

# PUBLIC EMPLOYMENT POLICIES IN A TWO-SECTOR MODEL WITH MATCHING FRICTIONS AND WAGE RIGIDITY.

## Abstract

The thesis is aimed to study the effects of public employment and public wages policies in a two-sectors model augmented with search and matching frictions along the lines of Mortensen and Pissarides (1995), in which job seekers direct their search between the two sectors, and private wages adjust rigidly to technology. Through simulations, I find that the public employment multiplier on total employment is negative (approximately -2.1) in the short run and positive (about 0.6 at the peak) in the long run (unlike in the model with random search). Unlike in Michailat (2014), also, the multiplier is only mildly counter-cyclical. When I model wages differently, the multiplier become very close to zero in steady state. I then show that a public wage shock slightly increases unemployment. Finally, similarly to Quadrini and Trigari (2007), I study the business cycle properties of the model according to different public employment and wages rules.

# 1 Introduction

In very recent years, there has been a huge debate in macroeconomics about what fiscal policies should be used in times of recession<sup>1</sup>. However, most of the models that analyze fiscal policies in recessionary periods do not take into account unemployment dynamics<sup>2</sup>: in other words, they assume that in equilibrium unemployment is zero. Furthermore, the majority of these models assume that government spending takes the form of purchases from the private sector or tax cuts. Both are strong assumptions, the first one because unemployment is obviously a fundamental variable in the economy, and the more so in recession<sup>3</sup>; the second one because government purchases from the private sector are a small part of government expenditures, which are composed by more than half from compensation to employees<sup>4</sup>.

Two seminal contributions have shown that different forms of government spending can have a significantly different impact from a macroeconomic point of view. Baxter and King (1993) show how purchases of goods from the private sector can have a qualitatively different effect than government investment on private output and employment; Finn (1998) shows that a shock to government employment and an increase in the purchases of goods from the private sector can have opposite effects on private employment, output, and investment.

In light of these facts, a body of research has recently focused on the purpose of understanding the consequences of modeling an increase in government spending as a shock to public employment (or wages). The first work aimed to study the consequences of modeling a public labor market separately from the private labor market - and, more specifically, to evaluate the effects of fiscal shocks on the model economy - is Quadrini and Trigari (2007). Starting from the latter, several authors (among which Gomes (2010) and, more recently, Michailat (2014)) have tried to address this issue. Despite having significantly contributed to the debate on fiscal policy directed to stimulate employment, these works have potential limitations. Specifically, in Quadrini and Trigari (2007), the authors assume that wages adjust flexibly to public employment or technology shocks. However, several authors have emphasized the importance of wage rigidity for the understanding of the adjustment process of employment to a technology shock. For instance, Shimer (2005) has shown how wage rigidity can account for employment volatility observed in the data.

In his work, Michailat (2014) introduces wage rigidity and shows its implications for the effectiveness of fiscal policy, with a particular focus on the countercyclicality of the fiscal multiplier. In his model, however, job seekers apply randomly to each sector. Despite being a practical shortcoming, this can represent a potential limitation of the model - as shown by Quadrini and Trigari (2007).

In this thesis, the aim is to study fiscal policies in the form of public employment and wages shocks,

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<sup>1</sup>For some very important contributions, see for instance Christiano, Eichenbaum and Rebelo(2011), Eggerston and Krugman(2012), Eggertson and Woodford (2006).

<sup>2</sup>Examples of New Keynesian models with equilibrium unemployment are for instance Walsh (2003), Trigari (2009), Blanchard and Galí (2010), and Monacelli et al. (2010).

<sup>3</sup>During the 2008 financial crisis, US unemployment rate roughly doubled from early 2008 through mid 2010. Also, many other countries suffer high and persistent unemployment.

<sup>4</sup>Many public services - like healthcare and educational services - are provided by the government to the population free of charge. The National Income and Product Accounts (NIPA) value these services at their cost of production; according to the NIPA, during the postwar period, 54.8% of these production costs are compensation of government employees, while only 37.5% is the share of purchase of intermediate goods and services.

and to investigate their effectiveness when the economy is hit by a negative technology shock to simulate a recessionary period. In order to do this, I extend the model of Michaillat (2012) and Michaillat (2014)<sup>5</sup> and allow public sector jobs to differ from private sector jobs. With this characterization, job seekers direct their search toward a specific sector and this, as I will show, can substantially improve our understanding of the role of public employment policies in this model, especially in the short run. In general, in fact, the more the two sectors are modeled differently, the more a public employment shock can have a negative effect on total employment in the short run. Viceversa, in the long run, the effect on employment is always positive.

Another limitation of the model by Michaillat (2014) is that, despite taking into account the rigid adjustment of wages in response to technology shocks, public sector policies have no effect on private labor demand but through an increase in hiring costs. Therefore, I show how a wage specification that takes into account both rigidity and the conditions on the public labor market can substantially alter the conclusions about the effectiveness of fiscal policy.

As a final step, I analyze the role of wage rigidity for employment fluctuations driven by technology shocks. In order to do so, I consider several fiscal rules for public employment and wages. In line with Quadrini and Trigari (2007), I find that the best policy tools are procyclical public employment and wages; I also show that a public employment shock has a much larger effect than public wages.

The thesis is structured as follows. In section 2 I make a review the literature about employment fiscal policies with a focus on public employment stimuli. In section 3 I set up the model; in section 4 I explain the main mechanism behind the introduction of directed search and diminishing marginal returns to labor. In section 5 I calibrate the parameters and in section 6 I simulate the baseline model and consider some extensions. I perform a sensitivity analysis in Section 7 in order to explain the intuition underlying the results. In section 8, I illustrate the business cycle properties of the model and the role of wage rigidity. Section 9 concludes.

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<sup>5</sup>Since these models already embed equilibrium unemployment and the concept of job rationing, they are particularly suitable in order to study the cyclical behavior of unemployment fiscal policies.

## 2 Literature review

Several works have examined the role of fiscal policy to stimulate employment, both theoretically and empirically. I will begin by considering the most important theoretical results that are related to my work.

Monacelli et al. (2010) estimate the effects of fiscal policy in the US labor market. Using a VAR methodology, they find that a 1% increase in GDP generates an unemployment multiplier of 0.6 at the peak. They also simulate an increase in government spending - in the form of government purchases of goods - in a baseline Real Business Cycle model and a New Keynesian model both augmented with labor market frictions à la Mortensen and Pissarides (1994). In their model, an increase in government spending affects the hiring rate through three channels. The first one is given by the fact that, after an government spending shock, taxes rise as well; this lowers the value of non-working activity, which raises the surplus and the hiring rate. There are, however, two channels that work in the opposite direction. The interest rate goes up, driven by the rise in the shadow value of wealth, and this discourages hiring. The last channel works through capital accumulation: a lower expected future capital shock implies a lower marginal product of labor, which decreases hiring. They find that only the Neo Keynesian model (assuming complementarity in utility between consumption and labor) matches the results of the empirical analysis. Despite the fact that in my model the channels through which an increase in government spending affects private employment are different, their paper is one of the first at analyzing the role of fiscal policy in stimulating unemployment<sup>6</sup>.

Ardagna (2007), in the context of a dynamic general equilibrium model with a unionized labor market, studies the effect of government spending in the form of public employment - as I assume in my model. She finds that public employment (as well as public wages and unemployment benefits) lead to lower employment because they increase the wage bargained by unions. However, her model does not take into account the mechanisms that arise when the model is embedded with search and matching frictions. For this reason, it is not directly comparable to my model.

Quadrini and Trigari (2007) study the implications of several exogenous rules for public employment and public wages on the cyclical behavior of the economy within a two-sector model augmented with search and matching frictions. They show that when the government sets acyclical public wages policies, the presence of a large public sector amplifies the effects of a technology shock. This happens because public employment opportunities stabilize the flow value of being unemployed and the threat value in the bargaining game for private wages. As a consequence, the volatility of employment decreases. An important driver of the results in their model is that job seekers direct their search between the two sectors; if the economy is hit by a negative technology shock, firms reduce vacancies and private wages go down; if public wages remain constant, more job seekers will search in the public sector. As a consequence, the vacancy filling probability in the private sector goes down further, and firms posts even less vacancies. This, as we will see, will be an important mechanism present in my model as well.

Gomes (2010) builds, in line with Quadrini and Trigari (2007), a dynamic stochastic general equilibrium model with search and matching frictions to study the effects of fiscal shocks in the form of separation rates shocks, public vacancies shock and public wages shocks. He finds that after a negative public separation

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<sup>6</sup>A previous attempt to study the role of fiscal policy in stimulating employment is Burnside et al. (2004). Their model, however, does not include search and matching frictions.

rate<sup>7</sup> shock unemployment decreases, while a public vacancies shock can increase or decrease unemployment, depending on the steady state value of the public wage premium. A public wages shock always increases unemployment, because while after each shock more unemployed search in the public sector (thus making more difficult for private firms to hire) and the private wage increases because of the increase in the value of unemployment (which further crowds out private employment), a public wage shock is not accompanied by an increase in public employment. He then examines the cyclical properties of the model when the latter is subject to productivity shocks by examining several exogenous wage and public employment rules: he shows that the optimal government policy (i.e., the one that corresponds to a lower volatility of employment) is to set counter-cyclical vacancies and pro-cyclical public wage. This happens because in the latter model the wage premium is exogenously set, while in Gomes (2010) it is determined optimally: in fact, depending on the steady state level of public wages, the crowding out of the public sector can more than compensate the increase in public employment.

Michaillat (2012) introduces a labor market model with search and matching frictions in which jobs are rationed: unemployment arises not only because of matching frictions, but because (since diminishing marginal returns and rigid responses of private-sector wages to productivity shocks are assumed) the marginal returns to labor of the last workers are below the wage, thus making it inefficient for the firm to hire these workers. Importantly, the latter effect is stronger in recession since in this case the private-sector wage is high relative to marginal productivity of labor, and more workers are rationed. Viceversa, since unemployment is higher (and tightness is lower) in recession and recruiting costs depend on the level of technology, frictional unemployment is high in expansion and lower in recession. Michaillat (2014) develops this framework by including in it a public sector. He studies the effect of a public employment shock and shows that, while the latter always crowds out private employment, the overall effect on employment is positive: consequently, the fiscal multiplier is always positive (more precisely, he is able to obtain a close form expression for the fiscal multiplier; with his assumptions, the latter is always positive and counter-cyclical by construction; dynamic simulations confirm this result). Furthermore, the latter is higher in recession since the crowding out effect is lower when the level of technology is lower; this happens because recruiting cost are a small part of the marginal cost of hiring a worker, and the increase in labor market tightness that arises after a public employment shock has a smaller effect on the firms optimal employment choice. It is important to notice that, while this is a very suitable model to study fiscal policy in recessions, wage are not the outcome of a bargaining game as in Gomes (2010) and Quadrini and Trigari (2007), but only depends on technology. This eliminates the crowding out effect that arises after a public employment shock when wages are negotiated between firms and workers described above. Another important limitation is that in his model, Michaillat assumes that private and public sector wages and separation rates (as well as job finding and job filling probabilities) are equal<sup>8</sup>. As I will show in the chapter devoted to the calibration of my model, this is a very strong assumption, and I will drop it in my model.

From the empirical side, it is important to take into account the stage of the business cycle to estimate the fiscal multiplier since as it is assumed in Michaillat (2014), the size of the latter can vary depending on the technology and unemployment levels. Auerbach and Gorodnichenko (2012a, 2012b) estimate gov-

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<sup>7</sup>In his setting, a public negative separation rate shock means that at the time of the shock, the government reduces the separation of jobs in the public sector, thus increasing public employment.

<sup>8</sup>With this assumptions, it is safe to assume also that job-seekers apply randomly to each sector as in Michaillat (2014).

ernment purchase multipliers for several countries; in their work, multipliers are allowed to vary smoothly according to the state of the economy. They find that GDP multipliers as well as the multipliers of other key macroeconomic variables are higher when the economy is slack.

Using the projection method developed by Jorda (2005), Owyang et al.(2013) find evidence that multipliers can more than double in recessionary periods of the Canadian economy from 1921 to 2011<sup>9</sup>, while they find no evidence using US data for the period 1890-2010.

The data seem to suggest that the use of fiscal policy in recession is a reliable candidate for substituting monetary policy, especially when the latter is ineffective. The aim of this thesis, though, is not to evaluate the effectiveness of fiscal policies in the data, but rather to provide the contribution about the public employment multiplier across different stages of the business cycle. Other models that consider increasing public employment are Quadrini and Trigari (2007), Gomes (2010) and Michaillat (2014). My model is different to the latter because of the important assumption of directed search<sup>10</sup> which, as we will see, will have important effects on the effectiveness of the fiscal policy considered.

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<sup>9</sup>In their work, they use a threshold model in which a recessionary period is denoted by a high level of unemployment (6.5% for US, 7% for Canada).

<sup>10</sup>Which is present in Quadrini and Trigari (2007) and Gomes (2010).

