu F(\bar{\omega})} \tag{21}$$

If the growth of asset prices and/or amount of assets traded remains high, the investors wealth, amount of borrowing, investment and economic activity exceeds the Bernanke *et al.* (1999) benchmark. We label this case an asset price and credit boom. As soon as the growth of market volume slows down, there is a asset price bust and a credit crunch. The real effects of the presence of the non-fundamental component in asset prices are positive when the asset market keeps growing at a sufficient rate, improving credit availability, increasing investment and real activity. When the growth of asset market looses momentum, the real effects reverse.

6 Other Sectors of the New Keynesian General Equilibrium Model

Now we embed the contracting problem described above in a general equilibrium model. We follow the framework of Bernanke *et al.* (1999) closely. In addition to investors and and firms, there are retailers, households, the central bank and government.

6.1 Investors and Banks

The investors' households are risk-neutral, but leave the system with a rate of γ. This ensures that investors always demand credit and do not accumulate enough wealth to be eventually fully self-financing. After departure, they consume the remaining part of their wealth. The investors accumulate wealth according to

$$N_{t+1} = \gamma V_t + W_t^e \tag{22}$$

where V_t is the value of investment in firms' shares as defined above by eq. 13. When the investor's household dies, it consumes all its wealth and departs the scene. This process creates the investors consumption C_t^e.

The demand price of assets P_t was defined by eq. 5, and the supply side by eq. 10. Dividing the banks' participation constraint (eq. 2) by B_{t+1} and using eq. 1 for substitution of the term inside the integral of after-default asset recovery, we can express the risk premium (the difference between Z_{t+1} and R_{t+1}) as a function of the default threshold $\bar{\omega}$:

$$\frac{R_{t+1}}{Z_{t+1}} = 1 - F(\bar{\omega}) + \frac{1-\mu}{\bar{\omega}} \int_0^{\bar{\omega}} \omega dF(\omega) \equiv \Psi(\bar{\omega}) \tag{23}$$

where $\frac{\partial \Psi(\bar{\omega})}{\partial \bar{\omega}} < 0$. Similarly, the demand price of risky assets is a function of the aggregate capital returns R^K_{t+1}, the contractual rate Z_{t+1} and the default threshold $\bar{\omega}$ (eq. 5), and can be transformed such that

$$\frac{P_t Z_{t+1}}{R^K_{t+1}} \equiv \Theta(\bar{\omega}) \tag{24}$$

where $\frac{\partial \Theta(\bar{\omega})}{\partial \bar{\omega}} > 0$, i.e. the price of the risky asset increases with the risk, which is a result of investors elevated risk preference induced by the limited liability. Combining these two relationships, one can see that the wedge between the risk-free rate R_{t+1} and capital returns R^K_{t+1} can be expressed as a function of $\bar{\omega}$ and risky asset price P_t, which in turn is determined by the supply-side increasing marginal costs, which link it to S_{t+1} (eq. 10).

Finally, a market clearing condition links the financial sector to the production sector:

$$S_t = Q_t + K_t \tag{25}$$

6.2 Firms

Representative firm produces output Y_t using the production function with capital K_t and aggregate labor L_t as inputs.

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{26}$$

where A_t is a stochastic total factor productivity following an autoregressive process. Capital share is denoted byα. Labor consists of workers' labor H_t and investors' labor H^e_t (consider venture capital):

$$L_t = H_t^{\Omega} (H^e_t)^{1-\Omega} \tag{27}$$

where Ω is the share of workers' labor. New capital creation involves installment costs, old capital depreciates at a rate δ.

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) - (1-\delta)K_t \tag{28}$$

The installment costs can be thought of as a competitive sector of capital producers, who purchase investment and rent capital stock to produce new capital using the production function $\Phi\left(\frac{I_t}{K_t}\right)$ to sell it for price Q_t. FOC to their problem determines the "replacement cost" component to the price of capital: $Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}$.

Firms produce wholesale goods, which are sold to monopolistically competitive retailers for a relative price $\frac{1}{X_t}$. The firms sector is assumed to be competitive. Return on capital R^K is equal to marginal product of capital times the price of produced wholesale goods, augmented by the change in value of capital(consisting of change in Q_t and depreciation). In expectation terms:

$$E[R_{t+1}^K] = E\left[\frac{\frac{1}{X_{t+1}}\frac{\alpha Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)}{Q_t}\right] \tag{29}$$

