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On the contact pressure oscillations of an isogeometric contact-impact algorithm

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Abstract. *Artificial oscillations in the contact pressure due to non-smooth contact surface are treated by the isogeometric analysis (IGA). After a brief overview of the B-splines and NURBS representations, an explicit finite element (FE) contact-impact algorithm is presented in a small deformation context. Contact constraints are regularized by the penalty method. The contact-impact algorithm is tested by means of dynamic Hertz problem. The classic FEA solution is compared to the IGA solution while different mass lumping techniques are considered.*

Keywords: contact-impact; isogeometric analysis; finite element method.

1 INTRODUCTION

The main difficulty in contact analysis is non-smoothness. It arises from inequality constraints as well as geometric discontinuities inducted by the spatial discretization. Contact analysis based on traditional finite elements utilizes element facets to describe contact surfaces. The facets are C^0 continuous so that the surface normals can experience jumps across facet boundaries leading to artificial oscillations in contact force and pressure. There were attempts to treat these geometric discontinuities by smoothing the contact surfaces using splines interpolation. These remedies, however, introduce an additional geometry on the top of the existing finite element mesh. This adds an additional layer of data management and increasing computational overhead. Details and further references can be found in [1].

Another remedy to the geometric discontinuity provides isogeometric analysis (IGA). The fundamental idea is to accurately describe a physical domain by proper representation (e.g. NURBS) and then to utilize the same basis for analysis. This is in contrast with the classical finite element method where the basis is given in advance by the element type. Consequently the physical domain could be approximated inaccurately. A more detailed description can be found in [2]. Isogeometric NURBS-based contact analysis has some additional advantages: preserving geometric continuity, facilitating patch-wise contact search, supporting a variationally consistent formulation, and having a uniform data structure for the contact surface and the underlying volumes.

Geometric basis and formulation for frictionless isogeometric contact were given in [6]. Sharp corners or C^0 edges that can exist on the interface of patches present a challenge to contact detection. A strategy to seamlessly deal with sharp corners was proposed in this reference. The contact constraints were regularized by the penalty method and the contact virtual work was discretized by the finite strain surface-to-surface contact element. Both one-pass and two-pass algorithms were tested.

In reference [7], the finite deformation frictionless quasi-static thermomechanical contact problem was considered. Two penalty-based contact algorithms were studied. The former was called knot-to-surface (KTS) algorithm. It is the straightforward extension of the classical node-to-surface (NTS) algorithm. It was shown that this approach is over-constrained and therefore not acceptable if a robust formulation with accurate tractions is desired. The latter was called mortar-KTS algorithm. In this algorithm a mortar projection to control pressures was employed to obtain the correct number of constraints.

The penalty-based mortar-KTS algorithm was extended to the frictional contact in [8, 10]. The mortar-KTS algorithm was also studied in conjugation with the augmented Lagrangian method in [11]. Isogeometric frictionless contact analysis using the non-conforming mortar method in the two-dimensional linear elasticity regime was presented in [9].

In this paper, we present a frictionless isogeometric contact algorithm. After brief overview of B-splines and NURBS representation in Section 2, the contact algorithm is presented in Section 3. The algorithm is studied by means of dynamic Hertz problem in Section 4.

2 B-SPLINES AND NURBS

This section gives an overview of the B-spline and NURBS representations. For a comprehensive description as well as efficient algorithmization see, e.g. [3]. Throughout this paper we use p to indicate the polynomial degree, n to indicate the number of basis functions, d_p to indicate the number of parametric dimensions, and d_s to indicate the number of spatial dimensions.

Let $\boldsymbol{\Xi}^i$, $i = 1, \ldots, d_\mathrm{p}$ be the open non-uniform knot vector associated with i^th parametric dimension of a patch

$$\boldsymbol{\Xi}^i = \left\{ \underbrace{\xi^i_1, \ldots, \xi^i_{p_i+1}}_{p_i+1 \text{ equal terms}}, \xi^i_{p_i+2}, \ldots, \xi^i_{n_i}, \underbrace{\xi^i_{n_i+1}, \ldots, \xi^i_{n_i+p_i+1}}_{p_i+1 \text{ equal terms}} \right\}, \quad i = 1, \ldots d_\mathrm{p} \tag{1}$$

The knot vector is a non-decreasing sequence of parametric coordinates. The knot vector is said to be non-uniform if the knots are unequally spaced in the parametric space. If the first and the last knot values appear $p_i + 1$ times, the knot vector is called open. The B-spline basis functions, $N_{j,p}(\xi)$, are defined by Cox-de Boor recursion formula. For $p = 0$ it is defined as

$$N_{j,0}(\xi) = \begin{cases} 1 & \xi \in [\xi_j, \xi_{j+1}), j = 1 \ldots n \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and for $p > 0$

$$N_{j,p}(\xi) = \frac{\xi - \xi_j}{\xi_{j+p} - \xi_j} N_{j,p-1}(\xi) + \frac{\xi_{j+1+p} - \xi}{\xi_{j+1+p} - \xi_{j+1}} N_{j+1,p-1}(\xi) \tag{3}$$

