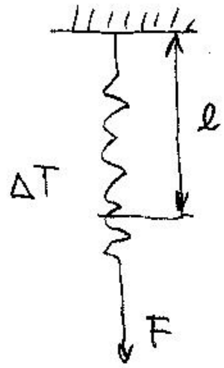
TERMODYNAMIKA PŘI MALÝCH DEFORMACÍCH

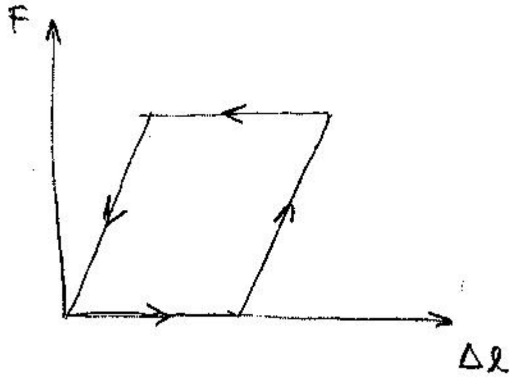
Př: Höschl



α = součinitel teplotní roztažnosti

c = součinitel tepelné kapacity

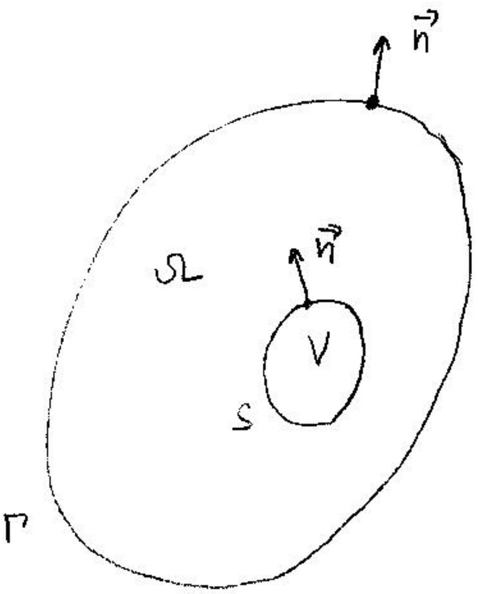
k = tuhost



$\Delta W = -F l \alpha \Delta T$ tepelný stroj

Perpetuum mobile ?

Značení



$\Omega(V)$: otevřená souvislá oblast

$\Gamma(S)$: Lipschitz

$\bar\Omega = \Omega \cup \Gamma$, $\bar V = V \cup S$

$\vec n$ vnější normála $\|\vec n\| = 1$

Př

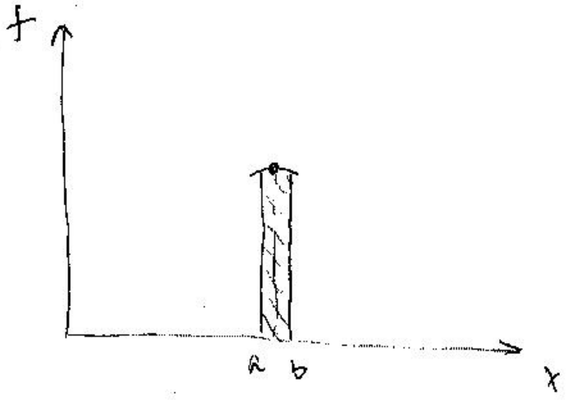
trhlina (není L.)

Př

$$\int_0^{2\pi} \sin x \, dx = 0$$

$\forall a, b \in \mathbb{R}$, $f \in C^0\langle a, b\rangle$

$$\int_a^b f(x)\, dx = 0 \iff f(x) \equiv 0$$



Platí i pro 3D úlohy.