Wages in workers' and investors' sectors of labor market are competitive and follow marginal products of labor:

$$W_t = (1-\alpha)\Omega\frac{1}{X_t}\frac{Y_t}{H_t} \tag{30}$$

$$W_t^e = (1-\alpha)(1-\Omega)\frac{1}{X_t}\frac{Y_t}{H_t^e} \tag{31}$$

6.3 Retailers and households

In addition to the agents involved in the contraction problem, the model features standard New Keynesian general equilibrium setup. Monopolistically competitive retailers buy wholesale goods from the producers and costlessly diversify products to establish market power. Retailers set prices according to Calvo pricing, where only a fraction of retailers change prices each period. The final product is sold to households. Monopolistically competitive retailers face the Dixit-Stiglitz demand functions for the final product varieties

$$Y_t(z) = \left(\frac{P_t^C(z)}{P_t^C}\right)^{\epsilon} \tag{32}$$

where $Y_t(z)$ is a demanded quantity and $P_t^C(z)$ is a price of consumption good z and ϵ is the elasticity of substitution. The consumption goods are aggregated to final consumption bundles using

$$Y_t = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon}dz\right]^{\epsilon/(\epsilon-1)} \tag{33}$$

And the consumption price index is

$$P_t^C = \left[\int_0^1 P_t^C(z)^{(1-\epsilon)}dz\right]^{1/(1-\epsilon)} \tag{34}$$

Each period, only a fraction θ of retailers chooses prices $P_t^\star$ to maximize expected profits until the next expected price change. Retailers transfer profits back to workers households.

$$P_t^C = [\theta {P_{t-1}^C}^{1-\epsilon} + (1-\theta)(P_t^{C\star})^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{35}$$

The optimal price-setting of monopolistically competitive retailers leads to a New Keynesian Phillips curve. In log-linear form, where lower-case letters define log-deviations from steady state:

$$\pi_t = E_{t-1}[\kappa(-x_t) + \beta\pi_{t+1}] \tag{36}$$

where β is workers' households discount factor and $\kappa = (1-\theta)(1-\theta\beta)/\theta$.

Households derive utility from consumption C_t, leisure $1-H_t$ and real money holdings $\frac{M_t}{P_t^C}$.

This gives rise to demand for consumption (Euler equation), labor supply and demand for real money balances. The household savings are deposited (D_t) at a risk-free rate R_t in banks, which use it to lend to investors ($D_t = B_t$). The expected utility

$$E_t \sum_{k=0}^{\infty} \beta^k [\ln(C_{t+k}) + \zeta \ln(\frac{M_{t+k}}{P^C_{t+k}}) + \xi \ln(1 - H_{t+k})] \tag{37}$$

where ζ is a preference parameter of real money holdings and ξ is a preference parameter of leisure, is maximized subject to the budget constraint

$$C_t = W_t H_t - T_t + \Pi_t R_t D_t - D_{t+1} + \frac{(M_{t-1} - M_t)}{P^C_t} \tag{38}$$

where T_t are taxes. The first order conditions of this problem form the Euler equation, labor supply and money demand:

$$\frac{1}{C_t} = E_t\{\beta \frac{1}{C_{t+1}} R_{t+1}\} \tag{39}$$

$$\frac{W_t}{C_t} = \xi \frac{1}{1 - H_t} \tag{40}$$

$$\frac{M_t}{P_t} = \zeta C_t \left(\frac{R^n_{t+1} - 1}{R^n_{t+1}} \right) \tag{41}$$

6.4 Government policies and resource constraint

Government consumes fraction of output, financing it by collected taxes and received seignorage.

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t \tag{42}$$

Central bank sets the nominal interest rate according to a inflation-targeting monetary policy rule.

$$R^n_{t+1} = (R^n_t)^{\rho} \, \Pi^{\psi}_t \varepsilon^{R^n}_{t+1} \tag{43}$$

where ε^{rn}_t is a monetary policy shock.

The resource constraint represents the national accounts identity in closed-economy setting

$$Y_t = C_t + I_t + G_t + C^e_t + \phi^y_t \tag{44}$$

where ϕ^y_t represents the resources devoted to monitoring costs, which are lost. The complete log-linearized model can be found in Appendix C with a special attention to the description of financial sector block.

