

***MODIFIKACE TVARU
INTERPOLAČNÍCH NURBS KŘIVEK***

Ing. Ivana LINKEOVÁ, Ph.D.

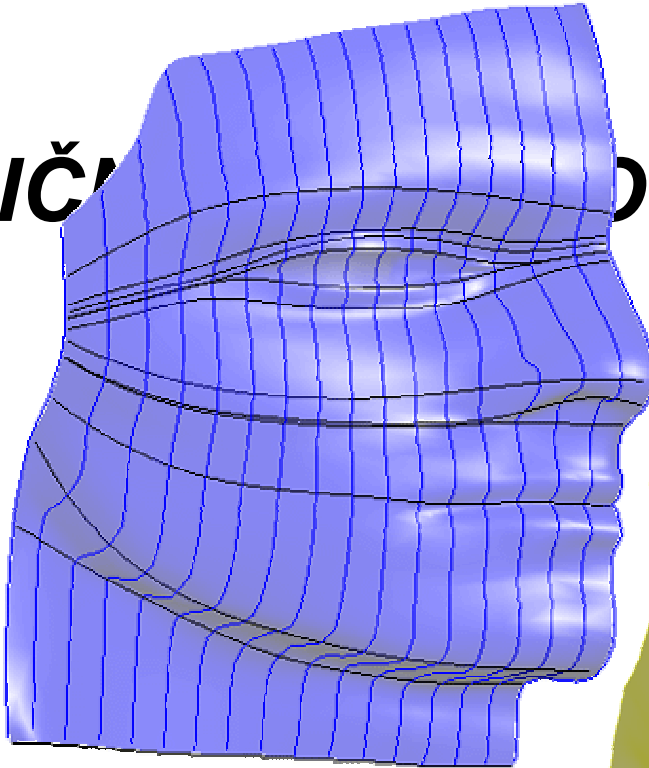
Ústav technické matematiky

Fakulta strojní, ČVUT v Praze

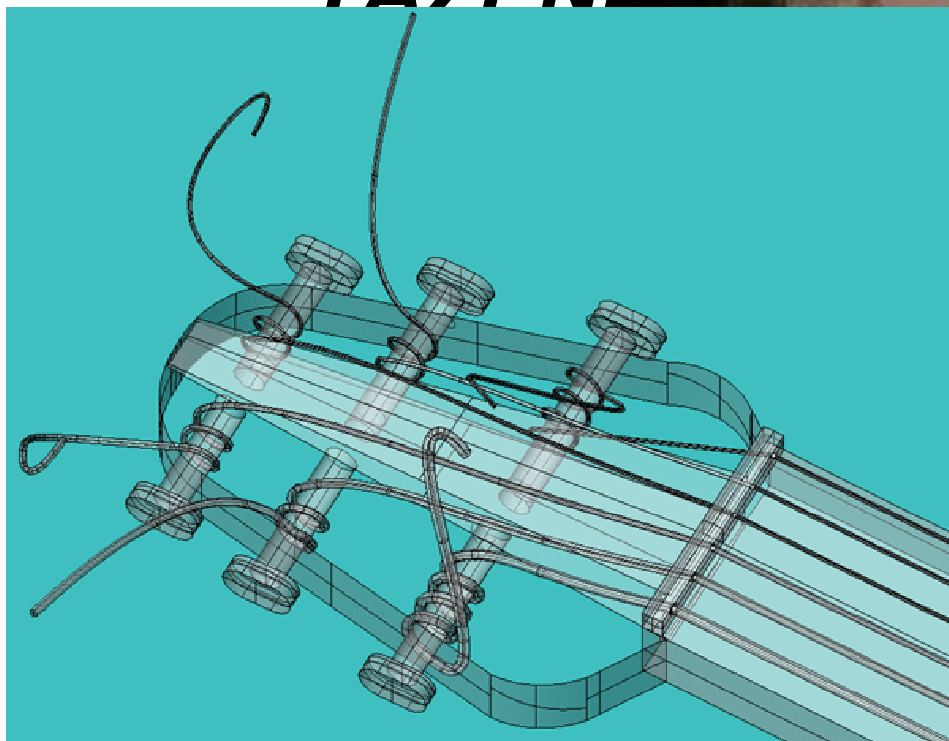




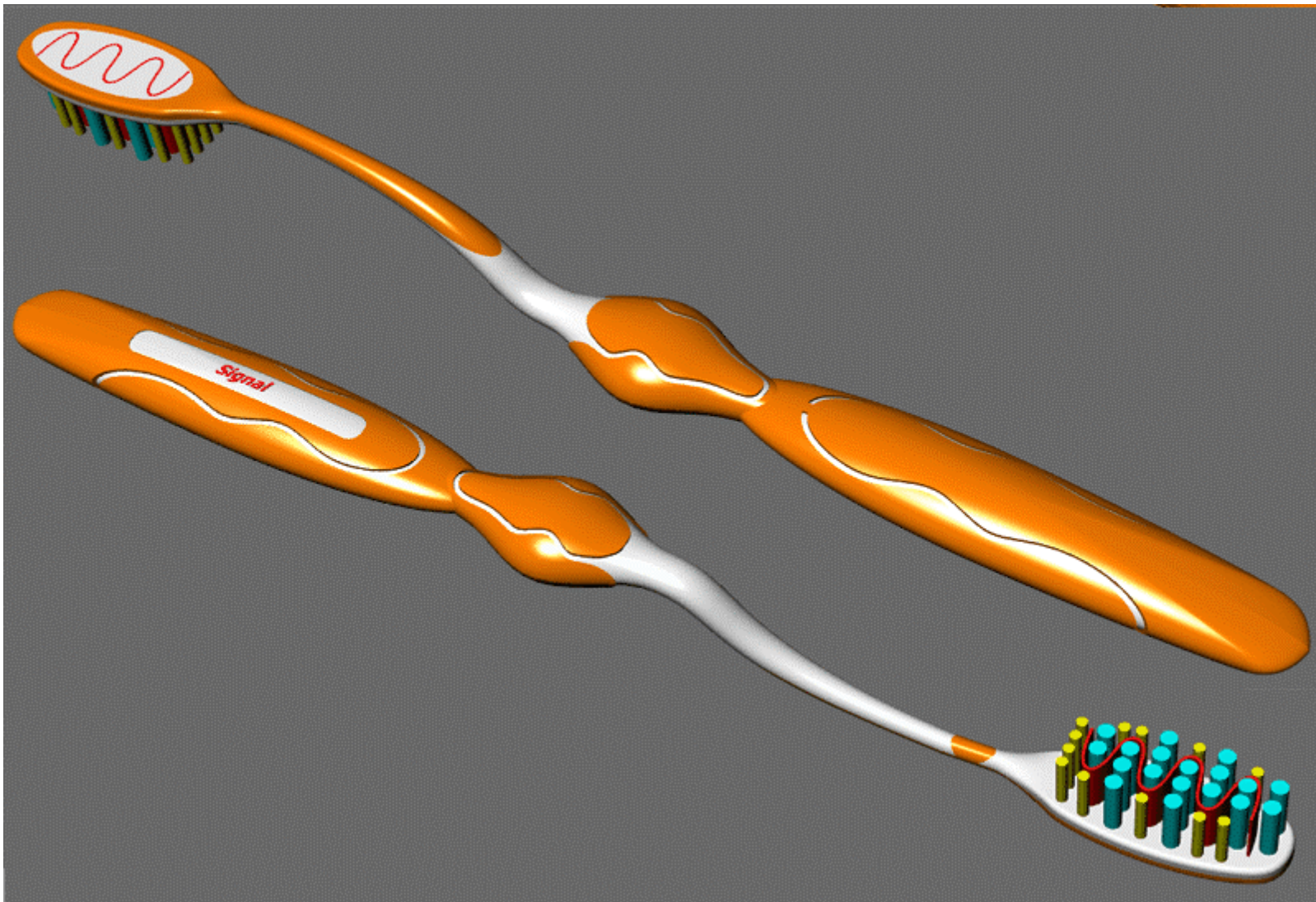
VIČNĚ DCHY



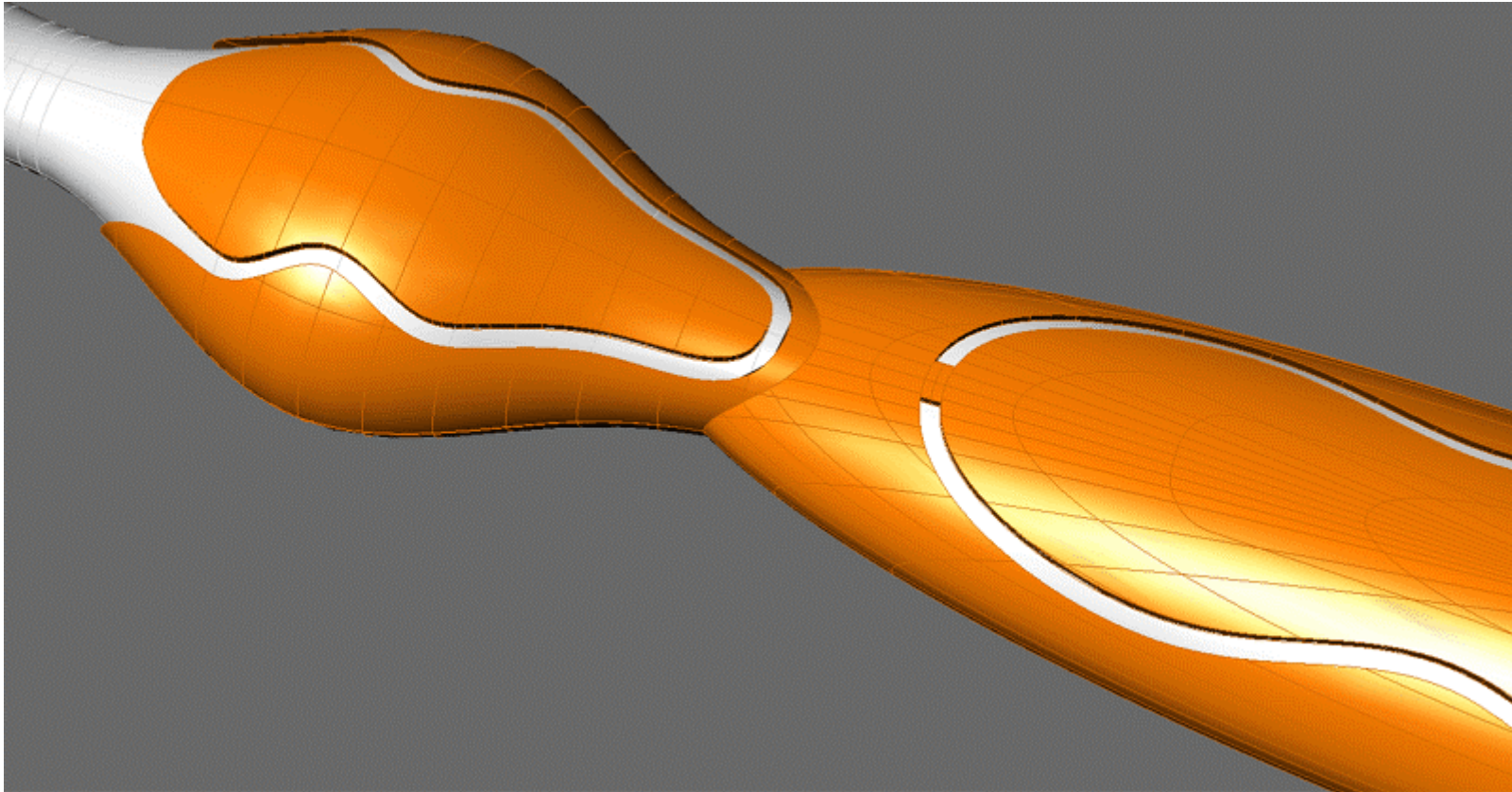
TAŽEN



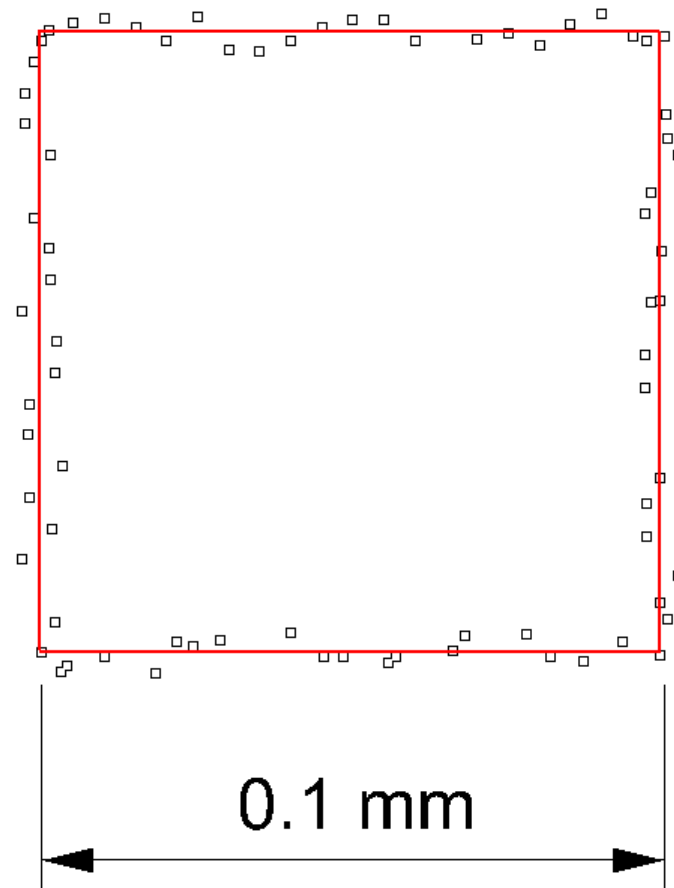
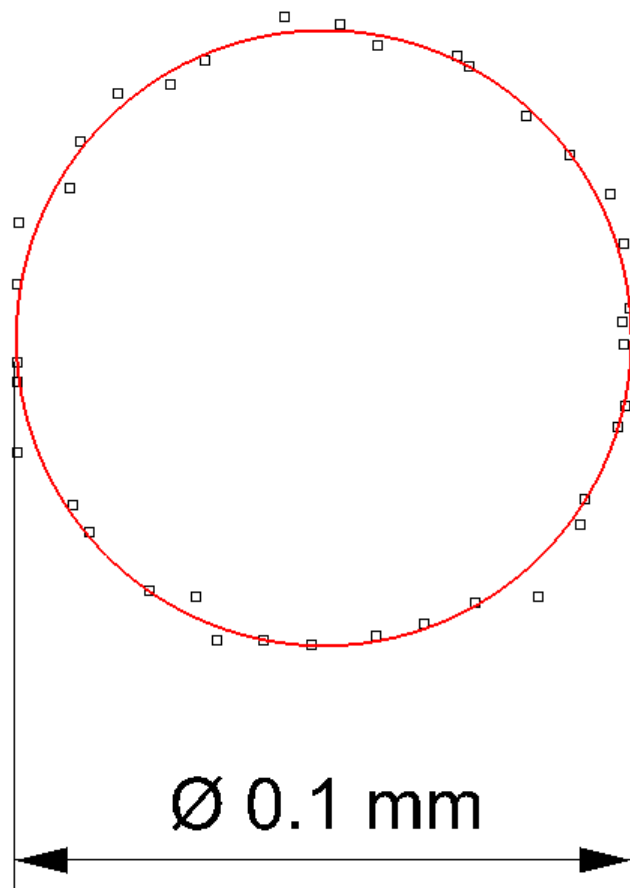
Michal JELÍNEK, Geometrie pro CAD, 2004 – 2005