Greenova věta

$$u \in C^1(\overline{V}) \qquad \int_V \frac{\partial u}{\partial x_i}\, dV = \int_S u\, n_i\, dS$$

Gaussova věta (divergenční teorém)

$$\int_V \operatorname{div} \vec{v}\, dV = \int_V \frac{\partial v_i}{\partial x_i}\, dV = \int_V \frac{\partial v_1}{\partial x_1}\, dV + \int_V \frac{\partial v_2}{\partial x_2}\, dV + \int_V \frac{\partial v_3}{\partial x_3}\, dV =$$

$$= \int_S v_1 n_1\, dS + \int_S v_2 n_2\, dS + \int_S v_3 n_3\, dS = \int_S v_i n_i\, dS = \int_S \vec{v}\cdot\vec{n}\, dS$$

Definice veličin

vnitřní energie $\quad U(V) = \int_V u\, dV \qquad u\ [J/m^3]$

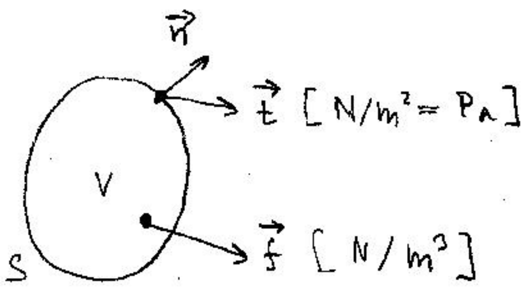
Helmholtz $\quad \Lambda(V) = \int_V \psi\, dV \qquad \psi\ [J/m^3]$

entropie $\quad S(V) = \int_V \eta\, dV \qquad \eta\ [J/m^3 K]$

výkon $\quad \dot{W}(V) = \int_V \dot{w}\, dV \qquad \dot{w}\ [W/m^3]$

tepelný výkon $\quad \dot{Q}(V) = \int_V \dot{q}\, dV \qquad \dot{q}\ [W/m^3]$

Výkon vnějších sil



$$t = \sigma n \qquad t_i = \sigma_{ij} n_j$$

$$\sigma_{ij,j} + f_i = 0$$

$$\sigma^T = \sigma \qquad \sigma_{ij} = \sigma_{ji}$$

$$\dot W(V) \overset{def}{=} \int_V \vec f \cdot \vec{\dot u}\, dV + \int_S \vec t \cdot \vec{\dot u}\, dS$$

$$\dot W(V) = \int_V f_i \dot u_i \, dV + \int_S t_i \dot u_i \, dS$$

$$\int_S t_i \dot u_i \, dS = \int_S \sigma_{ij} n_j \dot u_i \, dS = \int_V (\sigma_{ij} \dot u_i)_{,j} \, dV =$$

$$= \int_V \sigma_{ij,j} \dot u_i \, dV + \int_V \sigma_{ij} \dot u_{i,j} \, dV$$

$$\dot W(V) = \int_V \underbrace{(f_i + \sigma_{ij,j})}_{0} \dot u_i \, dV + \int_V \sigma_{ij} \dot u_{i,j} \, dV$$

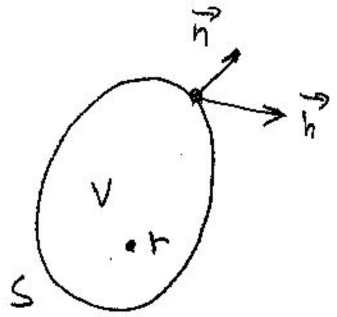
$$\sigma_{ij} \dot u_{i,j} = \tfrac{1}{2} \sigma_{ij} \dot u_{i,j} + \tfrac{1}{2} \sigma_{ji} \dot u_{i,j} = \tfrac{1}{2} \sigma_{ij} \dot u_{i,j} + \tfrac{1}{2} \sigma_{ij} \dot u_{j,i} =$$

$$= \sigma_{ij} \tfrac{1}{2} (\dot u_{i,j} + \dot u_{j,i}) = \sigma_{ij} \tfrac{1}{2} (u_{i,j} + u_{j,i})^{\cdot} = \sigma_{ij} \dot \varepsilon_{ij}$$

$$\boxed{\dot W(V) = \int_V \sigma_{ij} \dot \varepsilon_{ij} \, dV = \int_V \dot w \, dV}$$

$$\dot w = \sigma_{ij} \dot \varepsilon_{ij} \qquad [W/m^3]$$

Tepelný výkon



$\vec{h}$ [W/m^2]

r [W/m^3]

$$\dot{Q}(V) \overset{def}{=} \int_V r\,dV - \int_S \vec{h}\cdot\vec{n}\,dS$$

$$\int_S \vec{h}\cdot\vec{n}\,dS = \int_V \mathrm{div}\,\vec{h}\,dV$$

$$\boxed{\dot{Q}(V) = \int_V (r - \mathrm{div}\,\vec{h})\,dV = \int_V \dot{q}\,dV}$$