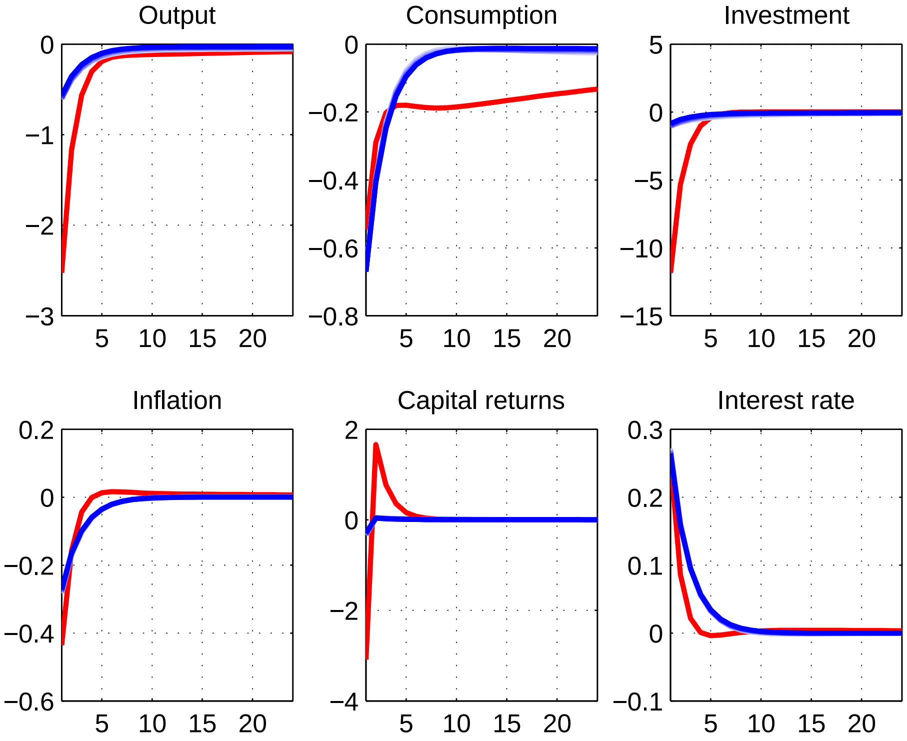

Figure 4: Impulse responses of macro variables to a restrictive MP shock

7 Model Simulations

7.1 Calibration

The parameters are calibrated according to Bernanke *et al.* (1999). Quarterly discount factor β is 0.99 and the labor supply elasticity η is 3. Capital share α is 0.35, the workers labor share is 0.64 and investors' labor share is the remaining 0.01. Capital depreciates at a quarterly rate δ of 0.025. The steady-state share of government expenditures G/Y is 0.2. Autocorrelation coefficients for technology, government expenditure, monetary policy and asset price shocks are, respectively, 0.999, 0.95, 0.9 and 0.9. Investment price elasticity to the ratio of new investment is 0.25. Similarly to Bernanke *et al.* (1999), the assumption here is that the business failure rate is 3 per cent annually and that the capital to net worth ratio is 2. The investors' quarterly departure rate γ equals 0.0272. The recovery costs μ are 0.12 and the idiosyncratic risk ω is log-normally distributed with variance 0.28. The Calvo parameter θ is 0.75 (a retailer changes price once a year on average). Monetary policy sensitivity parameter to inflation is 0.11.

7.2 Policy simulations

Figures 4 and 5 show the impulse responses of model variables (expressed on in log-deviations from steady state values) to a 0.1 percentage point increase of nominal monetary policy rate. In all graphs to come, the red line shows the responses of the benchmark Bernanke *et al.* (1999)

