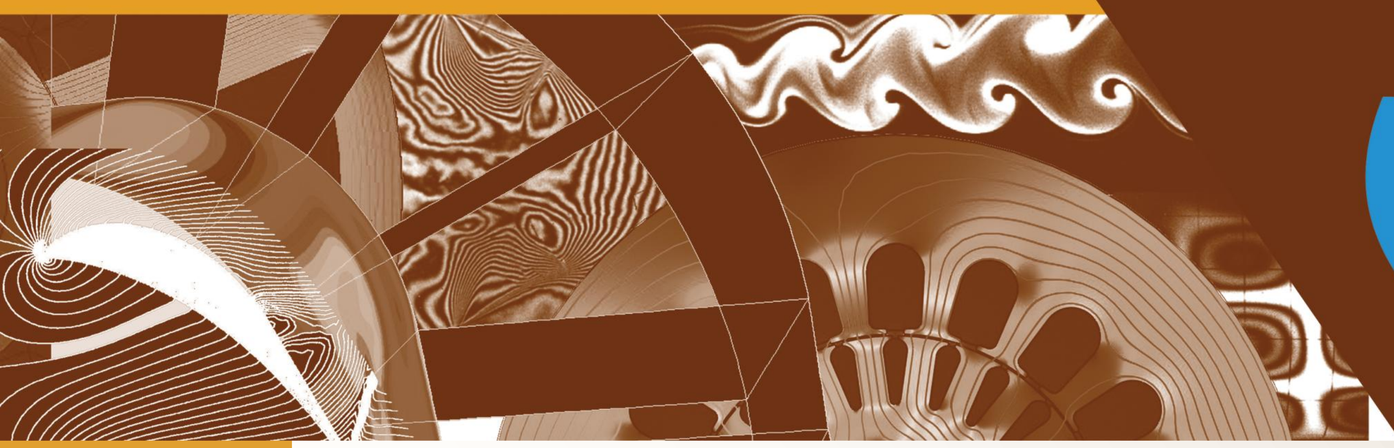

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EUROMECH Colloquim 540 – Advanced Modelling of Wave Propagation in Solids

COMPUTATION OF THE DISTANCE BETWEEN A POINT AND A QUADRATIC SURFACE WITH APPLICATION IN CONTACT MECHANICS

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Summary

In contact problems, local searching requires the calculation of the closest point projection of a contactor point onto a given master segment. Since the analytical solution does not exist for a general curved segment, an iterative numerical procedure must be adopted. To this end, several methods for the solution of non-linear algebraic systems are tested: the Newton-Raphson method, the least square projection, the steepest descent method, the Broyden method, the BFGS method, and the simplex method. The effectiveness of these methods is tested by means of a proposed benchmark problem which involves the bending of two rectangular plates over a cylinder. It is concluded that it is the simplex method that significantly increases the robustness of the local contact search procedure and, consequently, even improves the effectiveness of the global solution.

Formulation

Let us consider a slave point $\mathbf{x}_s$. A master point $\mathbf{x}_m$ satisfying

$$\mathbf{x}_m(\xi) = \arg\min_{\mathbf{x}\in\Gamma} \|\mathbf{x}(\xi) - \mathbf{x}_s\|$$

is the closest projection of the slave point $\mathbf{x}_s$ onto the surface Γ.