$$\dot{q} = r - \mathrm{div}\,\vec{h} \quad [W/m^3]$$

1. zákon termodynamiky

$$V: \quad \Delta Q + \Delta W = U_2 - U_1$$

$$\dot{Q} + \dot{W} = \dot{U}$$

Libovolná podoblast V:

$$\int_V (\dot{q} + \dot{w} - \dot{u}) \, dV = 0$$

$$\boxed{\dot{q} + \dot{w} = \dot{u}}$$

2. zákon termodynamiky

$\exists\ \eta$ $[J/m^3K]$ entropie = stavová veličina

$$S(V) = \int_V \eta \, dV$$

Clausius – Duhem

$$\dot{S}(V) \geq \int_V \frac{r}{T}\, dV - \int_S \frac{\vec{h}\cdot\vec{n}}{T}\, dS$$

Pozn: Funkci η je nutno naměřit pro konkrétní mat. model.

Lokální tvar CD – nerovnosti

$$\int_V \dot{\eta}\, dV \geq \int_V \frac{r}{T}\, dV - \int_V \operatorname{div}\left(\frac{\vec{h}}{T}\right) dV$$

$$\dot{\eta} \geq \frac{r}{T} - \operatorname{div}\left(\frac{\vec{h}}{T}\right)$$

$$\operatorname{div}\left(\frac{\vec{h}}{T}\right) = \left(\frac{h_i}{T}\right)_{,i} = \frac{h_{i,i}T - h_i T_{,i}}{T^2} = \frac{\operatorname{div}\vec{h}}{T} - \frac{\vec{h}\cdot \operatorname{grad} T}{T^2}$$

$$\dot{\eta} \geq \frac{r}{T} - \frac{\operatorname{div}\vec{h}}{T} + \frac{\vec{h}\cdot\operatorname{grad} T}{T^2}$$

$$\boxed{\dot{\eta} \geq \frac{\dot{q}}{T} + \frac{1}{T^2}\,\vec{h}\cdot\operatorname{grad} T}$$

Disipační nerovnost

Idea: $\dot q$ se nahradí $\dot w$ a $\dot u$

CD + 1. z. $\quad T\dot\eta \ge \dot u - \dot w + \frac{1}{T}\vec h \cdot \operatorname{grad} T$

$$\dot\eta T = (\eta T)^{\cdot} - \eta \dot T \qquad \text{(stavové veličiny)}$$

$$-\eta\dot T + \dot w - \frac{1}{T}\vec h\cdot\operatorname{grad}T = \dot u - (\eta T)^{\cdot} = \underbrace{(u-\eta T)}_{\psi\ [J/m^3]}{}^{\cdot}$$

Legendre: $\quad \psi \overset{def}{=} u - \eta T \quad$ Helmholtz (volná energie)

$$\boxed{-\eta\dot T + \dot w - \frac{1}{T}\vec h\cdot\operatorname{grad}T \ge \dot\psi}$$

Pomalé zatěžování: $\quad \dot T = 0,\ \operatorname{grad} T = 0 \quad \Rightarrow \quad \dot w \ge \dot\psi$

$$\left.\begin{array}{ll} \text{zatížení:} & \Delta w_z \ge \psi_2 - \psi_1 \\ \text{odlehčení:} & \Delta w_o \ge \psi_1 - \psi_2 \end{array}\right\} \quad \Delta w_z \ge \psi_2 - \psi_1 \ge -\Delta w_o$$

předpoklad stability materiálu: $\quad \Delta w_z > 0,\ \Delta w_o < 0$

$$|\Delta w_z| \ge \psi_2 - \psi_1 \ge |\Delta w_o| \quad \Rightarrow \quad \text{def. energie}$$

Pozn: $\quad \Delta q + \psi_2 - \psi_1 = u_2 - u_1, \quad \Delta q \simeq 10\,\Delta w \quad$ (Hlad: $\Delta q \pm \Delta w = \Delta u$)

Pozn: $\quad -\eta\dot T$ způsobuje tepelný chaos. Naopak $\operatorname{grad} T$ je zdrojem uspořádané entropie.