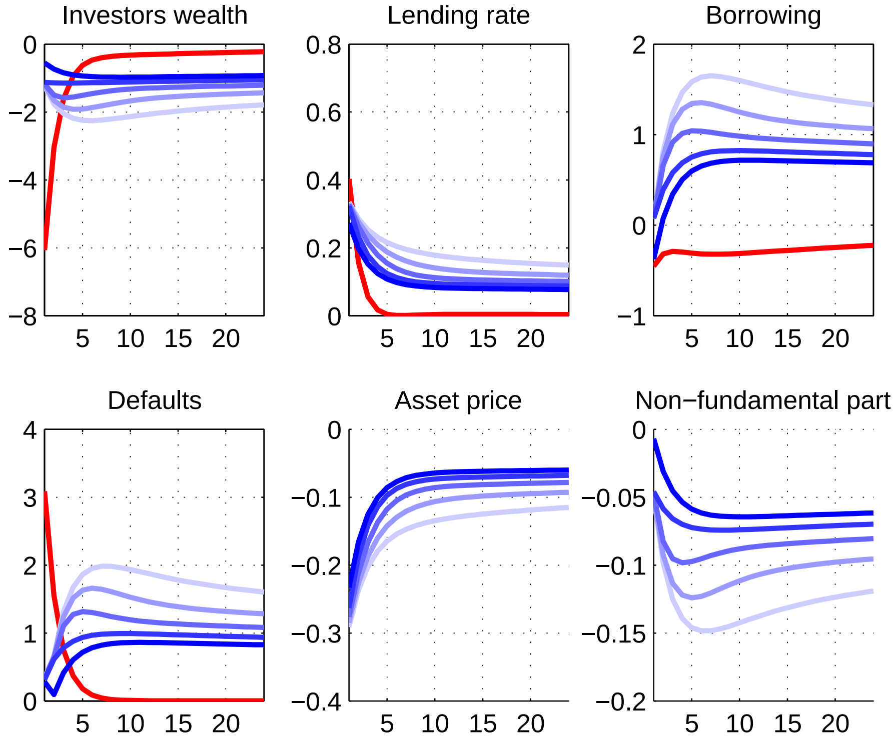

Figure 5: Impulse responses of financial variables to a restrictive MP shock

model, the darker blue line shows the responses of single-period assets version of the present model, lighter blue lines relate to longer maturities in the multiple-period assets version of the model. Because of the standard New Keynesian features of the model (monopoly power of retailers, price rigidities), the nominal interest rate hike transfers to an increase of the real interest rate. Consumption falls as households' optimal allocation shifts towards savings. Investment falls as well, but in a much smaller magnitude in comparison to the Bernanke *et al.* (1999) benchmark. The reason is that large part of the shock is absorbed by the financial sector, most notably by the prices of risky assets, while capital returns and investment are affected much less. In a sense, financial sector in this model works as a shock absorber, but at the same time the impact is transformed into a longer-lasting financial cycle. In a response to the restrictive monetary policy shock, output falls as well. Inflation drops as marginal costs decrease.

With nominal interest rate hike, the financing costs of loan rise, inducing an elevated default threshold. The asset prices fall (as they are discounted by the lending rate), including their non-fundamental component. As the cost of borrowing increases, investors' wealth falls. Unlike the benchmark model, these reactions are weaker on impact but more persistent, giving rise to a momentum of the financial and credit cycle. In the present model, borrowing reacts by an increase after several quarters, as the investors' wealth falls and the remaining funds are borrowed, partially absorbing the shock and smoothing the investment cycle. This effect occurs because elevated idiosyncratic risk makes investors demand more risky assets (which are claims

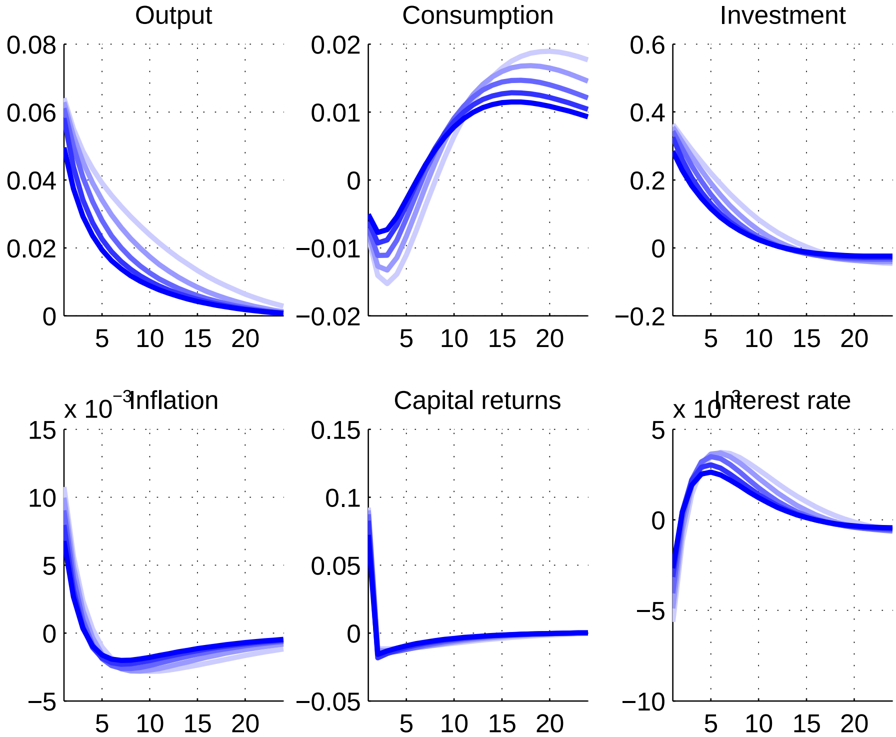

Figure 6: Impulse responses of macro variables to asset price shock

on installed capital) and thus mitigates the fall in investment observed in the benchmark model.

Figures 6 and 7 show the responses of model variables to a shock to the non-fundamental component of asset prices. As asset prices rise, investors' wealth increases, inducing more investment. Consumption temporarily falls as it is optimal to postpone consumption and invest. Because of higher marginal costs, inflation rises too. The responses of lending rate, amount of borrowing and default threshold heavily depend on if asset prices are treated as collateral as a part of investors' wealth, and for how long does the price stay in the portfolio. When the portfolio turnover is fast and assets mature quickly, the wealth increases only temporarily and expenditure on the more expensive risky assets is financed by borrowing, which increases default threshold and lending rate. When the longer maturities dominate, wealth rises more, lending rate falls, along with the amount of borrowing and the default threshold.