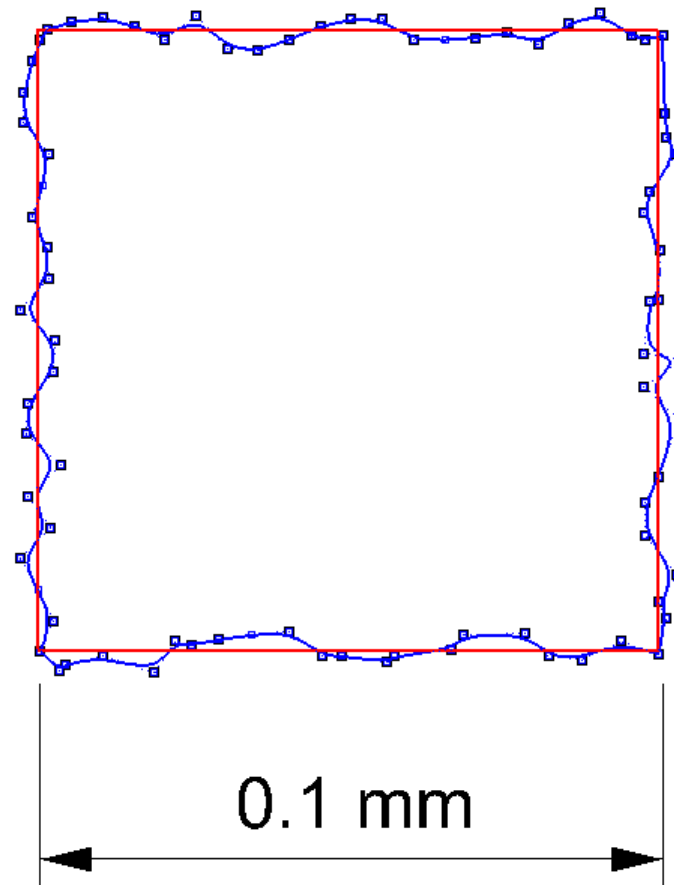
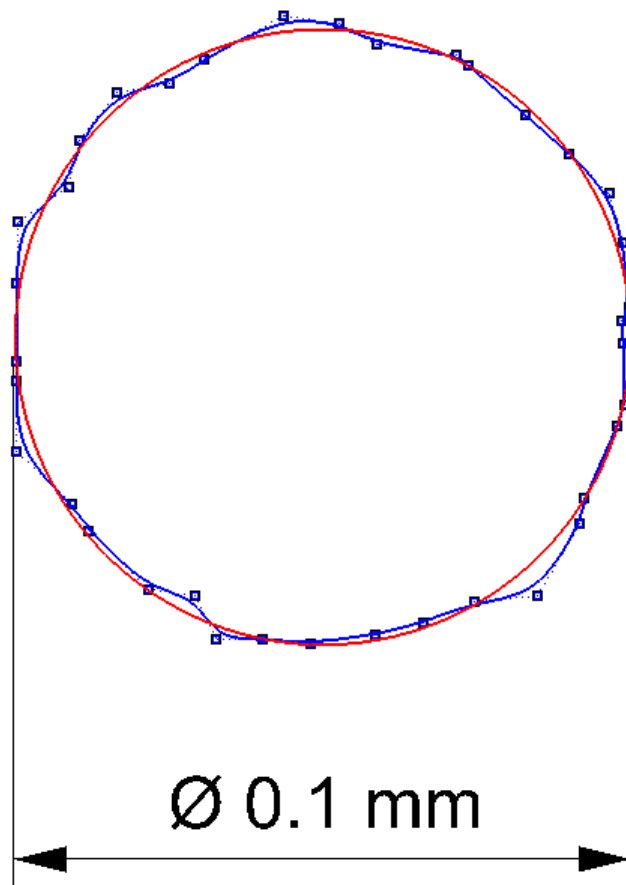


Petr HERALT, Geometrie pro CAD, 2004 – 2005



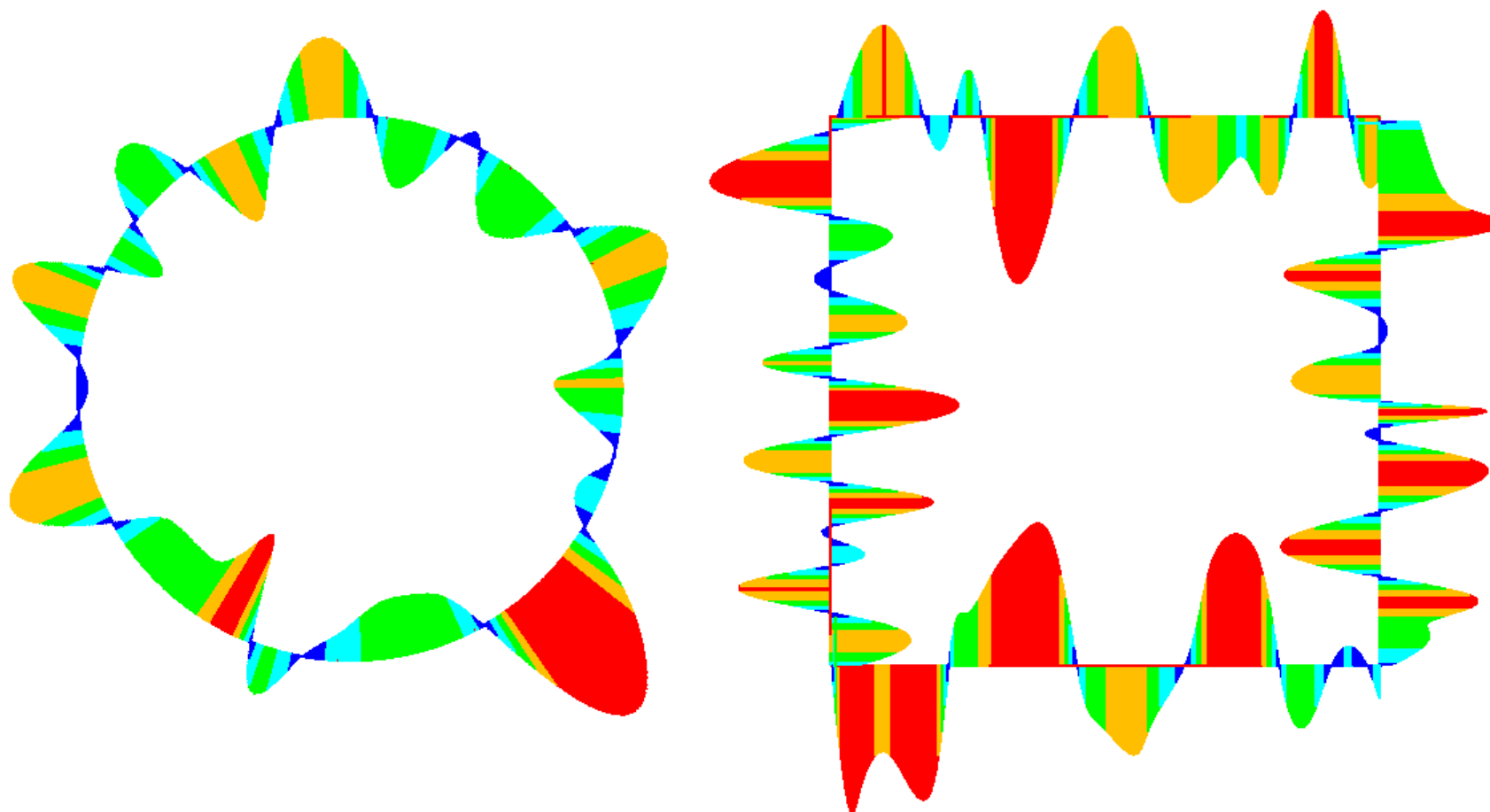
Petr HERALT, Geometrie pro CAD, 2004 – 2005





ZPRACOVÁNÍ NAMĚŘENÝCH DAT

ÚCHYLKA TVARU



APROXIMAČNÍ NURBS KŘIVKY



Ivana LINKEOVÁ, Logo konference SIGA 2011

APROXIMAČNÍ NURBS KŘIVKY

TVAROVACÍ NÁSTROJE:

ŘÍDICÍ BODY

P_0, P_1, \dots, P_n

STUPEŇ

p

UZLOVÝ VEKTOR

$U = (u_0, u_1, \dots, u_m)$

VÁHY ŘÍDICÍCH BODŮ

w_0, w_1, \dots, w_n

$$m = n + p + 1$$



DEFINICE NURBS KŘIVKY

$$C(u) = \sum_{i=0}^n P_i R_{i,p}(u), u \in [u_p, u_{m-p}]$$

UZLOVÝ VEKTOR

$$U = (u_0, u_1, \dots, u_m)$$

B-SPLINE BÁZOVÉ FUNKCE

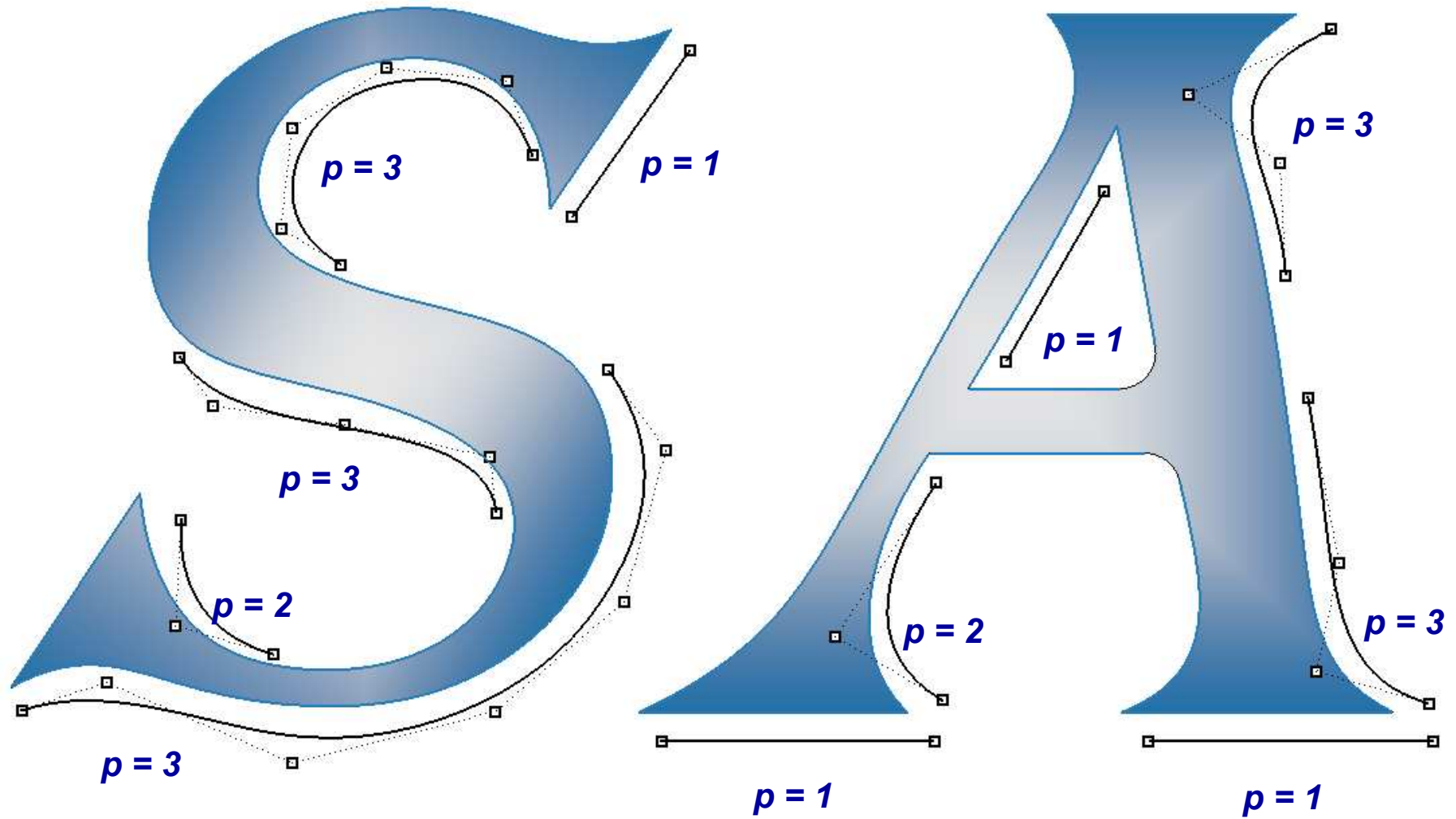
$$N_{i,0}(u) = \begin{cases} 0 & u \in [u_i, u_{i+1}), \\ 1 & u \notin [u_i, u_{i+1}), \end{cases}$$

$$N_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} N_{i+1,k-1}(u), k = 1, 2, \dots, p$$