Termoelasticita

$\varepsilon_{ij}, T: \quad \sigma_{kl}(\varepsilon_{ij}, T), \quad \psi(\varepsilon_{ij}, T), \quad \eta(\varepsilon_{ij}, T), \quad u(\varepsilon_{ij}, T)$

diss. nerovnost: $\quad -\eta \dot{T} + \sigma_{ij}\dot{\varepsilon}_{ij} - \frac{1}{T}\vec{h}\cdot \operatorname{grad} T \geq \frac{\partial \psi}{\partial \varepsilon_{ij}}\dot{\varepsilon}_{ij} + \frac{\partial \psi}{\partial T}\dot{T}$

$$-\left(\eta + \frac{\partial \psi}{\partial T}\right)\dot{T} + \left(\sigma_{ij} - \frac{\partial \psi}{\partial \varepsilon_{ij}}\right)\dot{\varepsilon}_{ij} - \frac{1}{T}\vec{h}\cdot \operatorname{grad} T \geq 0$$

$\dot{T} = 0, \quad \operatorname{grad} T = 0 \qquad$ (pomalé zatěžování)

$$\underbrace{\left(\sigma_{ij} - \frac{\partial \psi}{\partial \varepsilon_{ij}}\right)}_{0}\dot{\varepsilon}_{ij} \geq 0$$

Důkaz $\qquad A_{ij}\dot{\varepsilon}_{ij} \geq 0, \qquad \dot{\varepsilon}_{ij} = -\alpha A_{ij}, \quad$ kde $\alpha > 0$

$$-\alpha A_{ij} A_{ij} \geq 0 \quad \Rightarrow \quad A_{ij} = 0$$

$$\boxed{\sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}}}$$

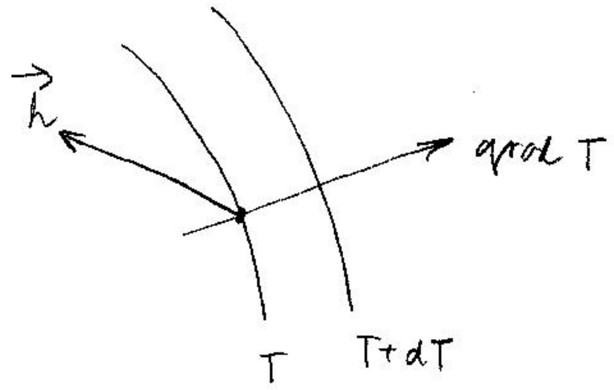
$$-\left(\eta + \frac{\partial \psi}{\partial T}\right)\dot{T} - \frac{1}{T}\vec{h}\cdot \operatorname{grad} T \geq 0$$

$\operatorname{grad} T = 0 \qquad$ (symetrické ochlazování válce)

$$-\left(\eta + \frac{\partial \psi}{\partial T}\right)\dot{T} \geq 0 \quad \Rightarrow \quad \boxed{\eta = -\frac{\partial \psi}{\partial T}}$$

$$-\frac{1}{T}\vec{h}\cdot \operatorname{grad} T \geq 0 \quad \Longrightarrow \quad \boxed{\vec{h}\cdot \operatorname{grad} T \leq 0}$$

Fourierova nerovnost:



Př $\vec{h} = -\lambda \operatorname{grad} T$, $\lambda\ [W/mK]$, $\lambda > 0$

λ může být i + def. matice (anisotropní mat.)

Vnitřní energie

$$\dot{u} = (\psi + \eta T)^{\cdot} = \underbrace{\frac{\partial \psi}{\partial \varepsilon_{ij}}}_{\sigma_{ij}} \dot{\varepsilon}_{ij} + \underbrace{\frac{\partial \psi}{\partial T}}_{-\eta} \dot{T} + \dot{\eta} T + \underline{\eta \dot{T}}$$

$$\left.\begin{aligned} \dot{u} &= \sigma_{ij}\dot{\varepsilon}_{ij} + \dot{\eta} T = \dot{w} + \dot{\eta} T \\ \underline{1.\,z.}\quad \dot{u} &= \dot{w} + \dot{q} \end{aligned}\right\} \Longrightarrow \dot{q} = \dot{\eta} T$$

$$\boxed{\dot{\eta} = \frac{\dot{q}}{T}}$$ definice entropie pro "vratné" děje

Pozn $\dot{\eta} = \frac{\dot{q}}{T} \ \wedge\ \frac{1}{T^2}\vec{h}\cdot \operatorname{grad} T \leq 0 \Longrightarrow \dot{\eta} \geq \frac{\dot{q}}{T} + \frac{1}{T^2}\vec{h}\cdot \operatorname{grad} T$