7.3 Should monetary policy react to asset prices?

In short, no. For the purpose of this exercise, the assumption of strict inflation-targeting monetary authority was eased, allowing the central bank to react also to output gap and asset prices. A grid search among various combination of reaction parameters to inflation, output gap and asset prices was conducted, locating the combinations of the lowest implied standard deviations of the two target variables, inflation and output gap. The combinations of minimized standard deviations define a monetary policy efficiency frontier, from which the central bank

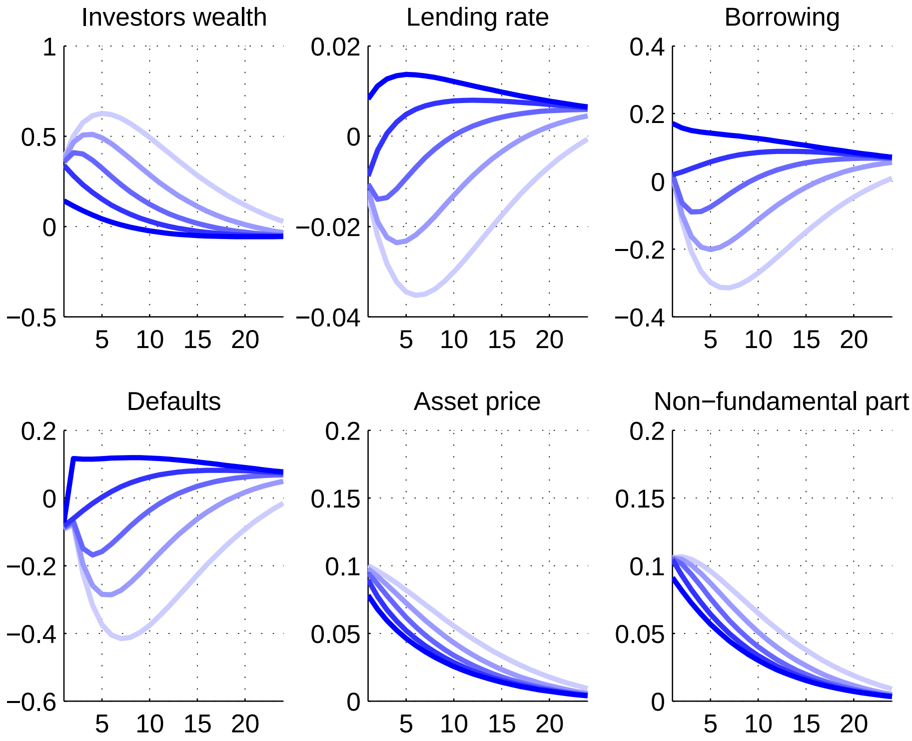

Figure 7: Impulse responses of financial variables to an asset price shock

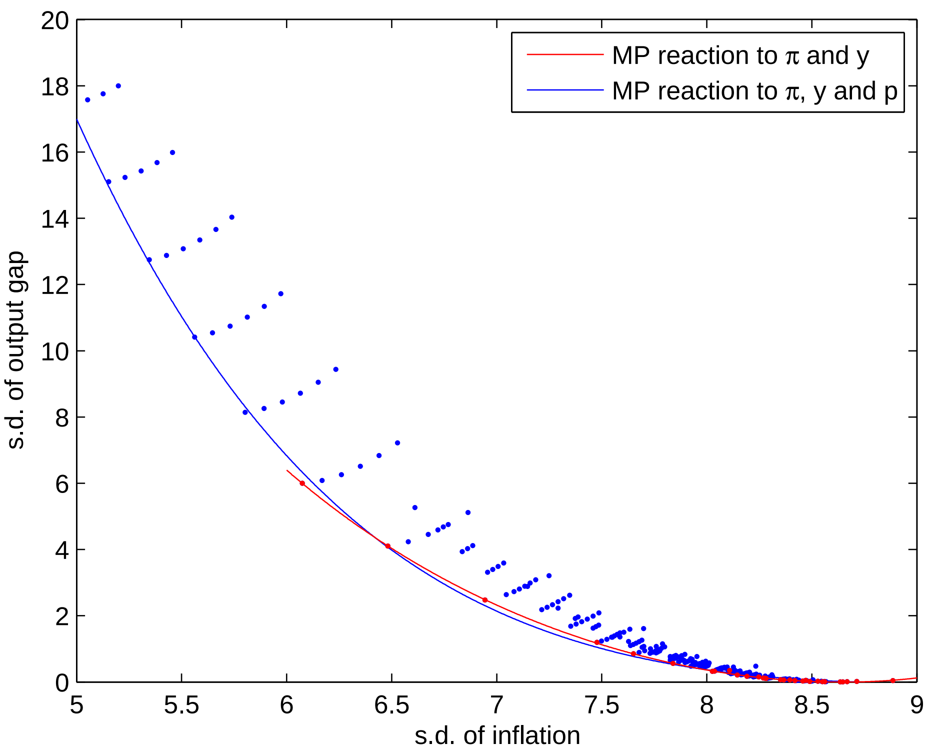

Figure 8: Monetary policy efficiency frontiers with single-period assets

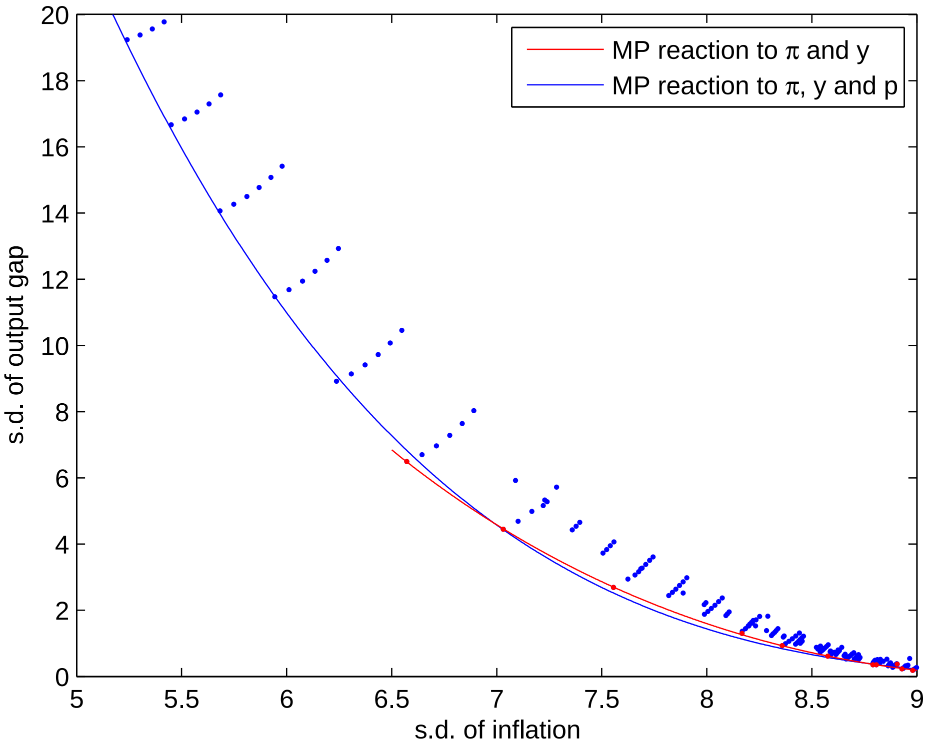

Figure 9: Monetary policy efficiency frontiers with multiple-period assets

can choose its optimal reaction function depending on its relative disutility from output and inflation fluctuations. Figure 8 depicts this monetary policy frontier when reacting to inflation and output gap only (in red) in comparison the outcome when monetary policy can react also to asset prices (blue). The simulations suggest that there is no gain from reacting to asset prices, as the lower envelope of minimized combinations of the standard deviations of inflation and output gap are achieved when the central bank does not react to asset prices at all. The reaction to the non-fundamental component of asset prices was also tested with a similar result: monetary policy reaction to asset price bubbles does not lead to lower volatility of output and inflation.

