SEMINAR
ON ORDINARY DIFFERENTIAL EQUATIONS AND INTEGRATION THEORY

Special session in honor of the 90th birthday of

Jaroslav Kurzweil

The main lecture hall of the Institute of Mathematics CAS,
Žitná 25, Praha 1

Program

Thursday, May 12

09.30	Opening	
10.00	P. Brunovský:	Dichotomy, the closed range theorem and optimal control
10.30	M. Pituk:	Linear differential equations with small delays
11.00	Coffee break	
11.30	A. Slavík:	Well-posedness and maximum principles for lattice reaction-diffusion equations
12.00	J. G. Mesquita:	Neutral functional dynamic equations on time scales
12.30	Lunch	
14.00	P. Krejčí:	The Kurzweil integral in financial markets
14.30	V. Recupero:	Convex valued geodesics and unilateral dynamics
15.00	B. R. Satco:	Kurzweil-Stieltjes integration for operators with applications
15.30	Coffee break	
16.00	M. Federson:	Overview and trends in generalized ODEs
16.30	J. Haluška	On integration in complete vector lattices

Friday, May 13

09.30	J. Mawhin	Four steps to Kurzweil's integral
10.00	A. Lomtatidze:	Periodic boundary value problem for second-order ordinary differential equations: resonance like case
10.30	Coffee break	
11.00	R. Hakl:	Periodic solutions to indefinite singular equations
11.30	M. Fabian:	On coincidence of Mc Shane and Pettis integrals
12.00	P. P. Hájek:	The work of Jaroslav Kurzweil on approximation in Banach spaces and its developments
12.30	Lunch	
14.00	P. Drábek:	Sturm-Liouville problem with weights via the Hardy inequality
14.30	G. A. Monteiro:	Convergence results for the abstract Kurzweil-Stieltjes integral: a survey
15.00	Coffee break	
15.30	M. Cichoń:	On regulated functions and selections
16.00	J. Malý:	Kurzweil integral on BV sets
17.30	Banquet	

Abstracts

Pavel Brunovský
(Comenius University, Bratislava, Slovakia)

Dichotomy, the closed range theorem and optimal control

Necessary conditions of optimality for discrete time optimal controls problems can be extended from finite to infinite horizon provided the range of a shift operator on a functional space is closed. It will be shown that this is the case if the dynamics of the problem is hyperbolic but may fail to be so if it is not.

Mieczysław Cichoń
(Adam Mickiewicz University, Poznań, Poland)

On regulated functions and selections

In many differential and integral problems, discontinuous functions should be treated as solutions. We will investigate the space of regulated functions G on a finite interval. It consists of functions having finite one-side limits at every point (i.e. with discontinuities of the first kind) and then it contains the space of continuous functions. It is worth noting that regulated functions are not necessarily of bounded variation.

The key point of the talk is to propose a common treatment for continuous and regulated functions. Note that non-separable Banach spaces cannot be embedded isometrically in the separable space $C([0,1])$, but for every Banach space X, one can find a compact Hausdorff space K and an isometric linear embedding j of X into the space $C(K)$ of scalar continuous functions on K. We propose to investigate the space of regulated functions as an isometric copy on some space of continuous functions, which simplifies all previous considerations about the considered space.

Let X be a Banach space. A function $u : [0,1] \to X$ is said to be regulated if there exist limits $u(t^+)$ and $u(s^-)$ for any point $t \in [0,1)$ and $s \in (0,1]$. The set of discontinuities of a regulated function is at most countable. Regulated functions are bounded and the space $G([0,1],X)$ of regulated functions on $[0,1]$ into the Banach space X is a Banach space too, endowed with the topology of uniform convergence, i.e. with the norm $\|u\|_\infty = \sup_{t\in[0,1]}\|u(t)\|$. The space $G([0,1],X)$ is not separable, contains, as a proper subsets, the spaces of continuous functions and functions with bounded variation.

In the talk we concentrate on some properties of sets in $G([0,1],X)$ being useful in studying differential and integral equations. We present some compactness and weak compactness criteria, based on the isometry between this space and a constructed space $C(K)$. Moreover, we introduce moduli of equi-regularity and a measure of noncompactness in this space. Based on the same idea we study the existence of selections for multivalued mappings, fulfilling the gap in this theory. The talk will be supplemented by some examples of applications for the obtained results.