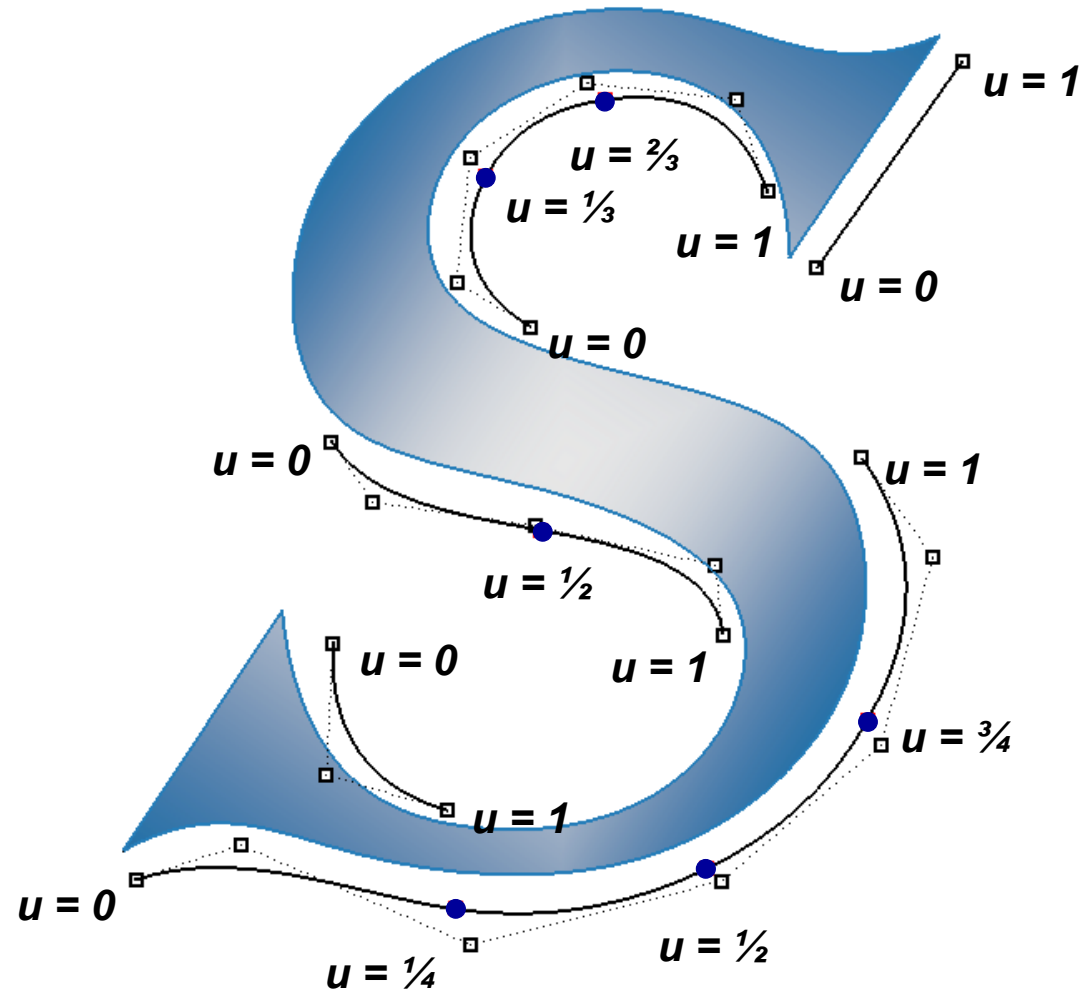
RACIONÁLNÍ BÁZOVÉ FUNKCE

$$R_{i,0}(u) = \frac{w_i N_{i,p}(u)}{\sum_{j=0}^n w_j N_{j,p}(u)}$$

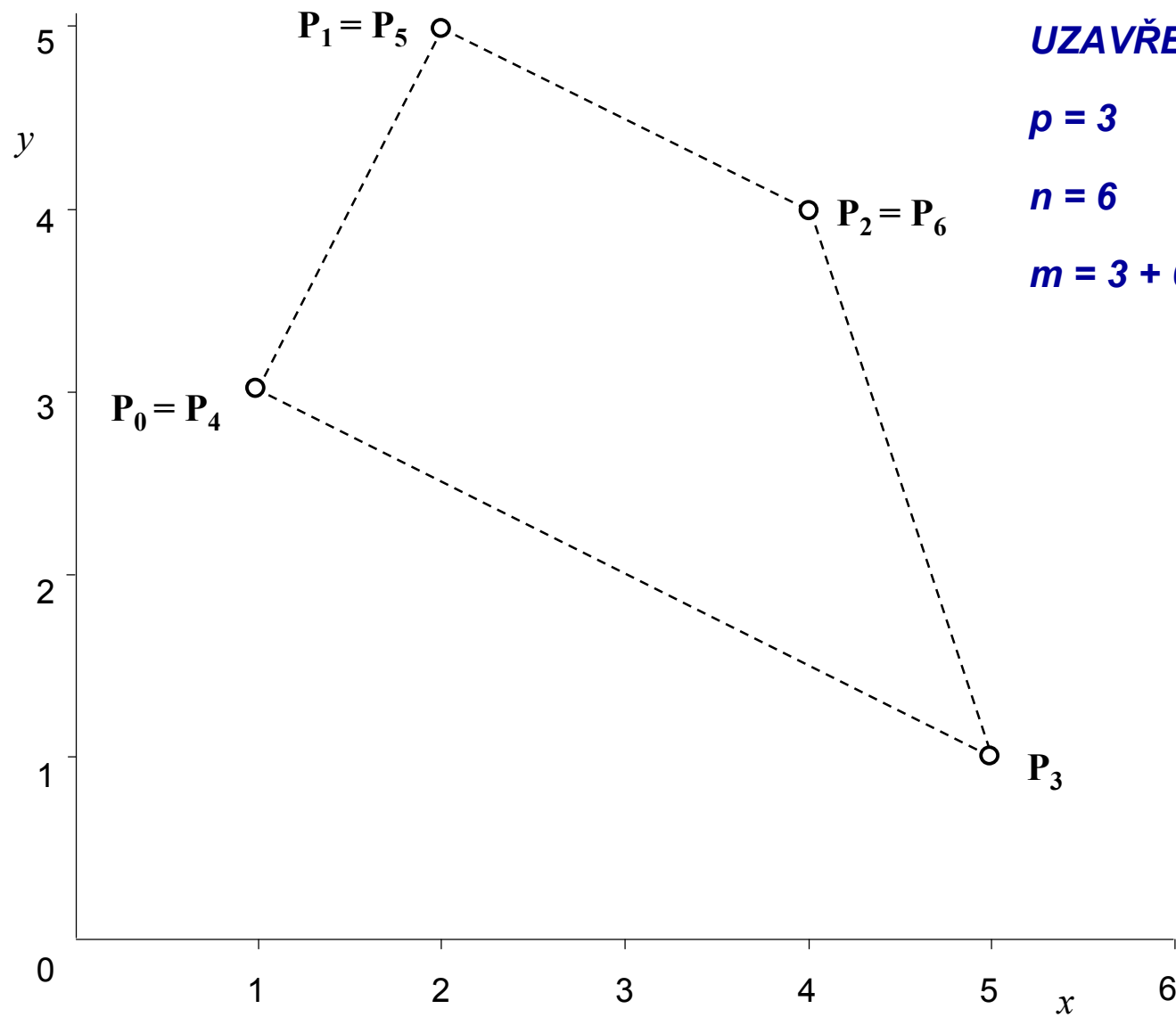
ŘÍDICÍ BODY, STUPEŇ



SEGMENTACE, UZLOVÝ VEKTOR



VSTUPNÍ DATA



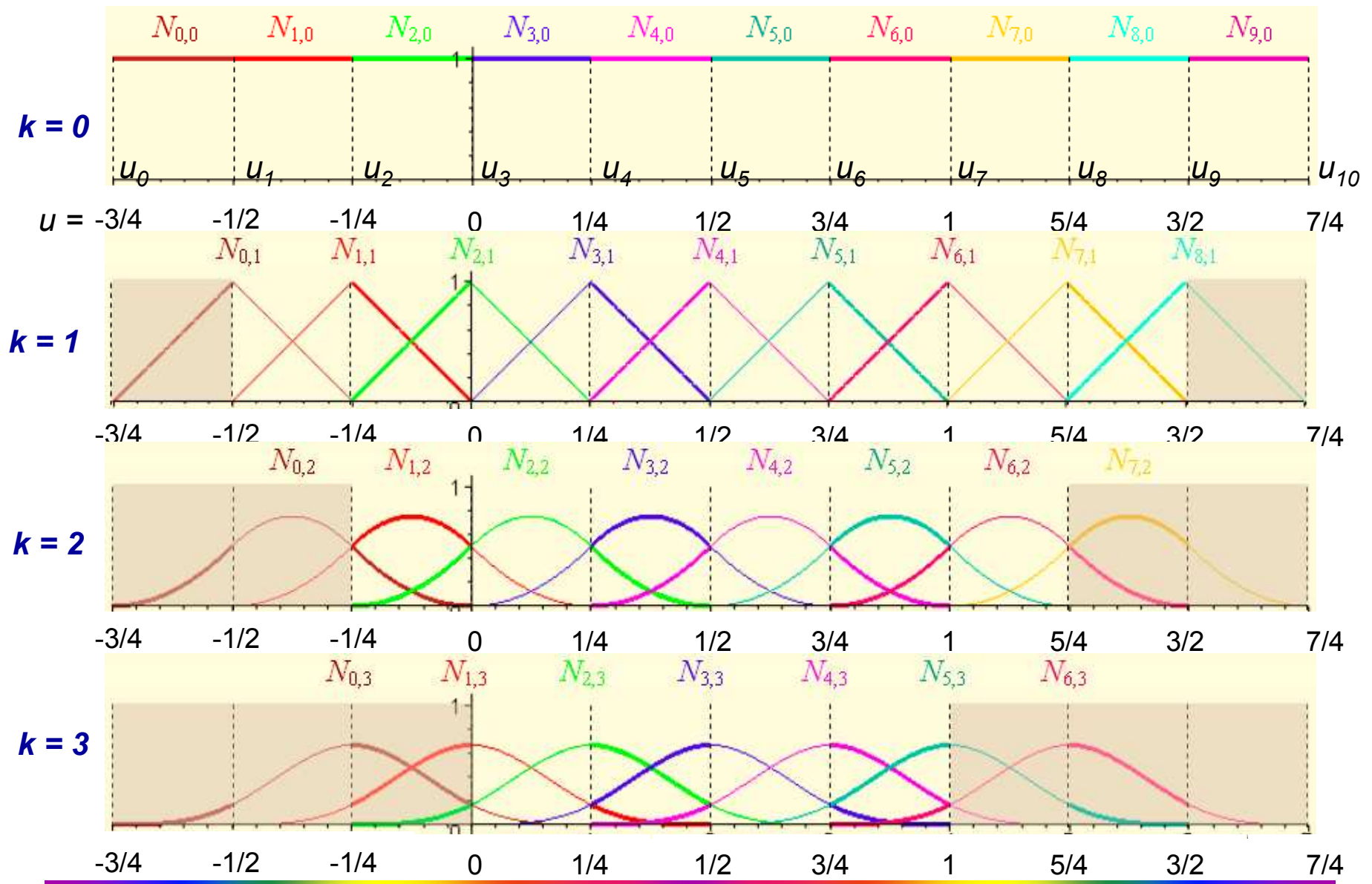
UZAVŘENÁ KŘIVKA

$$p = 3$$

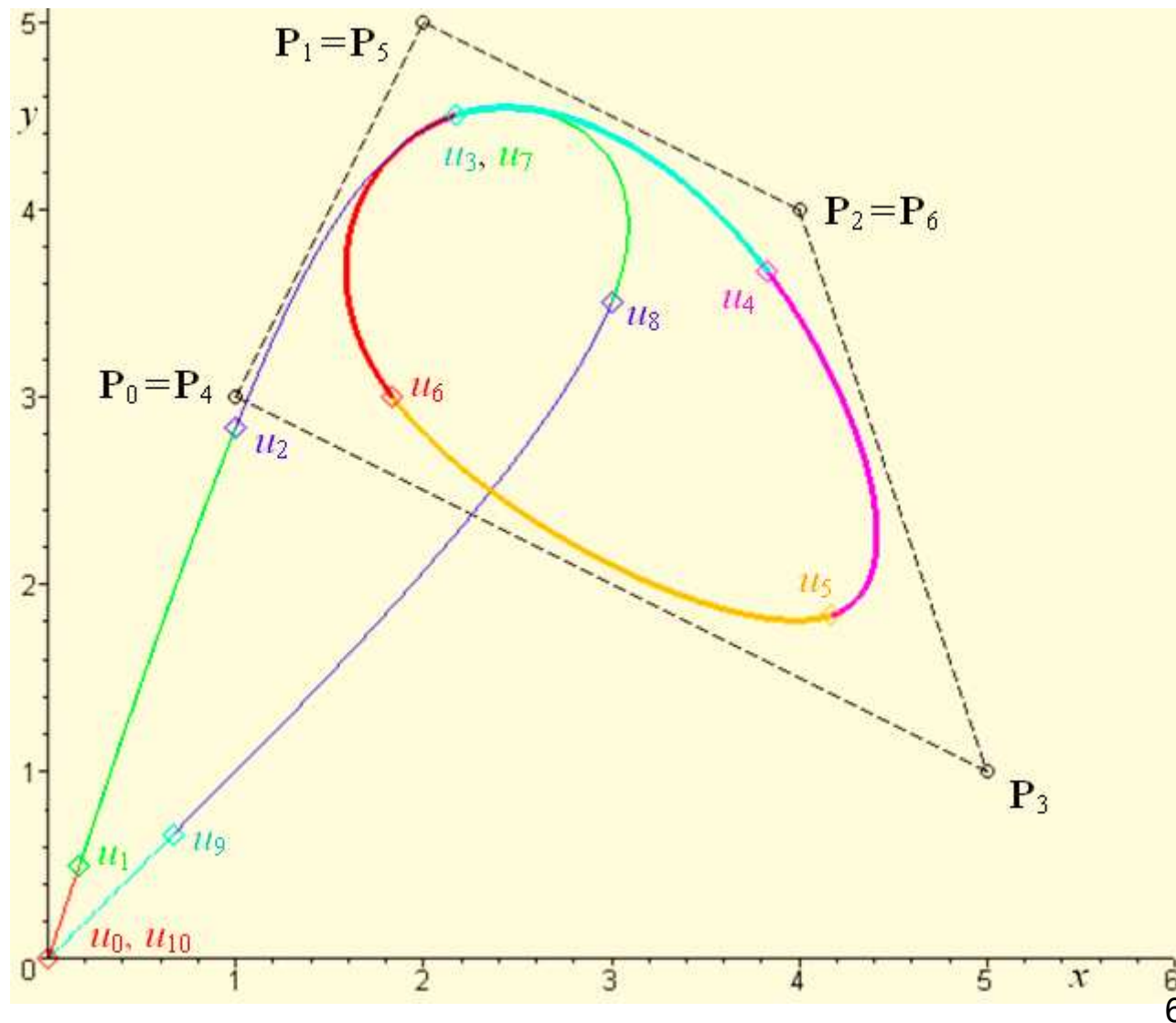
$$n = 6$$

$$m = 3 + 6 + 1 = 10$$

