DETERMINATION OF THE FUNDAMENTAL ROUGH WALL TURBULENT BOUNDARY LAYER CHARACTERISTICS

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Abstract

In the paper they are described: the evaluation of the shift of mean velocity profile zero level, the friction velocity, the wake function and the roughness function from the mean velocity profile in a rough wall turbulent boundary layer.

The no-slip condition cannot be applied in the layer on a rough surface because the roughness grains attached to the ground plane generate wakes that push out the mean flow velocity zero level from the ground plane. Thus in addition to standard boundary layer characteristic, we must determine the shift of the origin of the velocity profile into the layer of roughness elements $\tilde{\varepsilon}$. In the presented investigations, the plane of the roughness elements crests was defined as the ground plane, $y' = 0$.

Wall shear stress $\tau_w(x)$ [Pa] was evaluated from mean velocity profiles $U(x,y)$ either from the slope interpolated very near the surface or from the interpolation of the wake-law. The estimate of the velocity derivative on wall works quite satisfactory in pseudo-laminar layer and in boundary layer during early transition. The estimated error of calculated τ_w is between two and three percent (better accuracy with a smooth surface). The error estimate increases with the viscous sub-layer thinning.

Further unknowns join the previous one after finishing the transition process, in turbulent boundary layer on a rough surface. Namely the shift of the mean velocity profile Δu^+ (the roughness function) in the overlap region and the wake function $\omega(y/\delta)$ with the strength of the wake, Π (Coles,1956) must be determined when the surface is rough. The effect of roughness appears in the formulae for the dimensionless mean velocity (normalized by the friction velocity u_τ)

$$u^+ = \frac{1}{\kappa}\ln\left(y^+\right) + B - \Delta u^+ + \frac{2\Pi}{\kappa}\omega(\eta) \qquad (1)$$

where the dimensionless distances from the wall are y^+ and η and following relations are introduced

$$y^+ = \frac{y\,u_\tau}{\nu};\ \eta = \frac{y}{\delta};\quad y = y' + y_1 + \varepsilon;\quad \delta = y'_\delta + y_1 + \varepsilon;\quad u_\tau = \sqrt{\frac{\tau_w}{\rho}} \qquad (2)$$

The boundary layer thickness δ is measured in the distance from the wall where $U(x,\delta) = 0.99U_e$, the real normal distance from the surface of the velocity zero level is y, the traverse reading and the dead travel denote y' and y_1. Next the kinematics viscosity is ν and u_τ denotes the friction velocity.

Representation of the mean velocity profile in terms of the Velocity Defect Law is more suitable for fitting the measurements than the Log Law

$$u_e^+ - u^+ = \frac{2\Pi}{\kappa}\left[\omega(1) - \omega(\eta)\right] - \frac{1}{\kappa}\ln(\eta) \tag{3}$$

Certain experimental evidence indicates departures from the Coles wake function in the boundary conditions characterized: by a smooth wall and external turbulent flow and by a rough wall and low turbulence external flow. It was observed that using the Coles wake function and the Hama (1954) approximation (proposed farther from the wall, e.g. Rotta , 1962) the estimates of the friction velocity were higher than calculations from the momentum balance or by the estimate of the velocity profile derivative on the wall. Bradshaw (1987) attributed this effect to the underestimated value of the strength of the wake implied in Hama's approximation. Krogstad et al. (1992) analysed this problem in detail and concluded that a formulation allowing optimise also the strength of the wake Π in the wake function is necessary. Numerous authors recommend the use of the wake function

$$\omega(\eta) = \frac{\eta^2}{2\Pi}\left[(1+6\Pi) - (1+4\Pi)\eta\right] \tag{4}$$

Introducing formula (4) into the equation (3), we obtain after some formal adaptations

$$\frac{U(\eta)}{U_e} = 1 + \frac{1}{\kappa}\frac{u_\tau}{U_e}\left[-2\Pi + \ln\eta + (1+6\Pi)\eta^2 - (1+4\Pi)\eta^3\right] \tag{5}$$

