Nonlinear multiscale dynamics of the atmosphere and climate:

Cross-frequency interactions in the air temperature

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Abstract

- Interactions between dynamics on different temporal scales in about a century long records of daily mean surface air temperature from various European locations have been detected using conditional mutual information together with the Fourier-transform and multifractal surrogate data methods.
- Information transfer from larger to smaller scales has been observed as the influence of the phase of slow oscillatory phenomena with periods around 6–11 years on amplitudes of the variability characterized by smaller temporal scales from a few months to 4–5 years.
- The overall effect of the slow oscillations on the inter-annual temperature changes within the range 1–2°C has been observed in large areas of Europe.

Analytic signal approach for multiscale dynamics

- Natural complex systems exhibit oscillations and fluctuations on a wide range of time scales.
- Let us analyze interactions across temporal scales by quantifying dependence among instantaneous amplitudes and phases of oscillatory components related to different time scales.
- For an arbitrary time series $s(t)$ the **analytic signal** $\psi(t)$ is a complex function of time defined as

$$\psi(t) = s(t) + i\hat{s}(t) = A(t)e^{i\phi(t)}. \quad (1)$$

- The instantaneous phase $\phi(t)$ of the signal $s(t)$ is then

$$\phi(t) = \arctan\frac{\hat{s}(t)}{s(t)}. \quad (2)$$

- The instantaneous amplitude is

$$A(t) = \sqrt{s(t)^2 + \hat{s}(t)^2}. \quad (3)$$

- In this study a continuous complex wavelet transform (CCWT) is applied directly to experimental time series $s(t)$. At each scale (frequency) the complex wavelet coefficients can directly be used in Eq. (2) and (3) for the estimation of the phase $\phi(t)$ and the amplitude $A(t)$, respectively.

