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Interaction of a channel flow and moving bodies

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Introduction

The subject of this paper is the numerical simulation of the interaction of two-dimensional incompressible viscous flow through a channel (wind tunnel) and a vibrating airfoil. A solid airfoil with two degrees of freedom, which can rotate around the elastic axis and oscillate in the vertical direction, is considered. The numerical simulation consists of the finite element solution of the Navier-Stokes equations coupled with the system of ordinary differential equations describing the airfoil motion. We discuss the discretization of the problem and present some computational results.

Formulation of the problem

The two-dimensional non-stationary flow of viscous, incompressible fluid is considered in the time interval $[0, T]$, where $T > 0$. The symbol Ω_t denotes the computational domain occupied by the fluid at time t. The flow is characterized by the velocity field $\boldsymbol{u} = \boldsymbol{u}(x, t)$, and the kinematic pressure $p = p(x, t)$, for $x \in \Omega_t$ and $t \in [0, T]$. Further, $\alpha(t)$ and $h(t)$ denote the rotation angle and displacement of the airfoil.

The fluid flow is described by the Navier-Stokes system written in the ALE (Arbitrary Lagrangian–Eulerian) form (see, e.g. [1])

$$\frac{D^A}{Dt}\,\boldsymbol{u} + [(\boldsymbol{u} - \boldsymbol{w}) \cdot \nabla]\,\boldsymbol{u} + \nabla p - \nu \Delta \boldsymbol{u} = 0, \quad \operatorname{div} \boldsymbol{u} = 0 \quad \text{in } \Omega_t,$$

where $\nu > 0$ denotes the kinematic viscosity of the fluid, $\frac{D^A}{Dt}$ is the ALE derivative and $\boldsymbol{w}$ is the ALE velocity. (See, e.g. [1].)

The equations for the moving profile were derived from the Lagrange equations for the generalized coordinates h and α ([1]). In the matrix calculus these equations have the form

$$\widehat{\boldsymbol{K}}\boldsymbol{d}(t) + \widehat{\boldsymbol{B}}\dot{\boldsymbol{d}}(t) + \widehat{\boldsymbol{M}}\ddot{\boldsymbol{d}}(t) = \widehat{\boldsymbol{f}}(t), \tag{1}$$

where the stiffness matrix $\widehat{\boldsymbol{K}}$, the viscous damping $\widehat{\boldsymbol{B}}$ and the mass matrix $\widehat{\boldsymbol{M}}$ have the form

$$\widehat{\boldsymbol{K}} = \begin{pmatrix} k_{hh} & k_{h\alpha} \\ k_{\alpha h} & k_{\alpha\alpha} \end{pmatrix}, \quad \widehat{\boldsymbol{B}} = \begin{pmatrix} D_{hh} & D_{h\alpha} \\ D_{\alpha h} & D_{\alpha\alpha} \end{pmatrix}, \quad \widehat{\boldsymbol{M}} = \begin{pmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{pmatrix}$$

and the vector of the force $\widehat{\boldsymbol{f}}$ and the generalized coordinates $\boldsymbol{d}$ read

$$\widehat{\boldsymbol{f}}(t) = \begin{pmatrix} -L_2(t) \\ \mathcal{M}(t) \end{pmatrix}, \quad \boldsymbol{d} = \begin{pmatrix} h(t) \\ \alpha(t) \end{pmatrix}.$$

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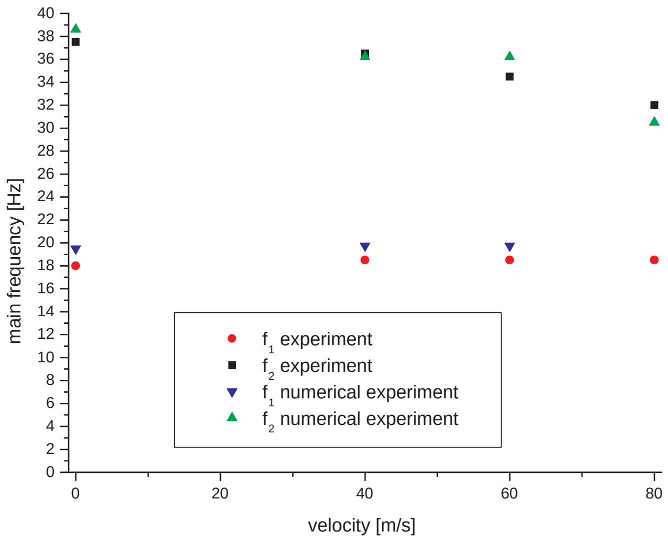

Figure 1: The main frequencies of the signals

The symbol L_2 stands for the component of the force acting on the profile in the vertical direction, $\mathcal{M}$ is the torsional moment of the force.

The above flow and structural equations are equipped by suitable initial and boundary conditions.

The time discretization of flow equations if carried out by a second order backward difference formula and the space discretization uses the finite element method stabilized by the SUPG and grad-div technique. The nonlinear discrete system is solved by the Oseen iterations. The structural system is solved numerically by the Runge-Kutta method.

Results

As a result we obtain the pressure and velocity fields and also the position of the moving profile. From this information we derive the frequency characteristics. The computational results carried out on the basis of data from [2] are compared with the experiment described in [3]. Figure 1 shows a good agreement of computational and experimental results.

References

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