This is a non-linear equation in three unknowns u_τ, ε and Π . Let us apply the method of least squares for the computation of the statistical estimates of the unknowns. We introduce new functions for the sake of the notation transparency. The value calculated from experiment is

$$f_i = \left(U(\eta_i)/U_e\right) \tag{6}$$

and the value calculated after the formula (5) is

$$\frac{U(\eta)}{U_e} = F_i = F(\eta_i, u_\tau, \varepsilon, \Pi) \tag{7}$$

Then we derive the sum of the squared deviations

$$S = \sum_{i=1}^{m}(f_i - F_i)^2 = \sum_{i=1}^{m}\left(f_i^2 - 2f_iF_i + F_i^2\right) \tag{8}$$

The least squares fit is found by the estimates of values u_τ, ε and Π that make zero the partial derivatives

$$\partial S/\partial u_\tau = \partial S/\partial\varepsilon = \partial S/\partial\Pi = 0 \tag{9}$$

Let us specify these equations

$$\frac{\partial S}{\partial u_\tau} = 2\sum_{i=1}^{m}\left[(F_i - f_i)\frac{\partial F_i}{\partial u_\tau}\right] = 0 \tag{10}$$

$$\frac{\partial S}{\partial\varepsilon} = 2\sum_{i=1}^{m}\left[(F_i - f_i)\frac{\partial F_i}{\partial\varepsilon}\right] = 0 \tag{11}$$

$$\frac{\partial S}{\partial\Pi} = 2\sum_{i=1}^{m}\left[(F_i - f_i)\frac{\partial F_i}{\partial\Pi}\right] = 0 \tag{12}$$

These three equations have to valid in the outer layer (y^+ > 50 or y/δ > 0.1)

$$0.1 \le \eta_i \le 1; \quad \eta_i = \frac{y_i}{y_\delta} \cong \frac{y_i'}{y_\delta'} + \frac{y_\delta' - y_i'}{y_\delta'^2}\left(\varepsilon + y_1'\right) \tag{13}$$

The partial derivatives of the function F_i with respect to the unknowns are given by formulas

$$\frac{\partial F_i}{\partial u_\tau} = \frac{1}{\kappa U_e}\left[\ln \eta_i - 2\Pi + \left(1 + 6\Pi\right)\eta_i^2 - \left(1 + 4\Pi\right)\eta_i^3\right] \tag{14}$$

$$\frac{\partial F_i}{\partial \varepsilon} = \frac{1}{\kappa}\frac{u_\tau}{U_e}\frac{\left(y_\delta - y_i\right)}{y_\delta^2}\left[\frac{1}{\eta_i} + 2\left(1 + 6\Pi\right)\eta_i - 3\left(1 + 4\Pi\right)\eta_i^2\right] \tag{15}$$

$$\frac{\partial F_i}{\partial \Pi} = \frac{1}{\kappa}\frac{u_\tau}{U_e}\left[-2 + 6\eta_i^2 - 4\eta_i^3\right] \tag{16}$$

The solution must fulfil some limiting conditions following from the physics of the problem and from the mode of measurement. Cathetometer is pointed at the upper edge of the flattened Pitot probe with the dimension 0.18 mm in the y-direction and the dead travel of the traverse is less than 0.2 mm. Thus the sum of the dead travel and the shift of the velocity zero level must meet the inequalities

$$0.2\ \mathrm{mm} \le \varepsilon + y_1 \le \left(0.2 + \mathrm{s}\right)\ \mathrm{mm} \tag{17}$$

where s is the height of the roughness grains.