### 3 The model

#### 3.1 General setting

In this section, I build a labor market model with search frictions along the line of Mortensen and Pissarides (1994), and Pissarides (2000). The general setting shares some features from Michailat(2014), Gomes(2010) and Quadrini and Trigari (2007). The labor force is made by a measure 1 of identical workers. There are two sectors: the government sector and the private sector. A worker can be unemployed  $u_t$ , or employed either in the private ( $l_t$ ) or in the public sector ( $g_t$ ):

$$1 = g_t + l_t + u_t \tag{1}$$

The process of hiring takes place through searching and matching. At the end of period  $t - 1$ , a constant fraction  $\lambda$  of the existing worker-job matches is exogenously destroyed. Unemployed who find a job start working in period  $t$ , together with the share of job-matches that are not destroyed in periods  $t - 1$ , given by  $(1 - \lambda) \cdot n_{t-1}$ , where  $n_t$  is total employment level at time  $t$ . Workers who lose their job can apply for a new job immediately. This gives us the fraction of unemployed workers searching for a job at the beginning of period  $t$ :

$$u_t^p = 1 - (1 - \lambda^p) \cdot l_t,$$

in the private sector and

$$u_t^g = 1 - (1 - \lambda^g) \cdot g_t$$

In the setting in which public wages are equal to private wages, it is reliable to assume that job-seekers apply to each sector randomly. Here, since public wages differ from private wages, I assume that job-seekers direct their search towards the private *or* the public sector. Both firms and government post vacancies  $v_t$  in order to hire workers. Following the literature, I assume that the number of matches is given by a Cobb-Douglas function, given by:

$$h_t^i = \mu^i \cdot (u_t^i)^{\eta^i} \cdot (v_t^i)^{1-\eta^i},$$

where  $i = p, g$  and the parameters  $\mu^i$  and  $\eta^i \in (0, 1)$  give us, respectively, the efficiency of the matching process and the elasticity of the matching function with respect to unemployment. Henceforth, we define labor market tightness (i.e. the number of vacancies per job-seeker) in each sector as  $\theta_t^i \equiv v_t^i/u_t^i$ . The probability of finding a job in each sector is defined as  $f(\theta_t^i) = h_t^i/u_t^i$ , while the probability of filling a vacancy is defined as  $q(\theta_t^i) = h_t^i/v_t^i$ .

#### 3.2 The household

We assume that all workers belong to a large household that consumes a final good  $c_t$  (purchased from firms) and a public good  $z_t$  (provided free of charge by the government). Household utility takes the form:

$$E_0 \sum_{t=0}^{+\infty} \beta^t \cdot [\ln(c_t) + \varphi \cdot \ln(z_t)],$$

where  $E_0$  is the expectation operator conditional on information at time 0, and the parameter  $\varphi$  measures the taste for the public good. Consumption is financed by income, which is the sum of two elements: real income  $w_t^g \cdot g_t + w_t^p \cdot l$ , and real profits  $\Pi_t$ , which are distributed by firms to households since it owns them. I follow the standard formulation introduced in Merz (1995) that workers pool their income before choosing consumption and savings. With the matching process we defined above, employment in each sector evolves according to the following equation:

$$l_t = (1 - \lambda^p) \cdot l_{t-1} + u_t^p \cdot f(\theta_t^p), \quad (2)$$

in the private sector and

$$g_t = (1 - \lambda^g) \cdot g_{t-1} + u_t^g \cdot f(\theta_t^g), \quad (3)$$

in the public sector.

### 3.3 Workers

I now define the share of unemployed workers searching in the public sector as

$$\sigma_t = \frac{u_t^g}{u_t}.$$

Since some of the results are driven by the fact that job seekers direct their search between the two sectors, this is a key variable in my model. I then write the value of being unemployed and of being employed in each sector. These values are needed to determine the equilibrium value of  $\sigma_t$ . In most models with search and matching frictions, these values are also needed to determine equilibrium wage. This happens because in the latter models, wages are usually the outcome of a bargaining game (see for instance Pissarides (2000) or Shimer (2005)); this is due to the fact that a positive surplus (the difference between the marginal product of labor or the value of a job, that determines the job creation condition, and the flow value of unemployment) must be shared between workers and firms. As we will see in section 3.5, I assume the real wage to be exogenously set, so the only reason why I need to specify the value of unemployment and of a job in each sector is that they determine  $\sigma$ .

I now derive the value function explicitly. Since the derivation is of course the same in the two sectors, I will explain it in a more general case.

I start from deriving the value of unemployment. The problem is to maximize the function:

$$\Omega_t^u = u(u_t) + \beta \cdot E_t(u_{t+1}),$$

subject to the family's budget constraint



$$c_t = n_t \cdot w_t + \Pi_t$$

and the law of motion of total employment  $n_t$ , given by:

$$n_{t+1} = (1 - \lambda)n_t + f(\theta_t) \cdot u_t$$

where unemployment is defined as  $u_t = 1 - n_t$ .

The FOC for the problem is:

$$\tilde{\mathcal{U}}_t(u) = u'_{u,t} + \beta \cdot E_t [\tilde{\mathcal{U}}_t(u)], \quad (4)$$

where I defined  $\tilde{\mathcal{U}}_t = \frac{\partial \Omega_t^u(u_t)}{\partial u_t}$  and  $u'_{u,t} = \frac{\partial u(u_t)}{\partial u_t}$ . Now rewrite equation (4) as:

$$\tilde{\mathcal{U}}_t(u) = \beta \cdot E_t [\tilde{\mathcal{U}}_t \cdot (1 - f(\theta_t)) + \tilde{\mathcal{W}}_t \cdot f(\theta_t)] \quad (5)$$

since by assumption the utility from unemployment is simply zero. Discounted back at time  $t$ , equation (5) becomes:

$$\mathcal{U}_t = \beta_{t,t+1} \cdot E_t [\mathcal{U}_t \cdot (1 - f(\theta_t)) + \mathcal{W}_t \cdot f(\theta_t)]$$

The derivation of the value of employment follows the same process, except that now the term  $u'_{n,t}$  is equal to the wage  $w_t$ .

Therefore, I can write the value of being employed in each sector is given as:

$$\mathcal{W}_t^i = w_t^i + E_t \beta_{t,t+1} [(1 - \lambda^i) \mathcal{W}_{t+1}^i + \lambda^i \mathcal{U}_{t+1}^i], \quad i = p, g$$

In words, the value of being employed in a given sector is given by the sum of the wage earned in that sector and the continuation value of the job: with probability equal to  $(1 - \lambda^i)$  the job will not be destroyed in the next period; viceversa, with probability  $\lambda^i$  the job will be destroyed and the member becomes unemployed. The term  $\beta_{t,t+1}$  is the stochastic discount factor and is equal to  $\beta \frac{u'_c(c_{t+1})}{u'_c(c_t)}$ . The value of being unemployed is given by:

$$\mathcal{U}_t^i = E_t \beta_{t,t+1} [(1 - f_t^i) \mathcal{U}_{t+1}^i + f_t^i \mathcal{W}_{t+1}^i], \quad i = p, g$$

where  $f_t^i = f(\theta_t^i)$  and is on the probability of finding a job in that sector ( $f_t^i$ ) as well as, through the term  $\mathcal{W}_{t+1}$ , on the separation rate in that sector ( $\lambda^i$ ) and on the wage in that sector ( $w_t^i$ ).

Since workers can search in each sector without restrictions, optimality implies that the values of searching in the private or the public sector must be equal, i.e.:

$$\mathcal{U}_t^g = \mathcal{U}_t^p = \mathcal{U}_t.$$

From this equality, we have that

$$\frac{h_t^p E_t [x_{t+1}^p]}{1 - \sigma_t} = \frac{h_t^g E_t [x_{t+1}^g]}{\sigma_t}, \quad (6)$$

where  $x_{t+1}^i = \mathcal{W}_{t+1}^i - \mathcal{U}_{t+1}$ .

An increase in the value of being employed in the public sector - driven for instance by an increase in the wage premium over the private sector wage - has the effect of raising  $\sigma_t$ : more unemployed search in that sector because now they can earn more than before the shock.  $\sigma_t$  then increases until the equality in equation (6) is restored.

Therefore, we can write the job flows dynamics for each sector as

$$l_t = (1 - \lambda^p) \cdot l_{t-1} + (1 - \sigma_t) \cdot u_t \cdot f(\theta_t^p), \quad (7)$$

in the private sector and

$$g_t = (1 - \lambda^g) \cdot g_{t-1} + \sigma_t \cdot u_t \cdot f(\theta_t^g), \quad (8)$$

in the public sector.

### 3.4 Firms

Firms produce the final good and sell it on a perfectly competitive market. I assume that the representative firm only uses labor  $l_t$  to produce output  $y_t$ ; following Michailat (2014), I assume that the production function has diminishing marginal return to labor (we will see later how this represents a fundamental assumption in the model, in order to have rationing unemployment); hence, the production function takes the form  $y_t = a_t \cdot l_t^\alpha$  where  $\alpha \in (0, 1)$ . Firms pay a real wage  $w_t^p$  to its employees, but incurs a per-period cost to keep a vacancy open given by  $r \cdot a_t > 0$ , expressed in units of final good; we assume that there is no randomness at the firm level, i.e. in period  $t$  the firm hires  $[l_t - (1 - \lambda^p) \cdot l_{t-1}]$  workers; the expected time needed to fill a vacancy is given by  $1/q(\theta_t^p)$ ; therefore, the expected vacancy cost is given by the term  $r \cdot a_t/q(\theta_t^p)$ . Firm's real profits are defined as:

$$a_t \cdot l_t^\alpha - w_t^p \cdot l_t - \frac{r \cdot a_t}{q(\theta_t^p)} \cdot [l_t - (1 - \lambda^p) \cdot l_{t-1}] \quad (9)$$

Given the processes  $\{\theta_t\}_{t=0}^{+\infty}$  and  $\{w_t^p\}_{t=0}^{+\infty}$ , the firm maximizes the discounted sum of real profits. We can now write the firm's labor demand, which is given implicitly by:

$$\alpha \cdot a_t \cdot l_t^{\alpha-1} = w_t^p + \frac{a_t \cdot r}{q(\theta_t^p)} - \beta \cdot (1 - \lambda^p) \cdot \frac{a_{t+1} \cdot r}{q(\theta_{t+1}^p)}, \quad (10)$$

and is obtained by maximizing (9).

### 3.5 Wages

The presence of a positive surplus to be shared between workers and firm implies that more wages may be consistent with equilibrium; in other words, private wages are privately efficient as long as both parties (workers and firms) get a positive surplus. This problem is commonly resolved using the Nash bargaining solution. Henceforth, I assume that the private real wage is given by an exogenous wage rule. In the literature, some examples of the use of this assumption are given by Michailat (2012) and Hall (2005).

Following Blanchard and Galí (2010), I assume that the real wage is a function of technology:

$$w_t^p = \omega \cdot a_t^\gamma, \quad (11)$$

where  $\omega$  is a parameter and measures the steady state level of the real private wage, while  $\gamma$  measures the degree of rigidity of the latter in response to a technology shock. Wages can remain rigid for several reasons. Labor market institutions, for instance, can impede the wage-adjusting process (Gorodnichenko et al., 2012); firms' managers may not want to cut wages in order to avoid antagonizing workers (for a survey, see Bewley (1999)). Campbell and Kamlani (1997) conduct a survey in order to find reason that could explain wage rigidity; among these, explanations concerned with adverse selection and the effect of wages on effort seem the most likely to explain this rigidity. Baker et al. (1994) find evidence that wages are only partly adjusted with respect to external labor market conditions, which justifies the fact that  $\gamma$  is lower than one. Public wages are set to be higher than private sector wages. Therefore, I assume that public sector wages are a positive and constant function of private sector wages, of the form:

$$w_t^g = \pi \cdot \bar{w}^p, \quad (12)$$

for every  $t$ , where  $\bar{w}^p$  is the steady state level of private sector wages. This is equivalent to say that while private sector wages respond to technology shocks, public sector wages are completely fixed. This, as we will see, will have an important impact on the results.

The public wage premium  $\pi$  is an arbitrary parameter, assumed to be constant over the cycle. A discussion of the calibration of this parameter is provided in section 5.

### 3.6 Government and resource constraints

I follow Michailat (2014) and assume that the government produces a public  $z_t$  using the production function  $z_t = \varsigma \cdot g_t^\alpha$ , where  $\varsigma$  is the productivity of the government and  $g_t$  is the public employment level. The government balances its budget each period:

$$g_t \cdot w_t^g + [g_t - (1 - \lambda^g) \cdot g_{t-1}] \cdot \frac{r \cdot a_t}{q(\theta_t^g)} = \tau_t \quad (13)$$

The left hand side of equation (13) is government expenses, i.e. the sum of compensation to public employees and the cost of hiring public workers, analogous to that of the private sector. On the right hand side we find lump sum taxes  $\tau_t$  used to finance the wage bill and the hiring costs.

Using the household's budget constraint, the definition of profits and equation (13), I can write the economy's resource constraint as<sup>11</sup>:

$$y_t = c_t + \frac{r \cdot a_t}{q(\theta_t^p)} \cdot [l_t - (1 - \lambda^p) \cdot l_{t-1}] + \frac{r \cdot a_t}{q(\theta_t^g)} \cdot [g_t - (1 - \lambda^g) \cdot g_{t-1}], \quad (14)$$

which simply says that output is either consumed, or devoted to hiring costs in the two sectors.