This talk is based on a joint research with Bianca Satco (Suceava, Romania), Kinga Cichoń (Poznań, Poland) and Mohamed Metwali (Damanhour, Egypt).

Pavel Drábek
(West Bohemian University, Plzeň, Czech Republic)

Sturm-Liouville problem with weights via the Hardy inequality

In this talk we consider Sturm-Liouville problems with weights having degenerations or singularities. We employ Hardy's inequality and Muckenhoupt's conditions and prove an analogue of the classical Sturm-Liouville theorem. An application to the radial problem on the ball and on the entire space will be given. In particular we show how the Hopf's and Vazquez's maximum principles are affected by the degeneration or singularity of the weights.

Marián Fabian
(Institute of Mathematics CAS, Czech Republic)

On coincidence of Mc Shane and Pettis integrals

McShane integral is a younger close relative of Kurzweil-Henstock integral. Let f be a scalarly null function from $[0,1]$ into a Banach space X. R. Deville and J. Rodríguez recently showed that if X is a Hilbert (generated) space, then f must be also McShane integrable. While A. Avilés, G. Plebanek and J. Rodríguez found a weakly compactly generated space X and a scalarly null $f : [0,1] \to R$ which is not McShane integrable. In the lecture we focus on a detailed study of this phenomenon. We show several other counterexamples and raise relevant open questions. We follow our paper "On the coincidence of the Pettis and McShane integrals", Czechoslovak Math. J. **65** (140) (2015), no. 1, 83–106.

Márcia Federson
(ICMC University of São Paulo, Brazil)

Overview and trends in generalized ODEs

We will present an outlook of the theory of generalized ODEs as well as new trends.

Petr P. Hájek
(Institute of Mathematics CAS, Prague, Czech Republic)

The work of Jaroslav Kurzweil on approximation in Banach spaces and its developments

We will describe the fundamental results of Jaroslav Kurzweil in the area of smooth approximations in Banach spaces. We will outline the subsequent development in this direction of research.

Robert Hakl
(Institute of Mathematics CAS, Prague, Czech Republic)

Periodic solutions to indefinite singular equations

Efficient conditions guaranteeing the existence of T-periodic solution to the second order differential equation

$$u'' = h(t)\, g(u)$$

are established, where $g : \mathbb{R}_+ \to \mathbb{R}_+$ is a decreasing continuous function with strong singularity at zero and the weight $h \in L(\mathbb{R}/T\mathbb{Z})$ is a sign-changing function. A relation between the degree of zeroes of the weight function and the order of the singularity of the nonlinear term is set. The approach is based on the Leray-Schauder degree, proving that no T-periodic solution of a certain homotopy appears on the boundary of an unbounded open set during the deformation to an autonomous problem. This is a joint work with Manuel Zamora.

Ján Haluška
(Mathematical Institute SAS, Košice, Slovakia)

On integration in complete vector lattices

For the Archimedean vector lattice $\mathbf{X}$, the complete vector lattice $\mathbf{Y}$ and the positive cone $\mathbb{L}$ of the vector lattice of all linear regular operators $L : \mathbf{X} \to \mathbf{Y}$, a Riemann-type construction of integral for $\mathbb{L}$-valued measures is discussed. Moreover, a convergence theorem is presented for the case that $\mathbf{Y}$ is almost regular.

Pavel Krejčí
(Institute of Mathematics CAS, Prague, Czech Republic)

The Kurzweil integral in financial markets

Decisions about whether to buy or sell an asset in financial markets depend upon the history of trading. The traders do not react to price fluctuations continuously, but have differing approaches to risk-taking and market forecasting. Standard market strategies can be described by a system of rate independent variational inequalities which are well known from plasticity modeling. The price evolution is driven by an exogenous information stream which is defined by many factors such as changing production costs, transportation costs, natural catastrophes, etc. The market price, however, follows in a substantial way the overall market sentiment which depends on the actual market activities and explains rapid and/or large price movements independent of the exogenous stream due to coupling and cascading effects of the information exchange between market participants. The model thus leads to an implicit system of variational inequalities. We show that it becomes unstable if small groups of traders have a dominant influence on the market sentiment. This may initiate a cascade, and discontinuities are likely to occur. The mathematical tools for describing these phenomena include discontinuous Prandtl-Ishlinskii operators in the Kurzweil integral setting with respect to both the time and the memory, and the main result is the well-posedness proof for the model in the space of right continuous regulated functions as a joint work with H. Lamba, S. Melnik, G. A. Monteiro, and D. Rachinskii.

