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Abstract: In this paper we study the self-propelled motion of a single deformable body in a viscous compressible fluid which occupies whole 3-dimensional Euclidean space. The considered governing system for the fluid is the isentropic compressible Navier-Stokes equations. The main result of this paper is the existence of a weak solution on a time interval $(0, +\infty)$.

Keywords: Self-propelled motion, compressible fluid, deformable structure, unbounded domain

1 Introduction

This paper is devoted to a self-propelled motion of a body S in a viscous compressible fluid which fills out the whole space $\mathbb{R}^3$.

The problem of self-propelled motion or self-propulsion is a common means of locomotion of macroscopic objects. Typical examples are motions performed by birds, fishes, airplanes, rockets and submarines. In the microscopic world, many minute organism, like flagellates and ciliates move by self-propulsion, were studied by many authors.

A number of animals have evolved aerial locomotion, either by powered flight or by gliding. Flying and gliding animals have evolved separately many times, without any single ancestor. Flight has evolved at least four times, in the insects, pterosaurs, birds, and bats. Gliding has evolved on many more occasions. Usually the development is to aid canopy animals in getting from tree to tree, although there are other possibilities. Gliding, in particular, has evolved among rainforest animals, especially in the rainforests in Asia (most especially Borneo) where the trees are tall and widely spaced. Several species of aquatic animals, and a few amphibians have also evolved to acquire this gliding flight ability, typically as a means of evading predators.

Animal aerial locomotion can be divided into two categories powered and unpowered. In unpowered modes of locomotion, the animal uses on aerodynamics forces exerted on the body due to wind or falling through the air. In powered flight, the animal uses muscular power to generate aerodynamic forces. Animals using unpowered aerial locomotion cannot maintain altitude and speed due to unopposed drag, while animals using powered flight can maintain steady, level flight as long as their muscles are capable of doing so.

The understanding of swimming or flying is one of the main challenges in fluid dynamics. This problem has been considered by many scientists for a long time: for instance around 350 BC, Aristotle was already writing observations on fish and cephalod locomotion. Much later, during the 17th century, Borelli [3] started the study of swimming and flying by using mathematics to confirm his theories. In the 20th century, a zoologist, James Gray, introduced (see [25]) a paradox - Gray's paradox - suggesting than the undulating way of swimming of dolphin is much more efficient that a conventional propeller for underwater motion. Even if this paper is controversial, it has led to many studies in order to contradict or understand this paradox. Many other works

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were dedicated to the understanding of fish locomotion: Taylor [41], Lighthill [28], Childress [4], Sparenberg [40], etc.

The system composed by a swimming or flying creature can be considered as a fluid-structure system. In the recent years, many mathematical works have been published in the field of fluid–structure interaction problems, many of them tackling the well-posedness of the corresponding equations of motion. The main difficulties to obtain well-posedness of such systems are the non-linearity coming from the fluid equations (the Navier-Stokes or the Euler equations), the coupling between the equations of the fluid and the equations of the structure and the fact that the spatial domain of the fluid is moving and unknown. The last problem is simpler in the case of a rigid body for the structure since in that case, the motion of the structure is completely described by the rotation and the translation of the structure. In the case where the structure is deformable, for instance for an elastic structure, the existence of weak solutions could be very difficult to obtain: if the displacement of the structure is not regular, neither is the domain of the fluid. In [8] and [1], some approximated models are considered for the motion of an elastic structure in a viscous incompressible fluid. More precisely, the equations of the elasticity are modified in order to gain some regularity for the elastic deformation. Note that in the case of plate equations, it is possible to obtain the existence of weak solution without these approximations (see [23]). A vast majority of works concerns a rigid solid moving in a viscous incompressible Newtonian fluid whose behavior is described by the equations of Navier–Stokes (historically, the weak formulation of the problem of the motion of rigid bodies in viscous fluids has been introduced and studied in [44], and further in [7], [10], [9], [24], [26], [27], [35], [36], [43], [42] for existence of weak or strong solutions). Note that, in these cases, the displacement of the structure remains regular enough and that we have a parabolic-ODE coupling. Concerning an elastic structure evolving in incompressible flow, we can refer to [8], [5], [6] and [2] where the structure is described by a finite number of eigenmodes or to [31], [30] for an artificially damped elastic structure.

The problem of existence of a weak solution to the self-propelled motion in viscous fluid was studied by Starovoitov in [37]. In [33] authors provide an existence result of equation describing self-propelled motion of a body in an incompressible fluid with prescribed deformation of body. The problem of existence of the strong solution of self-propelled motion was studied by Galdi, Silvestre see [38, 39, 21, 22]. In [32] authors studied self-propelled motion in viscous compressible fluids in bounded domains.

Concerning the mathematical theory of compressible fluids the fundamental results on New-tonian case were obtained in the last two decades by P. L. Lions [29] (barotropic case with $p(\rho) = \rho^\gamma$) and by E. Feireisl et al. [19] (generalization to a larger class of exponents γ), E. Feireisl [14] and E. Feireisl, A. Novotný [17] (heat conductive fluids, singular limits). Based on the entropy inequality, the concept was further generalized to the notion of dissipative solutions and of the weak–strong uniqueness, see [16, 18].

In this paper we provide an existence result for a system describing self-propelled motion in an unbounded three dimensional domain. We use the same approach as in [32]. In our case we observe that in the case of unbounded domain we need that $\gamma \geq 3$ or $\rho_\infty = 0$ in case $\gamma \geq 3/2$.

1.1 Preliminaries

In a time interval $t \in (0, \infty)$ a body occupies a domain $\mathcal{S}_t \subset \mathbb{R}^3$. The body is surrounded by a viscous compressible fluid. We denote $\mathcal{F}_t = \mathbb{R}^3 \setminus \mathcal{S}_t$.