Figure 9 shows the same frontiers when holdings of assets for multiple periods are allowed, i.e. when the asset prices enter the investors' wealth and can be used as collateral. The results are very similar: the efficient combinations of inflation and output standard errors on the monetary policy frontier are achieved when monetary policy does not react to asset prices at all. The same holds for the reaction to the non-fundamental component of asset prices: the present model implies that monetary policy should not react to asset price bubbles.

8 Concluding Remarks

This paper has analyzed the role of asset prices, and their non-fundamental component in the dynamics of financial and credit cycle. We have presented a model of financial intermediation, where limited liability of investors gives rise to an overpricing on the market of risky assets, such as shares of productive firms. The model is based on the established framework of Bernanke *et al.* (1999), but altered substantially to include a stock market, where stocks of firms with stochastic idiosyncratic returns are traded by limited-liable investors. We show a number of results. First, the prices of assets on this market exceed their fundamental values (which are defined as in the case of absence of the principal-agent problem, e.g. if the investors would not borrow but only use their own funds for trading). Second, the investors prefer to face the idiosyncratic, sector-specific risk to portfolio diversification, because of their limited liability. Third, we show that when the nominal amount of traded assets (i.e. a product of asset price and the amount of traded assets) maintains sufficient growth momentum, there is a boom also in the investors' wealth, credit, investment and output. When the growth of asset prices or amounts slows down, the wealth, credit, investment and output fall below the benchmark allocations, which we call a credit crunch. Finally, the described financial sector behaves counter-cyclically as a shock absorber, as in the periods of elevated risk the non-fundamental component of asset prices rises as investors prefer the higher risk, which stabilizes investors' wealth and encourages funding for capital investment.