CD – nerovnost

Termoelasticita – souhrn

$\exists\, \psi(\varepsilon_{ij}, T) \qquad \sigma_{ij} = \dfrac{\partial \psi}{\partial \varepsilon_{ij}} \qquad \eta = -\dfrac{\partial \psi}{\partial T}$

$\dot{\eta} = \dfrac{\dot{q}}{T} \qquad \dot{q} = r - \operatorname{div} \vec{h} \qquad \vec{h} \cdot \operatorname{grad} T \leq 0$

Vedení tepla

Fourierův zákon $\quad \vec{h} = -\lambda \operatorname{grad} T$

$$\dot{q} = r - \operatorname{div} \vec{h} = r + \operatorname{div}(\lambda \operatorname{grad} T)$$

$$\dot{q} = T\dot{\eta} = T \frac{\partial \eta}{\partial \varepsilon_{ij}} \dot{\varepsilon}_{ij} + T \frac{\partial \eta}{\partial T} \dot{T}$$

$c_\varepsilon \overset{def}{=} T \dfrac{\partial \eta}{\partial T}, \quad c_\varepsilon = c_V \simeq c_p \simeq c$ (měří se experimentálně)

$$\boxed{r + \operatorname{div}(\lambda \operatorname{grad} T) = c_V \dot{T} + T \frac{\partial \eta}{\partial \varepsilon_{ij}} \dot{\varepsilon}_{ij}}$$

termoelastická vazba

λ není funkcí x, y, z $\Rightarrow$ $\operatorname{div}(\lambda \operatorname{grad} T) = \lambda \Delta T$

$\tilde{r} = r - T \dfrac{\partial \eta}{\partial \varepsilon_{ij}} \dot{\varepsilon}_{ij}$

$$\boxed{\lambda \Delta T = c_V \dot{T} - \tilde{r}}$$

Duhamel – Neumann (1D)

$$\varepsilon = \frac{\sigma}{E} + \alpha \Delta T \qquad \alpha\,[1/K]\ ,\ \Delta T = T - T_0$$

$$\sigma = E(\varepsilon - \alpha \Delta T) \qquad \text{DN – zákon}$$

$$\alpha, E = \text{konst.}$$

$$\sigma = \frac{\partial \Psi}{\partial \varepsilon} \quad \Rightarrow \quad \Psi = \tfrac{1}{2} E \varepsilon^2 - E \alpha \Delta T \varepsilon + f(T)$$

$$\eta = -\frac{\partial \Psi}{\partial T} = E \alpha \varepsilon - f'(T)\ , \qquad f' = \frac{df}{dT}$$

$$\eta = \alpha E(\varepsilon - \alpha \Delta T) + \alpha^2 E \Delta T - f'(T) =$$

$$= \alpha \sigma + g(T)$$

$$\dot q = T \dot\eta = \alpha T \dot\sigma + T g'(T) \dot T\ , \qquad g' = \frac{dg}{dT}$$

součinitel tepelné kapacity $C_\sigma \overset{\text{def}}{=} T g'(T)$ při konstantním napětí. Lze dosadit $C_\sigma = C_p$.

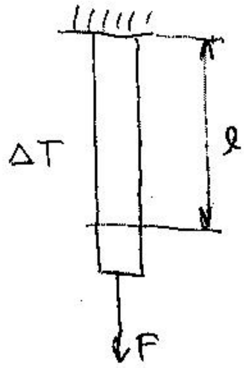
$$\boxed{\dot q = \alpha T \dot\sigma + \dot T}$$