Alexandre Lomtatidze
(Institute of Mathematics CAS & Technical University, Brno, Czech Republic)

PERIODIC BOUNDARY VALUE PROBLEM FOR SECOND-ORDER ORDINARY DIFFERENTIAL EQUATIONS: RESONANCE-LIKE CASE

For the problem

$$u'' = f(t,u); \quad u(0) = u(\omega),\ u'(0) = u'(\omega), \tag{1}$$

where $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is an ω-periodic function with respect to the first variable, new solvability results will be presented. Obtained results are applied to the solvability of the resonance-like problem

$$u'' = p(t)\,u + h(t)\,|u|^{\lambda}\,\mathrm{sgn}u + q(t); \quad u(0) = u(\omega),\ u'(0) = u'(\omega).$$

Here, $\lambda \in]0,1[$ and the homogeneous problem

$$u'' = p(t)\,u; \quad u(0) = u(\omega),\ u'(0) = u'(\omega)$$

possesses a nontrivial sign-constant solution.

Jan Malý
(Charles University, Prague, Czech Republic)

KURZWEIL INTEGRAL ON BV SETS

It is by no means clear what should be the right multidimensional version of Kurzweil integral. There are a variety of non-equivalent definitions in literature. From the point of view of stability (e.g. with respect to lipeomorphic change of variables), it seems that Pfeffer's definition is the most successful. However, this definition does not return the Kurzweil integral when applied to the onedimensional case. We modify the original Pfeffer's definition of BV integral and obtain a new one which gives a wider class of integrable functions. In particular, its one-dimensional version is exactly the Kurzweil integral. The new integral shares all pleasant properties with the previous ones and some of them can be obtained easier. This is a joint work with Washek F. Pfeffer.

Jean Mawhin
(Catholic University of Louvain, Belgium)

FOUR STEPS TO KURZWEIL'S INTEGRAL

Celebrating Jaroslav Kurzweil by a short historically oriented lecture about several steps in the first half of the XXth century leading to his integral $\int_a^b \mathrm{D}\,U$ for functions $U\colon [a,b] \times [a,b] \to \mathbb{R}$.

Jaqueline G. Mesquita
(University of Brasília, Brazil)

Neutral Functional Dynamic Equations on time scales

The goal of this paper is to study the neutral functional dynamic equations on time scales, using the theory of generalized ODEs. We prove a correspondence between the solutions of the generalized ODEs and the solutions of the measure neutral functional differential equations. After that, we prove a correspondence between the solutions of these last equations and the solutions of the neutral functional dynamic equations on time scales. Finally, we prove results concerning existence and uniqueness of solutions and continuous dependence for these equations.

This is a joint work with Márcia Federson, Miguel Frasson and Patrícia Tacuri.

Giselle A. Monteiro
(Mathematical Institute SAS, Košice, Slovakia)

Convergence results for the abstract Kurzweil-Stieltjes integral: a survey

In the past few years, the theory of generalized differential equations in Banach spaces has gained in popularity for it facilitates the study of continuous and discrete systems from a common standpoint. Such a theory was introduced by J. Kurzweil and is connected to the integration process named after him. In the particular case of generalized linear differential equations, an integral of Stieltjes type appears. Knowing that convergence theorems play a key role in the investigation of continuous dependence for equations, in this work we discuss some convergence results for the abstract Kurzweil-Stieltjes integral. More precisely, we analyse the conditions which ensure that a sequence of operator-valued functions, $\{F_n\}$, satisfies

$$\int_a^b F \, \mathrm{d}g = \lim_{n\to\infty} \int_a^b F_n \, \mathrm{d}g,$$

where F is the limit (in some sense) of F_n and g takes values in a Banach space.

Acknowledgement: Supported by the SASPRO Programme, co-financed by the European Union and the Slovak Academy of Sciences.

Mihály Pituk
(University of Pannonia, Veszprém, Hungary)

Linear differential equations with small delays

Ryabov, Driver and Jarník and Kurzweil showed that if the delays are small, then every solution of a system of linear delay differential equations approaches some special solution at infinity. In this talk we will show that the special solutions satisfy a system of linear ordinary differential equations whose coefficients can be described in terms of the delays and the coefficients of the original delay equation. As a consequence we obtain explicit stability criteria and asymptotic formulas for the solutions of the given linear delay differential equation.