The velocity derivative $\left(dU/dy\right)_w$ on the surface, calculated from the estimate of the friction velocity must be less than the slope of the straight line fitted through [y = 0, U = 0] and [y_1, U_1]

$$\frac{U_1}{y_1} \ge \left(\frac{dU}{dy}\right)_w = \frac{\rho u_\tau^2}{\mu} \quad \square \quad u^+ \le y^+ \tag{18}$$

The roughness function is non-negative

$$\Delta u^+ \ge 0 \tag{19}$$

An example of the solution is shown in following figures. We consider the best fit of experiment as the minimum of the sum

$$\left|\frac{\partial S}{\partial u_\tau}\right| + \left|\frac{\partial S}{\partial \varepsilon}\right| + \left|\frac{\partial S}{\partial \Pi}\right| \equiv \mathrm{TEST} \tag{20}$$

and/or as the minimum of the average of relative perturbations

$$\sqrt{\frac{1}{m}\sum_1^m\left(\frac{F_i - f_i}{f_i}\right)^2} \equiv \pm\sigma \tag{21}$$

and

$$\left(\frac{\partial S}{\partial u_\tau}\right)\left(\frac{\partial S}{\partial \varepsilon}\right)\left(\frac{\partial S}{\partial \Pi}\right) \equiv \mathrm{Product} \tag{22}$$

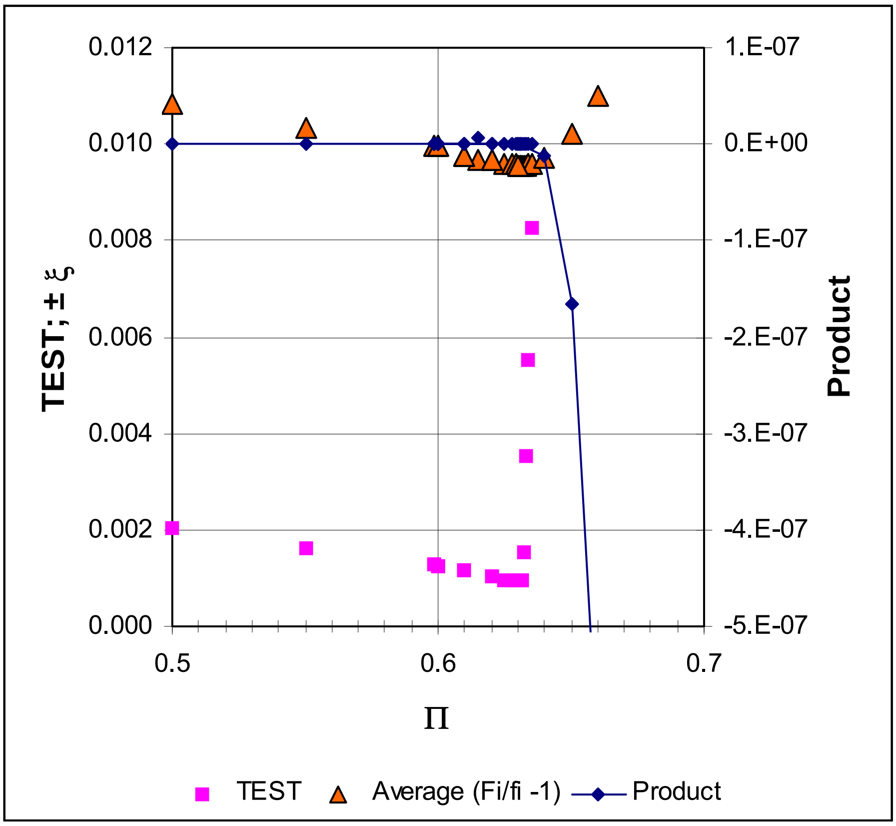

Figure 1: Distributions of the criteria (20), (21) and (22) on the chosen strength of the wake, Π.

The problem solution is found after the compliance of the criteria presented above. This is shown in the Figures 2, 3 and 4.