### 3.7 Equilibrium

**Definition** Given an initial level for total employment  $n_{-1} = l_{-1} + g_{-1}$  and the stochastic processes for  $\{a_t, g_t\}_{t=0}^{+\infty}$ , the equilibrium of the economy is defined as a set of stochastic processes for nine variables  $\{l_t, u_t, \theta_t^p, \theta_t^g, c_t, y_t, w_t^p, w_t^g, \sigma_t\}_{t=0}^{+\infty}$  that satisfy nine relationships:

- aggregate labor demand,  $l_t + g_t = n_t$ ;
- the law of motion of private employment, equation (7);
- the law of motion of public employment, equation (8);
- the production function,  $y_t = a_t \cdot l_t^\alpha$ ;
- the firm's labor demand, equation (10);
- the economy's resource constraint, equation (14);
- the equality between the values of searching in each sector, equation (6)
- the private wage schedule, equation (11);
- the public wage schedule, equation (12);

Importantly, the level of public employment  $g_t$  is exogenously determined by the government. In section 6 I will define explicitly when and how a public employment shock takes place; notice that, if the government does not increase or decrease the level of public employment  $g_t$ , the latter is assumed to be constant at  $g_t = \bar{g}$  and independent of the level of private employment.<sup>12</sup>

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<sup>11</sup>The explicit derivation of the resource constraint is provided in appendix B.

<sup>12</sup>Quadrini and Trigari (2007), by contrast, consider different levels of cyclical public employment: in other words, they allow the latter to be a positive function of private employment.

## 4 Properties of the model

### 4.1 Steady state

In this section, I depict the equilibrium when the economy is in steady state. For simplicity, assume that the real wage  $w^p$  and public employment  $g$  are fixed. Firstly, I determine aggregate labor demand  $n^d(w^p, g, \theta^p, \theta^g)$  is the sum of public employment  $g$  and firm's labor demand  $l^d$ , which can be found by maximizing (9) and which in steady state is given by:

$$\alpha \cdot l^{\alpha-1} = w^p + [1 - \beta \cdot (1 - \lambda^p)] \cdot \frac{r}{q(\theta^p)} \quad (15)$$

or, in explicit form:

$$l^d(w^p, \theta^p) = \left[ \frac{1}{\alpha} \cdot \left\{ w^p + [1 - \beta \cdot (1 - \lambda^p)] \cdot \frac{r}{q(\theta^p)} \right\} \right]^{\frac{-1}{1-\alpha}} \quad (16)$$

Quasi-labor supply  $n_t^s$  is given by the sum of  $l_t$  and  $g_t$  given, respectively, by (7) and (8).

The steady state equilibrium consists of two variables ( $n, \theta$ ) and two equilibrium relationships, which are given by  $n^s(\theta^p, \theta^g, \sigma) = n^d(w^p, g, \theta^p)$  and

$$n = n^d(w^p, g, \theta^p)$$

where

$$n^d = g + l^d(w^p, \theta^p, a)$$

Notice that in this model, job are *rationed* in the sense of Michailat (2012). Job rationing is an important feature of the model and deserves to be explored: in this framework, it arises as the sum of two elements - wage rigidity and diminishing marginal returns to labor. The mechanism leading to job rationing is the following. Suppose, for the moment, that there are no recruiting expenses ( $r = 0$ ). In this case, the firms' optimal employment choice - equation (16) - becomes

$$a \cdot \alpha \cdot l^{(\alpha-1)} = w$$

which simply says that the marginal product of labor equals the real wage (which adjust only partially to technology). Suppose now there is a negative technology shock: while the marginal product of labor adjusts proportionally to  $a$ , real wage adjusts only partially because of the assumed wage rigidity. Given that, because of the assumed diminishing marginal returns to labor, the marginal product of labor is a negative function of employment  $l$ , the marginal product of the least productive workers falls below the wage: these workers are not hired by the firms, and rationing unemployment arises. Notice that this happens even in the case in which  $r = 0$ , so that matching frictions are absent and therefore frictional unemployment (unemployment that arises because of matching frictions) is zero. To understand why both real wage rigidity and diminishing marginal returns to labor are necessary to generate rationing unemployment, suppose that the real wage adjust completely to a technology shock. In this case, the wedge between marginal product of

labor and real wage after the technology shock would not arise: the aggregate labor demand is invariant to technology shocks. As a consequence, rationing unemployment would be zero. Now, suppose instead that real wage is rigid, but that marginal returns to labor are constant. When the technology level is high enough, the wage remains above the marginal product of labor (which, with constant marginal returns, is simply the technology level  $a_t$ ), and we have no unemployment, since it is profitable for firms to hire workers: the equilibrium level of employment is 1. Viceversa, when technology level is low enough, the wage is above the marginal product of labor, and we have rationing unemployment. In this way, we can see that rationing unemployment arises if the level of technology is low, as it happens in recessionary periods<sup>13</sup>. To sum up, when technology level is low enough and the recruiting costs are different from zero, unemployment in the economy is the sum of frictional and rationing unemployment.

## 4.2 A static version of the multiplier

I now analyze the properties of the model in its static version. For simplicity, in this section I will make some assumptions that make it possible to write the multiplier in a simple and tractable form and analyze graphically the steady state effects of an increase in public employment  $g_t$ . Assumptions 1 and 2 are useful to gain the intuition, but are unrealistic and I will drop them when I will evaluate the dynamic responses of the model.

**Assumption 1** For any technology level  $a$ , the government policy is to set public employment as a constant fraction of total equilibrium employment  $n$ ,  $g = \xi \cdot n$ , and  $0 < \xi < 1$ .

**Assumption 2** Separation rates in the two sector are equal, i.e.  $\lambda^p = \lambda^g = \lambda$ .

Assumption 2 allows us to write the labor supply in a simple form:

$$n^s(\theta, \sigma) = \frac{[1 + \sigma \cdot (\psi - 1)] \cdot f(\theta)}{\lambda + (1 - \lambda) \cdot [1 + \sigma(\psi - 1)] \cdot f(\theta)}, \quad (17)$$

where I defined  $\psi$  as the ratio of the job-finding probabilities  $\frac{f(\theta^g)}{f(\theta^p)}$ . It is now easy to write the multiplier in closed form:

$$\frac{\partial n}{\partial g} = \frac{\partial n^d}{\partial g} \cdot \left[ 1 - \frac{1}{1 + (\frac{\epsilon^s}{\epsilon^d})} \right] + \frac{1}{1 + (\frac{\epsilon^s}{\epsilon^d})} \cdot \frac{\partial n^s}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial g}, \quad (18)$$

where I defined the elasticities to tightness of labor supply and labor demand as  $\epsilon^s \equiv (\theta/n^s) \cdot (\partial n^s / \partial \theta) > 0$  and  $\epsilon^d \equiv -(\theta/n^d) \cdot (\partial n^d / \partial \theta) > 0$  (normalized to be positive). The first one is counter cyclical, while the second one is pro cyclical.

I begin by considering the effect of an increase in public employment  $g$  on total labor demand  $n^d$  (suppose for a second that the quasi labor supply does not move after an increase in  $g$ ) in a  $(n, \theta)$  plot, where  $\theta$  is defined as total labor market tightness  $\frac{v}{u}$ . Of course, this has the effect of shifting outward the aggregate

<sup>13</sup>For a formal definition, see Proposition 3 and 4 in Michaillat (2012).

labor demand. The quasi labor supply falls short of the aggregate labor demand: as a consequence, tightness increases to reach the new equilibrium.<sup>14</sup>

Because of the increase in tightness due to the fact that there are less unemployed available on the labor market and that government posts more vacancies, the vacancy filling rate falls and the cost of hiring an extra worker for the private firm rise, which lead firm to reduce employment. Therefore, the effect of a positive public employment shock is the crowding out of private employment. Now the question is: by how much does an increase in  $g$  crowds out  $l$ ? This is equivalent to asking ourselves if the shock in  $g$  causes an increase in the aggregate labor demand  $n^d$ . To answer this question it is enough to look at what would happen if the decrease in  $l$  would be quantitatively equal to the increase in  $g$ : in that case, the new equilibrium (that is, the equilibrium that results as the outcome of the increase in  $g$ ) would be characterized by the same level of labor market tightness but lower private employment. Given that the tightness is the same as before, the marginal cost of labor (which in turns depends on the job filling rate  $q(\theta_t^p)$ ) would be also unchanged. With a lower level of private employment  $l$ , though, the marginal product of labor would be higher; this means that the firm's optimal employment choice - equation (15) - would be violated. Hence, private employment must decrease<sup>15</sup>. This first channel is captured by the first term on the right-hand-side of equation (18). Since  $\epsilon^s$  is counter cyclical and  $\epsilon^d$  is pro cyclical, this first term is counter cyclical.

By assuming that public wages differ to private sector wages (and that the probabilities of finding a job in the two sectors are different), we are assuming that job seekers direct their search toward either the public or the private sector. This introduces another channel to the model when the government increases  $g$ . To understand why, recall that in the mechanism described above, we have assumed that labor supply does not respond to an increase in  $g$ . This is, however, what would happen if I assumed that job seekers apply to each sector randomly, with no active search decision - as in Michailat (2014). On the contrary, the labor supply now does respond to an increase in public employment or public wages through the reaction of  $\sigma_t$ . The mechanism of this additional channel is very simple and is made of two steps: after an increase in  $g_t$  or  $w_t^g$ , the share of unemployed that direct their search toward the public sector,  $\sigma_t$ , mechanically increases (see equation (6)), either because now the probability to find a job is higher, or they can get a higher wage than before. The reaction of the labor supply  $n_t^s$  to an increase in  $\sigma_t$  depends on the probabilities of finding a job in each sector. Indeed  $\psi_t = \frac{f(\theta_t^p)}{f(\theta_t^s)} > 1$  implies that  $\frac{\partial n^s}{\partial \sigma} > 0$ . The intuition is simple: if more job seekers search for a job in the public sector, where the probability of finding a job is higher, more job seekers are hired and total labor supply increases. Viceversa, if  $\psi_t < 1$ , a larger number of workers remain unemployed. The latter mechanism is captured by the second term on the right-hand-side of equation (18), which is present if and only if we assume directed search among job seekers. Because of the characteristics of  $\epsilon^s$  and  $\epsilon^d$ , this term is pro cyclical.

It is useful to analyze graphically what happens to the economy when a public employment occurs, taking into account both channels through which the economy reacts. In figure 1 and 2, I consider the cases in which the economy is in expansion and recession in a  $(\theta, n)$  plane. After an increase in  $g$ , aggregate labor demand

<sup>14</sup>Notice that here I assumed that the government can directly increase  $g$ , and the equilibrium is reached through the adjusting of vacancies. I could have easily reversed my speech by assuming that the government directly posts the number of public vacancies  $v^g$ . In the latter case,  $g$  adjusts to reach the equilibrium through the equation  $g_t = (1 - \lambda^g) \cdot g_{t-1} + u_t^g \cdot f(\theta_t^g)$  which, in steady state, simply becomes  $g = \frac{u^g \cdot f(\theta_g)}{\lambda^g}$ .

<sup>15</sup>Notice that the assumption of diminishing marginal returns to labor is necessary for this last effect to take place.

shifts to the right because - as we have seen - the increase in public employment more than compensate the negative reaction of private employment  $l$ . As a consequence, tightness and total employment increase. Assuming that  $f(\theta^g)$  is lower than  $f(\theta^p)$ ,  $n^s$  shifts to the left after the public employment shock. As a result, equilibrium tightness further increases and equilibrium employment is lower than the value that arises after the reaction of  $n^d$ . The shift in aggregate labor demand causes a higher increase in labor market tightness (and therefore a higher crowding out effect on private employment) when the economy is in expansion (figure 1), because of the convexity of the labor supply. Viceversa, the shift in labor supply driven by the public employment shock is higher in expansion. The dynamic simulations in section 6 will confirm these theoretical results.