<u>Př</u> $\quad T = \text{konst.} \qquad \Delta q = \alpha T \Delta \sigma$

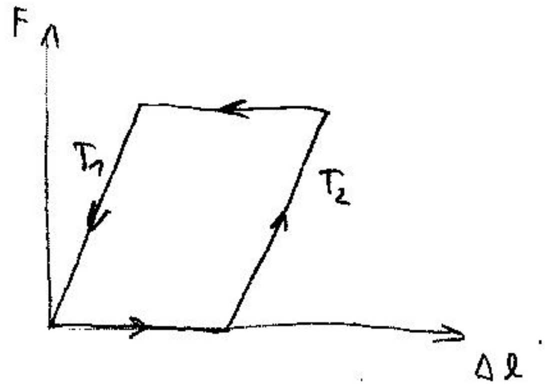
$$\Delta q \simeq 10^{-5} \times 300 \times 10^8 = 3 \times 10^5 \ [J/m^3]$$

$$\Delta w \simeq \frac{1}{2} \frac{(10^8)^2}{2 \times 10^{11}} = 0.25 \times 10^5 \ [J/m^3]$$

Př



Dáno: $E,\ \alpha,\ C_\sigma = C,\ A =$ průřez



1) $\Delta W_1 = 0 \qquad \Delta Q_1 = C \Delta T A \ell$

2) $\Delta W_2 = \frac{1}{2}\frac{F^2 \ell}{EA} \qquad \Delta Q_2 = \alpha T_2 \frac{F}{A} A \ell$

3) $\Delta W_3 = -F \ell \alpha \Delta T \qquad \Delta Q_3 = -C \Delta T A \ell$

4) $\Delta W_4 = -\frac{1}{2}\frac{F^2 \ell}{EA} \qquad \Delta Q_4 = -\alpha T_1 \frac{F}{A} A \ell$

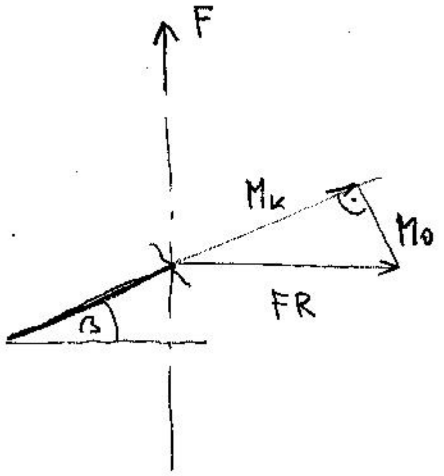
$$\Delta W = -F\ell\alpha\Delta T \qquad \Delta Q = F\ell\alpha\,(T_2 - T_1)$$

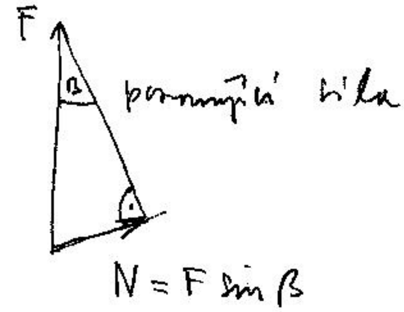
$$\text{účinnost} = \frac{|\Delta W|}{\Delta Q_2} = \frac{F\ell\alpha\Delta T}{\alpha T_2 F\ell} = \frac{\Delta T}{T_2} = 1 - \frac{T_1}{T_2} \quad (\text{Carnot})$$

$$1 - \frac{T_1}{T_2} \simeq 1 - \frac{300}{500} = 40\,\%$$

Pozn: elektrárny, letecké motory

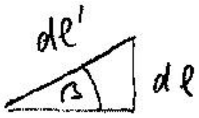
Vinutá pružina





$$\Delta Q = \alpha T \,\frac{F \sin\beta}{A}\, A l' = \alpha T F l' \sin\beta$$

l' = délka drátu l = výška pružiny



$$dl' \sin\beta = dl$$

integrací $\Delta Q = \alpha T F l$

Invarianty ε

$x_i, \varepsilon_{ij}, \varepsilon$ $\qquad$ $x'_i, \varepsilon'_{ij}, \varepsilon'$

def. invariantu: $f(\varepsilon_{ij}) = f(\varepsilon'_{ij})$

Př $\quad tr(\varepsilon) = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \varepsilon_{ii} = \varepsilon'_{ii} = tr(\varepsilon')$

Označme $\varepsilon^2 = \varepsilon\varepsilon \qquad \varepsilon^3 = \varepsilon\varepsilon\varepsilon$