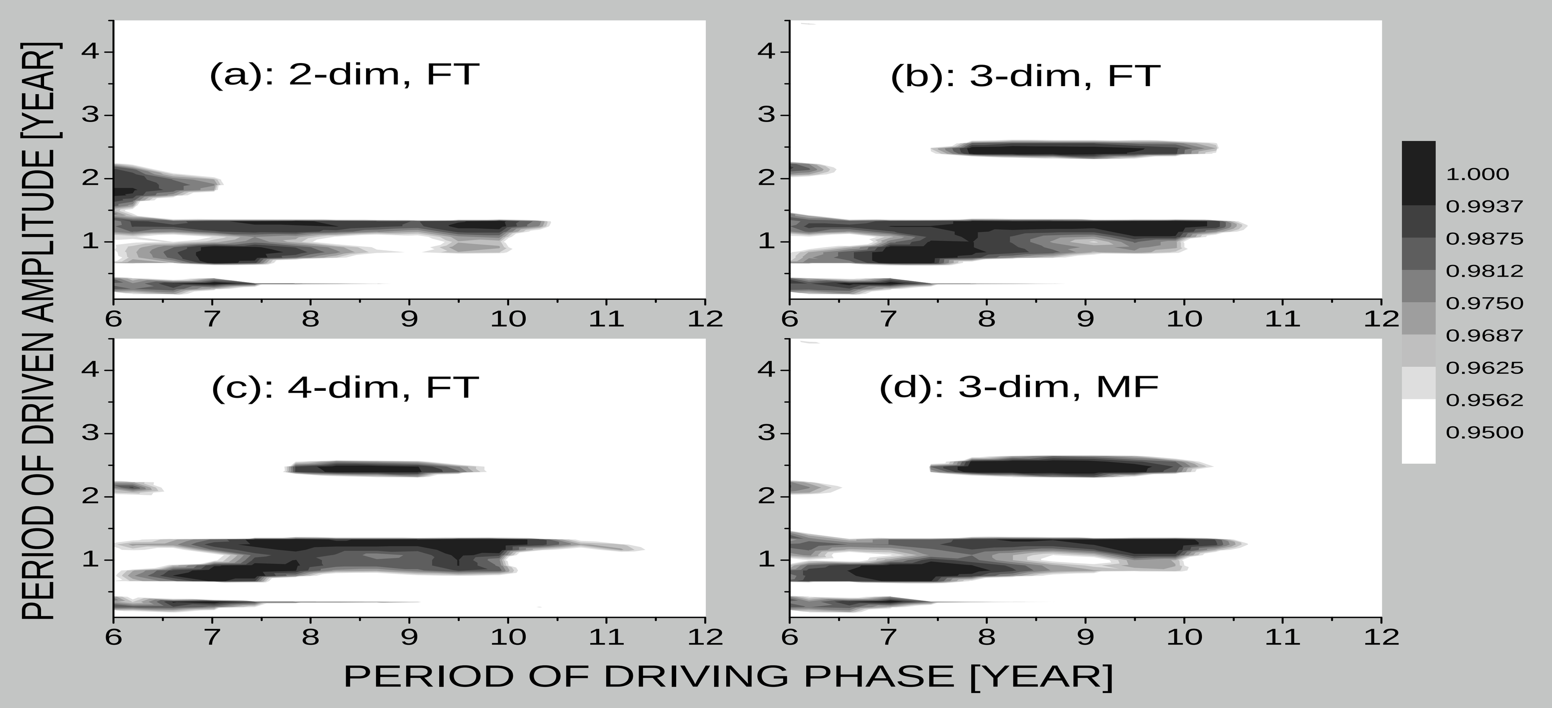
Directional interactions

- Information about the future X_τ of a process $\{X\}$, shifted τ time units forward, contained in the process $\{Y\}$ can be measured by the conditional mutual information (CMI) $I(Y; X_\tau|X)$, also known as the transfer entropy.
- In time series representation the functional $I(y(t); x(t+\tau)|x(t), x(t-\eta), \ldots x(t-m\eta))$ can be used for inference of causal (causality in the Granger sense) influence of $\{Y\}$ on $\{X\}$.
- We study a possible influence of the phase ϕ_1 of slow oscillations on the amplitude A_2 of higher-frequency variability of the same process, using the functional

$$I(\phi_1(t); A_2(t+\tau)|A_2(t), A_2(t-\eta), \ldots, A_2(t-m\eta)), \quad (4)$$

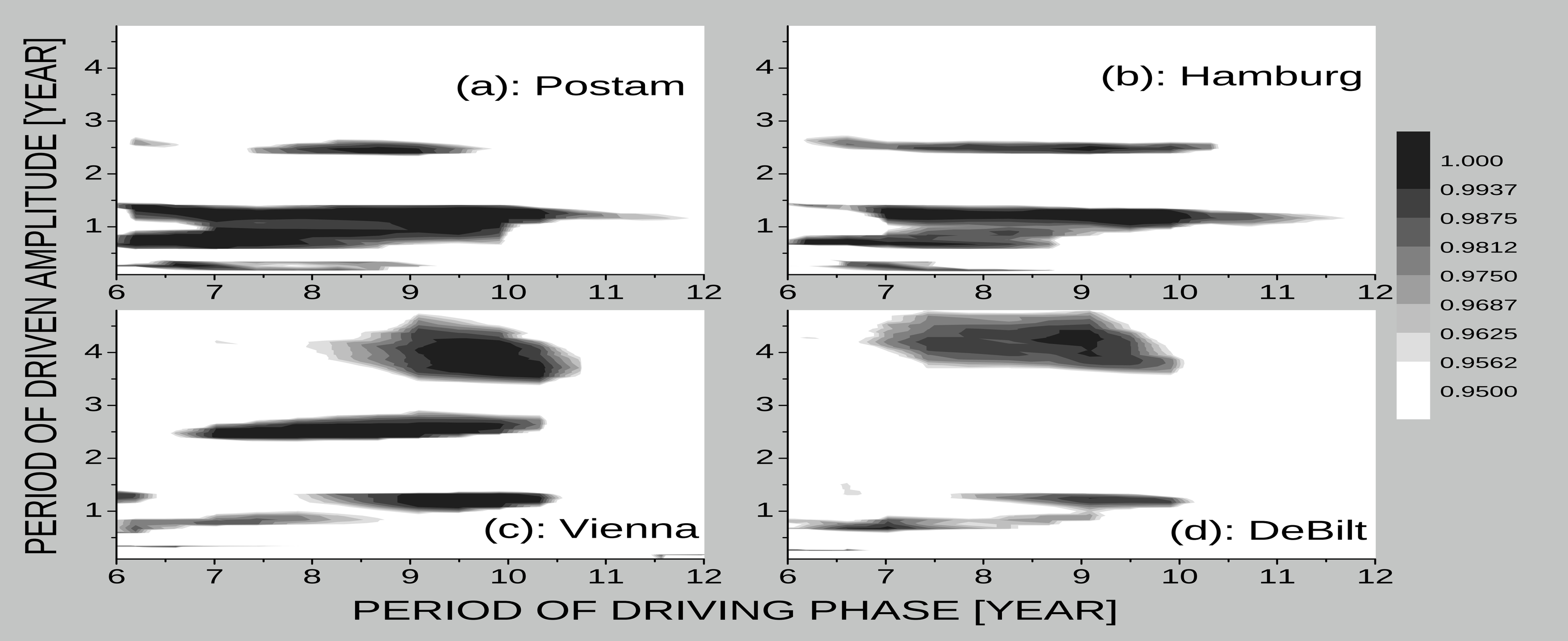
where τ is the forward time lag, η is the backward time lag in the $m+1$-dimensional condition.