The motion of the body consists of three elements: a translation, a rotation and a smooth deformation $\mathcal{A}_t : \mathbb{R}^3 \mapsto \mathbb{R}^3$. Every point $\mathbf{x} \in \mathcal{S}_t$ can be expressed as

$$\mathbf{x} = \eta[t](\mathbf{y}) = \mathbf{a}(t) + Q(t)\mathcal{A}_t(\mathbf{y}), \text{ i.e. } \mathcal{S}_t = \eta[t]\mathcal{S}_0,$$

where $\mathbf{y} \in \mathcal{S}_0$, $\mathbf{a}$ stands for a position of a center of gravity, Q for a rotation and $\mathcal{S}_0$ is an initial position of the body. The velocity of the point $\mathbf{x}$ is

$$\mathbf{x}'(t) = \eta'[t](\eta^{-1}[t](\mathbf{x})) = \mathbf{a}'(t) + Q'(t)\mathcal{A}_t(\mathbf{y}) + Q(t)\partial_t\mathcal{A}_t(\mathbf{y}) =$$
$$= \mathbf{a}'(t) + \omega(t) \times (x - \mathbf{a}(t)) + \mathbf{w}(t, \mathbf{x}), \quad (1.1)$$

where $\mathbf{w}(t, \mathbf{x}) = Q(t)\partial_t \mathcal{A}_t(\mathcal{A}_t^{-1}(Q^*(t)(\mathbf{x} - \mathbf{a}(t))))$ and

$$\mathbb{S}(\omega) = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix} \qquad \mathbb{S}(\omega(t)) = Q'(t)Q^T(t).$$

We use overlined letters for quantities related to the body, which is considered without any rotation and translation, i.e. in a deformed configuration. Namely:

$$\begin{aligned} \overline{\mathcal{S}}_t &= \mathcal{A}_t(\mathcal{S}_0), \\ \overline{\mathbf{w}}(t, \overline{\mathbf{x}}) &= \partial_t \mathcal{A}_t(\mathcal{A}_t^{-1}(\overline{\mathbf{x}})) \quad \forall \overline{\mathbf{x}} \in \overline{\mathcal{S}}_t. \end{aligned}$$

There exists a smooth solenoidal function with a compact support $\overline{\Lambda}$ which coincides with $\overline{\mathbf{w}}$ on a set $\overline{\mathcal{S}}_t$, i.e. $\overline{\Lambda}\chi_{\overline{\mathcal{S}}_t} = \overline{\mathbf{w}}$. Further, we define

$$\Lambda = Q(t)\overline{\Lambda}(t, Q^*(t)(x - a(t)). \tag{1.2}$$

We denote the density of the body by $\rho_S := \rho_S(t, .) : \mathcal{S}_t \mapsto (0, \infty)$, resp. $\overline{\rho}_S := \overline{\rho}_S(t, .) : \overline{\mathcal{S}}_t \mapsto (0, \infty)$. The density is given by

$$\begin{aligned} \rho_S(t, \mathbf{x}) &= \frac{\rho_S(0, \mathcal{A}_t^{-1}(Q^*(t)(\mathbf{x} - \mathbf{a}(t))))}{\det(\nabla \mathcal{A}_t \mathcal{A}_t^{-1}(Q^*(t)(\mathbf{x} - \mathbf{a}(t))))}, \\ \text{resp.} \quad \overline{\rho}_S(t, \mathbf{x}) &= \frac{\rho_S(0, \mathcal{A}_t^{-1}(\overline{\mathbf{x}}))}{\det(\nabla \mathcal{A}_t(\mathcal{A}_t^{-1}(\overline{\mathbf{x}})))}. \end{aligned}$$

Assumption 1. *We assume, that the given deformation $\mathcal{A}$ satisfies the following three assumptions:*

- **smoothness** *For every $t \geq 0$ the mapping $\mathbf{y} \mapsto \mathcal{A}(t, \mathbf{y})$ is a smooth diffeomorphism from $\mathcal{R}^3$ onto $\mathcal{R}^3$. Moreover, the mapping $t \mapsto \mathcal{A}(t, \mathbf{y})$ is smooth for every $\mathbf{y} \in \mathbb{R}^3$.*

- **total volume of the body remains constant**

$$|\overline{\mathcal{S}}_t| = |\mathcal{S}_0|.$$

- **interior forces cannot change a center of gravity and an angular momenta**

$$\int_{\overline{\mathcal{S}}_t} \overline{\rho}_S(t, \overline{\mathbf{x}})\overline{\mathbf{w}}(t, \overline{\mathbf{x}})d\mathbf{x} = 0,$$

$$\int_{\overline{\mathcal{S}}_t} \overline{\rho}_S(t, \overline{\mathbf{x}}) \left[\overline{\mathbf{x}} \times \overline{\mathbf{w}}(t, \overline{\mathbf{x}})\right] d\mathbf{x} = 0.$$

For $(t, \mathbf{x}) \in (0, \infty) \times \mathbb{R}^3$ we set[1]

$$\begin{aligned} \mathbf{u}(t, \mathbf{x}) &= \chi_{\mathcal{F}_t}\mathbf{u}_{\mathcal{F}}(t, \mathbf{x}) + \chi_{S_t} \left(\partial_t \eta[t]\right) \left(\left(\eta[t]\right)^{-1}(\mathbf{x})\right), \\ \rho(t, \mathbf{x}) &= \chi_{\mathcal{F}_t}\rho_{\mathcal{F}}(t, \mathbf{x}) + \chi_{S_t}\rho_S(t, \mathbf{x}), \end{aligned}$$

where $\mathbf{u}_{\mathcal{F}}$, $\rho_{\mathcal{F}}$ is velocity resp. density of the surrounding fluid. We assume that the following equations hold: Balance of mass:

$$\partial_t \rho_{\mathcal{F}} + \operatorname{div}(\rho_{\mathcal{F}}\mathbf{u}_{\mathcal{F}}) = 0 \text{ on } \mathcal{F}_t. \tag{1.3}$$

Balance of linear momentum:

$$\partial_t(\rho_{\mathcal{F}}\mathbf{u}_{\mathcal{F}}) + \operatorname{div}(\rho_{\mathcal{F}}\mathbf{u}_{\mathcal{F}} \otimes \mathbf{u}_{\mathcal{F}}) + \nabla p = \operatorname{div} T(\mathbf{u}) \text{ on } \mathcal{F}_t. \tag{1.4}$$

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[1]By χ_M we denote the characteristic function of a set M.