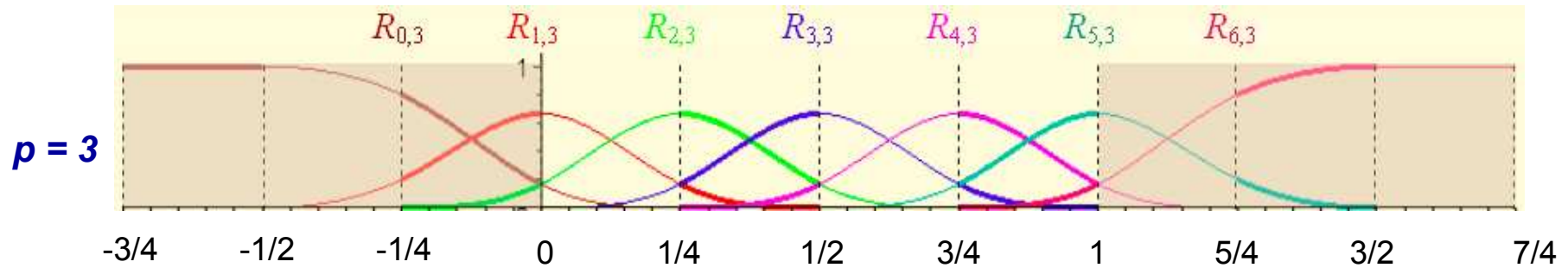
B-SPLINE BÁZOVÉ FUNKCE



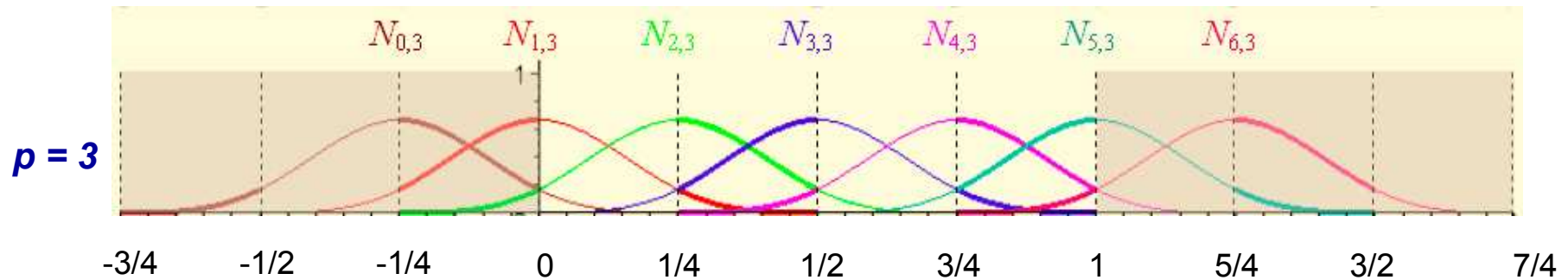
UZAVŘENÁ B-SPLINE KŘIVKA



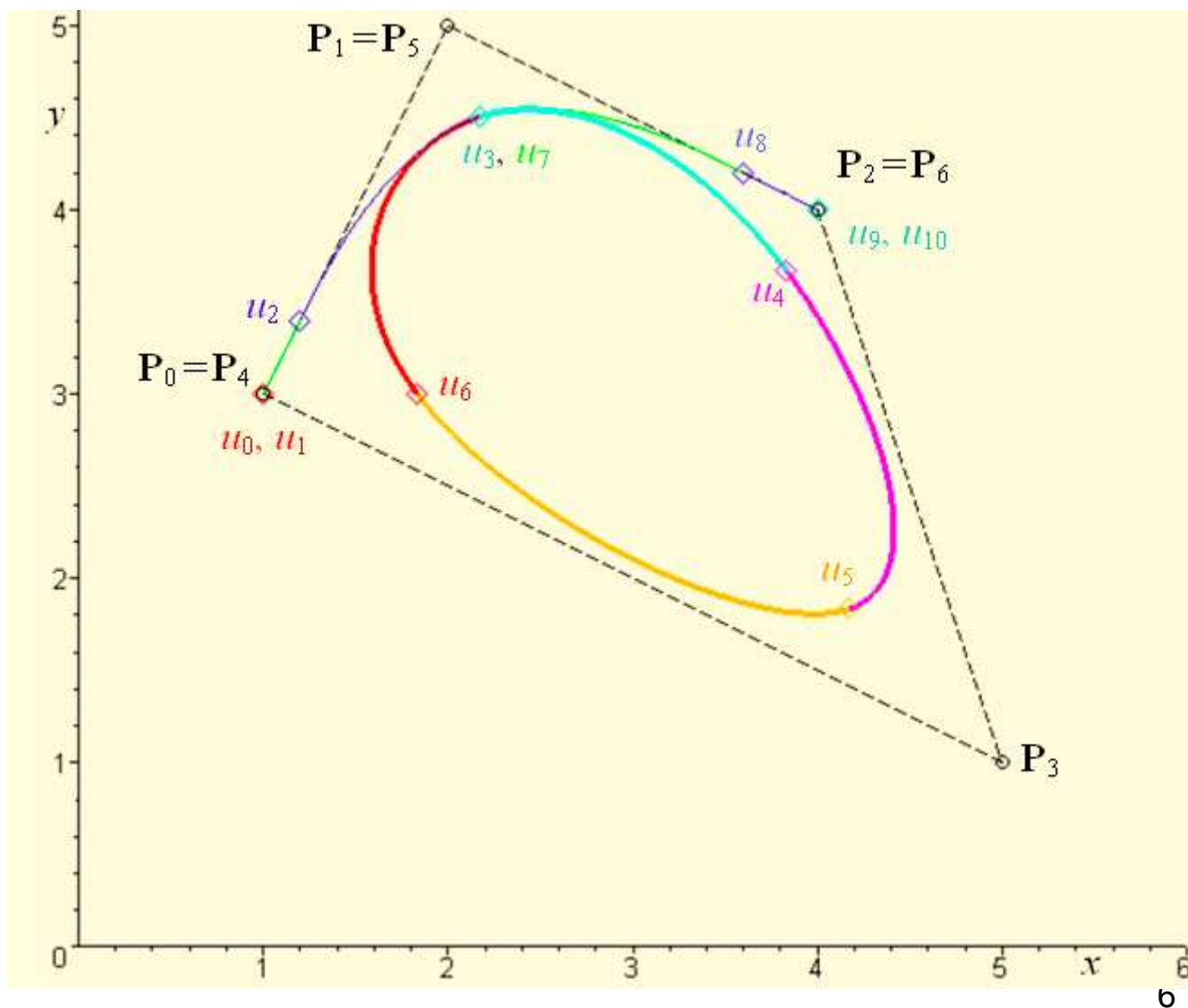
RACIONÁLNÍ BÁZOVÉ FUNKCE



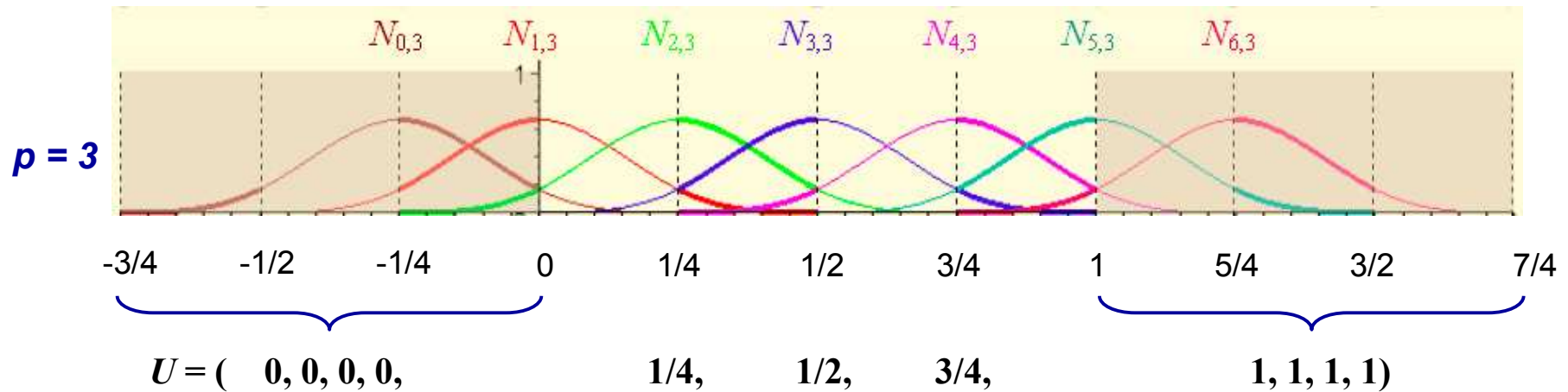
B-SPLINE BÁZOVÉ FUNKCE



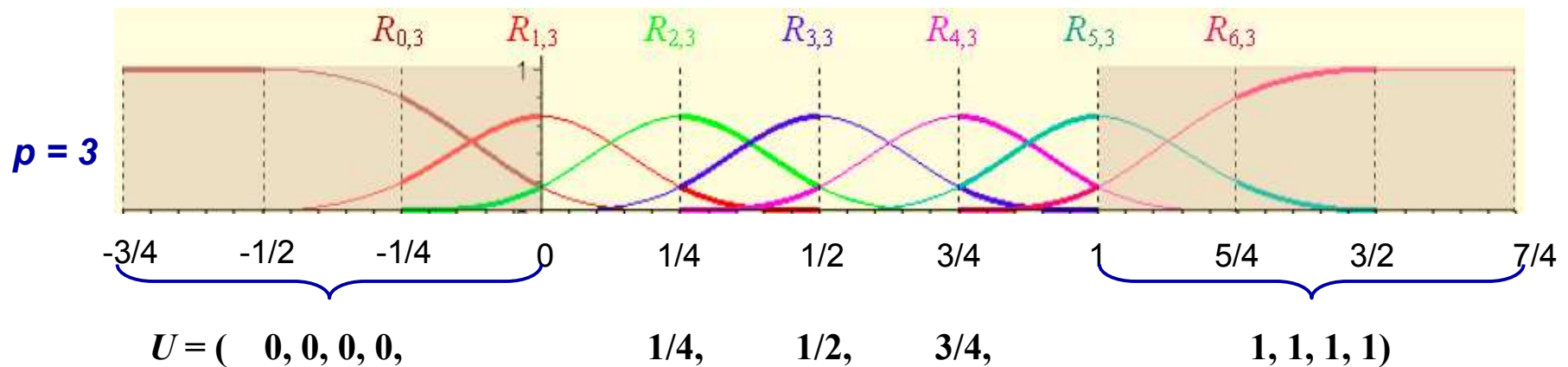
UZAVŘENÁ NURBS KŘIVKA



B-SPLINE BÁZOVÉ FUNKCE