Phase→amplitude interactions



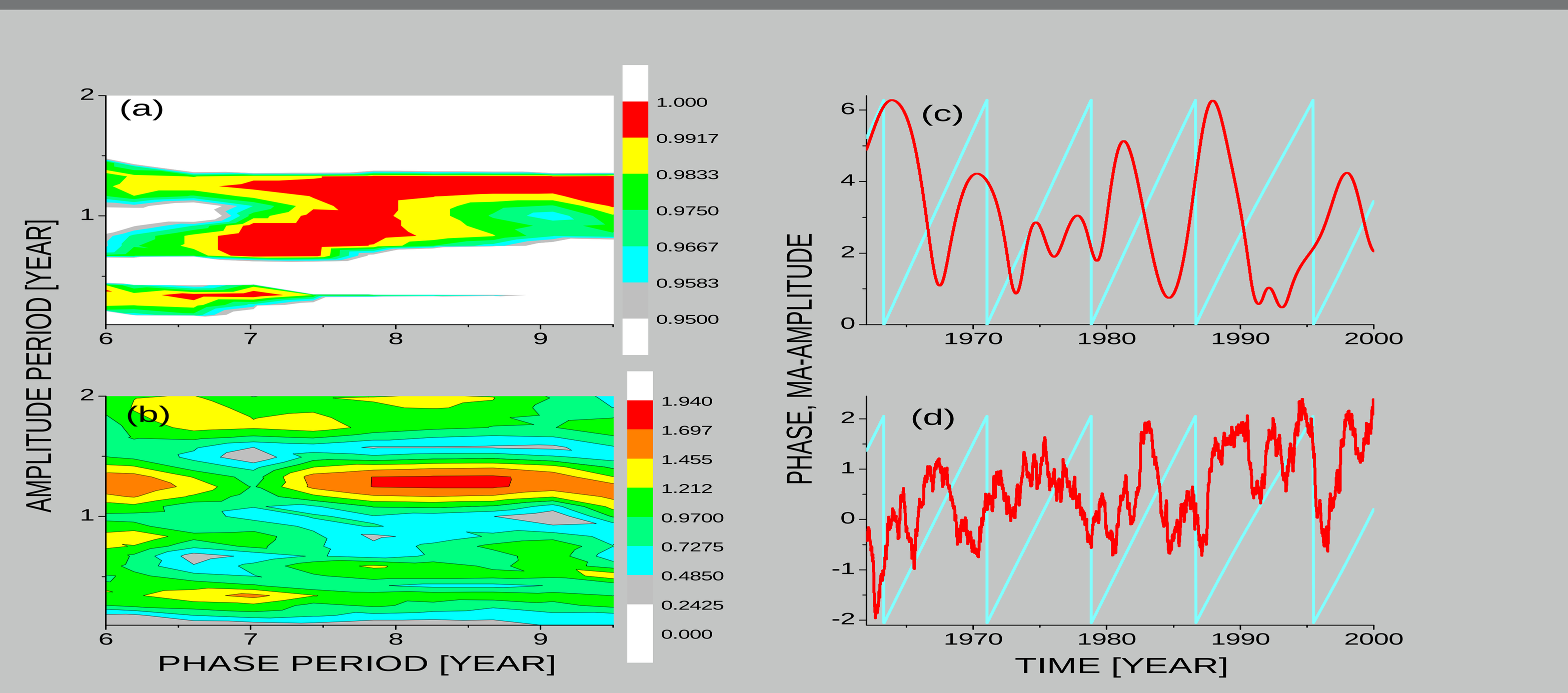
Causal influence of the phase of slower oscillations on the amplitude of faster fluctuations in the daily surface air temperature (SAT) from Prague-Klementinum. The significance levels (grey-coded if greater than 0.95) for the conditional mutual information (4) with (a) 2-dimensional, (b, d) 3-dimensional, and (c) 4-dimensional condition, obtained using (a,b,c) Fourier-transform (FT), and (d) multifractal (MF) surrogate data. CMI is estimated using 32768 daily SAT samples.