The stress tensor T is given via

$$T(\mathbf{u}) := 2\mu \mathbf{Du} + \lambda I \operatorname{div} \mathbf{u}, \tag{1.5}$$

where $2\mathbf{D} = \nabla + \nabla^T$ is a symmetric part of the stress tensor, $\mu \in (0, \infty)$, $\lambda \in \mathbb{R}$ and $\mu + \lambda \geq 0$, μ and λ are constant coefficients of viscosity. A pressure p is given by

$$p = \alpha \rho_{\mathcal{F}}^{\gamma}, \quad \alpha > 0, \tag{1.6}$$

with $\gamma \in \mathbb{R}$ restricted below. We consider the following boundary conditions

$$\begin{aligned}
\mathbf{u}_{\mathcal{F}}(t, \mathbf{x}) &= 0, \quad \mathbf{x} \in \partial \Omega, \\
\mathbf{u}_{\mathcal{F}}(t, \mathbf{x}) &= \mathbf{a}'(t) + \omega(t) \times (\mathbf{x} - \mathbf{a}(t)) + \mathbf{w}(t, \mathbf{x}) = \mathbf{u}_{\mathcal{S}}, \mathbf{x} \in \partial \mathcal{S}_t.
\end{aligned} \tag{1.7}$$

Since the motion $\overline{\mathcal{A}}_t$ is prescribed, we have to introduce equations for unknowns $a(t)$ and $\omega(t)$, which describe the movement of the body. Before we write down the equations, we set

$$\begin{aligned}
M &:= \int_{S_t} \rho_S, \\
J(t) &:= \int_{S_t} \rho_S(t, \mathbf{x}) \left(|\mathbf{x} - \mathbf{a}(t)|^2 - (\mathbf{x} - \mathbf{a}(t)) \otimes (\mathbf{x} - \mathbf{a}(t)) \right) d\mathbf{x}.
\end{aligned}$$

Finally, the functions $\mathbf{a}(t)$, $\omega(t)$ should satisfy

$$\begin{aligned}
M\mathbf{a}''(t) &= -\int_{\partial S_t} (T - pI)\, \mathbf{n}, \\
(J\omega)'(t) &= -\int_{\partial S_t} (\mathbf{x} - \mathbf{a}(t)) \times (T - pI)\mathbf{n}d\Gamma.
\end{aligned} \tag{1.8}$$

The initial state is described through

$$\begin{aligned}
\mathbf{a}(0) &= 0, & Q(0) &= I, & \mathcal{A}_0 &= Id, & \rho_S(0) &= \rho_{S0}, \\
\mathbf{a}'(0) &= \mathbf{a}_0, & \omega(0) &= \omega_0, & \rho_{\mathcal{F}}(0) &= \rho_{\mathcal{F}0}, & \rho(0)\mathbf{u}(0) &= \mathbf{m}_0.
\end{aligned} \tag{1.9}$$

For abbreviation, $\rho_0 = \chi_{\mathcal{F}_0} \rho_{\mathcal{F}0} + \chi_{\mathcal{S}_0} \rho_{S0}$. We also assume that

$$\mathbf{m}_0 = 0 \text{ a.e. on the set } \{\mathbf{x} \in \Omega, \rho_0(\mathbf{x}) = 0\}, \quad \frac{|\mathbf{m}_0|^2}{\rho_0} \in L^1(\Omega),$$

and that there exist $c_1, c_2 \in (0, \infty)$ such that

$$\rho_0 \chi_{\mathcal{S}_0} \in [c_1, c_2].$$

We define

$$\begin{aligned}
\mathcal{H}_\sigma(\mathcal{S}_t) &= \{\mathbf{v} \in L^2_{\text{loc}} \, ; \quad \mathbf{Dv} = 0 \quad \text{in } \mathcal{S}_t\}, \\
\mathcal{K}_\sigma(\mathcal{S}_t) &= \mathcal{H}_\sigma(\mathcal{S}_t) \cap D_0^{1,2}(\Omega).
\end{aligned}$$

We set

$$\rho(t, \mathbf{x}) = \begin{cases} \rho_{\mathcal{F}}(t, \mathbf{x}) & \text{if } \mathbf{x} \in \mathcal{F}_t, \\ \rho_{\mathcal{S}}(t, \mathbf{x}) & \text{if } \mathbf{x} \in \mathcal{S}_t, \end{cases} \quad \mathbf{u}(t, \mathbf{x}) = \begin{cases} \mathbf{u}(t, \mathbf{x}) & \text{if } \mathbf{x} \in \mathcal{F}_t, \\ \mathbf{a}'(t) + \omega(t) \times (\mathbf{x} - \mathbf{a}(t)) + \mathbf{w}(t, \mathbf{x}) & \text{if } \mathbf{x} \in \mathcal{S_t}. \end{cases}$$

For $R > 0$ we denote by B_R an open ball with center at 0 and radius R. We define a norm $\|\cdot\|_{L_2^\gamma(B_R)}$ as

$$\|f\|_{L_2^\gamma(B_R)} = \|\chi_{|f| \leq 1} f\|_{2, B_R} + \|\chi_{|f| > 1} f\|_{\gamma, B_R}$$

and an Orlicz space L_2^γ as a space of all measurable functions $f : B_R \mapsto \mathbb{R}$ with $\|f\|_{L_2^\gamma(B_R)} < \infty$. From (1.1) it follows that $\eta[t]$ is defined by the following ODE

$$\partial_t \eta[t](\mathbf{x}) = \mathbf{u}_R(t, \eta[t](\mathbf{x})), \quad \eta[0](\mathbf{x}) = \mathbf{x}, \quad \forall \mathbf{x} \in \mathcal{S}_0.$$

Definition 2. *Let $T > 0$, $\Omega \subset \mathbb{R}^3$ and $\rho_\infty > 0$. We say that a pair $(\rho, \mathbf{u}) \in L^\infty((0,T), L^\gamma) \times (L^2((0,T), W^{1,2})) \cap L^\infty((0,T), L^2))$ (resp. $(\rho - \rho_\infty, \mathbf{u}) \in L^\infty((0,T), L_2^\gamma(\Omega) \times L^2((0,T), D^{1,2}(\Omega))$ in case Ω is an unbounded domain) is a weak solution to (1.3) - (1.9) on an interval $(0,T)$ if*