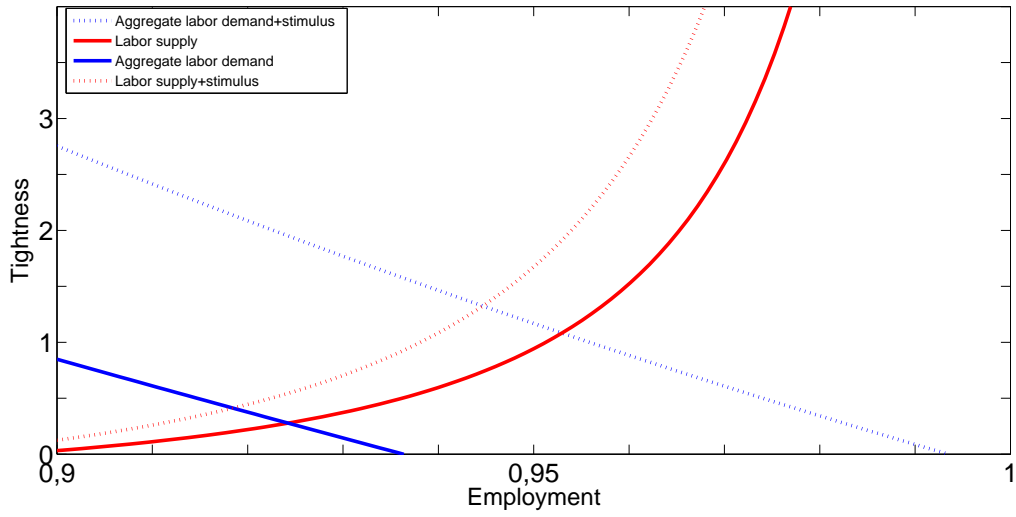


Figure 1: Equilibrium in steady state - recession scenario

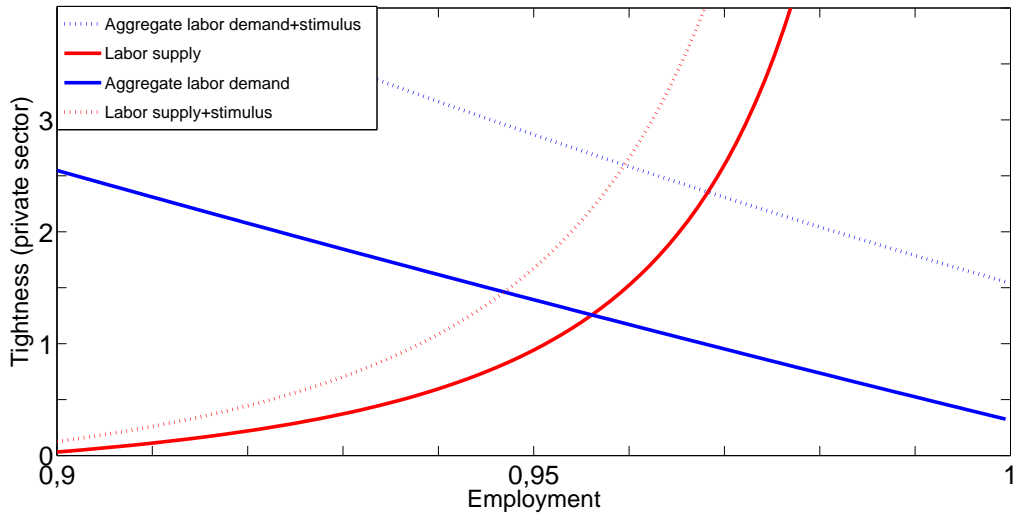


Figure 2: Equilibrium in steady state - expansion scenario

## 5 Calibration

In this section, I describe the assign a value to the parameters of the model. I calibrate the model in order to capture U.S. labor market data. Table 1 and Table 2 summarize, respectively, the calibration of the parameters and the steady state values. Plots of some of the time series used to calibrate the parameters are provided in the appendix.

I set the production function parameter  $\alpha$  to 0.66 (which implies diminishing marginal returns to labor). The discount factor parameter  $\beta$  is conventionally set to 0.999.

**Labor market parameters** - Firstly, I calibrate the elasticities of the matching function in both sectors with respect to unemployment,  $\eta^p$  and  $\eta^g$ . In their survey, Petrongolo and Pissarides (2001) suggest that (in a one sector model) this value should be set between 0.5 and 0.7. Gomes (2010) estimates his model using data from the US economy with Bayesian methods, and finds posterior means for  $\eta^p$  and  $\eta^g$  equal to 0.647 and 0.159, respectively. I therefore assign these values to the elasticities in the two sectors. Following Michailat(2014), I set the wage flexibility  $\gamma$  equal to 0.7. This value is taken from Pissarides (2009) and Haefke et al. (2008).

I use the standard notation  $\bar{x}$  for denoting the steady state value of a variable  $x$ . I estimate the steady state value for unemployment by taking the average of the unemployment rate during the postwar period, using data from the U.S. Bureau of Labor Statistics (BLS henceforth) . I find a value of  $\bar{u} = 0.066$  for the steady state level of unemployment. Then, I estimate the steady state level of private and (public) employment by taking the average of the fraction of the labor force employed in the private and (public sector), during the postwar period, using the same data. I find, respectively,  $\bar{l} = 0.781$  and  $\bar{g} = 0.153$ . I also set  $\pi$ , the steady-state public wage premium (defined as the ratio  $\frac{\bar{w}^g}{\bar{w}^p}$ ) at 1.03. This is an arbitrary value taken from the range between 0% and 10% of possible values that Gregory and Borland (1999) suggest.

Steady state target	Notation	Value	Source
Technology	$\bar{a}$	1	Convention
Unemployment	$\bar{u}$	0.066	BLS,1948-2013
Public employment	$\bar{g}$	0.153	BLS,1948-2013
Private employment	$\bar{l}$	0.781	BLS,1948-2013

Table 1: Steady state variables

Next, I take the average separation rate for the public and the private sector from BLS data, during 2001-2013, and find that they are  $\lambda^p = 0,00984$  and  $\lambda^g = 0,0035$ . Finally, for simplicity, I assume that the cost of posting a vacancy is fixed and equal in each sector, and is given by  $r = 0.32 \cdot \omega$ , where  $\omega$  is the steady state real wage level. This is in line with Michailat (2014), Barron et al. (1997) and Silva and Toledo (2009). Davis, Faberman and Haltinwanger (2010) estimate that the mean duration of a vacancy is 20 days for the private sector, and 30 days for the public sector. therefore, I set  $\bar{q}^p = 0.35$  and  $\bar{q}^g = 0.224$  in order to match their estimates at a weekly frequency.

**Other parameters** - Following the standard RBC literature, I assume that log-technology  $\log(a_t)$  follows

a standard  $AR(1)$  process of the type  $\log(a_{t+1}) = \rho_a \cdot \log(a_t) + \epsilon_t^a$ , where  $\epsilon_t^a$  is a *iid* random shock with zero mean and normally distributed. The auto-regressive parameter  $\rho_a$  is conventionally set to 0.95. I then normalize the steady state value of technology to  $\bar{a} = 1$ . The real wage level is recovered from the firm's labor demand in steady state: I find that  $\omega = 0,7108$ . With this calibration, I am also able to find the steady state value of one of the key variables in the model,  $\bar{\sigma}$ : I find a value of 0,19, which is slightly lower than the one found by Gomes(2010), where  $\bar{\sigma} = 2$ . Finally, I set the matching effectiveness in each sector so that it matches the equality between new hires in steady state and the matching function:  $h^i = \mu^i \cdot (\bar{u}^i)^{\eta^i} (\bar{v}^i)^{1-\eta^i}$ . I find, respectively,  $\mu^p = 0,234$  and  $\mu^g = 0,366$ .

Parameter	Notation	Value	Source
Production function parameter	$\alpha$	0.66	Convention
Discount factor	$\beta$	0.999	Convention
Elasticity of the matching function to $u_t^p$	$\eta^p$	0.647	Gomes(2010)
Elasticity of the matching function to $u_t^g$	$\eta^g$	0.159	Gomes(2010)
Separation rate (private sector)	$\lambda^p$	0.00984	BLS, 2001-2013
Separation rate (public sector)	$\lambda^g$	0.0035	BLS, 2001-2013
Public wage premium	$\pi$	1.03	Gregory and Borland (1999)
Real wage level (private sector)	$\omega$	0.7108	Matches steady state targets
Unemployed searching in the public sector	$\bar{\sigma}$	0.19	Matches steady state targets
Elasticity of real wage to technology	$\gamma$	0.5	Pissarides (2009), Haefke et al. (2008)
Matching effectiveness (private sector)	$\mu^p$	0.234	Matches steady state targets
Matching effectiveness (public sector)	$\mu^g$	0.366	Matches steady state targets
Recruitment cost	$r$	0.227	Michaillat (2014)
Autocorrelation of log-technology	$\rho_a$	0.95	Convention

Table 2: Baseline calibration of the model

## 6 Quantitative analysis

### 6.1 Public Employment shock

As a first step, I simulate the baseline model after a public employment shock and compute the public employment fiscal multiplier. The aim of this section is two-fold: first, I evaluate the impact of introducing two distinct sectors in the economy and, by consequence, the impact that the assumption of directed search (as opposed to random search) has not only on the size and the sign of the multiplier, but also on its cyclical properties; in order to do that, I build an extension of the model where job seekers apply randomly to each sector<sup>16</sup>. Second, I compare my model results with those obtained by Quadrini and Trigari (2007) and, starting from the latter, Gomes (2010)<sup>17</sup>. Recall that, in the latter models, the production function is assumed to be of the form:  $y_t = a_t \cdot l_t$ . As a consequence, the aggregate labor demand is flat since it does not depend on  $\theta$  (see the derivation of the firm's labor demand, Section 4), and there is no shift in the labor demand after a public employment shock. To sum up, in my model a public employment shock causes both a labor supply to shift (because of the assumption of directed search), and an aggregate labor demand shift (because of the assumption of diminishing marginal returns to labor). The multiplier will then embed these two effects<sup>18</sup>.

I simulate an approximation of the response of the calibrated model of section 4 to a public employment shock only. Notice that, as in Michailat (2014), one aim is to capture the non-linearities of the model when the economy departs from the steady state a shock<sup>19</sup>; I therefore cannot follow the conventional procedure of log-linearizing the model around a steady state. I will also assume that agents have perfect foresight: they perfectly anticipate the time path of all relevant variables after the shock takes place. The shock corresponds approximately to a hiring of 0.1% of the public labor force. The shock hits the economy in this way: assume that at time  $t = 0$ , the level of public employment is  $g^*$ . At time  $t = 1$ , the shock occurs, and the government hires more workers than the equilibrium level:  $g_1 = g^* + 0.00153$ , where 0.00153 is the 0.1% of the public labor force. Henceforth, I assume that the public wage is kept fixed by the government at its steady state level,  $w^g = \pi \cdot \bar{w}^p$ . At time  $t = 2$ , the government hires as many workers as before the shock.

A positive public employment shock crowds out private employment. This happens because the probability of getting a job in the public sector is now higher than before thanks to the new vacancies posted by the government, and the share of unemployed seeking for a job in the public sector jumps at the time of the shock; this reduces the firm's vacancy filling probability  $q(\theta_t^p)$  and increases hiring costs, thus lowering private employment - see the firm's optimal choice, equation (16). I plot in Figure 3 and Figure 4 the Impulse Response Functions of my model, compared to that of - respectively - the model with Random search and the model with constant marginal return to labor for a time horizon of 120 weeks. After the increase in public sector employment, private employment is crowded out because of the above mentioned mechanism. As a consequence, overall unemployment decreases. After the public employment shock at  $t = 1$ , the share of unemployed searching in the public sector  $\sigma_t$  decreases, and private employment quickly builds up and

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<sup>16</sup>The fully described model with random and directed search is provided in Appendix A.

<sup>17</sup>The only difference with these models is that they assume flexible wages; in my baseline model and in its extensions, I assume rigid wages à la Blanchard and Galí (2010)

<sup>18</sup>To my knowledge, my model is the first to study these two effects at the same time.

<sup>19</sup>These non-linearities arise because of the convexity of the quasi-labor supply: see section 4.

converges back to its steady state value. It takes more time for public employment to reach its steady state, and overall unemployment becomes *lower* than its steady state value of 6.6% after three quarters. Therefore, from the response of the model to a public employment shock only, we can conclude that the effect of the latter on total unemployment varies and depends on the time horizon after the shock.

Notice that my model differs from both Michailat (2014) and Quadrini and Trigari (2007). In the first one, he assumes random search among job seekers, therefore the crowding out effect on private employment is given by the increase in labor market tightness due to the increase in total vacancies<sup>20</sup>. In the latter, there is one additional channel through which private employment is crowded out, and is given by the fact that after a public employment shock, the overall job finding probability  $f(\theta_t) = \frac{h_t}{u_t}$  increases and so does the value of being unemployed; as a consequence, private wage increases because in that model wages are assumed to be a product of a Nash bargaining. In other words, the private sector wage increases flexibly after a public employment shock, and this further reduces the hiring activity in the private sector. Due to the assumed wage rigidity, in my model this channel is absent. The other difference between the baseline model and the model by Quadrini and Trigari (2007) is that they assume constant marginal returns to labor. The implications of this assumption have already been explained above.

In order to evaluate the effectiveness of the fiscal shock in raising employment, I now plot the public employment multiplier, and give intuition. In order to evaluate the effectiveness of fiscal policy, I begin by computing the instantaneous multiplier at time  $t$ , defined as:

$$\frac{(n_t^* - n_t)}{(g_t^* - g_t)}, \quad (19)$$

where  $n_t^*$  and  $g_t^*$  are total and public employment levels when the technology shock is accompanied by a public employment shock, and  $n_t$  and  $g_t$  are the responses of the latter variables after a technology shock alone<sup>21</sup>. The multiplier (19) measures the period-by-period effectiveness of the fiscal policy on total employment,  $n_t$ <sup>22</sup>. Figure 5 compares the multipliers of the baseline model and the extension with random search.

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<sup>20</sup>Indeed, when random search is assumed, firm's labor demand depends on total labor market tightness through the vacancy filling probability, which is given by  $q(\theta_t)$  where  $\theta_t = \frac{v_t}{u_t}$  is total labor market tightness.