The model illustrates that expansive monetary policy shock temporarily boosts on the financial market by decreasing the lending rate, the fraction of defaulting investors and increasing asset prices (including the non-fundamental component), which in turn inflates investors' wealth. A positive shock to asset prices gives rise to a financial cycle: first, the lending rate and rate of defaults decreases and wealth increases. After several quarters (depending on the maturity structure of portfolios) the lending rate and the rate of defaults rise as asset price fall and shrink the collateral values.

Finally, the monetary policy efficiency frontiers suggest that reacting to asset prices or the non-fundamental component does not help to achieve more favorable combinations of inflation and output gap volatilities.

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A Proof of the Overpricing in Risky Asset Price

Proof. We need to show that

$$P_t = \frac{1}{Z_{t+1}} \frac{\int_{\bar{\omega}}^{\infty} \omega R_{t+1}^K dF(\omega)}{(1 - F(\bar{\omega}))} \geq \frac{R_{t+1}^K}{R_{t+1}} = P_t^F \tag{45}$$

Now we define $\tilde{Z}_{t+1}$ and $\tilde{P}_t$ such that

$$\tilde{P}_t = \frac{1}{\tilde{Z}_{t+1}} \frac{\int_{\bar{\omega}}^{\infty} \omega R_{t+1}^K dF(\omega)}{(1 - F(\bar{\omega}))} = \frac{R_{t+1}^K}{R_{t+1}} = P_t^F \tag{46}$$

Now by showing that $\tilde{Z}_{t+1} > Z_{t+1}$ we will prove that $P_t > P_t^F$. Using that $\tilde{P}_t = P_t^F = \frac{R_{t+1}^K}{R_{t+1}}$:

$$\tilde{Z}_{t+1}(1 - F(\bar{\omega})) = \frac{R_{t+1}}{R_{t+1}^K} \int_{\bar{\omega}}^{\infty} \omega R_{t+1}^K dF(\omega) \tag{47}$$

Eliminating R_{t+1}^K and multiplying with B_{t+1}:

$$\tilde{Z}_{t+1} B_{t+1}(1 - F(\bar{\omega})) = R_{t+1} B_{t+1} \underbrace{\int_{\bar{\omega}}^{\infty} \omega dF(\omega)}_{>1} > R_{t+1} B_{t+1} \tag{48}$$

Now use the banks' participation constraint (2):

$$Z_{t+1} B_{t+1}(1 - F(\bar{\omega})) = R_{t+1} B_{t+1} \underbrace{-(1 - \mu) \int_{0}^{\bar{\omega}} \omega R_{t+1}^K S_{t+1} dF(\omega)}_{<0} < R_{t+1} B_{t+1} \tag{49}$$

therefore $\tilde{Z}_{t+1} > Z_{t+1}$ and $P_t > P_t^F$. □

Therefore, the risky asset is overpriced in comparison to the fundamental price.

B Investors Prefer Not to Diversify

We show that everything else equal, the investors prefer diversification over diversification. Therefore, the investors do not have incentives to deviate from the no-diversification equilibrium and the equilibrium is stable.

The expected return on shares net of financing costs under full diversification (no risk) equals

$$R_{t+1}^K - Z_{t+1} B_{t+1} = \int_{0}^{\infty} \omega R_{t+1}^K - Z_{t+1} B_{t+1} dF(\omega) \tag{50}$$

The expected return on shares net of expected financing costs when there is no diversification and the limited liable investor fully faces the idiosyncratic risk is

$$E[R^K_{t+1} - Z_{t+1}B_{t+1}|\omega > \bar{\omega}] = \int_{\bar{\omega}}^{\infty} \omega R^K_{t+1} - Z_{t+1}B_{t+1} dF(\omega) \tag{51}$$

Now its obvious that $E[R^K_{t+1} - Z_{t+1}B_{t+1}|\omega > \bar{\omega}] > R^K_{t+1} - Z_{t+1}B_{t+1}$ as

$$\underbrace{\int_0^{\infty} \omega R^K_{t+1} - Z_{t+1}B_{t+1} dF(\omega)}_{R^K_{t+1} - Z_{t+1}B_{t+1}} = \underbrace{\int_0^{\bar{\omega}} \omega R^K_{t+1} - Z_{t+1}B_{t+1} dF(\omega)}_{<0} + \underbrace{\int_{\bar{\omega}}^{\infty} \omega R^K_{t+1} - Z_{t+1}B_{t+1} dF(\omega)}_{E[R^K_{t+1} - Z_{t+1}B_{t+1}|\omega > \bar{\omega}]} \tag{52}$$

Therefore the investor prefers idiosyncratic risk over diversification. Because of the limited liability, the investor enjoys profits in case of success, while he does not internalize the costs in case of default. Limited liability shifts investors' demand for risk.