- *$\rho \geq 0$;*

- *$u - \mathbf{\Lambda} \in L^2((0,T), \mathcal{K}_\sigma(\mathcal{S}_t))$;*

- *A renormalized equation of the continuity equation holds in a weak sense, i.e.*

$$\partial_t b(\rho) + \operatorname{div}(b(\rho)\mathbf{u}) + (b'(\rho)\rho - b(\rho))\operatorname{div}\mathbf{u} = 0 \text{ in } \mathcal{D}'((0,T) \times \Omega), \qquad (1.10)$$

  where $b \in C^1(\mathbb{R})$;

- *Balance of linear momentum holds in a weak sense, i.e.*

$$\int_0^T \int_\Omega (\rho\mathbf{u})\partial_t\varphi + [\rho\mathbf{u} \otimes \mathbf{u}] : \mathbf{D}\varphi + p\operatorname{div}\varphi d\mathbf{x}dt =$$
$$\int_0^T \int_\Omega T(\mathbf{u}) : \mathbf{D}\varphi d\mathbf{x}dt + \int_\Omega \mathbf{m_0}\varphi(0,.)d\mathbf{x}, \quad \forall\varphi \in \mathcal{R}(S_t), \quad (1.11)$$

  where

$$\mathcal{R}(S_t) = \{\varphi \in C_0^\infty([0,T) \times \Omega), \mathbf{D}\varphi(x) = 0 \text{ on an open neighborhood of } S_t\};$$

- *The energy inequality*

$$\frac{1}{2}\int_\Omega \rho(\tau)|\mathbf{u}(\tau)|^2 + \frac{\alpha}{\gamma-1}\rho^\gamma(\tau)d\mathbf{x} + \int_0^\tau \int_\Omega 2\mu|\mathbf{D}\mathbf{u}|^2 + \lambda(\operatorname{div}\mathbf{u})^2 d\mathbf{x}dt \leq C(\rho(0), \mathbf{u}(0), \mathcal{A})$$

  (resp.

$$\int_\Omega \frac{\rho(\tau)|\mathbf{u}(\tau)|^2}{2} + \frac{\alpha}{\gamma-1}\rho^\gamma(\tau) + \alpha\rho_\infty^\gamma - \frac{\alpha\gamma}{\gamma-1}\rho_\infty^{\gamma-1}\rho(\tau)d\mathbf{x} + \int_0^\tau \int_\Omega 2\mu|\mathbf{D}(\mathbf{u})|^2 + \lambda(\operatorname{div}\mathbf{u})^2 d\mathbf{x}dt$$
$$\leq C(\rho(0), \mathbf{u}(0), \mathcal{A})$$

  in case of an unbounded domain) holds for a.e. $\tau \in [0,T]$;

- *The movement of the body $\mathcal{S}$ is compatible with $\mathbf{u}$ in following sense*

$$\mathbf{u}_\mathcal{F}(t,.) - \mathbf{u}_S(t,.) \text{ belongs locally to the space } W_0^{1,2}(\Omega \setminus S_t).$$

We will introduce lemma and theorem which we will use in the proof of the main theorem.

Lemma 3 (Invading domains – Lemma 6.6 in [34]). *Let $\{f_n\}$, $f_n \in L^p(I, L_{\operatorname{loc}}^q(\mathbb{R}^n))$ $(1 < p,q \leq \infty, N \geq 1)$, be a sequence such that $\|f_n\|_{L^p(I,L^q(B_M))} \leq K(M)$, $M = M_0, M_0 + 1, \ldots$. Then there exists a subsequence $\{n'\} \subset \{n\}$ such that $f_{n'} \to f$ weakly star in $L^p(I, L^q(B_R))$ for every $R > 0$.*

Theorem 4. *Let $0 \in \mathbb{R}^3$ be a center of gravity of a body $\mathcal{S}$. Let R be sufficiently large and $\Omega = B_R$. Then there exists a time $T_R \in (0,\infty)$ such that there exists a weak solution $(\rho_R, \mathbf{u}_R)$ to (1.3) – (1.9) (with $\rho_{R_0} = \rho_0|_{B_R}$ and $m_{R_0} = m_0|_{B_R}$) on a time interval $(0, T_R)$.*

Proof. See [32]. $\qquad\square$

1.2 Main result

Theorem 5. *Let $\mathcal{S}_0 \subset \mathbb{R}^3$ be a bounded connected open set, $\gamma > \frac{3}{2}$, $\rho_\infty \geq 0$, ρ_0 be such that $\rho_0 - \rho_\infty \in L_2^\gamma(\mathbb{R}^3)$, $\rho_0|_{\mathcal{S}_0} \in [C_1, C_2]$ for some $C_1, C_2 \in (0, \infty)$ and $\mathbf{a}(0) = 0$ where $\mathbf{a}$ is a center of gravity of $\mathcal{S}_0$ (i.e. $a(0) = \frac{1}{M} \int_{\mathcal{S}_0} \rho_0 \mathbf{x} d\mathbf{x}$). Further, let $\mathbf{m}_0 : \mathbb{R}^3 \mapsto \mathbb{R}^3$ fulfills*

$$\mathbf{m}_0|\{\rho_0 = 0\} = 0, \quad \frac{\mathbf{m}_0}{\rho_0} \in L^1(\mathbb{R}^3).$$

Let, moreover, one of these two conditions holds:

- *$\rho_\infty = 0$, or*

- *$\gamma \geq 3$.*

Then for an arbitrary $T \in (0, \infty)$ there exists a weak solution $(\rho, \mathbf{u})$ to (1.3)–(1.9) on a time interval $(0, T)$.