<sup>21</sup>When I simulate a public employment shock only,  $n_t^*$  and  $g_t^*$  are the levels of total and public employment when the shock takes place, and  $n_t$  and  $g_t$  are the steady state levels of total and public employment,  $\bar{n}$  and  $\bar{g}$ .

<sup>22</sup>Trivially, a positive multiplier signifies a positive effect of an increase of public employment on total employment.

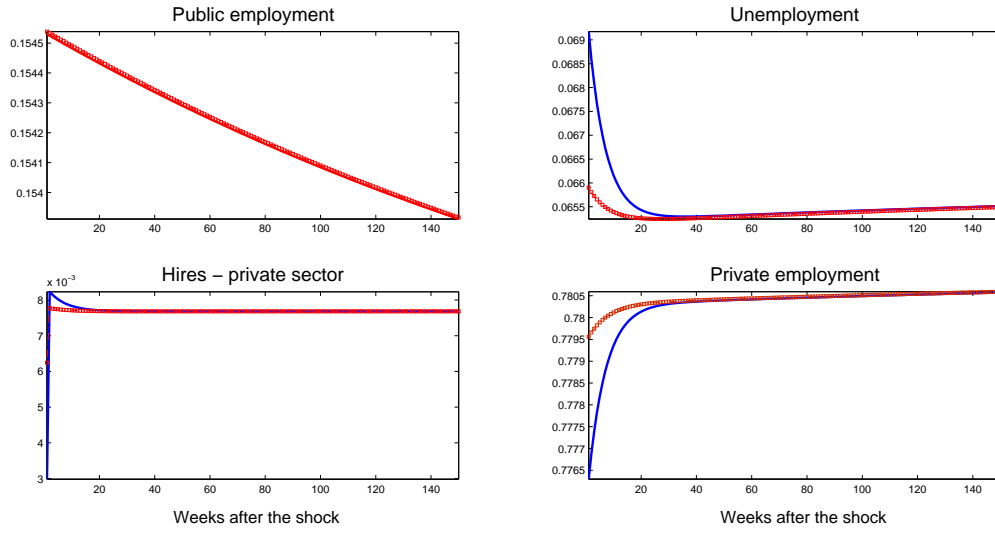


Figure 3: IRF after a 0.1% public employment shock. Note: blue line (baseline), red line (random search).

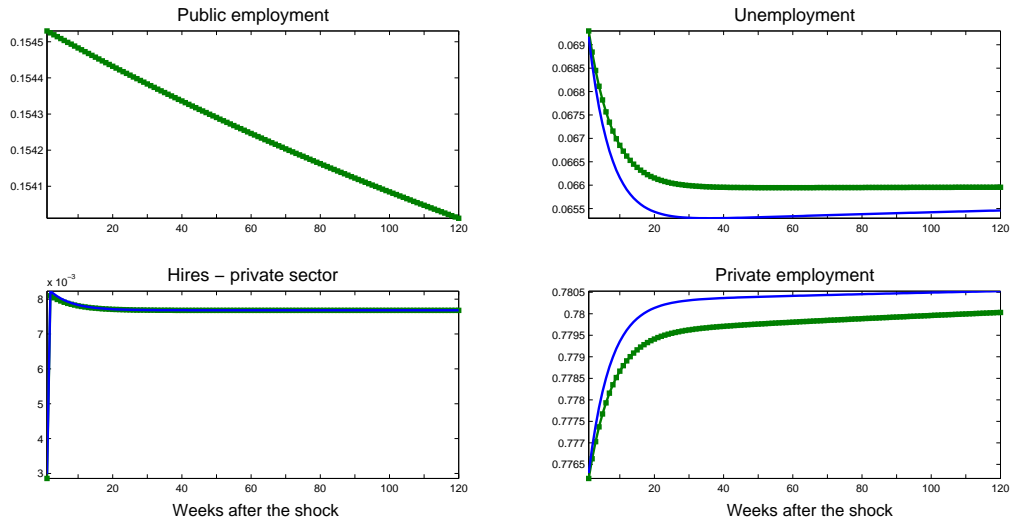


Figure 4: IRF after a 0.1% public employment shock. Note: blue line (baseline), green line (constant marginal returns to labor).

The first is always positive and reaches a steady state value of 0.56. By contrast, the latter is significantly negative at time  $t = 1$ , and is equal to  $-2.07$ ; it becomes positive only after three quarters, and it reaches a

steady state value of 0.53.

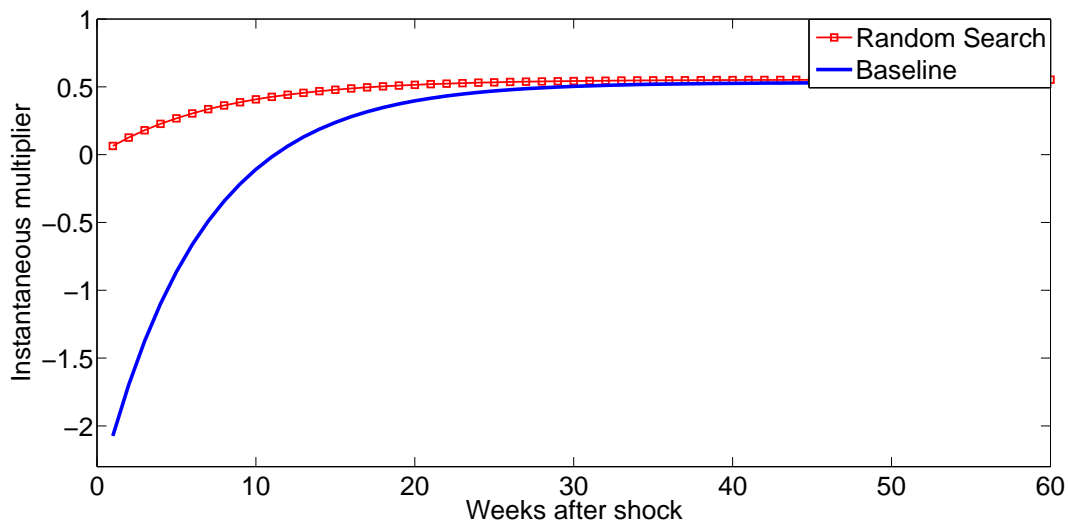


Figure 5: Fiscal multipliers with random and directed search (baseline model). Note: 0.1% public employment shock.

In Figure 6, I plot the baseline multiplier compared to the multiplier that arises from the version of the model with constant marginal return to labor. Compared to the multiplier constructed from the extension with constant marginal returns to labor, the baseline multiplier is significantly higher in steady state (by about 0.5%). From this first step of my analysis, we can already draw an interesting result. Indeed, the sign of the instantaneous multiplier can be both negative and positive, depending on the time horizon we want to consider. As we have seen, the short-run multiplier is strongly negative, while the long run multiplier is positive, and much smaller in absolute value.

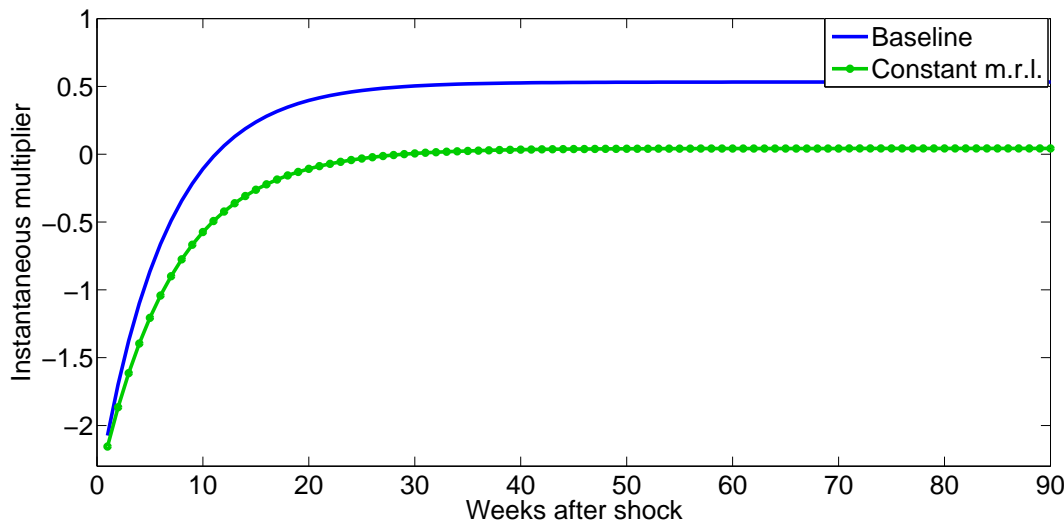


Figure 6: Fiscal multipliers with constant and diminishing marginal returns to labor (baseline model). Note: 0.1% public employment shock.

This is contrast with the findings of Michailat (2014), which introduces a positive multiplier at any time horizon. The disparity of the results depends entirely on the directed search assumption which has quite a significant impact on the multiplier in the short run.

## 6.2 Countercyclicity of the multiplier

Since the aim of my analysis is not only to evaluate the effectiveness of fiscal policy, but also to explore whether and how it changes across different stage of the business cycle, I now analyze the properties of the multiplier after both a positive and a negative technology shock of 0.1%.

To start with, as in the previous section, I briefly describe the multiplier in the model with directed search. In Appendix B I plot the public employment multiplier of the latter model. As expected, the latter is always positive and counter cyclical. In recession, the multiplier has a peak value of 0.663 after 38 weeks. In expansion, the multiplier grows slowly after the shock. After the technology shock, both multipliers reach a steady state level of 0.56. Importantly, both the pattern and the steady state values become quite different when I switch from a negative to a positive technology shock.

Figure 7 plots the pattern of the public employment multiplier after a positive and negative technology shock in the baseline model. The two multipliers are quite similar. However, because of the mechanisms explained in section 6.1, the impact multiplier of the model with directed search is largely negative in expansion ( $-2.15$ ) at time  $t = 1$ , and remains negative for 12 periods (which with my calibration correspond to three quarters). After 12 weeks, it becomes positive and reaches a steady state level of 0.53 after approximately 200 weeks. In a recession scenario, the impact multiplier is  $-1.94$  immediately after the shock. After 12 weeks, it also becomes positive and reaches its peak value of 0.639 after 43 weeks. After the peak, it converges back



to reach its steady state value.

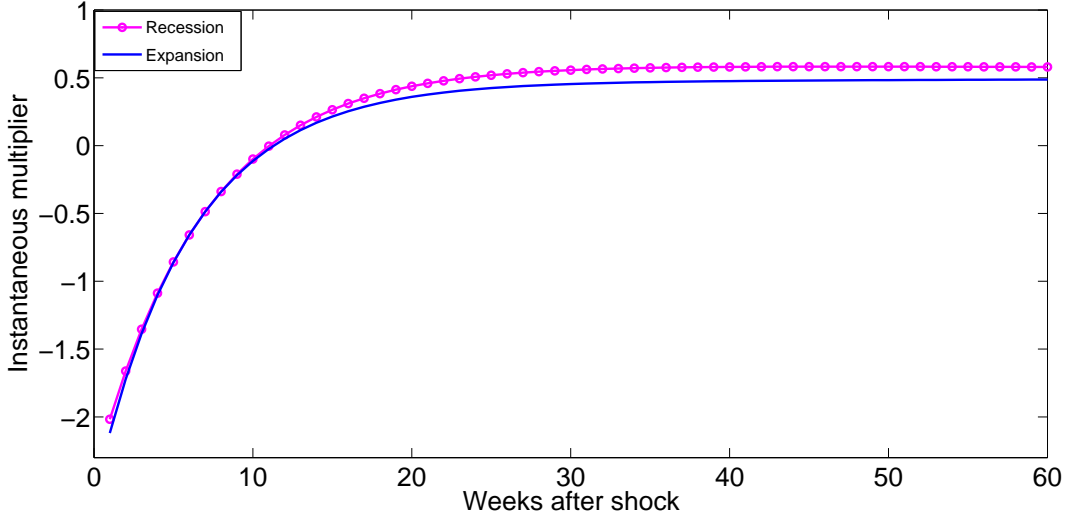


Figure 7: Fiscal multiplier after a public employment shock: recession and expansion scenarios

These findings deserve some comments. First, the counter cyclicality of the multiplier that arises in the model with random search also depends on the time horizon we want consider. In fact, in the model with random search the multiplier is higher in recession at any time  $t^{23}$ . In my model, instead, there is no difference in the impact multipliers at time  $t = 1$  between expansion and recession. This holds true until the effect of  $\sigma_t$  on labor supply vanishes. Indeed, in the long run the multiplier in recession is higher when it reaches the peak.

Second, the multiplier in steady state is only slightly lower in the model with directed search. This happens because the second term in the right-hand-side of equation (18), despite having an impact in the dynamics of the multiplier, has only a marginal effect on the size of the latter in the very long run (i.e., in steady state).

Third, these results seem to contradict the results by Michailat (2014): indeed, the multiplier in the latter work significantly varies over the cycle; on the contrary, the difference between the multipliers in recession and expansion is negligible, especially in the short run.