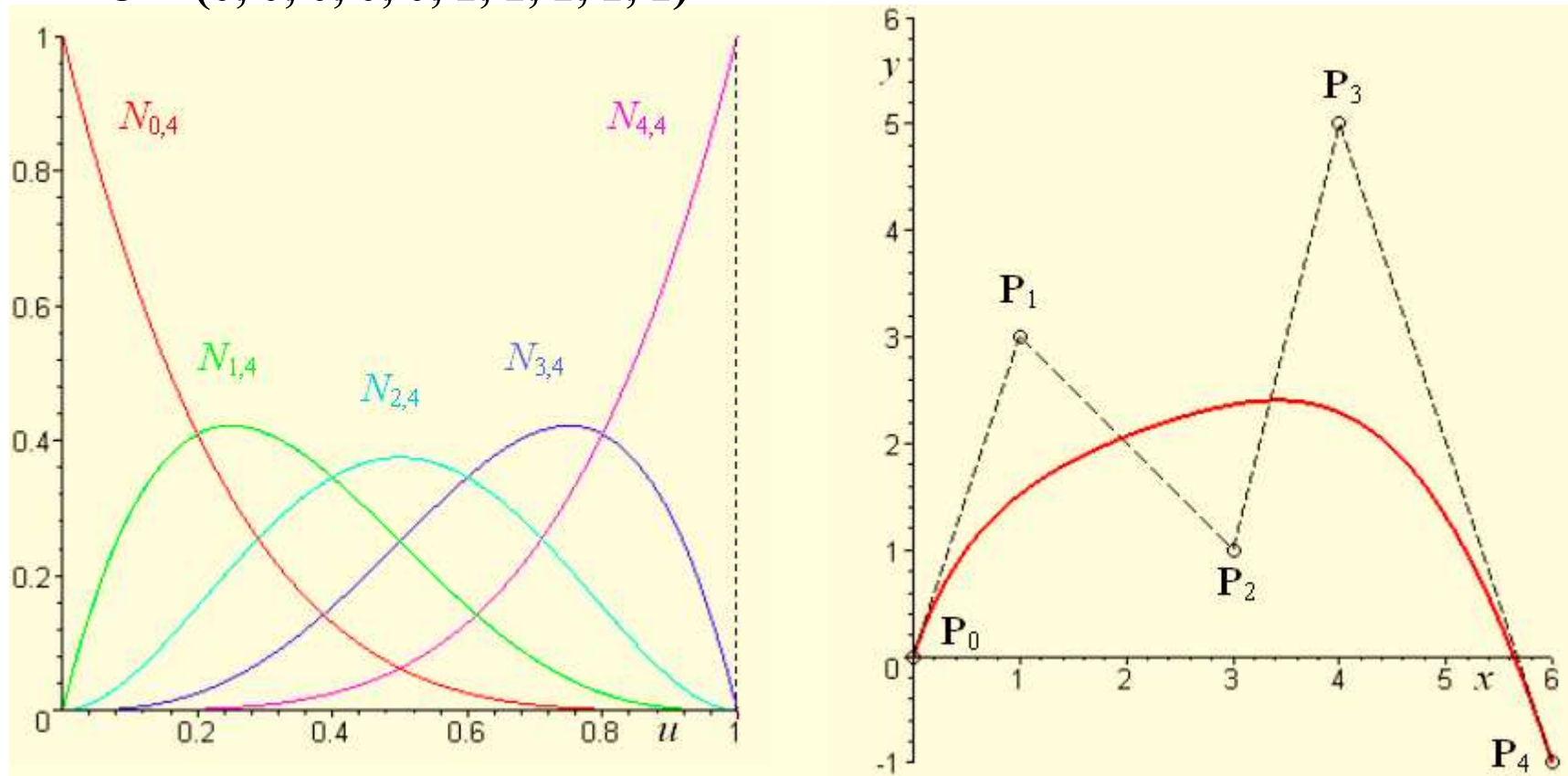


RACIONÁLNÍ BÁZOVÉ FUNKCE

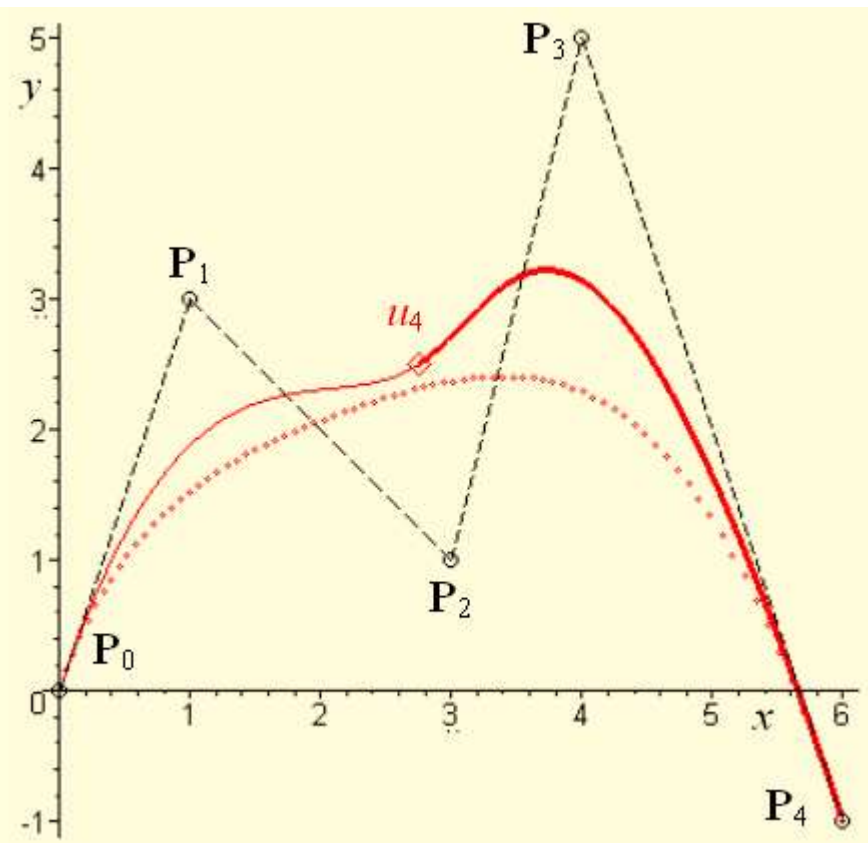
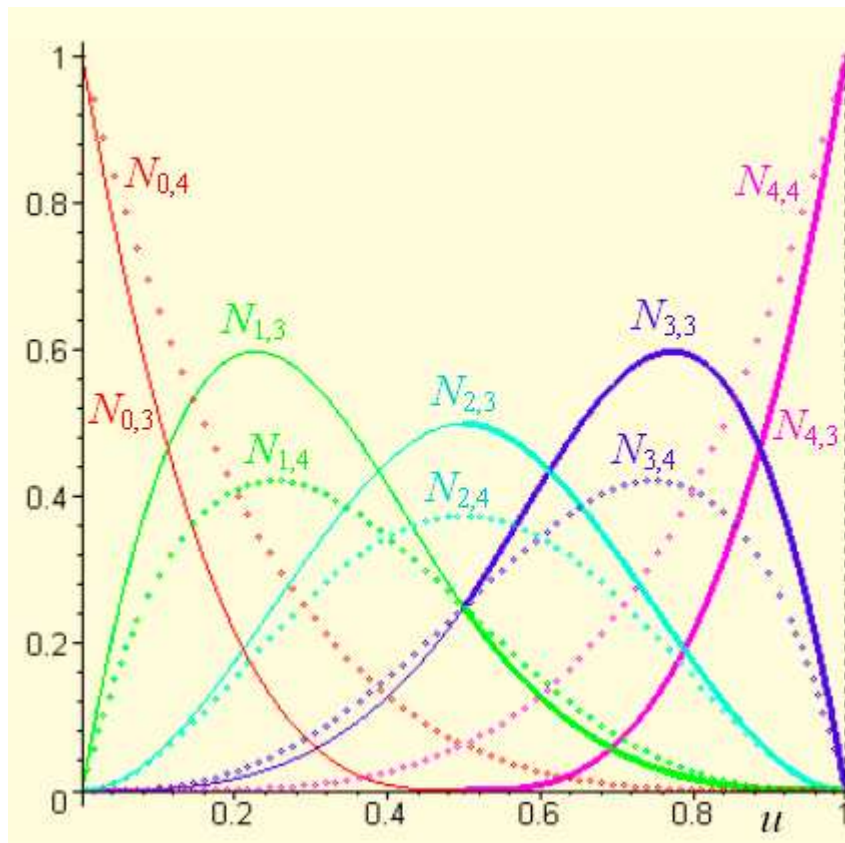


UKOTVENÁ B-SPLINE KŘIVKA

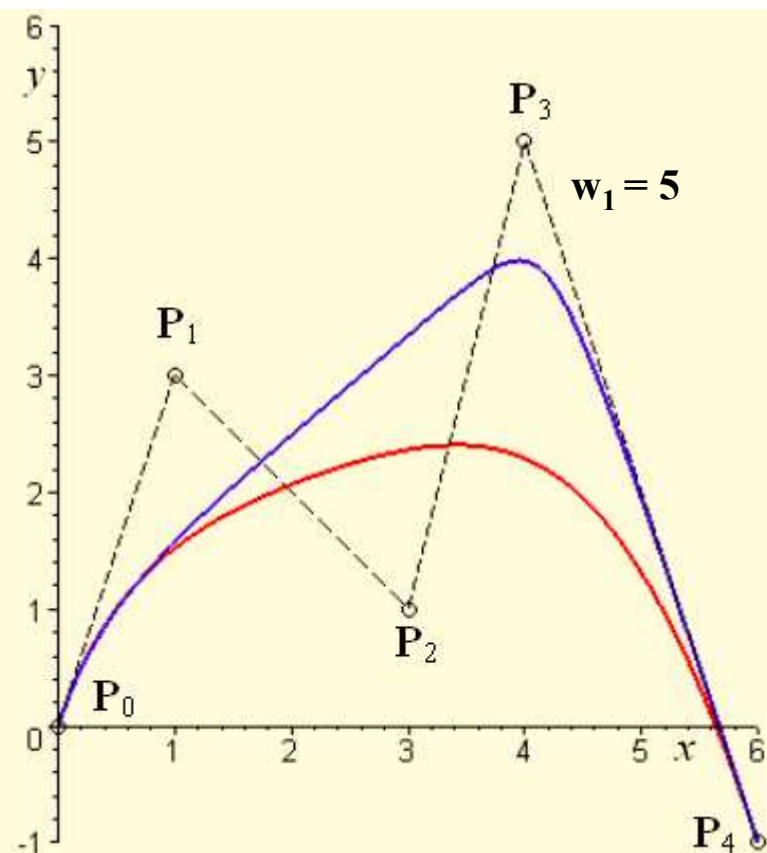
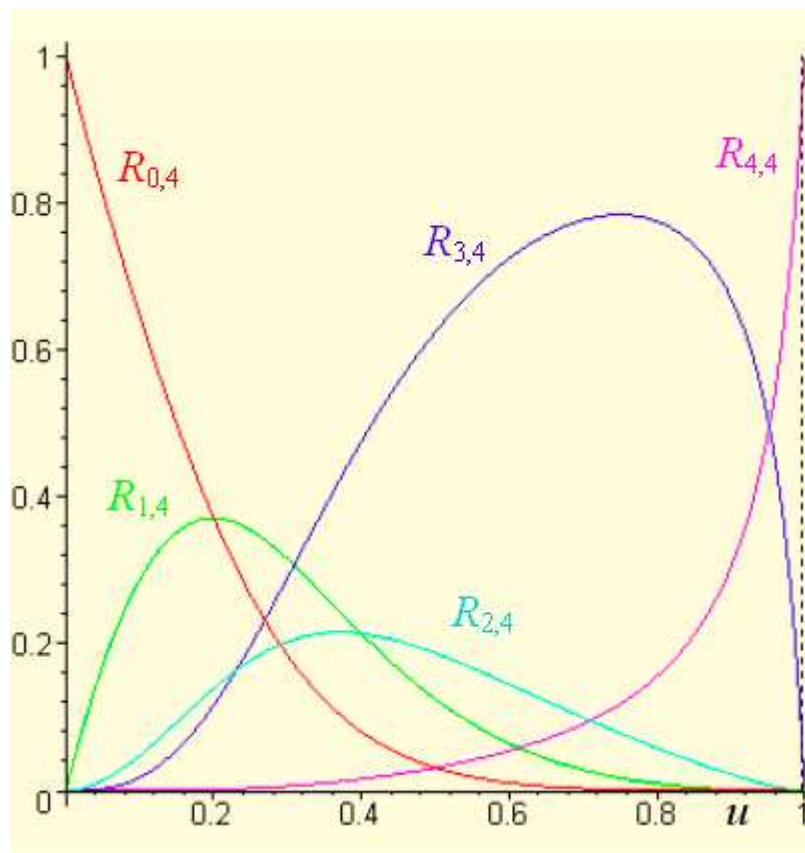
$$U = (0, 0, 0, 0, 0, 1, 1, 1, 1, 1)$$



STUPEŇ



VÁHY ŘÍDICÍCH BODŮ



INTERPOLAČNÍ NURBS KŘIVKY

TVAROVACÍ NÁSTROJE:

DEFINIČNÍ BODY

Q_0, Q_1, \dots, Q_n

STUPEŇ

p

UZLOVÝ VEKTOR

$U = (u_0, u_1, \dots, u_m)$

VEKTOR PARAMETRIZACE

$H = (h_0, h_1, \dots, h_n)$

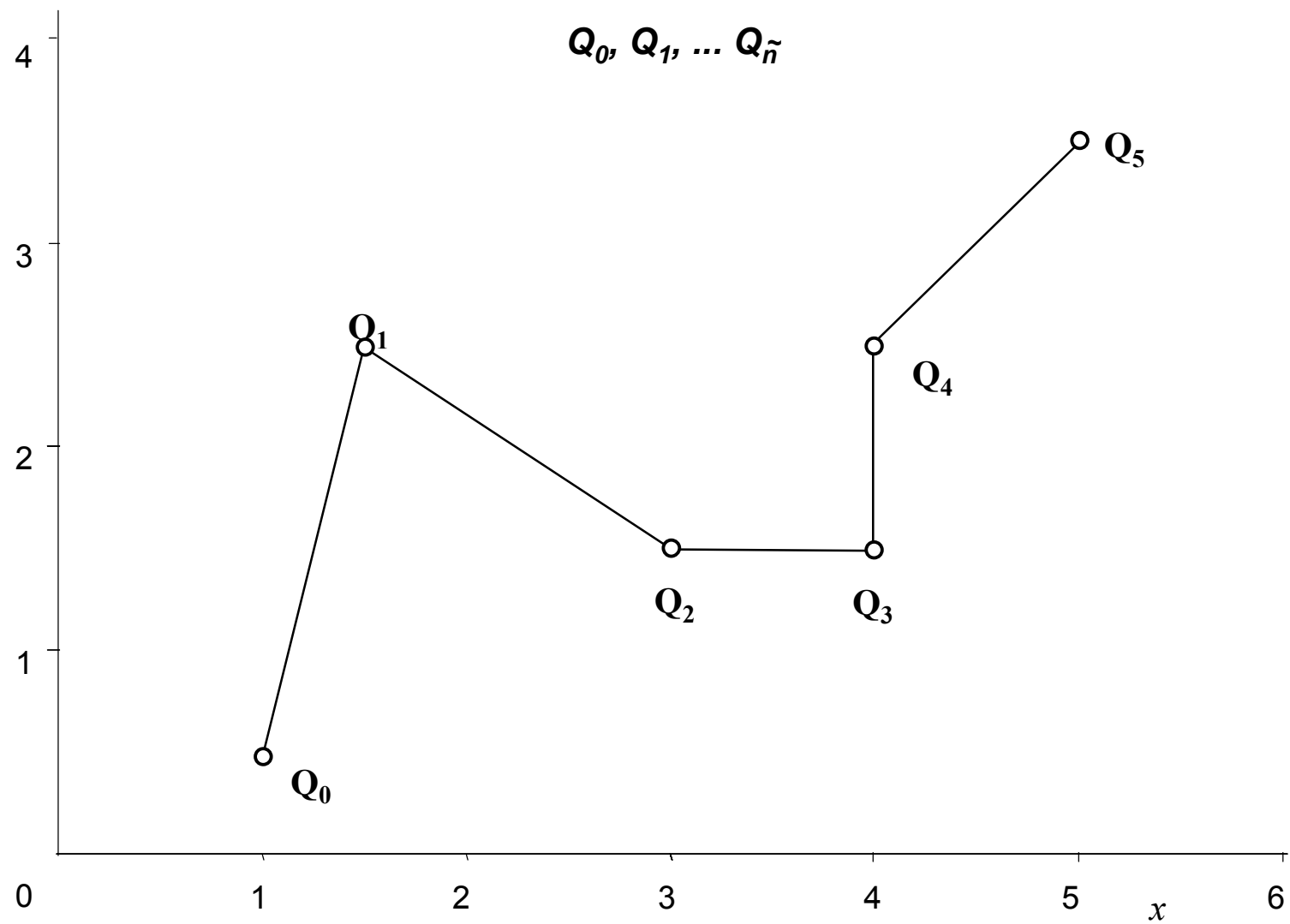
VÁHY ŘÍDICÍCH BODŮ

w_0, w_1, \dots, w_n

ZPŮSOB INTERPOLACE

PROSTÁ, UZLOVÁ

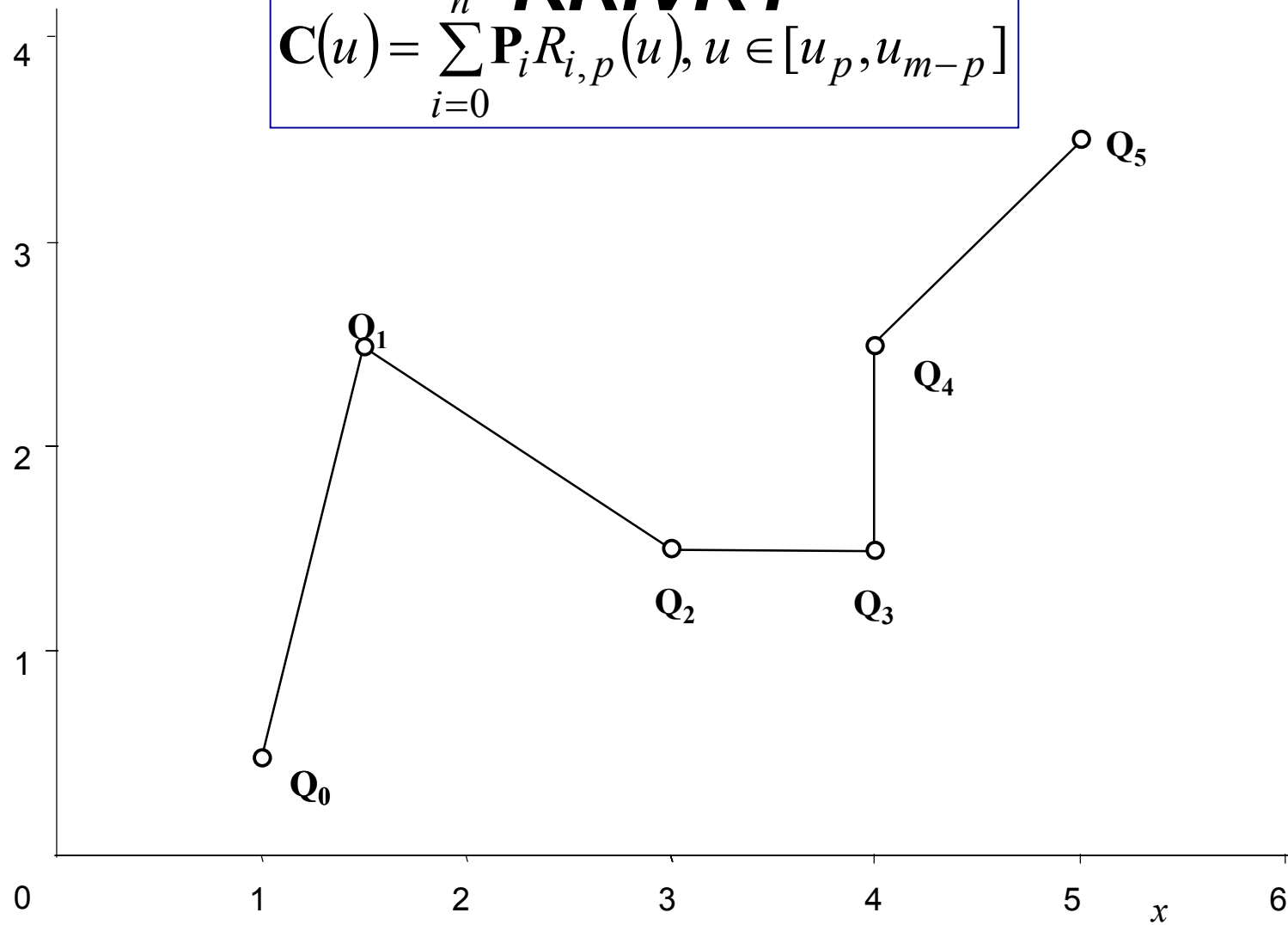
DEFINIČNÍ BODY



DEFINICE INTERPOLAČNI NURBS

KŘIVKY

$$C(u) = \sum_{i=0}^n P_i R_{i,p}(u), u \in [u_p, u_{m-p}]$$



PROSTÁ INTERPOLAČNÍ NURBS KŘIVKA

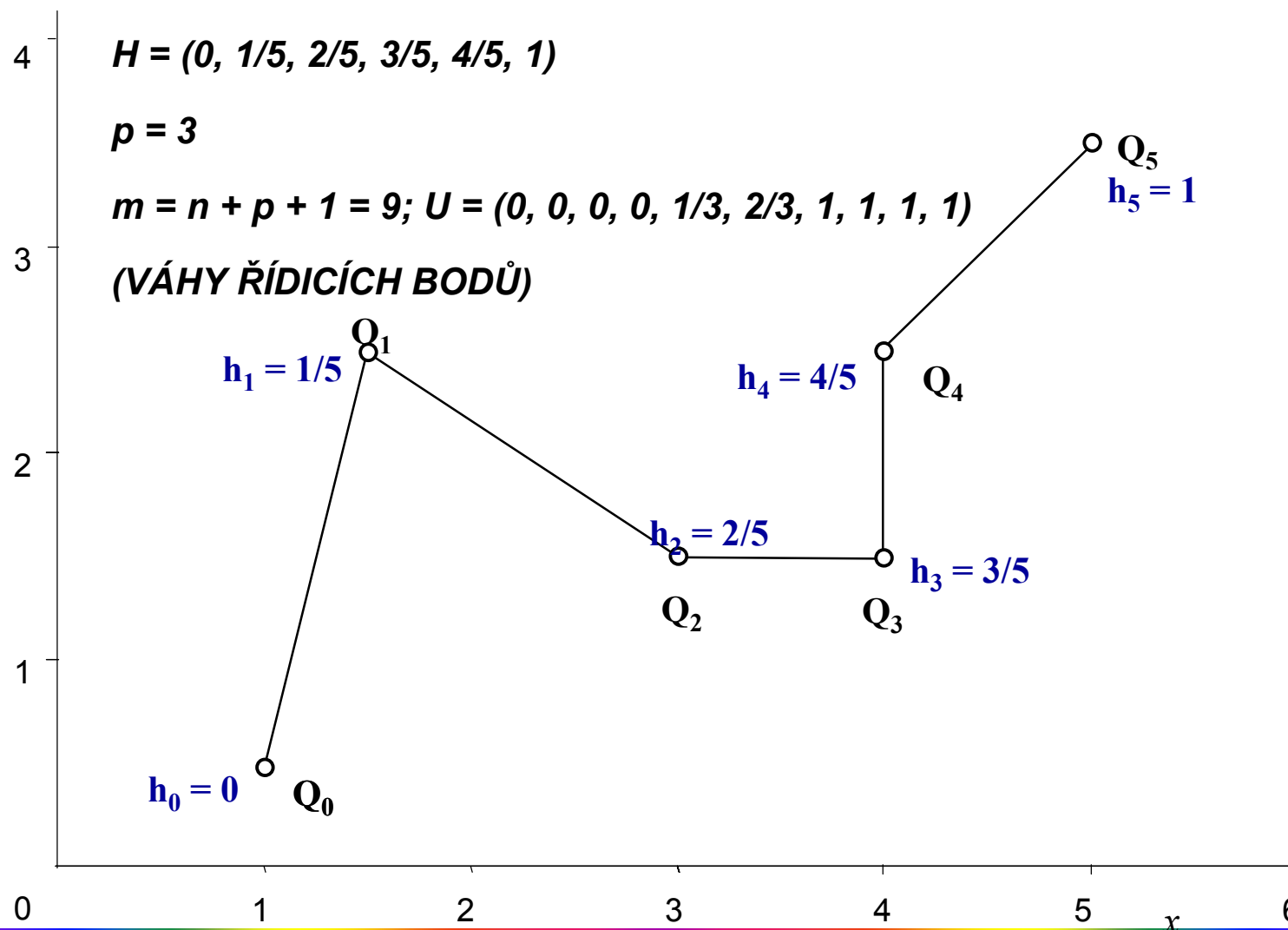
$$\tilde{n} = n = 5$$

$$H = (0, 1/5, 2/5, 3/5, 4/5, 1)$$

$$p = 3$$

$$m = n + p + 1 = 9; U = (0, 0, 0, 0, 1/3, 2/3, 1, 1, 1, 1)$$

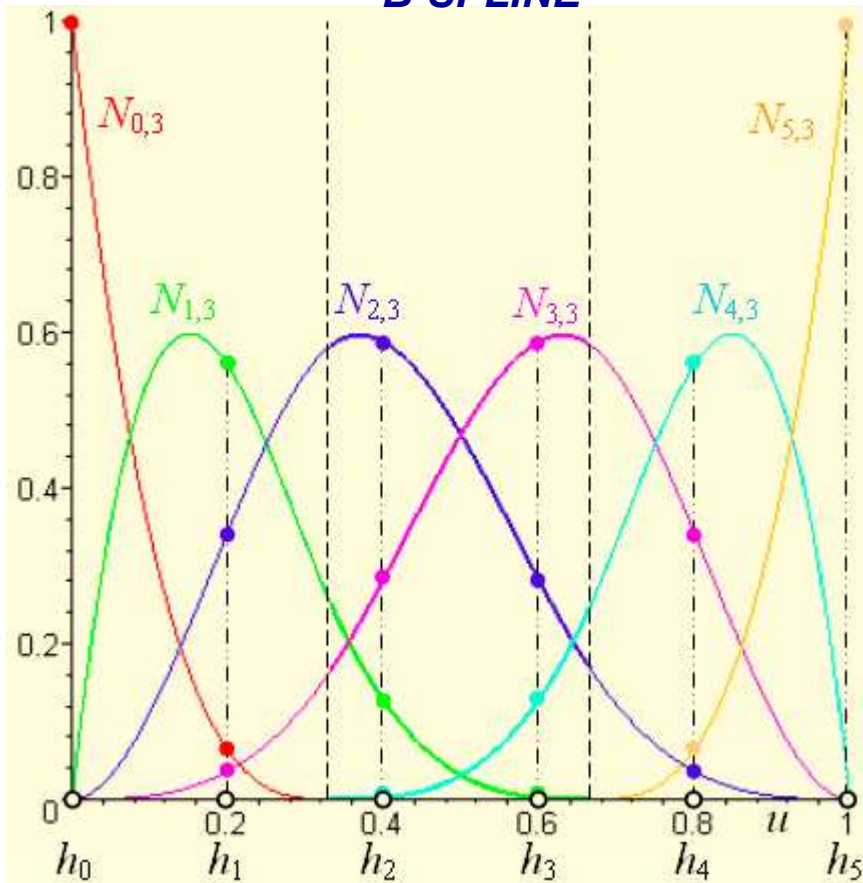
(VÁHY ŘÍDICÍCH BODŮ)



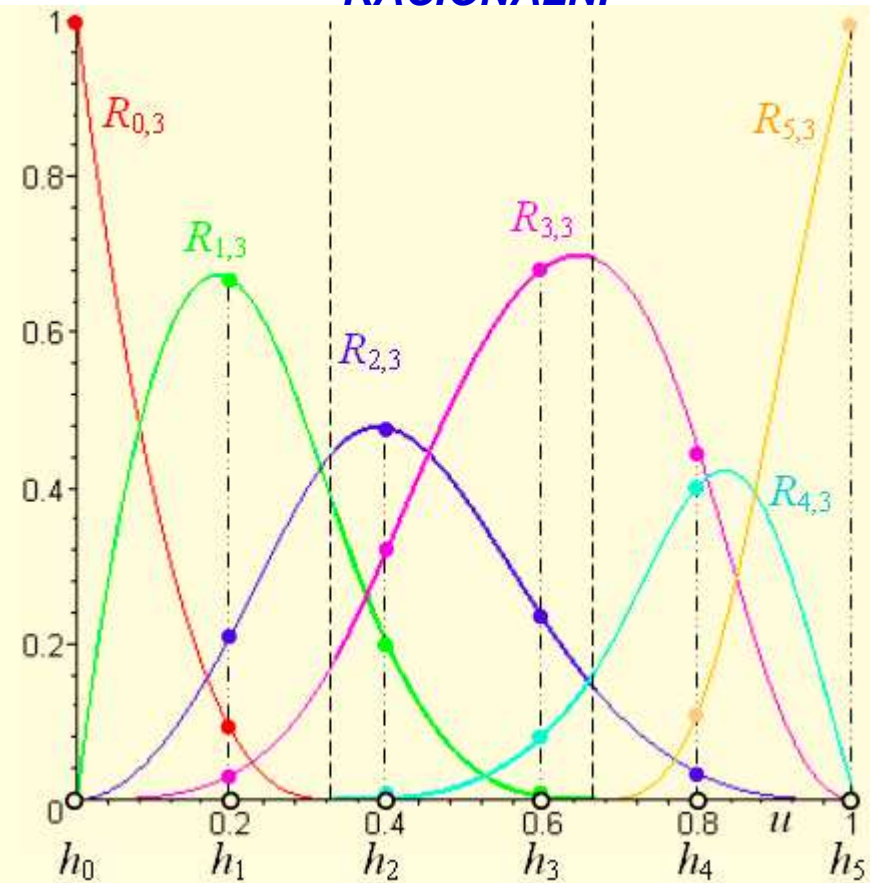
BAZOVÉ FUNKCE A JEJICH FUNKCNI

$$\mathbf{C}(h_i) = \sum_{j=0}^{\tilde{n}} R_{j,p}(h_i) \mathbf{P}_j = \mathbf{Q}_i, \quad i = 0, \dots, \tilde{n}$$

B-SPLINE



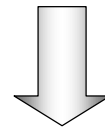
RACIONÁLNÍ



PROSTÁ INTERPOLAČNÍ NURBS KŘIVKA

$$\mathbf{C}(h_i) = \sum_{j=0}^{\tilde{n}} R_{j,p}(h_i) \mathbf{P}_j = \mathbf{Q}_i, \quad i = 0, \dots, \tilde{n}$$

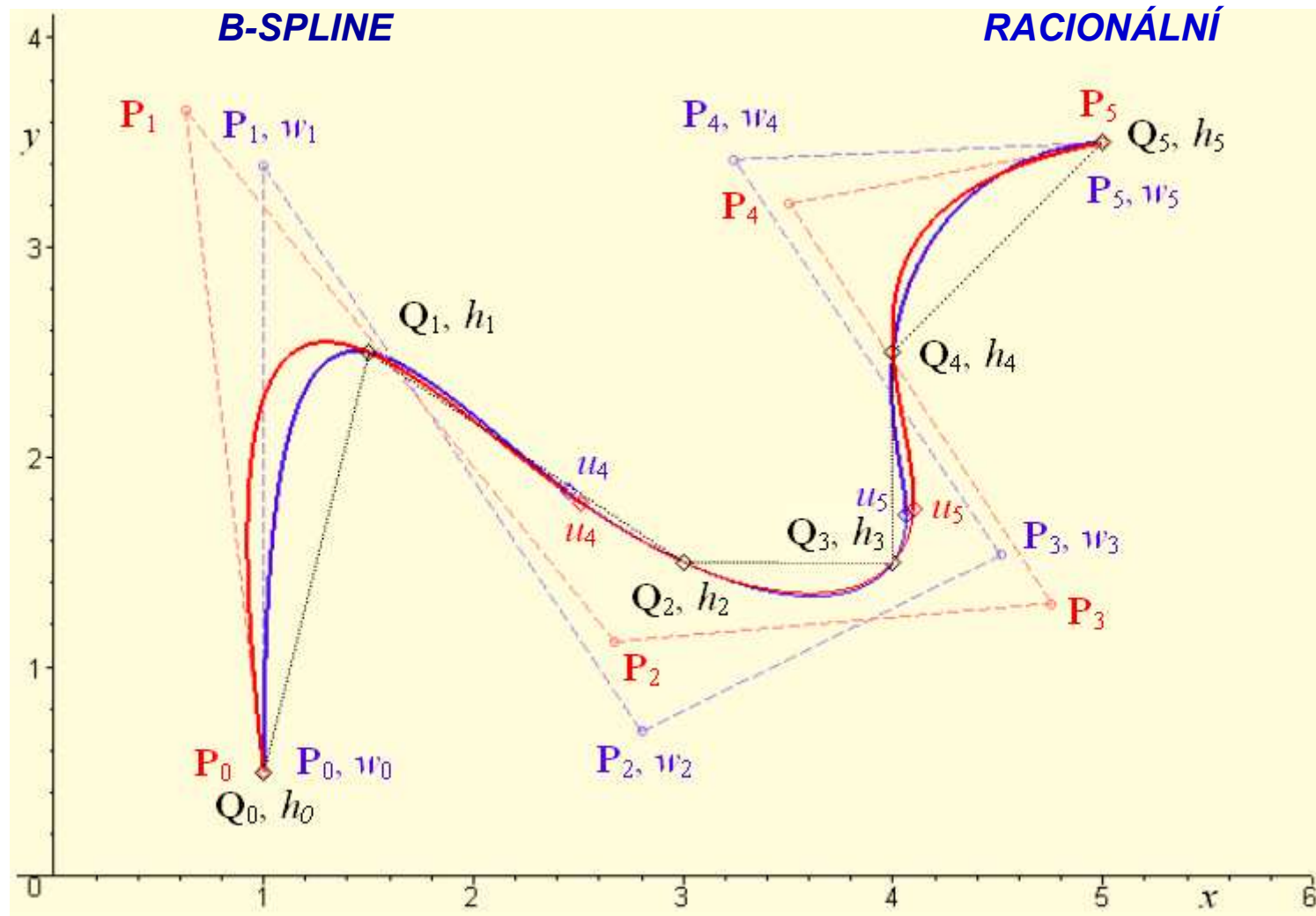
$$\begin{pmatrix} R_{0,p}(h_0) & R_{1,p}(h_0) & \cdots & R_{\tilde{n},p}(h_0) \\ R_{0,p}(h_1) & R_{1,p}(h_1) & \cdots & R_{\tilde{n},p}(h_1) \\ \vdots & \vdots & & \vdots \\ R_{0,p}(h_{\tilde{n}}) & R_{1,p}(h_{\tilde{n}}) & \cdots & R_{\tilde{n},p}(h_{\tilde{n}}) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{P}_0 \\ \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_{\tilde{n}} \end{pmatrix} = \begin{pmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{\tilde{n}} \end{pmatrix}$$