Pozn Násobení matic zachovává tensorový charakter

$$\left.\begin{aligned} I_1 &= tr(\varepsilon) \\ I_2 &= \tfrac{1}{2} tr(\varepsilon^2) \\ I_3 &= \tfrac{1}{3} tr(\varepsilon^3) \end{aligned}\right\} \quad I_n = \frac{1}{n} tr(\varepsilon^n)$$

Pozn: Jen 3 invarianty jsou nezávislé

Vlastní čísla jsou invarianty

$$\det|\varepsilon - \lambda I| = 0$$

$$\det|\varepsilon' - \lambda I| = 0$$

$\lambda_1, \lambda_2, \lambda_3 \in R \qquad \varepsilon_1, \varepsilon_2, \varepsilon_3$ hlavní deformace

Charakteristická rovnice

$$\lambda^3 - \bar{I}_1 \lambda^2 + \bar{I}_2 \lambda - \bar{I}_3 = 0$$

$$\bar{I}_1 = I_1$$

$$\bar{I}_2 = \frac{1}{2} I_1^2 - I_2$$

$$\bar{I}_3 = I_3 - I_1 I_2 + \frac{1}{6} I_1^3$$

Pozn: $I_n \to \bar{I}_n \to \varepsilon_n$

Izotropie $x_i' =$ hlavní osy ε

$$f(\varepsilon_{ij}) = f(\varepsilon_{ij}') = f(\varepsilon_1, \varepsilon_2, \varepsilon_3) = f(I_1, I_2, I_3)$$

Lineární tvar

$$I_1 = tr(\varepsilon) = \varepsilon_{ii}$$

$$A = \varepsilon^2 = \varepsilon \varepsilon \qquad I_2 = \frac{1}{2} tr(A)$$

$$A_{ik} = \varepsilon_{ij} \varepsilon_{jk} \qquad I_2 = \frac{1}{2} A_{ii} = \frac{1}{2} \varepsilon_{ij} \varepsilon_{ji}$$

$$I_3 = \frac{1}{3} \varepsilon_{ij} \varepsilon_{jk} \varepsilon_{ki}$$

Derivace invariantů

$$\frac{\partial I_1}{\partial \varepsilon_{11}} = \frac{\partial}{\partial \varepsilon_{11}}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) = 1 \quad \text{podobně pro } \varepsilon_{22}, \varepsilon_{33}$$

$$\frac{\partial I_1}{\partial \varepsilon_{ij}} = \begin{cases} 1 \text{ pro } i=j \\ 0 \text{ pro } i \neq j \end{cases} \quad \delta_{ij} \text{ kronecker}$$

$$\frac{\partial I_2}{\partial \varepsilon_{ij}} = \frac{1}{2}\frac{\partial}{\partial \varepsilon_{ij}}(\varepsilon_{kl}\varepsilon_{lk}) = \frac{1}{2}\frac{\partial \varepsilon_{kl}}{\partial \varepsilon_{ij}}\varepsilon_{lk} + \frac{1}{2}\varepsilon_{kl}\frac{\partial \varepsilon_{lk}}{\partial \varepsilon_{ij}} =$$

$$= \frac{1}{2}\delta_{ik}\delta_{jl}\varepsilon_{lk} + \frac{1}{2}\varepsilon_{kl}\delta_{il}\delta_{jk} =$$

$$= \frac{1}{2}\varepsilon_{ji} + \frac{1}{2}\varepsilon_{ji} = \varepsilon_{ji} = \varepsilon_{ij} \quad (\text{symetrie})$$

$\dfrac{\partial I_3}{\partial \varepsilon_{ij}}$ se spočítají podobně

$$\frac{\partial I_1}{\partial \varepsilon_{ij}} = \delta_{ij}$$

$$\frac{\partial I_2}{\partial \varepsilon_{ij}} = \varepsilon_{ij}$$

$$\frac{\partial I_3}{\partial \varepsilon_{ij}} = \varepsilon_{ik}\varepsilon_{kj}$$

Elasticita

T = konst. $\exists\, \Psi(\varepsilon_{ij})$ $\sigma_{ij} = \frac{\partial \Psi}{\partial \varepsilon_{ij}}$

$$d\sigma_{ij} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_{kl}} d\varepsilon_{kl} = \frac{\partial^2 \Psi}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}} d\varepsilon_{kl}$$

$$d\sigma_{ij} = C_{ijkl}\, d\varepsilon_{kl} \qquad C_{ijkl} = \frac{\partial^2 \Psi}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}}$$

$$(\dim)^{\text{řád}} = 3^4 = 81 \qquad i \leftrightarrow j,\ k \leftrightarrow l,\ ij \leftrightarrow kl$$

21 koeficientů

Pozn.: symetrie C v MKP.