Remark 6. *In order to prove the main theorem, we use an approach presented in [34]. Since the existence of a weak solution in a case of a bounded domain has been proven yet – see [32], we consider weak solutions in a domain B_R. In section 2 we tend with R to infinity and we proceed to a limit with all necessary quantities.*

1.3 Energy inequality

In what follows we derive the energy inequality for a system on a bounded domain $\Omega = B_R$. Since we want to proceed with R to infinity, we would like to derive estimates independent of R. In order to do, we multiply (formally) (1.11) by $\mathbf{u} - \Lambda$. We get

$$\int_\Omega \left(\frac{1}{2} \rho |\mathbf{u}|^2 + \underbrace{\frac{\alpha}{\gamma - 1} \rho^\gamma + \alpha \rho_\infty^\gamma - \frac{\alpha \gamma}{\gamma - 1} \rho_\infty^{\gamma - 1} \rho}_{P^{\rho_\infty}(\rho)} \right) (t) + \int_{Q_t} \left(\mu |\mathbf{Du}|^2 + \lambda |\operatorname{div} \mathbf{u}|^2 \right) \leq$$

$$\leq \int_\Omega \left(\frac{1}{2} \rho |\mathbf{u}|^2 + P^{\rho_\infty}(\rho) \right) (0) + \int_{Q_t} \partial_t(\rho \mathbf{u}) \Lambda - \int_{Q_t} (\rho \mathbf{u} \otimes \mathbf{u}) \mathbf{D} \Lambda$$

$$- \int_{Q_t} \alpha \rho^\gamma \operatorname{div} \Lambda + \int_{Q_t} \mu \mathbf{DuD} \Lambda + \int_{Q_t} \lambda \operatorname{div} \mathbf{u} \operatorname{div} \Lambda =: \sum_{i=1}^6 I_i.$$

Terms I_1, I_3, I_4, I_5 and I_6 can be handled in a well known way, see c.f. [34, Section 7]. Let us point out that $\|\Lambda\|_{L^\infty(W^{1,\infty})} \leq C$ (see [33, Lemma 5]). We also emphasize that $\|\partial_t(\Lambda)\|_{L^2(L^\infty)} \leq C(1 + \|u\|_{L^2(L^2(\operatorname{supp} \Lambda))})$ and $|\operatorname{supp} \Lambda| \leq C$. We write s_Λ instead of $\operatorname{supp} \Lambda$ in further calculations. We have

$$\int_0^t \int_\Omega \partial_t(\rho \mathbf{u}) \Lambda = \int_{s_\Lambda} \rho(t) \mathbf{u}(t) \Lambda(t) - \int_{s_\Lambda} \rho(0) \mathbf{u}(0) \Lambda(0) - \int_0^t \int_{s_\Lambda} \rho \mathbf{u} \partial_t \Lambda$$

$$\leq \int_{s_\Lambda} \sqrt{\rho(t)} \sqrt{\rho(t)} |\mathbf{u}(t)| \Lambda(t) + C + \int_0^t \|\partial_t \Lambda\|_\infty \int_{s_\Lambda} \rho |\mathbf{u}|$$

$$\leq \varepsilon \int_\Omega \rho(t)^\gamma + \varepsilon \int_\Omega \rho(t) |\mathbf{u}(t)|^2 + C_\varepsilon + \varepsilon \|\Lambda\|_{2,\infty} + C_\varepsilon \int_0^t \left(\int_{s_\Lambda} \rho |\mathbf{u}| \right)^2.$$

Under assumptions of Theorem 5, two possibilities may occur.
Either $\rho_\infty = 0$. In this case there exists a constant C independent of Ω such that $\int_\Omega \rho \leq C$. Thus it holds that

$$\int_0^t \left(\int_{s_\Lambda} \rho |\mathbf{u}| \right)^2 \leq \int_0^t \left(\int_\Omega \sqrt{\rho} \sqrt{\rho |\mathbf{u}|^2} \right)^2 \leq \int_0^t \int_\Omega \rho \int_\Omega \rho |\mathbf{u}|^2 \leq C \int_0^t \int_\Omega \rho |\mathbf{u}|^2.$$

Or $\gamma > 3$. In this case we may continue, using Young inequality and Sobolev embedding, as

$$\int_0^t \left(\int_{s_\Lambda} \rho |\mathbf{u}| \right)^2 \leq \int_0^t \int_{s_\Lambda} \rho^2 |\mathbf{u}|^2 \leq C_\varepsilon \int_0^t \int_{s_\Lambda} \rho^3 + \varepsilon \int_0^t \int_{s_\Lambda} |\mathbf{u}|^6 \leq C \int_0^t \int_\Omega P^{\rho\infty}(\rho) + \varepsilon \int_0^t \int_{s_\Lambda} |D\mathbf{u}|^2.$$

In both cases we get[2]

$$\int_\Omega \left(\frac{1}{2} \rho |\mathbf{u}|^2 + P^{\rho\infty}(\rho) \right)(t) + \int_{Q_t} \left(\mu |\mathbf{D}\mathbf{u}|^2 + \lambda |\operatorname{div}\mathbf{u}|^2 \right)$$

$$\leq C \left(1 + \int_0^t \int_\Omega \mathcal{P}^{\rho\infty}(\rho) + \int_0^t \int_\Omega \rho |u|^2 \right),$$

where the constant on the right hand side does not depend on a R.
According to the Gronwall inequality we get

$$\int_\Omega \left(\frac{1}{2} \rho |\mathbf{u}|^2 + P^{\rho\infty}(\rho) \right)(t) \leq C e^{t \max\{2, \frac{\alpha}{\gamma-1}\}},$$

and, consequently,

$$\int_\Omega \left(\frac{1}{2} \rho |\mathbf{u}|^2 + P^{\rho\infty}(\rho) \right)(t) + \int_{Q_t} \left(\mu |\mathbf{D}\mathbf{u}|^2 + \lambda |\operatorname{div}\mathbf{u}|^2 \right) \leq C(\Omega, t, \Lambda, \text{ initial conditions}). \quad (1.12)$$

Further, since $a'(t) = M^{-1} \int_{\mathcal{S}_t} \rho \mathbf{u}$, we get

$$\|a'(t)\|_\infty = M^{-1} \left\| \int_{\mathcal{S}_t} \rho \mathbf{u} \right\|_\infty \leq C \left\| \left(\int_{\mathcal{S}_t} \rho \right)^{1/2} \left(\int_{\mathcal{S}_t} \rho |u|^2 \right)^{1/2} \right\|_\infty \leq C, \quad (1.13)$$

with C independent of R.