Phase→amplitude interactions



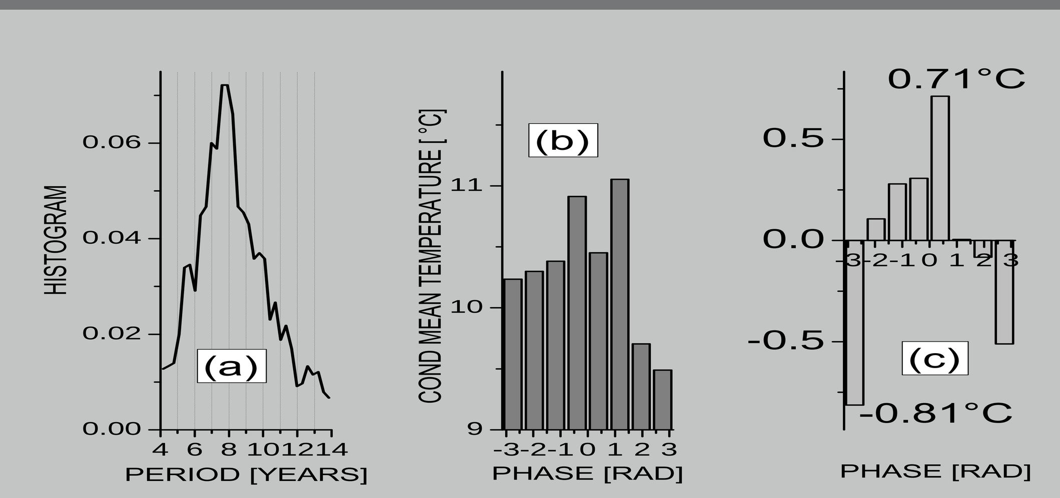
Causal influence of the phase of slower oscillations on the amplitude of faster fluctuations in the daily surface air temperature from (a) Potsdam, (b) Hamburg, (c) Vienna, and (d) DeBilt. The CMI significance levels are grey-coded if they are greater than 0.95.

Effect of the phase→amplitude interactions



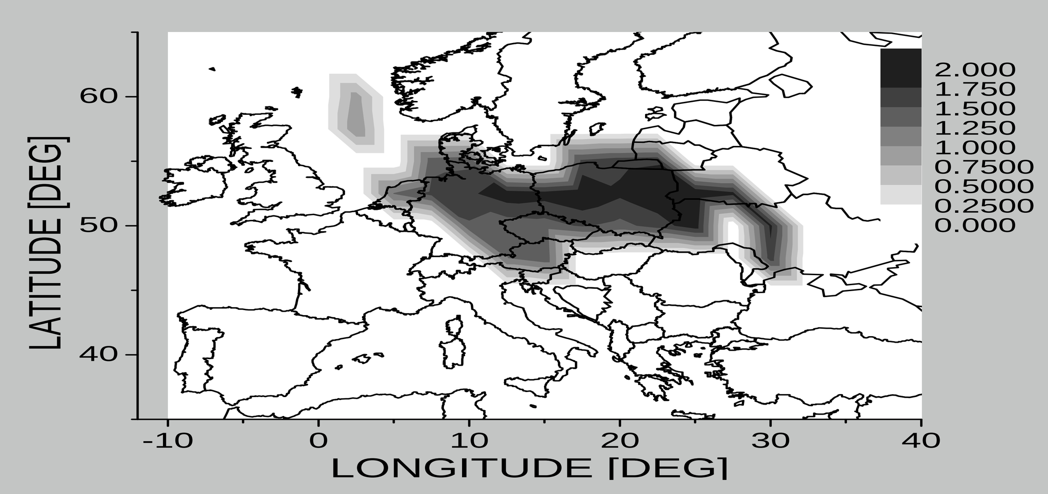
(a) Causal influence (color-coded CMI significance levels if greater than 0.95) of the phase of slower oscillations on the amplitude of faster fluctuations in the daily SAT. (b) Maximum differences (relative values) of conditional means of the amplitude A_2 (periods on the ordinate), conditioned on the phase ϕ_1 (periods on the abscissa). (c) The phase of the 8-year cycle (blue line, radians) and the moving-average amplitude (red curve, arbitrary units) of the 1.3-year cycle. (d) The phase of the 8-year cycle (blue line, rescaled values) and the moving-average amplitude (red curve, °C) of the total Prague SATA variability.

Slow oscillatory mode and its cross-scale effect



(a) Histogram of periods of the oscillatory mode extracted from the Prague SAT using the singular spectrum decomposition. (b, c) Conditional means of the Prague (b) SAT, and (c) SAT anomalies, computed conditionally on the phase of the SAT oscillatory mode extracted using the CCWT with the central wavelet period 8 years.

Cross-scale interactions and temperature variability



Significant maximum differences of the conditional means of ERA SAT anomalies (grey-coded in °C) in relation to the phase of the SAT oscillatory mode extracted using the CCWT with the central wavelet period 8 years.

Conclusion

- Quantitative evidence for the information transfer from larger to smaller scales in the atmospheric dynamics in the form of causal influence of the phase of slow oscillatory phenomena with periods around 6–11 years on amplitudes of the variability characterized by smaller temporal scales from a few months to 4–5 years in surface air temperature daily mean time series.
- Overall effect of the slow oscillations on the inter-annual temperature variability within the range 1–2°C has been observed in large areas of Europe.
- **Supported by Czech Science Foundation project No. P103/11/J068.**