Isotropní materiál

$$\Psi(\varepsilon_{ij}) = \Psi(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \Psi(I_1, I_2, I_3)$$

$$\sigma_{ij} = \frac{\partial \Psi}{\partial I_1}\frac{\partial I_1}{\partial \varepsilon_{ij}} + \frac{\partial \Psi}{\partial I_2}\frac{\partial I_2}{\partial \varepsilon_{ij}} + \frac{\partial \Psi}{\partial I_3}\frac{\partial I_3}{\partial \varepsilon_{ij}} =$$

$$= \frac{\partial \Psi}{\partial I_1}\delta_{ij} + \frac{\partial \Psi}{\partial I_2}\varepsilon_{ij} + \frac{\partial \Psi}{\partial I_3}\varepsilon_{ik}\varepsilon_{kj}$$

lineární + isotropní

$$\sigma_{ij} = \lambda I_1 \delta_{ij} + 2\mu \varepsilon_{ij} \qquad \text{Hooke (Voigt 1910)}$$

λ, μ = Lamého konstanty

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = G = \frac{E}{2(1+\nu)}$$

Deformační energie

$$\frac{\partial \Psi}{\partial I_1} = \lambda I_1 \qquad \frac{\partial \Psi}{\partial I_2} = 2\mu \qquad \frac{\partial \Psi}{\partial I_3} = 0$$

$$\boxed{\Psi = \frac{1}{2}\lambda I_1^2 + 2\mu I_2}$$

$$I_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \qquad I_2 = \frac{1}{2}(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2)$$

$$\Psi = \frac{1}{2}\lambda(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + 2\varepsilon_1\varepsilon_2 + 2\varepsilon_2\varepsilon_3 + 2\varepsilon_3\varepsilon_1) + \mu(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) =$$

$$= \left(\frac{1}{2}\lambda + \mu\right)(\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2) + \lambda(\varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_3\varepsilon_1)$$

<u>Předp. stability</u> $\quad \Psi \geq 0$

Ψ = kvadratická forma, Hessián: $H_{ij} = \dfrac{\partial^2 \Psi}{\partial \varepsilon_i \partial \varepsilon_j} = +$ def.

$$\frac{\partial^2 \Psi}{\partial \varepsilon_1^2} = \lambda + 2\mu \qquad \frac{\partial^2 \Psi}{\partial \varepsilon_1 \partial \varepsilon_2} = \lambda$$

$$H = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda \\ \lambda & \lambda + 2\mu & \lambda \\ \lambda & \lambda & \lambda + 2\mu \end{bmatrix}$$

vyšetřujeme pozitivní definitnost

$$H\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\lambda+2\mu\\3\lambda+2\mu\\3\lambda+2\mu\end{bmatrix} \quad (3\lambda+2\mu)\begin{bmatrix}1\\1\\1\end{bmatrix}$$ hydrostat. mód

$$H\begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}2\mu\\0\\-2\mu\end{bmatrix} = 2\mu\begin{bmatrix}1\\0\\-1\end{bmatrix}$$ deviátorní mód

$$H\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}2\mu\\-2\mu\\0\end{bmatrix} = 2\mu\begin{bmatrix}1\\-1\\0\end{bmatrix}$$ deviátorový mód

požadujeme $3\lambda + 2\mu = 3K > 0$, $2\mu = 2G > 0$

modul objemové pružnosti   modul pružnosti ve smyku

$$K = \frac{E}{3(1-2\nu)} \qquad G = \frac{E}{2(1+\nu)}$$

<u>$E < 0$</u>

$1+\nu < 0 \quad \wedge \quad 1-2\nu < 0$

$\nu < -1 \quad \wedge \quad \nu > \frac{1}{2}$

<u>$E > 0$</u>

$\nu > -1 \quad \wedge \quad \nu < \frac{1}{2}$

$E, K, G > 0 \qquad \nu \in (-1, 1/2)$