Vincenzo Recupero
(Polytechnic University of Torino, Italy)

CONVEX VALUED GEODESICS AND UNILATERAL DYNAMICS

In this talk we show how suitable geodesics in the space of closed convex sets allow to apply rate-independent techniques to the sweeping processes, a class of differential inclusions with convex constraint including the play operator as an important particular case.

Bianca-Renata Satco
(University Stefan Cel Mare of Suceava, Romania)

KURZWEIL-STIELTJES INTEGRATION FOR OPERATORS WITH APPLICATIONS

A study of semilinear measure driven problems

$$\mathrm{d}u = Au\,\mathrm{d}t + f(t,u)\,\mathrm{d}g,\ u(0) = u_0$$

will be presented in Banach spaces. The real function g is non-decreasing (or, more generally, regulated), while A is the infinitesimal generator of a C_0-semigroup of bounded linear operators on a Banach space.

The existence of bounded variation solutions will be provided via an appropriate fixed point theorem.

Antonín Slavík
(Charles University, Prague, Czech Republic)

WELL-POSEDNESS AND MAXIMUM PRINCIPLES FOR LATTICE REACTION-DIFFUSION EQUATIONS

The classical reaction-diffusion equation

$$\partial_t u(x,t) = k\,\partial_{xx} u(x,t) + f(u(x,t))$$

describes the evolution of chemical concentrations, temperatures, or populations. Motivated by problems from biology, chemistry, or kinematics, various authors have considered the lattice reaction-diffusion equation

$$\partial_t u(x,t) = k(u(x+1,t) - 2u(x,t) + u(x-1,t)) + f(u(x,t)), \quad x \in \mathbb{Z}, \quad t \in [0,\infty), \tag{1}$$

as well as the discrete reaction-diffusion equation

$$u(x,t+1) - u(x,t) = k(u(x+1,t) - 2u(x,t) + u(x-1,t)) + f(u(x,t)), \quad x \in \mathbb{Z}, \quad t \in \mathbb{N}_0. \tag{2}$$

Equations (1) and (2) are also interesting from the standpoint of numerical mathematics, since they correspond to semi- or full discretization of the original reaction-diffusion equation.

In order to study both (1) and (2) in a unified way, we use the language of the time scale calculus and consider the nonautonomous lattice reaction-diffusion equation

$$u^{\Delta}(x,t) = au(x+1,t) + bu(x,t) + cu(x-1,t) + f(u(x,t),x,t), \quad x \in \mathbb{Z}, \quad t \in \mathbb{T}, \tag{3}$$

where $a, b, c \in \mathbb{R}$, $\mathbb{T} \subseteq \mathbb{R}$ is a time scale, and u^Δ denotes the Δ-derivative with respect to time.

Special cases of Eq. (3) include the autonomous Fisher and Nagumo lattice equations, or nonautonomous logistic population models with a variable carrying capacity.

We focus on the well-posedness of bounded solutions to initial-value problems for Eq. (3), and also study continuous dependence of solutions on the underlying time scale. Our second goal is to establish the maximum principles for Eq. (3).

Some of our proofs rely on techniques from the Kurzweil-Stieltjes integration theory. An initial-value problem for Eq. (3) can be transformed to the form

$$U(t) = U(t_0) + \int_{t_0}^{t} \Phi(U(s), s)\, \mathrm{d}g(s), \tag{4}$$

where the function U takes values in the Banach space $\ell^\infty(\mathbb{Z})$, and the integral on the right-hand side is the Kurzweil-Stieltjes integral with respect to a function g which depends on the time scale $\mathbb{T}$. The problem of continuous dependence of solutions to Eq. (3) with respect to the choice of the time scale $\mathbb{T}$ then becomes a question of continuous dependence of U on the choice of g.

When the time scale $\mathbb{T}$ in Eq. (3) is discrete, the weak maximum principle can be proved by a simple induction argument. Using the continuous dependence of solutions on the time structure, we extend its validity to an arbitrary time scale.

This talk is based on a joint research with Petr Stehlík and Jonáš Volek (University of West Bohemia, Czech Republic).