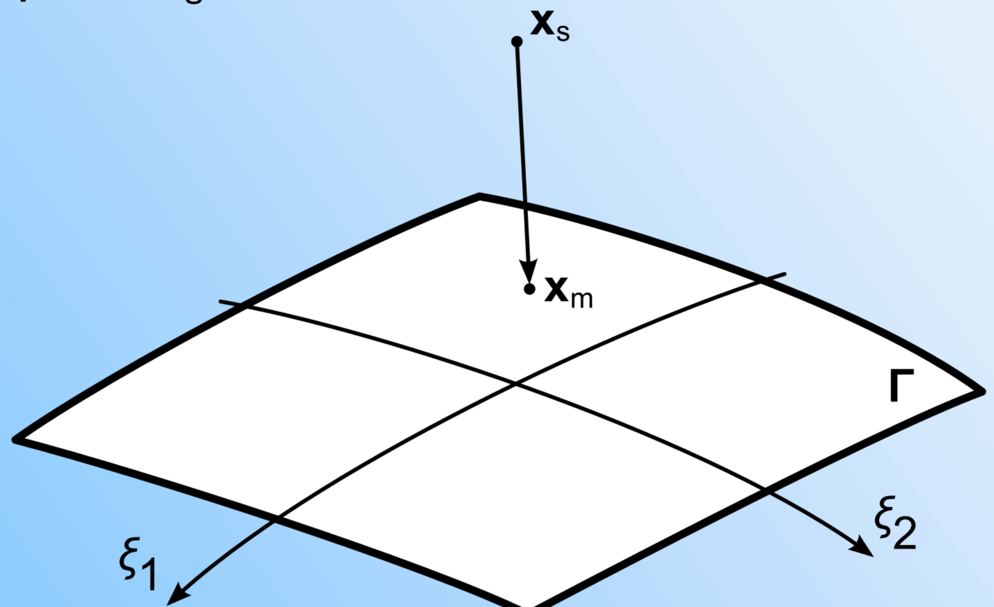

Fig. 1. Closest point projection.

The parametric coordinates of such a point may be obtained via the minimization of the squared distance function defined as

$$f(\xi) := \frac{1}{2}[\mathbf{x}(\xi) - \mathbf{x}_s] \cdot [\mathbf{x}(\xi) - \mathbf{x}_s]$$

At the stationary point the gradient of the squared distance function must vanish, thus

$$\nabla f(\xi) = \nabla\mathbf{x}(\xi)[\mathbf{x}(\xi) - \mathbf{x}_s] = \mathbf{0}$$

Tested methods

Various numerical methods of unconstrained optimization [3] are employed for the solution of the local contact searching problem:

- The Newton-Raphson method
- The least square projection method
- The methods of steepest descent
- The Broyden method
- The BFGS method
- The simplex method

Numerical example

The effectiveness of different methods is tested by means of the numerical example, originally introduced in [1], which involves bending of two elastic rectangular plates over an elastic cylinder (see Fig. 2). Since the applied loading caused a significant deflection of the plate, the effect of large displacements and rotations in total Lagrangian formulation is considered.