2   Proof of the main theorem

Let $(\rho_R, \mathbf{u}_R)$ be a weak solution to a system (1.3)–(1.9) on B_R emanating from initial conditions $\rho_{0R} = \rho_0|_{B_R}$ and $\mathbf{u}_{0R} = \mathbf{u}_0|_{B_R}$ and let $\mathcal{S}_{Rt}$ be a position of a body related to solution $\{\rho_R, \mathbf{u}_R\}$ in a time instant t. From Theorem 4 we know that the solution $(\mathbf{u}_R, \rho_R)$ exists till $\mathcal{S}_t \subset\subset \Omega$. Further, (1.13) yields

$$\sup(|\mathbf{x}|, \mathbf{x} \in \mathcal{S}_t) \leq \mathbf{a}(t) + \operatorname{diam}\overline{\mathcal{S}}_t \leq \int_0^t \mathbf{a}'(s)ds + \operatorname{diam}\overline{\mathcal{S}}_t \leq Ct + \operatorname{diam}\overline{\mathcal{S}}_t.$$

Consequently, $T_R \to 0$ as $R \to \infty$. Thus, for an arbitrary $T > 0$ we may find R_0 such that $T_R > T$ for all $R > R_0$ where T_R comes from Theorem 4.

From (1.12) we derive

$$\left\| \rho_R |\mathbf{u}_R|^2 \right\|_{L^\infty((0,T), L^1(B_R))} \leq C,$$
$$\left\| \nabla \mathbf{u}_R \right\|_{L^2((0,T), L^2(B_R))} \leq C,$$

and

$$\left\| \rho_R - \rho_\infty \right\|_{L^\infty((0,T), L_2^\gamma(B_R))} \leq C,$$

where all constants are independent of R.
We recall that $\mathbf{u}_R|_{\mathcal{S}_{Rt}} = \mathbf{a}'_R(t) + \omega_R(t)(\mathbf{x} - \mathbf{a}_R(t)) + \Lambda_R$. Therefore there exists a ball B with finite diameter such that $S_{Rt} \subset B$ for every R and $t \in (0, T)$ We consider a cut-off function $\xi \in C^\infty$ such

---

[2]Every possible ε is chosen such that relevant terms are absorbed to the left hand side.

that $\xi|_{2B} = 1$ and $\operatorname{supp}\xi$ is compact. Then functions $(\mathbf{a}'_R(t) + \omega_R(t)(\mathbf{x} - \mathbf{a}_R(t)) + \Lambda_R)\,\xi$ satisfy assumptions of Lemma 5.1 in [13], we get

$$S_{Rt} \xrightarrow{b} S_t \text{ as } R \to \infty \text{ uniformly in } t,$$

$$(\mathbf{a}'_R(t) + \omega_R(t)(\mathbf{x} - \mathbf{a}_R(t)) + \Lambda_R)\xi \to \mathbf{v} \text{ weakly star in } L^2(W^{1,\infty}),$$

where $S_t = \eta[t]S_0$ and η is given by

$$\partial_t\eta[t](\mathbf{x}) = \mathbf{v}(t, \eta[t](\mathbf{x})), \; \eta[0](\mathbf{x}) = \mathbf{x}.$$

Let $I \subset (0,T)$ be a compact interval and $K \subset B_R$ be a compact set such that

$$I \times K \cap \{(t,x) \in I \times B_R, x \in \mathcal{S}_t\} = \emptyset.$$

We define

$$\varphi(t) = \psi(t)\, \underbrace{B\left[b_k(\rho_R(t))\kappa - \fint_K b_k(\rho_R(t))\kappa\right]}_{=:\Phi}, \quad \psi \in \mathcal{D}(I),$$

where B stands for Bogovskii operator and $\kappa : B_R \mapsto \mathbb{R}$ is a smooth cut-off function with $\kappa\chi_K = 1$ and $\operatorname{supp}\kappa \cap \{(t,x) \in I \times B_R, x \in \mathcal{S}_t\} = \emptyset$. Using φ as a test function in (1.11) we derive (having in mind $p = \alpha\rho^\gamma$)

$$\int_0^T \int_Q \alpha\rho_R^\gamma b_k(\rho_R)\kappa = \int_0^T \int_Q \alpha\rho_R^\gamma \fint_Q b_k(\rho_R)\kappa - \lambda \int_0^T \int_Q \operatorname{div} u_R \operatorname{div} \Phi\psi$$

$$- \mu \int_0^T \int_Q \nabla u_R \nabla \Phi\psi - \int_0^T \int_Q \rho_R u_R \psi \partial_t \Phi - \int_0^T \int_Q \rho_R u_R \psi' \Phi$$

$$- \int_0^T \int_Q \psi \rho_R u_R \otimes u_R \nabla \Phi$$

$$=: \sum_{j=1}^6 I_j.$$

Due to already known estimates, we have

$$|J_1| \leq \alpha \int_0^T \int_Q \rho_R^\gamma \fint_Q b_k(\rho_R)\kappa) \leq c\|b_k(\rho_R)\kappa\|_{L^1L^1},$$

and

$$|J_2| + |J_3| \leq c\|\psi\|_\infty \|b_k(\rho_R)\kappa\|_{L^2L^2}.$$

Further,

$$|J_4| \leq \int_0^T \psi\|\mathbf{u}_R\|_6\|\rho_R\|_\gamma\|\partial_t\Phi\|_{\frac{6\gamma}{5\gamma-6}}.$$

Generalized continuity equation yields

$$\partial_t\Phi = B\left[\partial_t b_k(\rho_R)\kappa - \fint_Q \partial_t b_k(\rho_R)\kappa\right] =$$

$$B\left[((b'_k(\rho_R)\rho_R - b_k(\rho_R))\operatorname{div} \mathbf{u})\,\kappa - \fint_Q ((b'_k(\rho_R)\rho_R - b_k(\rho_R))\operatorname{div} \mathbf{u})\,\kappa\right]$$

$$- B\left[\operatorname{div}(b_k(\rho_R)\mathbf{u})\kappa - \fint_Q \operatorname{div}(b_k(\rho_R)\mathbf{u})\kappa\right] =: J_{41} - J_{42}.$$