### 6.3 Public Employment Shock with (Rigid) Bargained Wages

The private wage schedule (11) is theoretically valid because, in equilibrium, the real wage must fall between the marginal product of labor and the flow value of unemployment and in this sense it is privately efficient. However, for a complete understanding of the effectiveness of the public employment multiplier, I should

<sup>23</sup>In recession, at time  $t = 1$  it is equal to 0.068, while in expansion it is equal to 0.061. At the peak, they are respectively 0.663 and 0.53 (the steady state value).

make use of a wage schedule that captures the influence that conditions of public sector jobs have on private sector wages and, by consequence, on private labor demand. I will do so without dropping the assumption that wages adjust rigidly to technology shocks.

I henceforth assume that wages adjust according to the following rule<sup>24</sup>:

$$w_t^p = (1 - \chi) \cdot w_t^N + \chi \cdot w_{t-1}^p, \quad (20)$$

where  $\chi$  is a parameter that reflects the degree of wage rigidity ( $\chi = 0$  implies no rigidity), and  $w_t^N$  is the outcome wage of a Nash bargaining game between firms and workers.

The derivation of  $w_t^N$  is the following: given the bargaining power of workers  $\eta$ , private wage  $w_t^p$  now solves the maximization problem:

$$\max_{w_t^p} (\mathcal{W}_t^p - \mathcal{U}_t)^\eta \cdot (\mathcal{J}_t - \mathcal{V}_t)^{1-\eta}, \quad (21)$$

where  $\mathcal{J}_t$  and  $\mathcal{V}_t$  are, respectively, the value of a job and of an open vacancy for a firm measured in terms of current consumption of the final good. These are given explicitly by the following expressions:

$$\mathcal{J}_t = a_t \cdot \alpha \cdot l_t^{\alpha-1} - w_t^p + E_t \beta_{t,t+1} [(1 - \lambda^p) \cdot \mathcal{J}_{t+1}]$$

$$\mathcal{V}_t = -r + E_t \beta_{t,t+1} [q_t^p \mathcal{J}_{t+1} + (1 - q_t^p) \mathcal{V}_{t+1}].$$

Free entry implies that the value of posting a vacancy is zero:  $\mathcal{V}_t = 0$ . This implies that:

$$\frac{r}{q(\theta_t^p)} = \beta E_t \mathcal{J}_{t+1}. \quad (22)$$

The first-order condition of (21) is given by:

$$\eta \cdot \mathcal{J}_t = (1 - \eta) \cdot (\mathcal{W}_t^p - \mathcal{U}_t). \quad (23)$$

Simply using the expressions for the value functions, eq. (22) and (23) I obtain an expression for  $w_t^N$ :

$$w_t^N = \eta \cdot \alpha l_t^{\alpha-1} \cdot a_t + \eta \cdot \frac{r \cdot f(\theta_t^p) \cdot (1 - \lambda^p)}{q(\theta_t^p)}. \quad (24)$$

The rule (20) implies that, in steady state,

$$\bar{w}^p = \bar{w}^N.$$

While being a short cut to a micro founded specification for rigid wages, the aggregate wage norm (12) constitutes a plausible starting point for analyzing the impact of wage rigidities on the cyclical behavior of private employment according to different policy rules.

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<sup>24</sup>This specification has been used, for instance, by Monacelli et al. (2010) and Christoffel and Lanzert (2005).

The difference between this setting and the baseline model is that if private wages follow equation (24), they respond not only to a technology shock, but also to a public employment shock<sup>25</sup>. To see this, it is sufficient to look at equation (24): when  $g_t$  increases, tightness in the private sector increases through the reaction of  $\sigma_t$ , as we have already seen; besides the crowding out of  $l_t^d$  caused by the increase in hiring costs through  $q(\theta_t^p)$ , there is an additional effect on the response of the aggregate labor demand: since  $\theta_t^g$  increases, the private real wage increases and this further reduces the demand for labor, augmenting the baseline crowding out effect.

In figure 8, I plot the multiplier after a public employment shock equivalent to a 1% of the public labor force<sup>26</sup>. The multiplier is negative and remains below zero for 28 weeks; more importantly, however, the multiplier has a peak value very close to zero.

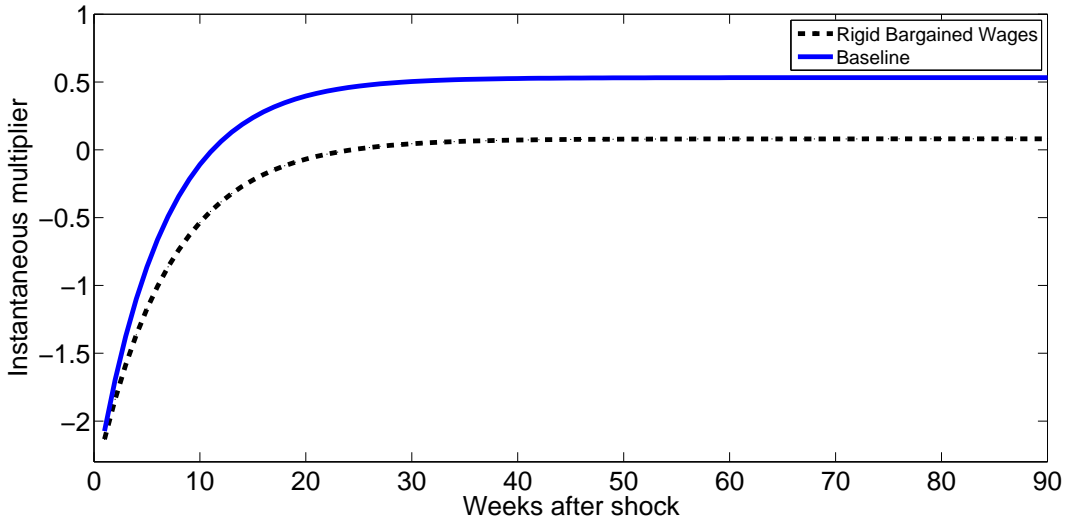


Figure 8: Fiscal multiplier after a 0.1% public employment shock

In other words, public employment almost entirely crowds out private employment when wages are the outcome of a Nash bargaining game between firms and workers, notwithstanding the assumed wage rigidity in the form of (20).

<sup>25</sup>Notice that, although dropping the assumption that wages adjust rigidly only to technology shocks, this does not imply that job rationing disappears in the model: the latter, indeed, relies on the fact that private wages are high enough so that the market does not converge to full employment. See Michailat (2012) and Michailat (2014) for a formal definition. In line with recent literature, I set the bargaining power parameter  $\eta$  at 0.5.

<sup>26</sup>Through the simulations, I choose arbitrarily the value of  $\chi$  and set it equal to 0.7. Since, in steady state,  $\bar{w}^p = \bar{w}^N$ , the choice of  $\chi$  does not affect the steady state value of the multiplier in Figure 7.

## 6.4 A Shock to Public Wages

So far, I have assumed that, after a public employment shock, public wages remained constant. As Quadrini and Trigari (2007) show, however, public wages can have a significant impact on the properties of the model economy in response to a shock. For this reason, I will simulate the response of the economy after a public wages shock.

The goal of this section is to evaluate the reaction of the model economy as such, and in comparison to the public employment shock I have analyzed in section 6.1.1. In order to isolate the effects of a public wages shock, I will assume that public employment is kept fixed by the government at its steady state level,  $\bar{g}$ . Public wages are assumed to follow the exogenous process given by

$$\log(w_t^g) = \log(\bar{w}^g) + \epsilon^{w^g},$$

where  $\bar{w}^g$  is the steady state level of public wages, assumed to be equal to  $\bar{w}^g = \pi \cdot \bar{w}^p$  as in the previous sections, and  $\epsilon^{w^g}$  is an auto-regressive process of order 1, with autocorrelation coefficient arbitrarily set equal to 0.9. (the size of this coefficient does not alter significantly the results). I simulate a 6.6% shock to public wages. The Impulse Response Functions of the baseline to a public wages shock are plotted in Figure 9<sup>27</sup>.

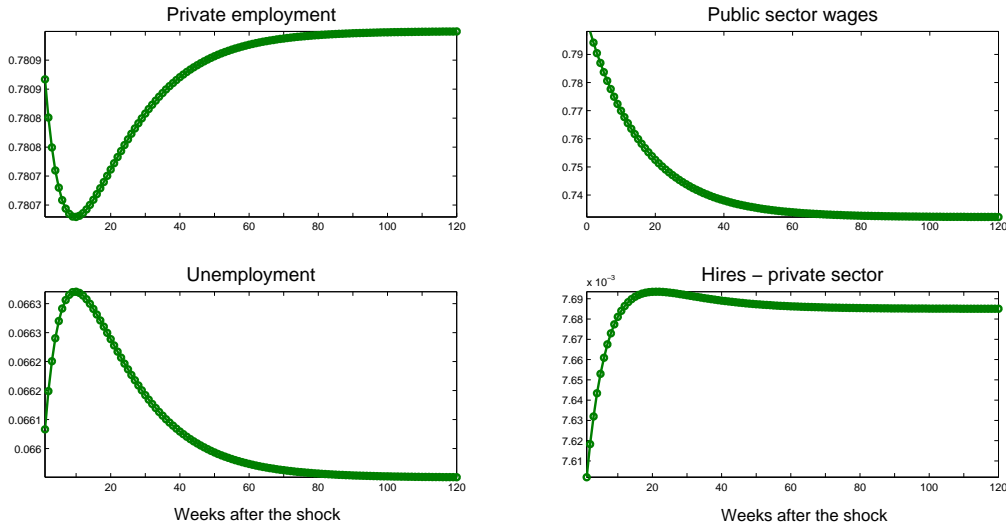


Figure 9: IRF to a Public Wages shock. Note: model simulated in response to a 6.6% shock.

Let us briefly see through which channel a shock to public wages operates. After a wages shock, more unemployed search for a job in the public sector - as it happens after a public employment shock. Unlike the latter case, however, a public wages shock does not cause an increase in public employment: the crowding out effect of private employment is not accompanied by an increase in hiring; as a consequence, unemployment

<sup>27</sup>In Appendix B, I plot the IRF after a public wage shock for other variables of the model economy.

rises above its steady state level, and converges back to the latter as soon as public wages do the same (see Figure 9).

Nonetheless, the most interesting result comes from the fact that the increase in unemployment after the wage shock is much smaller than after the public hiring shock; to understand why, I compare my result to those obtained by Gomes (2010). In his model, a public sector wage shock increases unemployment through one more channel: indeed, the public wage shock spills over to the private sector wage (since the latter depends on the conditions of public sector jobs through the reaction of  $\sigma_t$ ). The latter channel is absent in my model. After shocks of equal size to public employment and wages, unemployment increases by approximately 4% and 2%, respectively. In my model, by contrast, the increase in unemployment after a wage shock is much smaller compared to an employment shock. This means that, despite the directed search assumption plays a role through  $\sigma_t$  (which crowds out private employment through  $q(\theta_t^p)$ ), the increase in private wages plays a much more important role in driving unemployment fluctuations after a shock.

## 7 Sensitivity Analysis

Throughout Section 6, I assumed that the wages, the separation rates, and the job-finding probabilities in the two sectors are different. This assumption is crucial for directed search - which is a fundamental assumption in my model - since job seekers direct their search if the conditions on public and private labor market (summarized by these three variables) are equal <sup>28</sup>. In this section, I will show how different calibrations and assumptions about the separation rates and the public wage premium (and as a consequence, the job-finding probabilities) can affect the fiscal multiplier.

### 7.1 Separation rates, $\lambda$

I will start my sensitivity analysis by the separation rates in the two sectors. I will simulate the model assuming that separation rates in the public sector are equal to their private sector counterparts:  $\lambda^g = \lambda^p = 0.00984$ . I then repeat the simulation assuming that the separation rates are equal to the sum of the separation rates in the two sectors:  $\lambda^g = \lambda^p = 0.01331$ . Figure 10 plots the multiplier arising from these two simulations, along with the multiplier in the baseline model.

There are two interesting results to be noticed here. The first is that, when I set the separation rate equal in each sector, the multiplier is much higher (and becomes positive) in the short run, while it is lower in steady state. This happens because since I eliminate a difference between the two sectors (i.e, the separation rate) the effect of directed search is much lower in response to the shock (compare the red and the green line with the blue line in Figure 10).

Second, the multiplier is only marginally affected by the change in the separation rates (from 0.00984 to 0.01331). In other words, the biggest effect is achieved by setting the rates equal in each sector, rather than by the value itself.

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<sup>28</sup>In the United States, for instance, the separation rates in the public sector are almost three times lower than in the private sector, with the consequence that public sector jobs last much longer than in the private sector. See Appendix C.