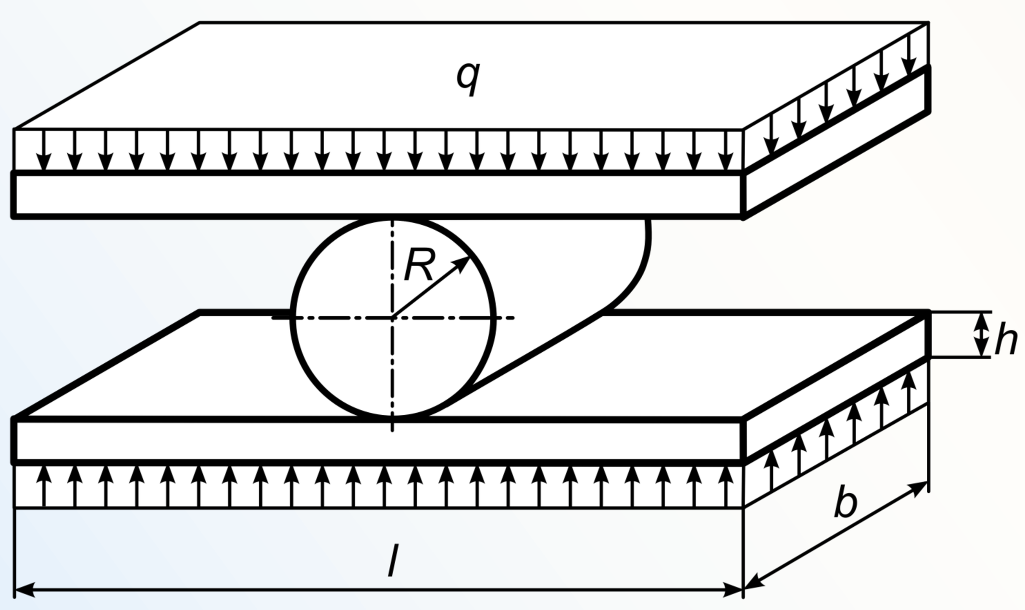

Fig. 2. Bending of two rectangular plates over a cylinder. R = 0.4 m, l = 2 m, b = 0.6 m, h = 0.08 m, Young's modulus E = 2.1e5 MPa, Poisson's ratio ν = 0.36, surface traction q = 22.5 MPa

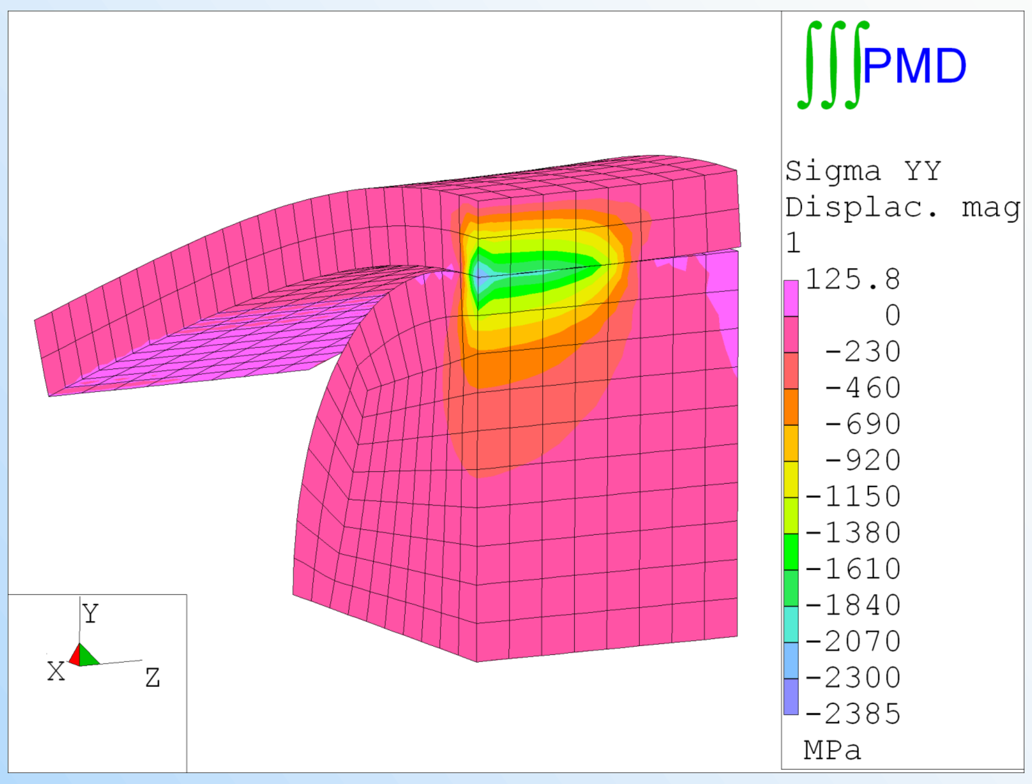

Fig. 3. Distribution of σ_{yy}-stress colour contours in deformed configuration of the model.