$\mathbf{P}_0, \mathbf{P}_1, \dots, \mathbf{P}_n$

$$\mathbf{C}(u) = \sum_{i=0}^n \mathbf{P}_i R_{i,p}(u), \quad u \in [u_p, u_{m-p}]$$

PROSTÁ INTERPOLAČNÍ NURBS KŘIVKA

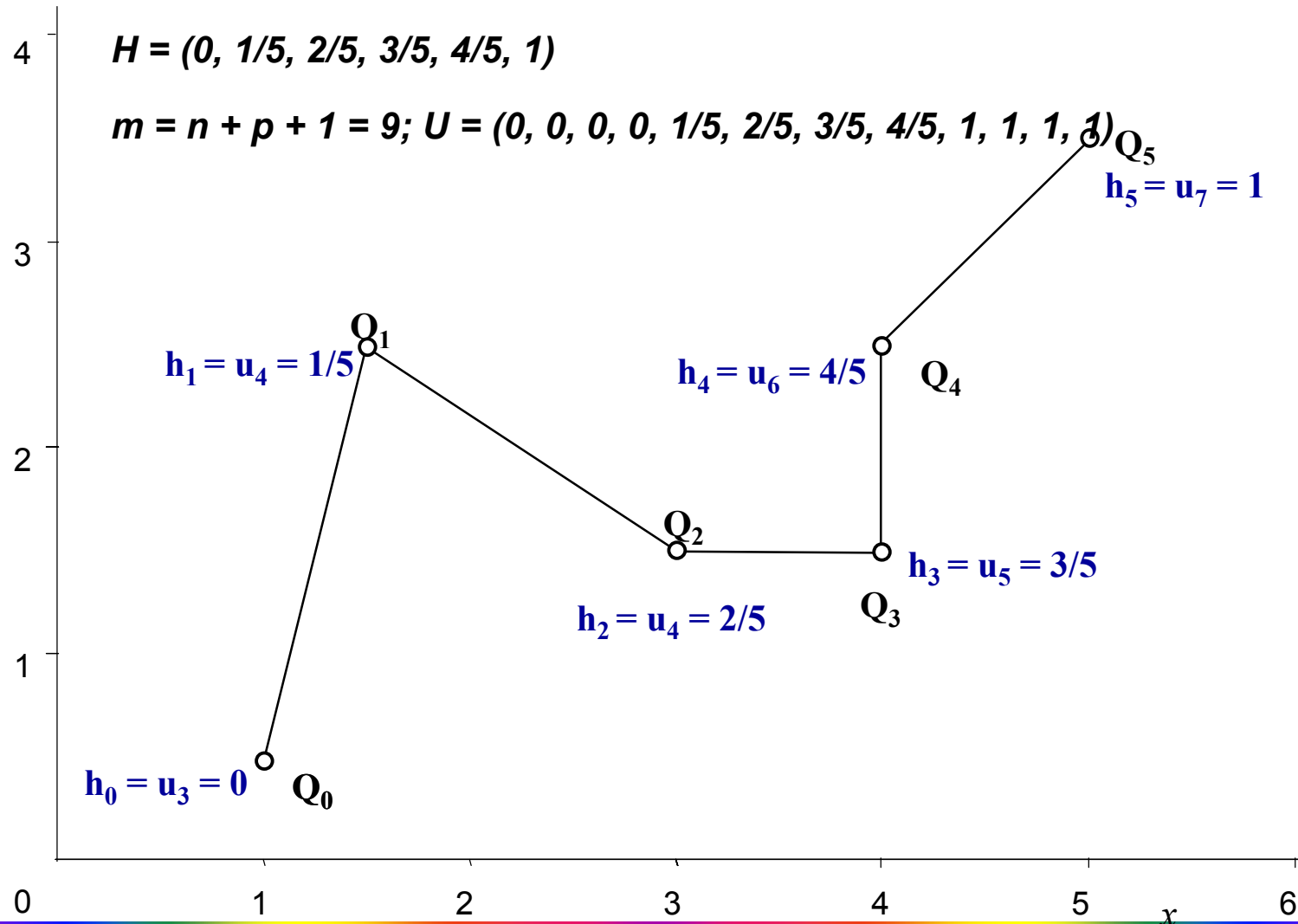


UZLOVÁ INTERPOLAČNÍ NURBS KŘIVKA

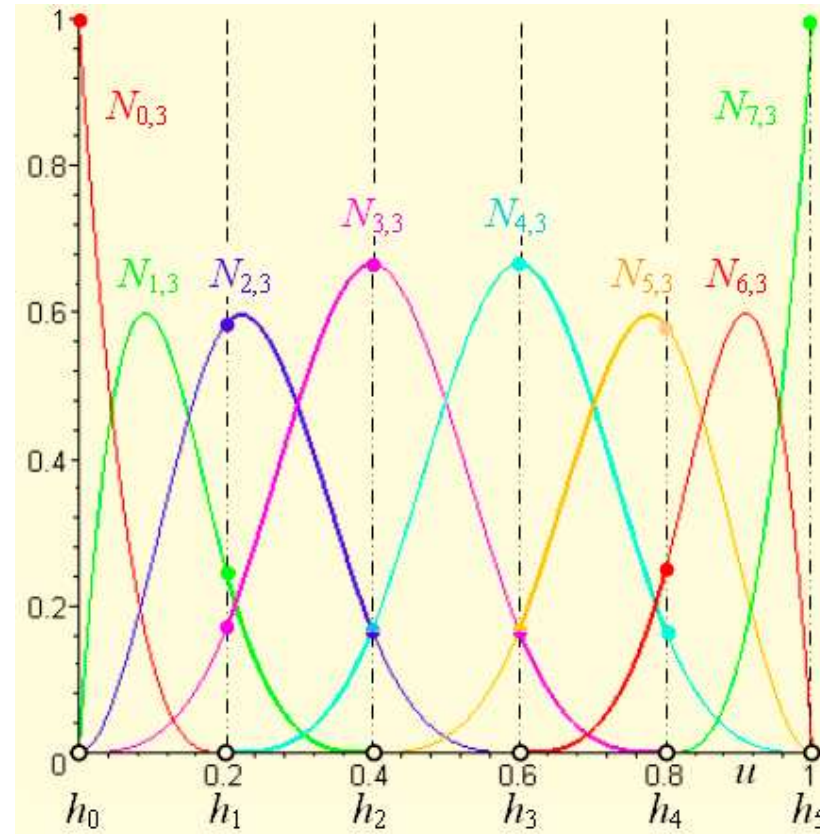
$$\tilde{n} = n + p - 1 = 5 + 3 - 1 = 7$$

$$H = (0, 1/5, 2/5, 3/5, 4/5, 1)$$

$$m = n + p + 1 = 9; U = (0, 0, 0, 0, 1/5, 2/5, 3/5, 4/5, 1, 1, 1, 1)$$



BAZOVÉ FUNKCE A JEJICH FUNKČNÍ HODNOTY



UZLOVÁ INTERPOLAČNÍ NURBS KŘIVKA

$$\mathbf{C}(h_i) = \sum_{j=0}^n N_{j,3}(h_i) \mathbf{P}_j = \mathbf{Q}_i, \quad i = 0, \dots, \tilde{n}$$

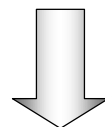
$$\mathbf{C}^{(k)}(h_0) = \sum_{j=0}^n N_{j,3}^{(k)}(h_0) \mathbf{P}_j = \mathbf{Q}_0^{(k)}$$

$$\mathbf{C}^{(k)}(h_{\tilde{n}}) = \sum_{j=0}^n N_{j,3}^{(k)}(h_{\tilde{n}}) \mathbf{P}_j = \mathbf{Q}_{\tilde{n}}^{(k)},$$



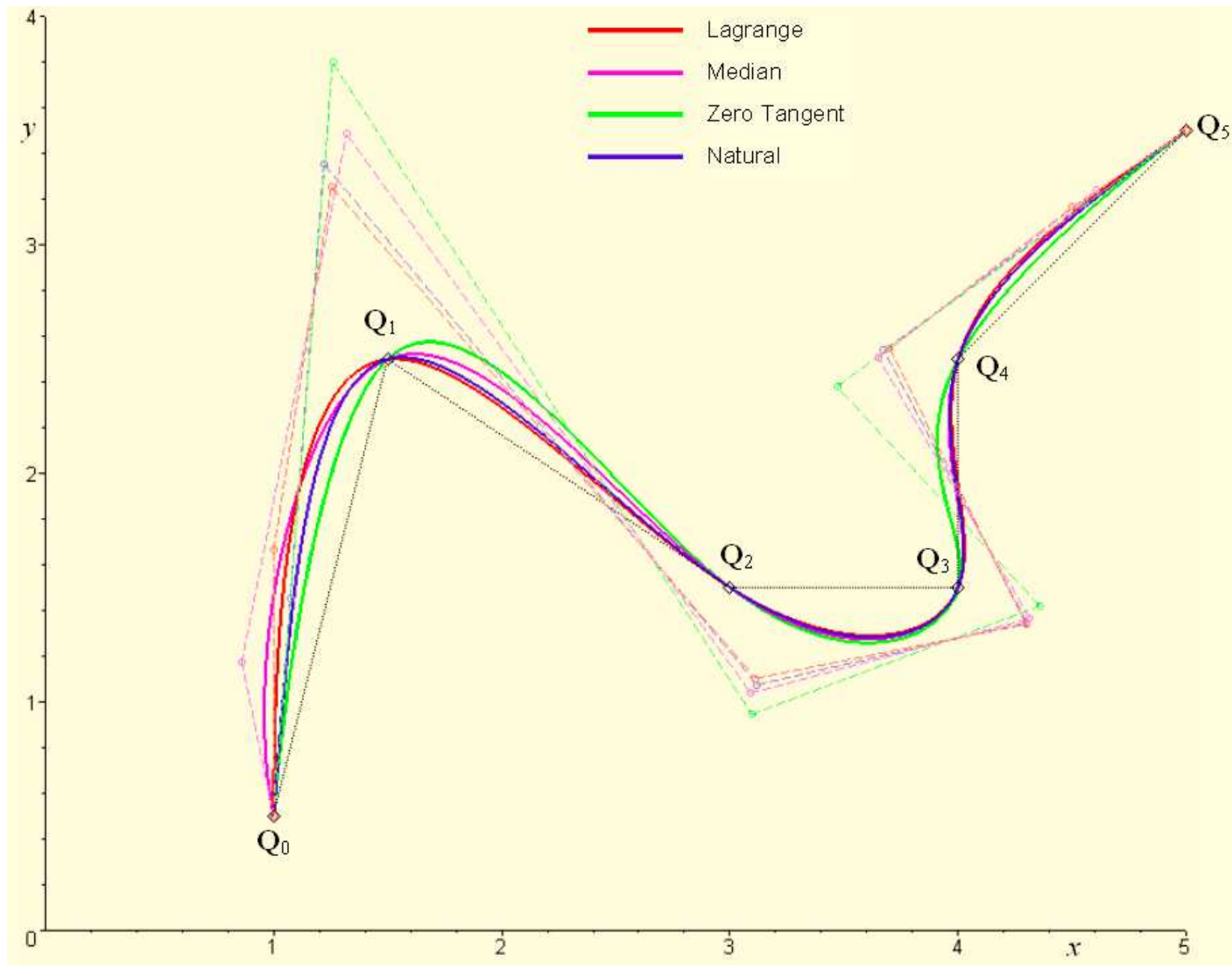
UZLOVÁ INTERPOLAČNÍ NURBS KŘIVKA

$$\begin{pmatrix} N_{0,3}(h_0) & N_{1,3}(h_0) & \cdots & N_{n,3}(h_0) \\ N_{0,3}(h_1) & N_{1,3}(h_1) & \cdots & N_{n,3}(h_1) \\ \vdots & & & \vdots \\ N_{0,3}(h_{\tilde{n}}) & N_{1,3}(h_{\tilde{n}}) & \cdots & N_{n,3}(h_{\tilde{n}}) \\ N_{0,3}^{(k)}(h_0) & N_{1,3}^{(k)}(h_0) & \cdots & N_{n,3}^{(k)}(h_0) \\ N_{0,3}^{(k)}(h_{\tilde{n}}) & N_{1,3}^{(k)}(h_{\tilde{n}}) & \cdots & N_{n,3}^{(k)}(h_{\tilde{n}}) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{P}_0 \\ \mathbf{P}_1 \\ \vdots \\ \mathbf{P}_{n-2} \\ \mathbf{P}_{n-1} \\ \mathbf{P}_n \end{pmatrix} = \begin{pmatrix} \mathbf{Q}_0 \\ \mathbf{Q}_1 \\ \vdots \\ \mathbf{Q}_{\tilde{n}} \\ \mathbf{Q}_0^{(k)} \\ \mathbf{Q}_{\tilde{n}}^{(k)} \end{pmatrix}$$