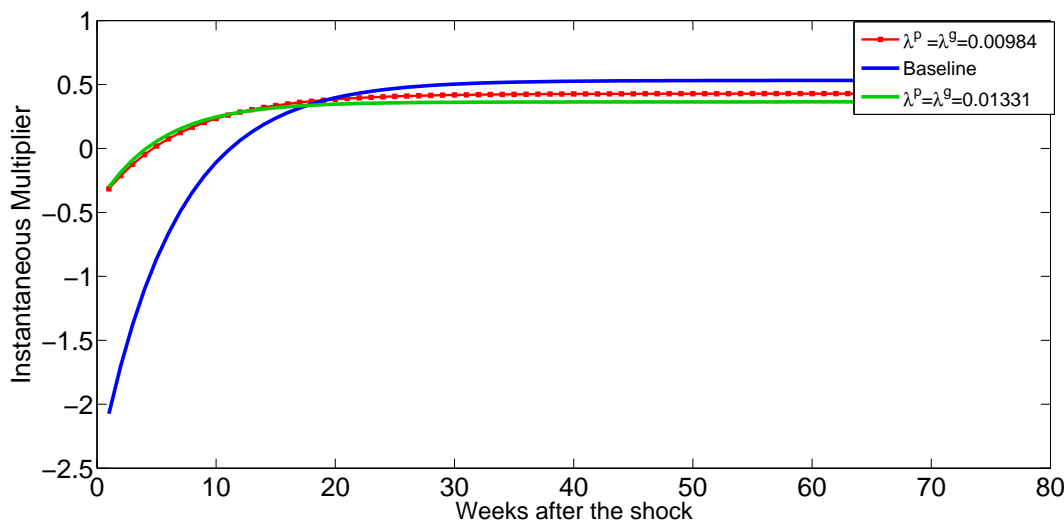


Figure 10: Fiscal multiplier with separation rates equal in each sector. Note: 0.1% Public Employment shock.

## 7.2 Public wage premium, $\pi$

In Section 6, I choose arbitrarily the public wage premium  $\pi$  and set it equal to 1.03. This parameter varies depending on the country considered, the sex and the education of a worker, among other variables. The afore mentioned survey by Gregory and Borland (1999) place the premium between 0% and 10%. In accordance with this evidence, Quadrini and Trigari place the premium at 3.75%. However, Neumann et al. (2010) show that, during the New Deal, relief work programs paid a lower (hourly) wage than private-sector counterparts. In light of this evidence, I now simulate the model assuming that public sector wages can be positive, negative or equal to private sector wages. Notice that, as long as separation rates in each sector differ, I can simulate the model even in the case in which  $\pi = 1$ . Indeed, if  $\lambda^p = \lambda^g$  (as in 7.1), private and public sector wages must differ since they must be compensated for the different job finding probabilities. For simplicity, I will perform my sensitivity analysis on  $\pi$  only in the latter case.

To start with, I have to choose one value for  $\lambda$ , and set it equal to 0.00984. Figure 11 plot the fiscal multiplier for three different values of  $\pi$ . As expected, as long as public sector wages are higher than private sector wages (i.e.,  $\pi > 1$ ), the multiplier is negative in response to the shock; the opposite holds true when  $\pi < 1$ . The multipliers are only slightly different in steady state. In other words, the wage premium parameter  $\pi$  has a significant effect in the short run, but almost negligible in the long run.



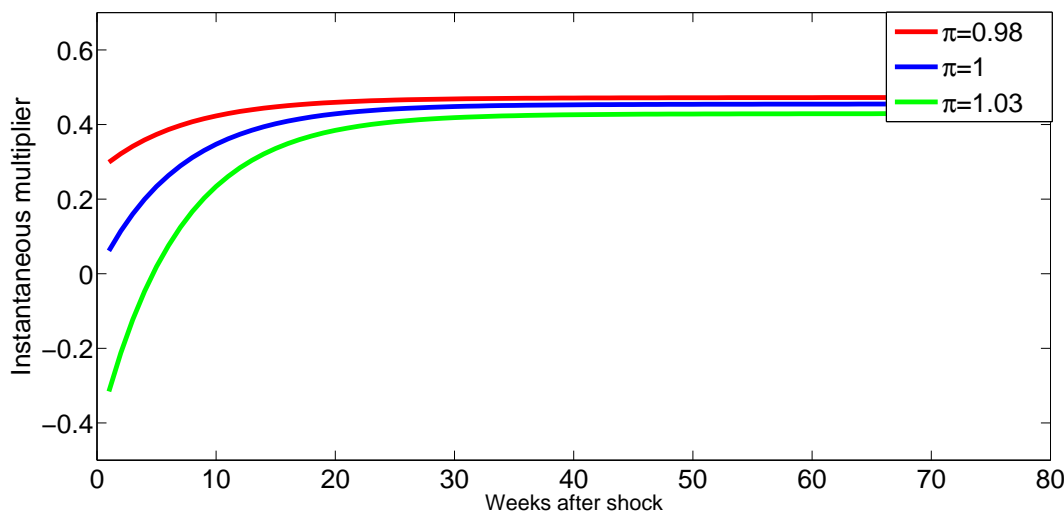


Figure 11: Fiscal multipliers with different values of the wage premium,  $\pi$ . Note: separation rates are equal in each sector, and assume to be 0.00984. The multiplier is obtained after a 0.1% public employment shock

Notice that, thanks to the results of 7.1, setting a different value for the separation rates would only marginally alter the result of Figure 11, since the two rates are equal. Figure 12 explains the intuition behind Figure 11: at the time the shock occurs, the multiplier is very sensitive to the value of  $\pi$ . Viceversa, in steady state, the latter only marginally amplifies (or dampens) the size of the multiplier (i.e., the multiplier is a -almost - constant function of the wage premium). Notice that the wage premium has qualitatively the same effect on the multiplier both in the short and in the long run (that is, higher wage premium leads to a lower multiplier). Another important result is that the value of  $\pi$ , as recalled, alter the results significantly in the short run; I can conclude that both the separation rates and the wage premium have a strong influence in the short run. Importantly, these parameters are related to the direct search assumption, since the latter relies on the fact that these parameters are different between the two sectors. This is a very important result, and it tells us that the shift in the labor supply (see Section 4), which in turn takes place because of the directed search assumption, is only able to affect the multiplier in the very short run (see Figure 12).

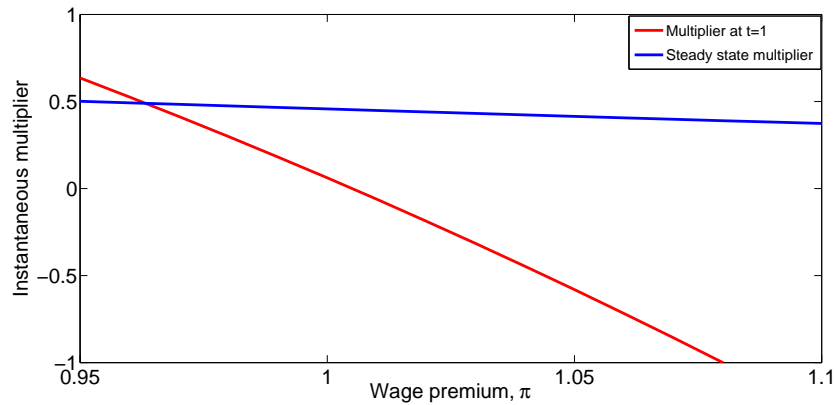


Figure 12: Fiscal multiplier at time 1 and in steady state, for different values of the wage premium parameter,  $\pi$ . Note: the multiplier is obtained after a public employment shock of 0.1%.

### 7.3 Marginal returns to labor, $\alpha$

As explained in Section 4 and 6, in extension of the model in which constant marginal returns to labor are assumed ( $\alpha = 1$ ), the labor demand does not depend on labor market tightness; as a consequence, no shift of the aggregate labor demand occurs when there is an increase in public employment: the only shift that occurs is the one of the labor supply. Motivated by the results of Figure 6, I perform a sensitivity analysis on the parameter  $\alpha$  (recall that, in the baseline case,  $\alpha = 0.66$ ). The aim is to show that, the higher  $\alpha$ , the lower the multiplier will be in steady state. I will therefore compare the multipliers that arise in the baseline case ( $\alpha = 0.66$ ), in the constant-marginal-to-labor extension, and in an arbitrary value in the middle of the two. Figure 13 shows the results.

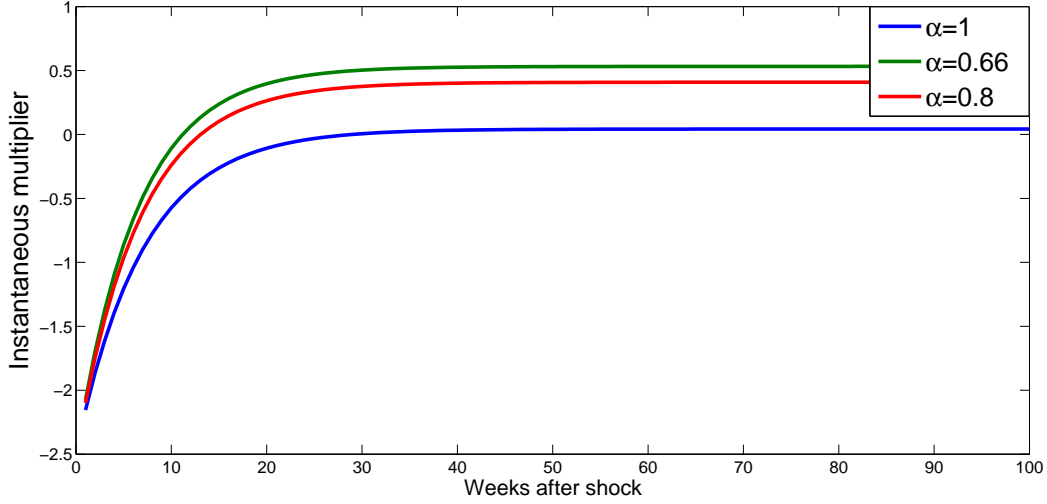


Figure 13: Fiscal multiplier for different values of  $\alpha$ . Note: The multiplier is obtained after a public employment shock of 0.1%

As expected, the multipliers with different values of  $\alpha$  are almost equal in response to the shock; on the contrary, they become more and more different in steady state. Since the parameter  $\alpha$  is the one that determines the slope of the labor demand, we can conclude that the assumption of diminishing marginal returns to labor (which implies that the first term on the right hand side of equation (18) is different from zero) determines the size of the multiplier in steady state, while it has a negligible effect in the very short run, as opposed to the directed search assumption (see 7.1 and 7.2).

#### 7.4 Long run and short run behavior of the multiplier

An interesting result that emerges from Section 6 is that while the directed search assumption has a strong effect on the multiplier only in the short run, the assumption of diminishing (as opposed to constant) marginal returns to labor seems to drive the size of the multiplier in the (very) long run. I will now give the intuition behind these results.

To begin with, let us assume that Assumption 1 and 2 in Section 4 hold true. In this case, I can write explicitly the total effect of an increase in public employment  $g$  on private employment  $l$  as<sup>29</sup>:

$$\frac{\partial l}{\partial g} = -\frac{1 - u \cdot (\psi - 1) \cdot \sigma \cdot [(1 - \xi)/\xi]}{1 + (\epsilon^s/\epsilon^d) + u \cdot (\psi - 1) \cdot \sigma}, \quad (25)$$

where  $\xi$ ,  $\epsilon^s$  and  $\epsilon^d$  have already been defined in Section 4.

In the two extensions in which constant marginal returns to labor and random search are assumed, equation (25) becomes, respectively:

<sup>29</sup>The proof can be found in Michaillat (2012b)

$$\frac{\partial l}{\partial g} = -\frac{1 - u \cdot (\psi - 1) \cdot \sigma \cdot [(1 - \xi)/\xi]}{1 + u \cdot (\psi - 1) \cdot \sigma}, \quad (26)$$

since  $\epsilon^d = +\infty$  (i.e., the aggregate labor demand is horizontal and perfectly elastic), and

$$\frac{\partial l}{\partial g} = -\frac{1}{1 + (\epsilon^s/\epsilon^d)}, \quad (27)$$

which corresponds to the multiplier in Michaillat (2014).

By means of (25), (26), and (27) we can now understand the behavior of the multiplier in the short and long run. For simplicity, let us begin by assuming constant marginal returns to labor. Since we have assumed that  $\lambda^g = \lambda^p = \lambda$ , a positive (negative) wage premium  $\pi$  implies a negative (positive) steady state ratio of job-finding probabilities,  $\bar{\psi}$ . In steady state, a higher wage premium means a lower ratio of job-finding probabilities; this, however, has only a marginally effect on the multiplier (see equation (25), and Figure 11). By contrast, in the short run, unemployment fluctuates in response to the public employment shock. As a consequence, the effects of a change in  $\psi_t$  in the short run are magnified by the fluctuation of unemployment. Notice that this effect works through the term  $u \cdot (\psi - 1)$ .

From the latter intuition, we have seen why the multiplier in the short run is almost entirely driven by the directed search assumption and the choice of the parameter  $\pi$ . In the long run, the multiplier changes significantly according to the value assigned to  $\alpha$ : a higher value assigned to  $\alpha$  implies a lower ratio of elasticities  $\frac{\epsilon^s}{\epsilon^d}$ , hence a lower multiplier (see (25) and (27), and Figure 13). When the effect of the directed search assumption - which, as we have seen above, works through unemployment fluctuations and the ratio of job-finding probabilities - vanishes, the effect of assuming diminishing instead of constant marginal returns to labor has a strong effect on the size of the multiplier. In other words, the effect of a change in the parameter  $\alpha$  in the short run is almost completely absorbed by the effect of directed search. These intuitions also confirms the results obtained in Section 6.