According to the Stokes theorem

$$J_{42} = B\left[\operatorname{div}(\kappa b_k(\rho_R)\mathbf{u}) - \nabla\kappa b_k(\rho_R)\mathbf{u} + \fint_Q \nabla\kappa b_k(\rho_R)\mathbf{u}\right] =$$

$$B\left[\operatorname{div}(\kappa b_k(\rho_R)\mathbf{u})\right] - B\left[\nabla\kappa b_k(\rho_R)\mathbf{u} - \fint_Q \nabla\kappa b_k(\rho_R)\mathbf{u}\right].$$

Due to properties of the Bogovski operator (see cf. Chapter 3 in [20]), we get

$$|J_4| \le c\|\psi\|_\infty \left(\|(\rho_R(b_k)'(\rho_R) - b_k(\rho_R))\operatorname{div}\mathbf{u}_R\|_{L^2 L^{\underline{p}}} + \|b_k(\rho_R)\mathbf{u}_R\|_{L^2 L^{\underline{p}}}\right),$$

with $p = \frac{6\gamma}{5\gamma - 6}$, $\underline{p} = \frac{3p}{p+3}$ and $\gamma > \frac{6}{5}$. Thus

$$|J_5| \le c\|\psi'\|_1 \|b_k(\rho_R)\kappa\|_{L^\infty L^{\frac{6\gamma}{5\gamma-3}}},$$

$$|J_6| \le c\|b_k(\rho_R)\kappa\|_{L^\infty L^{\frac{3\gamma}{2\gamma-3}}}.$$

Let us set $b_k = \begin{cases} s^\theta & \text{if } s \in [0, k) \\ k^\theta & \text{if } s \in [k, \infty) \end{cases}$. This yields

$$|s(b_k)'_+(s) - b_k(s)| = \begin{cases} |1-\theta|s^\theta & \text{if } s \in [0, k) \\ k^\theta & \text{if } s \in [k, \infty) \end{cases} \le \max\{1, |1-\theta|\}s^\theta.$$

Let $\psi = \psi_m \in \mathcal{D}(I)$ be such that $\psi_m\chi_{[\frac{1}{m}, 1-\frac{1}{m}]} = 1$, $0 \le \psi_m \le 1$. It follows that $\|\psi\|_\infty = 1$ and $\|\psi'\|_1 \le c$. Due to a priori bounds, we get for $\theta \le \min\{\frac{\gamma}{2}, \frac{2}{3}\gamma - 1\}$

$$|J_1| + |J_2| + |J_3| + |J_6| \le c.$$

Further, for $\theta \le \frac{5}{6}\gamma - \frac{1}{2}$ we have

$$|J_5| \le c,$$

and, with all above restrictions in mind, we also get

$$|J_4| \le c.$$

Tending with k and m to infinity, we get

$$\int_{K \times I} \rho_R^{s(\gamma)} \le C(\rho_0, \mathbf{m}_0, K, I), \quad s(\gamma) = \frac{5\gamma - 3}{3}. \tag{2.1}$$

We may proceed to a limit. From the previous considerations and from Lemma 3 it follows that there exists functions $\rho \in L^\infty(L^\gamma_{\text{loc}}(\mathbb{R}^3))$, $\mathbf{u} \in L^2(D^{1,2}(\mathbb{R}^3))$ and $\overline{\rho^\gamma} \in L^\infty(L^{s(\gamma)}_{\text{loc}}((0,T) \times \mathcal{F}_t))$ such that

$$\mathbf{u}_R \rightharpoonup \mathbf{u} \text{ in } L^2((0,T), L^6(\mathbb{R}^3)),$$

$$\nabla\mathbf{u}_R \rightharpoonup \nabla\mathbf{u} \text{ in } L^2((0,T), L^2(\mathbb{R}^3)),$$

$$(\rho_R - \rho_\infty) \overset{*}{\rightharpoonup} (\rho - \rho_\infty) \text{ in } L^\infty((0,T), L^\gamma_2(\mathbb{R}^3)),$$

$$\rho_R \rightharpoonup \rho \text{ in } L^{s(\gamma)}((0,T) \times \mathcal{F}_t))),$$

$$\rho_R^\gamma \rightharpoonup \overline{\rho^\gamma} \text{ in } L^{s(\gamma)/\gamma}((0,T) \times \mathcal{F}_t))).$$

Since $\|\rho_R u_R\|_{L^\infty((0,T), L^{\frac{2\gamma}{\gamma+1}})}$ is bounded , equation (1.10) yields the uniform continuity of ρ_R in $W^{-1, \frac{2\gamma}{\gamma+1}}(\mathbb{R}^3)$. Consequently $\rho_R \to \rho$ in $C^0((0,T), L^\gamma_{\text{weak}}(\mathbb{R}^3))$ and, using [34, Lemma 6.4] we get

$$\rho_R \to \rho \text{ in } L^p((0,T), W^{-1,2}(\mathbb{R}^3)).$$

Further,

$$\rho_R \mathbf{u}_R \overset{*}{\rightharpoonup} \rho \mathbf{u} \text{ in } L^\infty(L^{\frac{2\gamma}{\gamma+1}}),$$

$$\rho_R \mathbf{u}_R \rightharpoonup \rho \mathbf{u} \text{ in } L^2(L^{\frac{6\gamma}{\gamma+6}}).$$