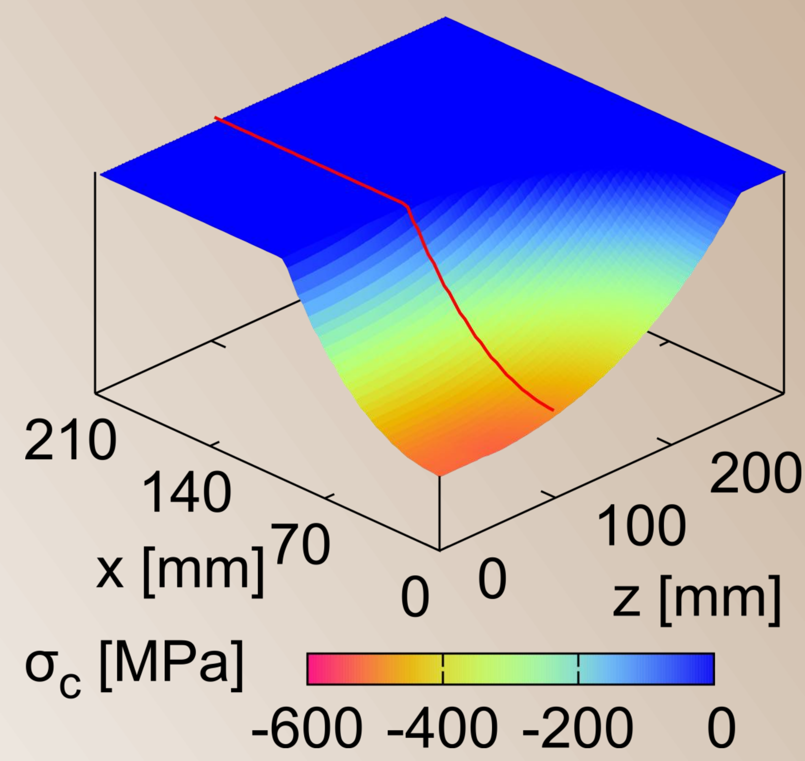

Fig. 4. Contact pressure distribution on the quarter of the plate for the penalty parameter ε = 10^{11} [N/m^3].

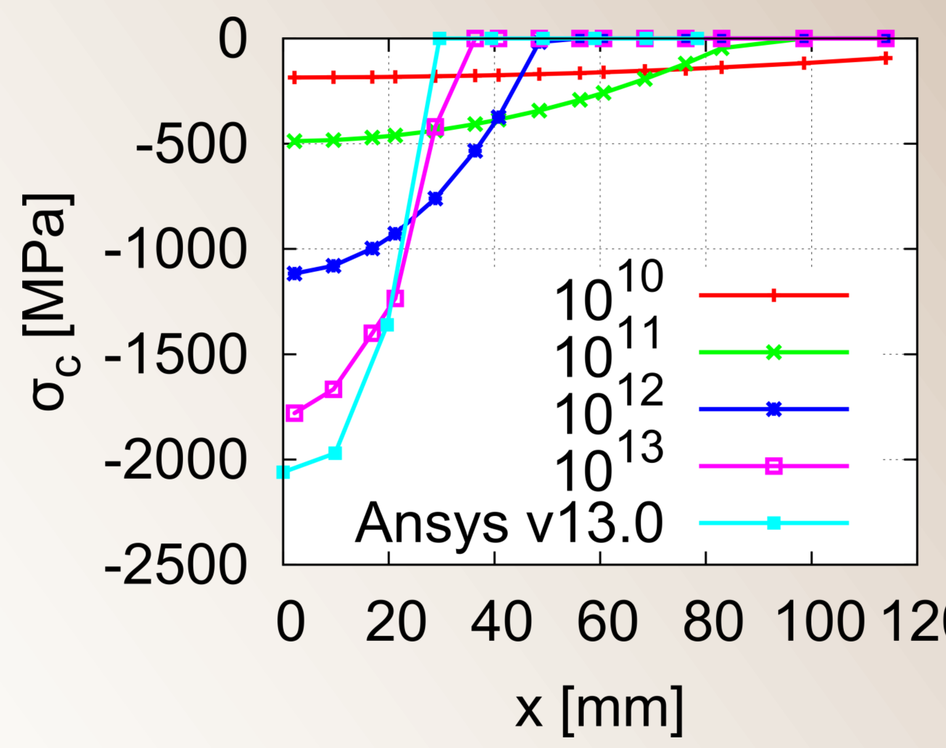

Fig. 5 Contact pressure distribution on the plate for increasing the penalty value (for a section drawn in Fig. 4 by the red curve).

Acknowledgment

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