B-splines as the polynomial functions are known to be unable to describe conic sections. The NURBS (Non-Uniform Rational B-Splines) was developed to extend interpolatory capability of the B-splines. A p^th degree NURBS basis function is defined by

$$R^p_j(\xi) = \frac{N_{j,p}(\xi) w_j}{\sum_{\hat{j}=1}^{n} N_{\hat{j},p}(\xi) w_{\hat{j}}} \tag{4}$$

where w_j is referred to as the j^th weight.

Multivariate NURBS objects can be constructed simply by tensor product of univariate NURBS basis functions (4). For $d_\mathrm{p} = 2$ it yields

$$R^{p_1,p_2}_{j_1,j_2}(\xi^1, \xi^2) = R^{p_1}_{j_1}(\xi^1) \otimes R^{p_2}_{j_2}(\xi^2) = \frac{N_{j_1,p_1}(\xi^1) N_{j_2,p_2}(\xi^2) w_{j_1,j_2}}{\sum_{\hat{j}_1=1}^{n_1} \sum_{\hat{j}_2=1}^{n_2} N_{\hat{j}_1,p_1}(\xi^1) N_{\hat{j}_2,p_2}(\xi^2) w_{\hat{j}_1,\hat{j}_2}} \tag{5}$$

and similarly for the higher parametric dimension. With NURBS basis functions at hand we can introduce surface discretization as

$$\boldsymbol{x}(\xi^1, \xi^2) = \sum_{j_1=1}^{n_1} \sum_{j_2=1}^{n_2} R^{p_1,p_2}_{j_1,j_2}(\xi^1, \xi^2) \mathbf{P}_{j_1,j_2} \tag{6}$$

where $\mathbf{P}_{j_1,j_2} \in \Re^{d_\mathrm{s}}$ is the control net, i.e., the array of coordinates of control points. Adopting the isogeometric concept, an analogous interpolation is used for the unknown displacement field and its variation.

3 EXPLICIT DYNAMIC CONTACT ALGORITHM

In this section an isogeometric treatment of the frictionless contact between two elastic deformable bodies is presented. An algorithm, originally proposed in [12], has been adapted to the isogeometric analysis and expanded to explicit dynamics. The main idea is as follow. The contact boundary value problem is formulated in the weak sense

$$\delta\boldsymbol{\Pi}_\mathrm{int,ext}(\mathbf{u}, \delta\mathbf{u}) + \delta\boldsymbol{\Pi}_\mathrm{c}(\mathbf{u}, \delta\mathbf{u}) = 0 \tag{7}$$

$$g_\mathrm{N}(\mathbf{u}) \geq 0 \tag{8}$$

where $\mathbf{u}$ and $\delta\mathbf{u}$ are the displacement field and its variation respectively, $\delta\mathbf{\Pi}_{\text{int,ext}}$ denotes the virtual work due to internal, external and inertial forces, $\delta\mathbf{\Pi}_{\text{c}}$ is the contact virtual work and g_{N} is the gap function. In reference [12], $\delta\mathbf{\Pi}_{\text{c}}$ was proposed in the form

$$\delta\mathbf{\Pi}_{\text{c}}(\mathbf{u},\delta\mathbf{u}) = -\int_{\Gamma_{\text{c1}}} \varepsilon_{\text{N}} g_{\text{N}} \delta\mathbf{u}\,\text{d}\Gamma - \int_{\Gamma_{\text{c2}}} \varepsilon_{\text{N}} g_{\text{N}} \delta\mathbf{u}\,\text{d}\Gamma \tag{9}$$

where ε_{N} is the penalty parameter. Note that the contact virtual work is integrated over both contact boundaries Γ_{c1} and Γ_{c2} so that the algorithm preserves symmetry. Consequently, after FE discretization the action-reaction principle is not explicitly fulfilled. However, it should be shown that the equilibrium is recovered during the mesh refinement.