C Log-Linearized Model

This section of appendix presents the log-linearized model including the extension of two-period assets, as it enters the simulations. The financial sector block is discussed in greater detail than other parts, which directly come from Bernanke *et al.* (1999). The lower-case letters denote log-deviations from steady state values. The ratios of capital letters denote the steady state values of the respective ratios. ϕ^i_t denote second order terms which do not affect the first order dynamics and are omitted from the simulations. Further, define nominal interest rate $r^n_{t+1} = r_{t+1} + E[\pi_{t+1}]$.

C.1 Firms

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t \tag{53}$$

$$k_{t+1} = \delta i_t + (1-\delta) k_t \tag{54}$$

$$r^k_{t+1} = (1-\epsilon)(y_{t+1} - k_{t+1} - x_{t+1}) + \epsilon q_{t+1} - q_t \tag{55}$$

$$q_t = \psi(i_t - k_t) \tag{56}$$

C.2 Retailers and households

$$\pi_t = E_{t-1}\left[\kappa(-x_t) + \beta \pi_{t+1}\right] \tag{57}$$

$$c_t = -r_{t+1} + E_t[c_{t+1}] \tag{58}$$

$$c^e_t = n_{t+1} + \psi^{c^e}_t \tag{59}$$

$$y_t - h_t - x_t - c_t = \eta^{-1} h_t \tag{60}$$

C.3 Financial intermediation

Although some of the log-linearized equations and variables could be eliminated in this block, the full set of equations is presented and the dynamics of all corresponding variables is shown as the financial sector is the key part of the general equilibrium model. In addition to the main text, we also introduce the shock to the (non-fundamental) asset price, υ_t, with a standard deviation of 0.1, corresponding to the empirical moment of log-deviation of quarterly Dow Jones index form its HP trend. The Ξ stands for the steady state value and ϖ for a log-deviation from a steady state of default threshold $\bar{\omega}$, while the $f(\Xi)$ is the the probability density function of idiosyncratic return distribution evaluated at steady-state default threshold.

$$z_{t+1} - r_{t+1} = \frac{1}{2}(1 - \mu f(\Xi))\Xi\varpi_{t+1}; \tag{61}$$

$$p_t = E\left[r^k_{t+1}\right] - z_{t+1} + f(\Xi)\Xi E\left[\varpi_{t+1}\right] + \upsilon_t; \tag{62}$$

$$p^f_t = p_t - (r^k_{t+1} - r_{t+1}) \tag{63}$$

$$\varpi_t + r^k_t + k_t = z_t + b_t \tag{64}$$

$$p_t + q_t + k_{t+1} = \frac{N}{QK}n_{t+1} + (1 - \frac{N}{QK})b_{t+1} \tag{65}$$

$$n_{t+1} = \frac{\gamma RK}{N}(r^k_t - r_t) + r_t + n_t \tag{66}$$

$$p_t = \iota(q_t + k_{t+1}) \tag{67}$$

Equation 61 is the bank participation constraint, linking the risk premium to the default threshold. Equation 62 is the log-linear version of the demand for risky assets, which together with the previous equations defines the wedge between risk-free rate and capital returns as a function of default threshold and asset prices. Examining the determinacy conditions of the model, it comes out that the term involving expectation of default threshold in equation 62 has to be lower than the right-hand side of equation 61 for the model to be determined, i.e. the default has to be a sufficiently tail event. Equation 63 defines the fundamental price, the equation 64 defines the default threshold and equation 65 defines borrowing. Equation 66 is a law of motion of investors' wealth. In the case where assets are held for multiple periods, this equation changes to

$$n_{t+1} = \frac{\gamma RK}{N}(r^k_t + p_t - r_t - pcost_{t-1}) + r_t + n_t \tag{68}$$

where the asset price dynamics enter the investors wealth. The cost of previously purchased assets $pcost_{t-1}$ equals p_{t-1} in the case of two-period assets, but can include longer lags of asset purchasing costs to create an arbitrary mix of asset maturities in the portfolio. Similarly to Bernanke *et al.* (1999), we assume that the composition of portfolios is stable over time and the shares of held assets (including different maturities) stays close to steady state values. In the plots presented in the main text, maturities from one quarter ahead to four quarters ahead are combined in respective portfolios with equal weights. Finally equation 67 describes the supply side of the investment asset market, linking its price to the produced quantity of investment opportunities via increasing marginal costs.

C.4 Monetary policy, resource constraint and shocks

$$r_t^n = \rho r_{t-1}^n + \psi \pi_{t-1} + \varepsilon_t^{r^n} \tag{69}$$

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c_t^e + \psi_t^y \tag{70}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{71}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{72}$$

$$\upsilon_t = \rho_p \upsilon_{t-1} + \varepsilon_t^p \tag{73}$$