## 8 Concluding remarks

Despite the fact that the main component of government consumption is compensation to public employees, the effect of public employment and wages policies for the cyclical behavior of the economy is not a very common topic in macroeconomic research. In my thesis, I have studied both the effect of public employment and wages on the economy and their effectiveness stimulating employment.

In order to do that, I build a general equilibrium model with search frictions à la Mortensen and Pissarides with a public and a private labor market in which job seekers direct their search among the two sectors. I assign different job finding probabilities, wages and separation rates to each sector, and assume (following Michailat(2014)) wage rigidity and diminishing marginal returns to labor.

The results can be summarized as follows. By comparing the multipliers in models with random and directed search, I show that in the latter is strongly negative in the short run, while the first is always positive; in the long run, however, their difference is quite small. Viceversa, when I assume constant marginal returns to labor, the multiplier significantly changes only in the long run. I then simulate the baseline model under both a positive and a negative technology shock, in order to study the cyclical properties of the multiplier. I find that, unlike Michailat(2014), the multiplier is only mildly countercyclical.

I then study the effects of fiscal shocks in the form of public wages. In my model, the latter have only a small negative effect on employment. This result is partly driven by the fact that rigid private sector wages only respond to technology shocks. While keeping the assumption of rigid wages, I study how the public employment multiplier varies if I allow private wages to respond markedly to conditions on public sector jobs. When this happens, the multiplier is significantly lower both in the short and in the longer run.

As a final step of my analysis, I study the business cycle properties of my model after a positive and a negative technology shock. In line with Quadrini and Trigari (2007), I find that the best policy rules in order to stabilize employment is to set pro cyclical public employment and wages. However, in my model, public employment plays a much larger role than public wages in affecting the volatility of total employment after a technology shock.

While contributing to the understanding of the effectiveness of public employment and wages, there are several limitations that suggest that further analysis on the topic is needed. I can think of at least two. The first one comes from the fact that the public employment multiplier is essentially a government spending multiplier. As such, it would be interesting to study the implications of assuming that a public employment shock is financed through debt financing. An interesting example is Monacelli et al. (2010).

The second limitation comes from the fact that, in order to model a rigid wage that responds to shocks in the public labor market, I have assumed a simple law of motion for private wages. However, in the literature several authors have proposed more sophisticated modelizations of private wages (see for instance the aforementioned Gertler and Trigari (2010) and Hall and Milgrom (2008)). The study of the fiscal multiplier with such a wage rule would be more accurate and would offer a more complete understanding of the business cycle properties of private employment according to different public employment rules. These extensions are left for future research.

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## Appendix

**A1 Model equations - Baseline** The model is fully described by the following equations:

$$w_t^p = \omega \cdot a_t^\gamma \quad (28)$$

$$p_t c_t + b_t = p_t \cdot l_t \cdot w_t^p + p_t \cdot g_t \cdot w_t^g + R_{t-1} b_{t-1} + p_t \cdot T_t \quad (29)$$

$$1 = \beta \cdot E_t \left[ \frac{c_t}{c_{t+1}} \right] \quad (30)$$

$$y_t = a_t \cdot (l_t)^\alpha \quad (31)$$

$$y_t = c_t + \frac{r \cdot a_t}{q(\theta_t^p)} [l_t - (1 - \lambda^p) l_{t-1}] + \frac{r \cdot a_t}{q(\theta_t^g)} [g_t - (1 - \lambda^g) g_{t-1}] \quad (32)$$

$$\alpha \cdot (l_t)^{\alpha-1} = \frac{w_t}{a_t} + \frac{r}{q(\theta_t^p)} - \beta \cdot (1 - \lambda^p) E_t \left[ \frac{a_{t+1}}{a_t} \frac{r}{q(\theta_{t+1}^p)} \right] \quad (33)$$

$$l_t = (1 - \lambda^p) l_{t-1} + h_t^p \quad (34)$$

$$g_t = (1 - \lambda^g) g_{t-1} + h_t^g \quad (35)$$

$$u_t = 1 - l_t - g_t \quad (36)$$

$$\ln(a_t) = \ln(\bar{a}) + \epsilon_t^a; \epsilon_t^a = AR(1) \quad (37)$$

$$f(\theta_t) = h_t^p / u_t (1 - \sigma_t) \quad (38)$$

$$f(\theta_t^g) = h_t^g / u_t \cdot \sigma_t \quad (39)$$

$$q(\theta_t^p) = h_t^p / v_t^p \quad (40)$$

$$q(\theta_t^g) = h_t^g / v_t^g \quad (41)$$

$$x_t^p = \mathcal{W}_t^p - \mathcal{U}_t^p \quad (42)$$

$$x_t^g = \mathcal{W}_t^g - \mathcal{U}_t^g \quad (43)$$

$$\frac{h_t^p}{1 - \sigma_t} E_t [x_{t+1}^p] = \frac{h_t^g}{\sigma_t} E_t [x_{t+1}^g] \quad (44)$$

$$h_t^p = \mu^p \cdot ((1 - \sigma_t)u_t)^{\eta^p} (v_t^p)^{1-\eta^p} \quad (45)$$

$$h_t^g = \mu^g \cdot (\sigma_t \cdot u_t)^{\eta^g} (v_t^g)^{1-\eta^g} \quad (46)$$

**A2 Model equations - Random search** In this version of the model, matches are given by their relative vacancies. Therefore, we drop equation 44 and equations 42,43. Equations 38,39, 45,46 become, respectively:

$$f(\theta_t^g) = h_t^g/u_t \quad (47)$$

$$f(\theta_t^p) = h_t^p/u_t \quad (48)$$

$$h_t^g + h_t^p = \mu \cdot (u_t)^\eta (v_t^p + v_t^g)^{1-\eta} \quad (49)$$

$$v_t^p h_t^g = v_t^g h_t^p \quad (50)$$

**A3 Model equations - Constant marginal returns to labor** What changes in this version of the model with respect to the baseline version is simply the production function (equation(31)) and, as a consequence, the firm's labor demand (equation (33)). They become, respectively:

$$y_t = a_t \cdot l_t \quad (51)$$

$$1 = \frac{w_t}{a_t} + \frac{r}{q(\theta_t^p)} - \beta \cdot (1 - \lambda^p) E_t \left[ \frac{a_{t+1}}{a_t} \frac{r}{q(\theta_{t+1}^p)} \right] \quad (52)$$

**B Other figures** Figure 14 and 15 compare the IRF after a 1% public employment shock and a wages shock in the model with direct and random search for other variables in the economy.

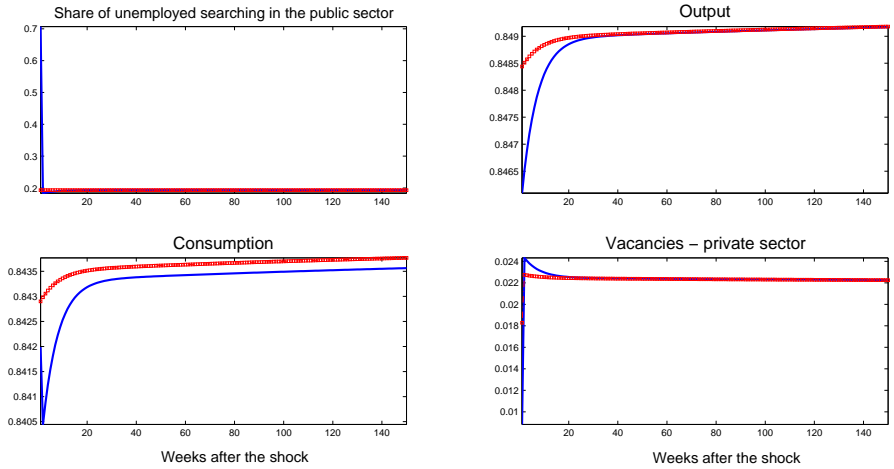


Figure 14: IRF to a Public employment shock. Note: blue line (baseline), red line (random search).

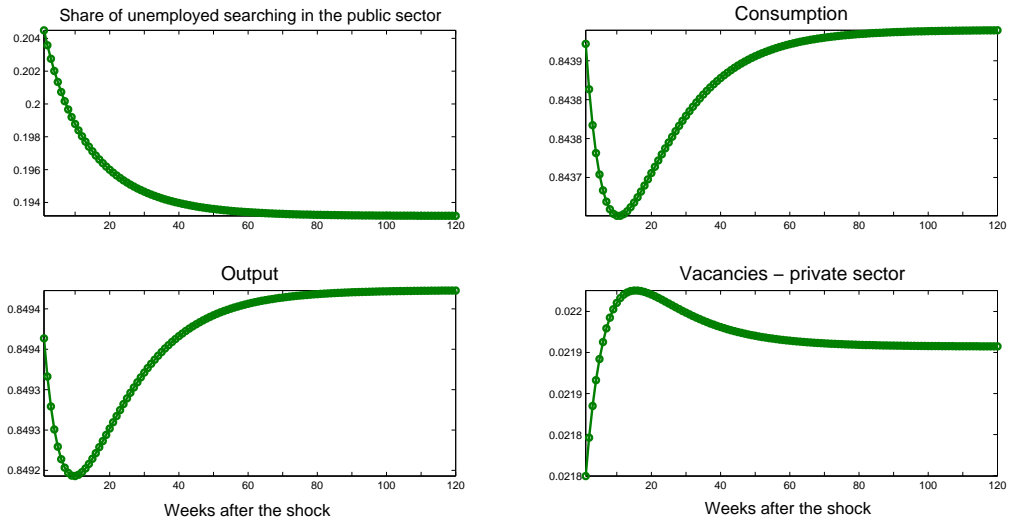


Figure 15: IRF to a Public wages shock, directed search.

I now plot the public employment multiplier in the model with random search, in expansion (Figure 16) and recession (Figure 17).

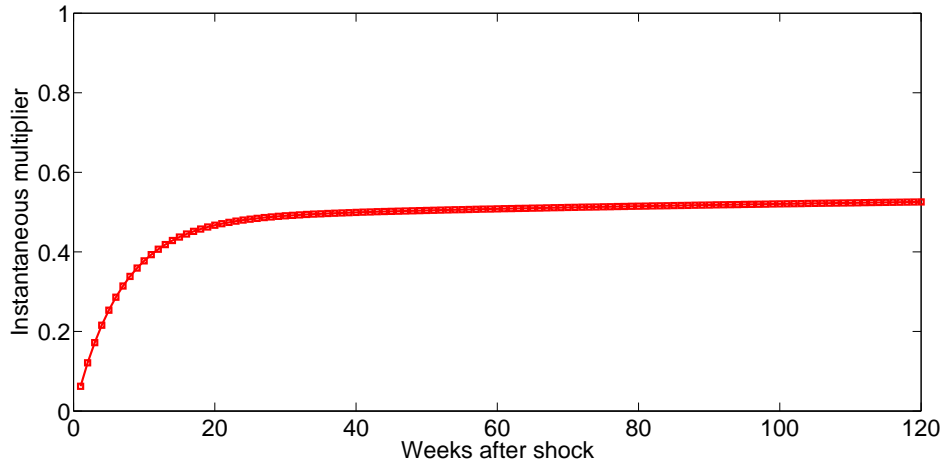


Figure 16: Fiscal multiplier in expansion - model with random search

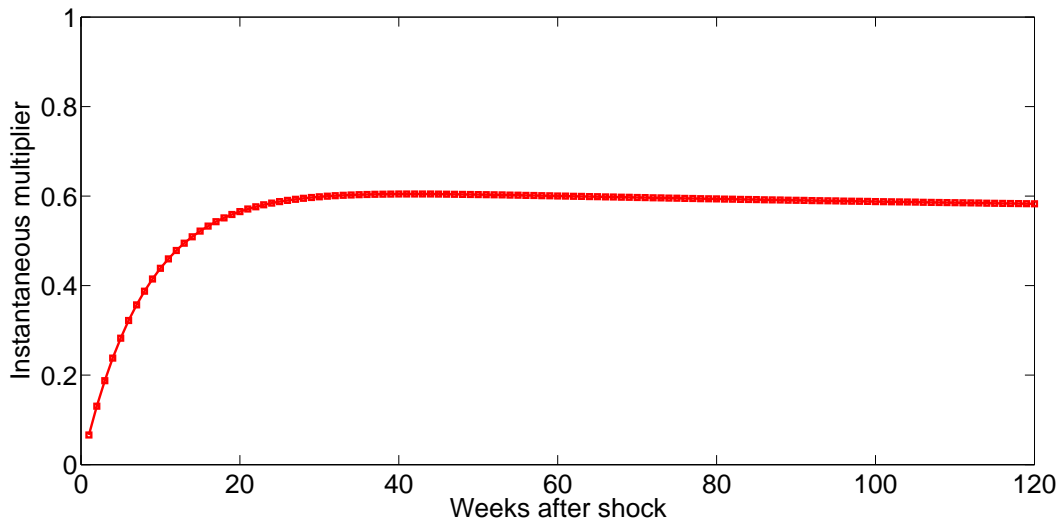


Figure 17: Fiscal multiplier in recession - model with random search