Applying the standard finite element procedures [4] to the weak form (8)-(9), the resulting system of ordinary differential equations (ODE) is obtained as

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{F}(\mathbf{U}) = \mathbf{R} + \mathbf{R}_{\text{c12}}(\mathbf{U}) + \mathbf{R}_{\text{c21}}(\mathbf{U}) \tag{10}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{U}$ is the displacement vector, $\mathbf{F}$ is the vector of internal forces, $\mathbf{R}$ is the vector of external forces and $\mathbf{R}_{\text{c12}}$, $\mathbf{R}_{\text{c21}}$ are vectors of contact forces. Two superimposed dots denote time derivates. The system of ODEs is integrated by the central difference method (CDM) [4] which yields

$$\mathbf{M}\mathbf{U}_{n+1} = \Delta t^2 \left[\mathbf{R} + \mathbf{R}_{\text{c12}}(\mathbf{U}_n) + \mathbf{R}_{\text{c21}}(\mathbf{U}_n) - \mathbf{F}(\mathbf{U}_n)\right] + \mathbf{M}(2\mathbf{U}_n - \mathbf{U}_{n-1}) \tag{11}$$

The stability of the integration process requires time step, Δt, to be smaller or equal to $2/\omega_{\max}$, where $\omega_{\max}$ is the maximal angular frequency of the FE mesh. The global mass matrix, $\mathbf{M}$, arises from its element counterpart, $\mathbf{M}_e$, by the standard FE assembly procedure. The element mass matrix arising from the variational formulation has the form

$$\mathbf{M}_e = \int_{\Omega_e} \rho \mathbf{H}^{\text{T}} \mathbf{H}\,\text{d}\Omega \tag{12}$$

where ρ is the density and $\mathbf{H}$ is the matrix of shape functions (5). This mass matrix is called consistent. The efficient solution of the linear system (11) requires diagonalization of M. The common techniques are the row sum method and HRZ method [12].

4 DYNAMIC HERTZ PROBLEM

In this section, an example is presented to illustrate the performance of the classic FEA and IGA contact-impact algorithm described in the previous section. The example deals with Hertz dynamic problem, a classical benchmark for which an analytical solution is available [13]. In the example, the effect of mass lumping is investigated. The analysis is limited to the second order elements. In particular, quadratic serendipity eight-node finite elements are used in case of FEA, and second order basis function in case of IGA.

The presented numerical example deals with frictionless impact of the cylinders of radius $R = 4\,\text{m}$. The material of each of the cylinders is linearly elastic with Young's modulus $E = 1000\,\text{MPa}$, Poisson's ratio $\nu = 0.2$, and density $\rho = 1\,\text{kg}\cdot\text{m}^3$. The initial velocity of the cylinders is $2\,\text{m}\cdot\text{s}^{-1}$. In the initial configuration the cylinders just touches each other in a point. Due to symmetry, only the half of each cylinder is considered. The penalty parameter is $\epsilon_{\text{N}} = 1\times10^5\,\text{N}\cdot\text{m}^{-2}$. The explicit time integration by CDM is performed for $0.9\,\text{s}$ with the time step $5\times10^{-4}\,\text{s}$. In order to evaluate the effect of mass lumping techniques on the oscillations of the contact forces and contact pressure distribution in IGA, further analyses are performed using consistent mass matrix and mass matrix lumped by the row sum method. Fig. 1 shows that consistent mass matrix delivers a more accurate contact pressure distribution than row sum and HRZ mass lumping techniques.

5 CONCLUSIONS

This paper addressed the utilization of the NURBS based isogeometric analysis in an explicit contact-impact algorithm. Two main conclusions may be drawn:

- For second order elements and mass matrix lumped by the HRZ method, IGA in comparison with classic FEA leads to a more oscillatory contact force and consequently also contact pressure.
- The oscillations of the contact forces in IGA are minimal for consistent mass matrix.

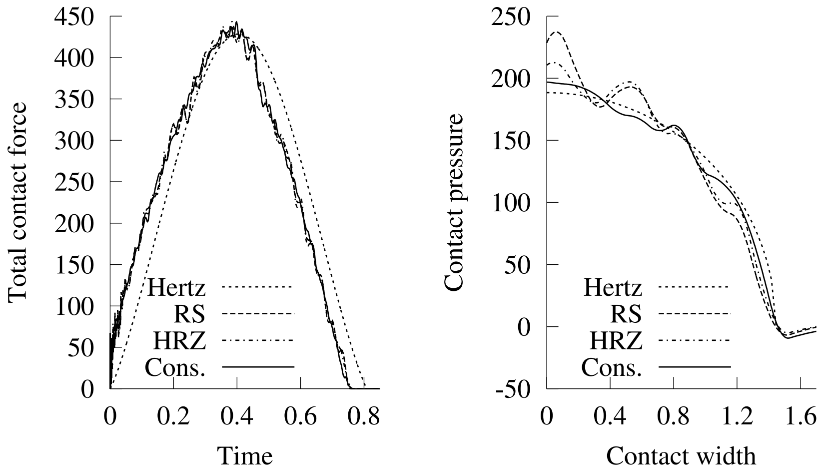

Figure 1: Influence of mass lumping techniques on contact forces (left) and contact pressures (right) for IGA.

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