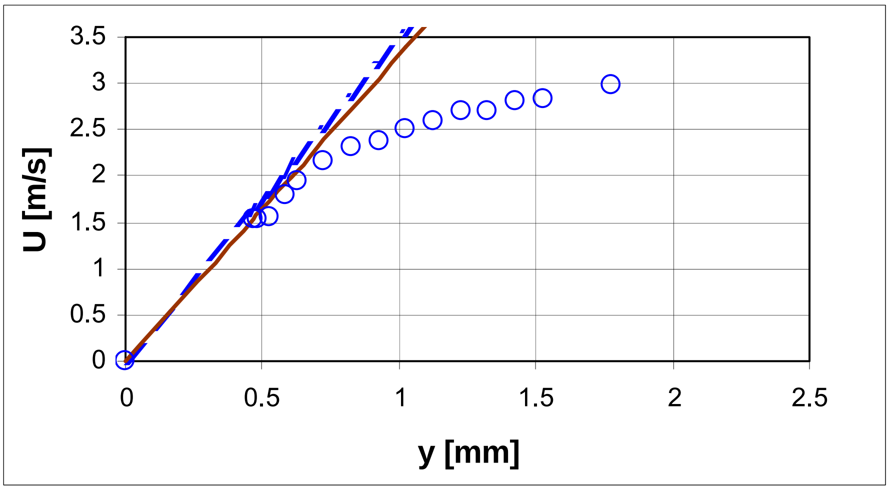

Figure 2: The requirement (18).

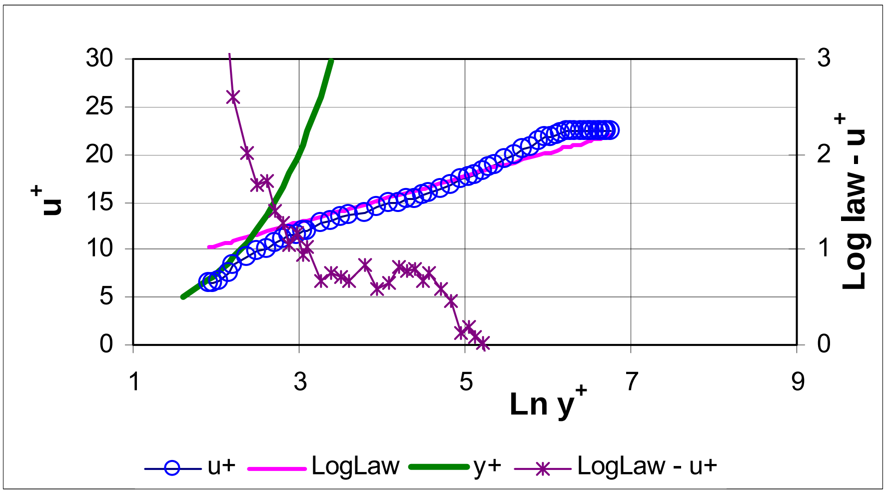

Figure 3: Log law and the evaluation of the roughness function.

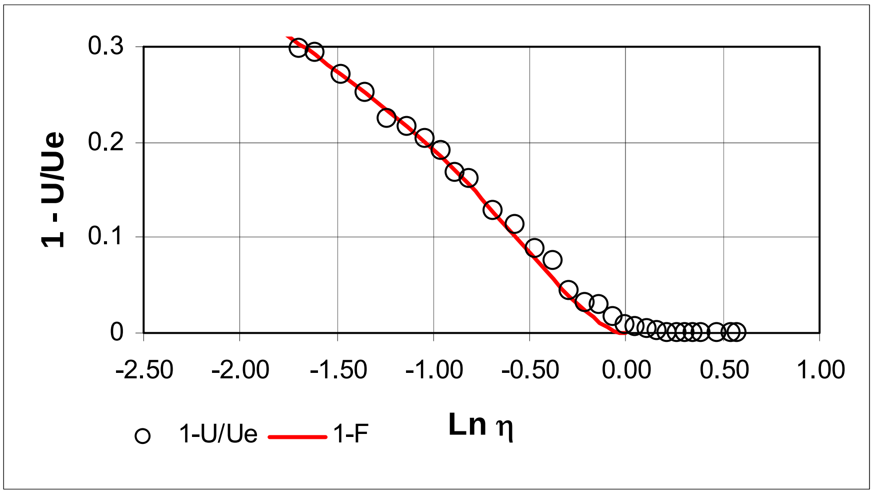

Figure 4: The interpolation of the decay law.

CONCLUSION

The procedure of the evaluation of the shift of mean velocity profile zero level, the friction velocity, the wake function and the roughness function from the mean velocity profile in a rough wall turbulent boundary layer is described in the paper.

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