Let I and K be chosen as before. From equation (1.11) we get uniform continuity of $\rho_R u_R$ on interval I in space $W^{-1,s}(K)$ where $s = \min\left\{\frac{6\gamma}{4\gamma+3}, \frac{s(\gamma)}{\gamma}\right\}$. Using the same arguments as before, we derive that

$$\rho_R \mathbf{u}_R \to \rho \mathbf{u} \text{ in } C^0(I, L^{\frac{2\gamma}{\gamma+1}}(K)),$$

and

$$\rho_R \mathbf{u}_R \to \rho \mathbf{u} \text{ in } L^p(I, W^{-1,2}(K)),$$

$$\rho_R \mathbf{u}_R \otimes \mathbf{u} \rightharpoonup \rho \mathbf{u} \otimes \mathbf{u} \text{ in } L^2(I, L^{\frac{6\gamma}{4\gamma+3}}(K)).$$

We are going to prove that $\overline{\rho^\gamma} = \rho^\gamma$. First of all, the renormalized continuity equation (1.10) holds also for a limit function. This can be proved by the same method as in [12], Section 7.
We introduce a sequence of functions

$$L_k(z) = \begin{cases} z\log(z), & 0 \le z \le k \\ z\log(k) + z - k, & z > k. \end{cases}$$

Further, $L_k'(z)z - L_k(z) = T_k(z) = \min\{z, k\}$. Thus

$$\partial_t L_k(\rho_R) + \operatorname{div}(L_k(\rho_R)\mathbf{u}_R) + T_k(\rho_R)\operatorname{div}\mathbf{u}_R = 0,$$

and

$$\partial_t L_k(\rho) + \operatorname{div}(L_k(\rho)\mathbf{u}) + T_k(\rho)\operatorname{div}\mathbf{u} = 0.$$

Subtracting these inequalities and passing to the limit, we get, for $\Phi \in \mathcal{D}(\mathbb{R}^3)$

$$\int_{\mathbb{R}^3}\left(\overline{L_k(\rho(t))} - L_k(\rho(t))\right)\Phi = \int_0^t\int_{\mathbb{R}^3}\left(\overline{L_k(\rho)} - L_k(\rho)\right)\mathbf{u}\nabla\Phi + \int_0^t\int_{\mathbb{R}^3}\left(T_k(\rho)\operatorname{div}\mathbf{u} - \overline{T_k(\rho)\operatorname{div}\mathbf{u}}\right)\Phi.$$

Consequently, we use $\Phi_m \in \mathcal{D}(\mathbb{R}^3)$, $\Phi_m \nearrow 1$ as a test function and we get

$$\int_{\mathbb{R}^3}\overline{L_k(\rho(t))} - L_k(\rho(t)) = \int_0^t\int_{\mathbb{R}^3}T_k(\rho)\operatorname{div}\mathbf{u} - \overline{T_k(\rho)\operatorname{div}\mathbf{u}}. \qquad (2.2)$$

All assumptions of Proposition 4.3 in [11] are satisfied due to (2.1) and

$$\lim_{n\to\infty}\int_0^T\int_{\mathbb{R}^3}(p(\rho_R) - (\lambda + 2\mu)\operatorname{div}\mathbf{u}_R)T_k(\rho_R)\varphi = \int_0^T\int_{\mathbb{R}^3}\left(\overline{p(\rho)} - (\lambda + 2\mu)\operatorname{div}\mathbf{u}\right)\overline{T_k(\rho)}\varphi.$$

Using the same considerations as in section 5.3 in [11] one may derive

$$\overline{T_k(\rho)\operatorname{div}u} \ge \overline{T_k(\rho)}\operatorname{div}u.$$

Finally, the right hand side of (2.2) can be estimated as

$$\int_0^t\int_{\mathbb{R}^3}T_k(\rho)\operatorname{div}\mathbf{u} - \overline{T_k(\rho)\operatorname{div}\mathbf{u}} \le \int_0^t\int_{\mathbb{R}^3}\left(T_k(\rho) - \overline{T_k(\rho)}\right)\operatorname{div}\mathbf{u}.$$

We tend with k to infinity. The right hand side of (2.2) tends to zero (see cf. section 9 in [13]) and thus

$$\overline{\rho\log(\rho)(t)} = \rho\log(\rho)(t) \quad \text{for all } t \in [0, T].$$

Using Theorem 10.20. from [15] we get

$$\rho_R \to \rho \text{ in } L^1([0,T] \times \mathbb{R}^3).$$

By interpolation and (2.1), we get $\rho_R \to \rho$ in $L^\alpha_{\mathrm{loc}}(Q^f)$ for all $1 \le \alpha < s(\gamma)$. As a consequence $\overline{\rho^\gamma} = \rho^\gamma$.

We are in position to show that $\mathbf{u}|_{\mathcal{S}_t} = \mathbf{a}' + \omega(t)(\mathbf{x} - a(t)) + \Lambda(= v|_{\mathcal{S}_t})$ where Λ is defined as in (1.2). Since $\mathbf{a}'_R, \omega_R \in L^\infty$ uniformly in R, we get

$$\mathbf{a}'_R(t) \stackrel{*}{\rightharpoonup} \mathbf{a}(t) \text{ in } L^\infty(0,T),$$
$$\omega'_R(t) \stackrel{*}{\rightharpoonup} \omega(t) \text{ in } L^\infty(0,T),$$
$$\mathbf{a}_R(t) \to \mathbf{a}(t) \text{ pointwisely in } (0,T).$$

Further, from

$$
\begin{aligned}
|Q_R(t) - Q(t)| &= \left| \int_0^t \mathbb{S}(\omega_R(s))Q_R(s) - \mathbb{S}(\omega(s))Q(s)ds \right| \\
&= \left| \int_0^t \left(\mathbb{S}(\omega_R(s)) - \mathbb{S}(\omega(s)) \right) Q(s)ds + \int_0^t \mathbb{S}(\omega_R(s))(Q_R(s) - Q(s))ds \right| \\
&\le \left| \int_0^t \left(\mathbb{S}(\omega_R(s)) - \mathbb{S}(\omega(s)) \right) Q(s)ds \right| + C \int_0^t |Q_R(s) - Q(s)|ds.
\end{aligned}
$$

We derive, using Gronwall lemma, $Q_R(t) \to Q(t)$ pointwisely in $(0,T)$. Since Λ is continuous, we may conclude $\Lambda_R \to \Lambda$ pointwisely in $(0,T)$.

The energy inequality comes easily from already known estimates.

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