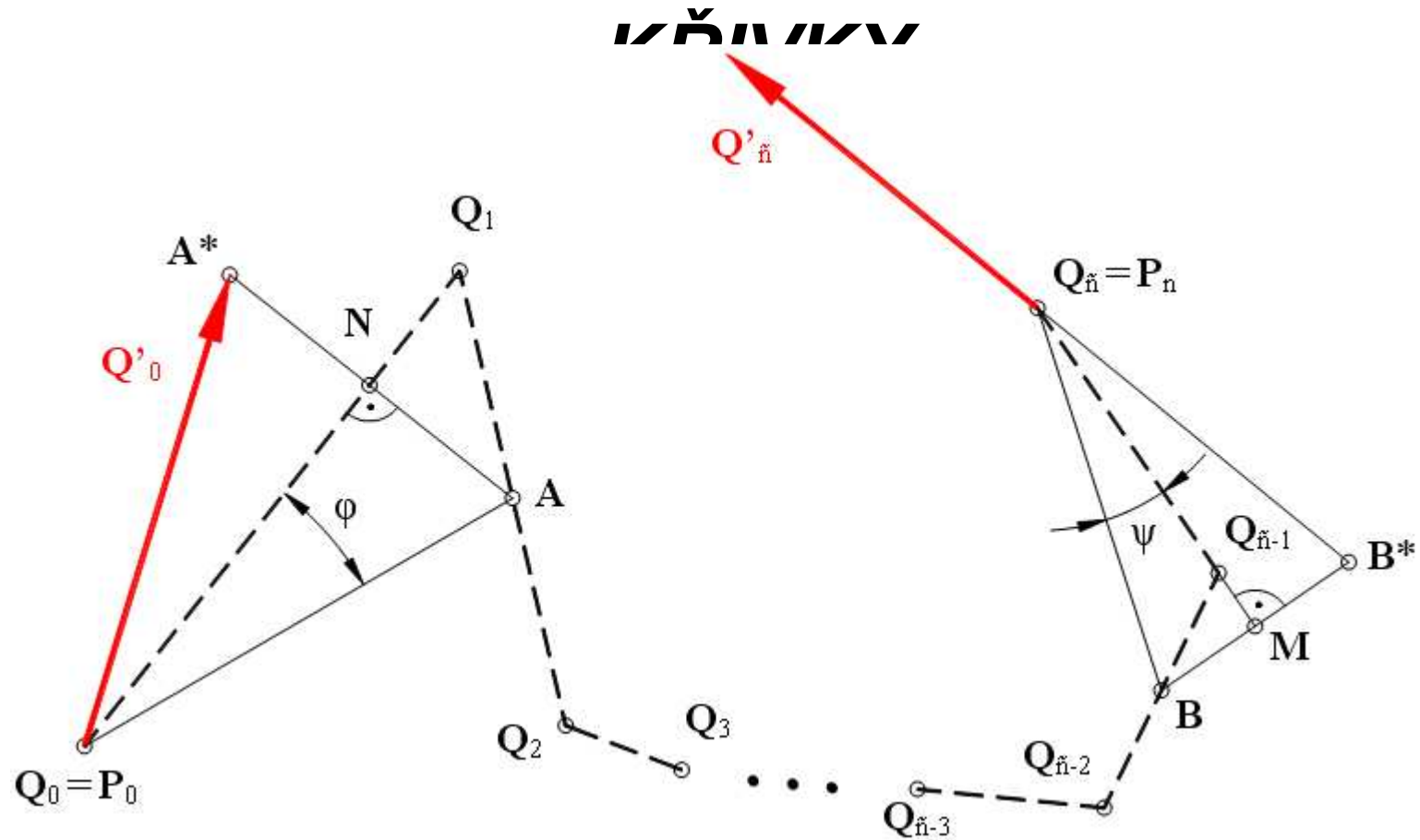


$$\mathbf{C}(u) = \sum_{i=0}^n \mathbf{P}_i R_{i,p}(u), u \in [u_p, u_{m-p}]$$

UZLOVÁ INTERPOLAČNÍ NURBS KŘIVKA



V KRAJNÍCH BODECH INTERPOLAČNÍ



NURBS PLOCHA

$$\mathbf{S}(u, v) = \sum_{i=0}^m \sum_{j=0}^n R_{i,j}(u, v) \mathbf{P}_{i,j}, \quad 0 \leq u \leq 1, \quad 0 \leq v \leq 1$$

$$R_{i,j}(u, v) = \frac{N_{i,p}(u) N_{j,q}(v) w_i}{\sum_{i=0}^m \sum_{j=0}^n N_{i,p}(u) N_{j,q}(v) w_i}$$

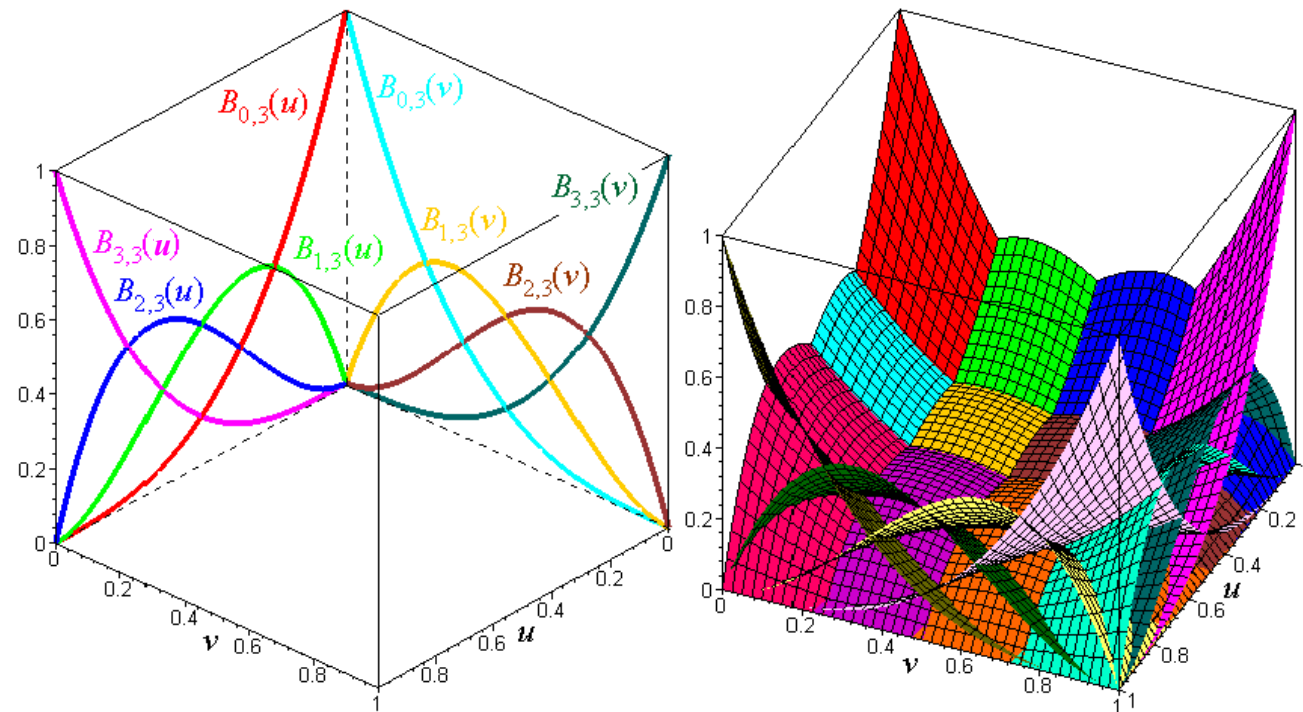
$$N_{i,0}(u) = \begin{cases} 1 & u_i \leq u < u_{i+1} \\ 0 & \text{jinde} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

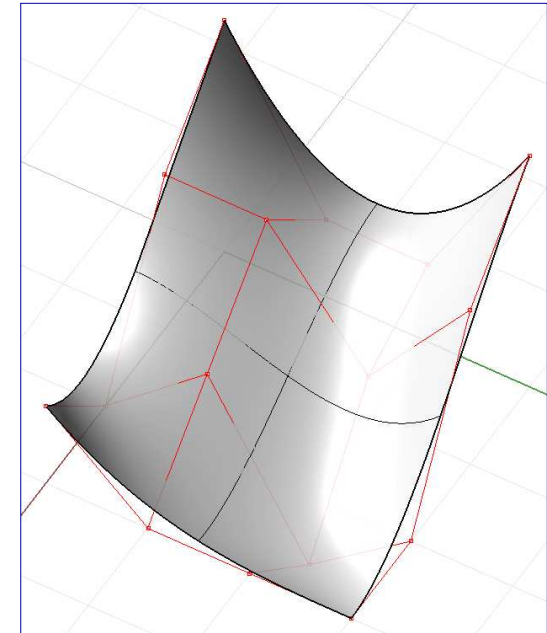
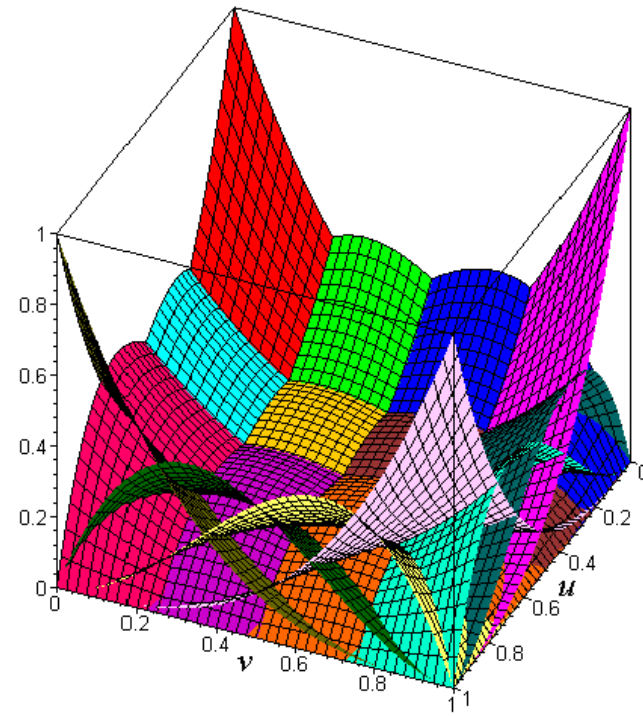
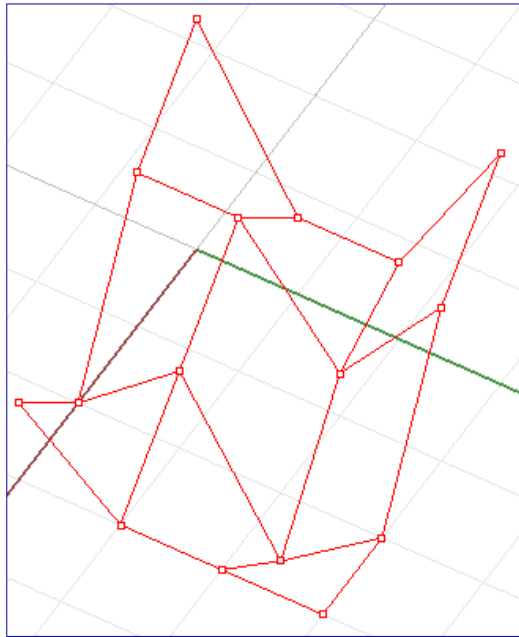
$$N_{j,0}(v) = \begin{cases} 1 & v_j \leq v < v_{j+1} \\ 0 & \text{jinde} \end{cases}$$

$$N_{j,p}(v) = \frac{v - v_j}{v_{j+q} - v_j} N_{j,q-1}(v) + \frac{v_{j+q+1} - v}{v_{j+q+1} - v_{j+1}} N_{j+1,q-1}(v)$$

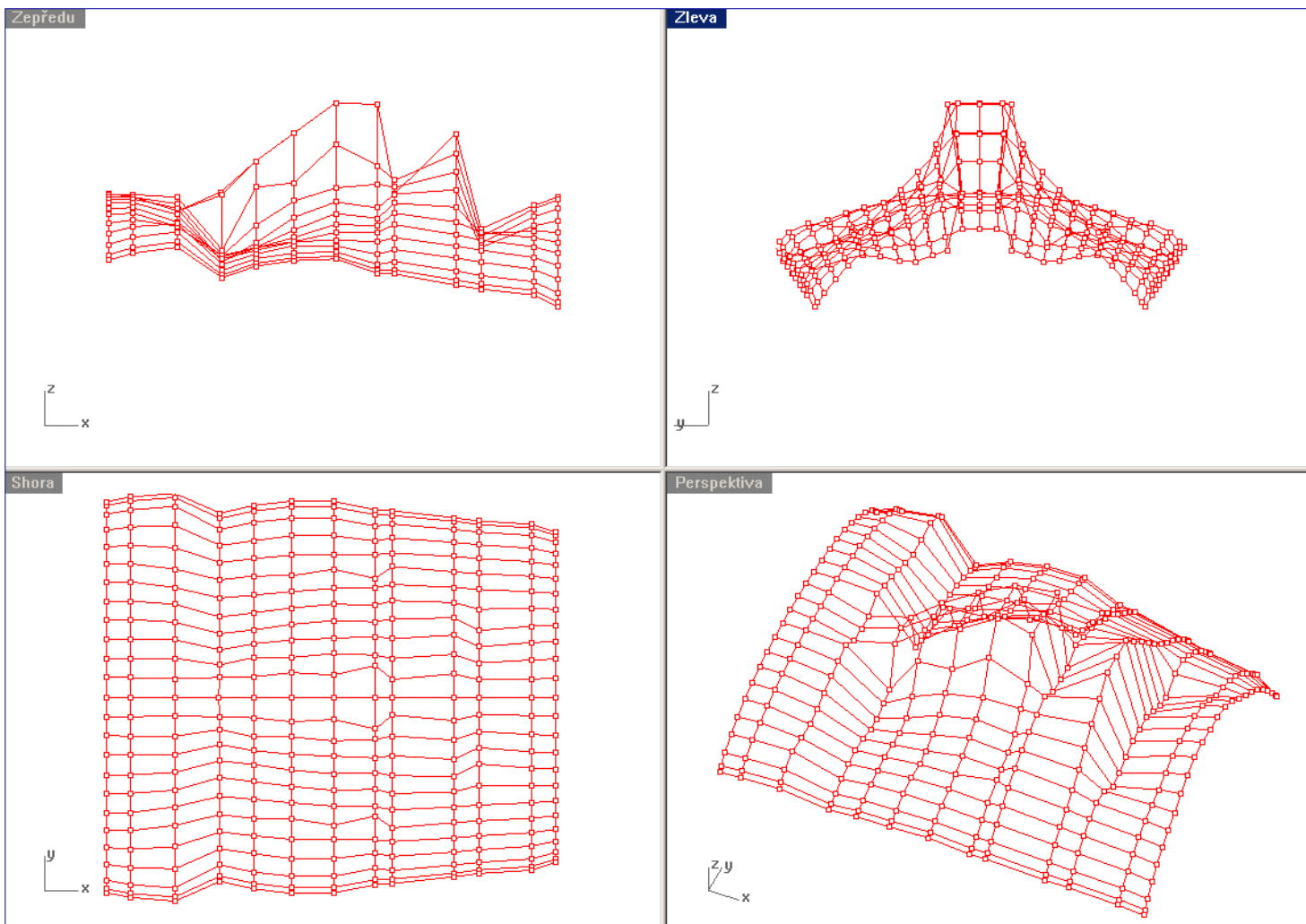
DVOUPARAMETRICKÉ BÁZOVÉ FUNKCE



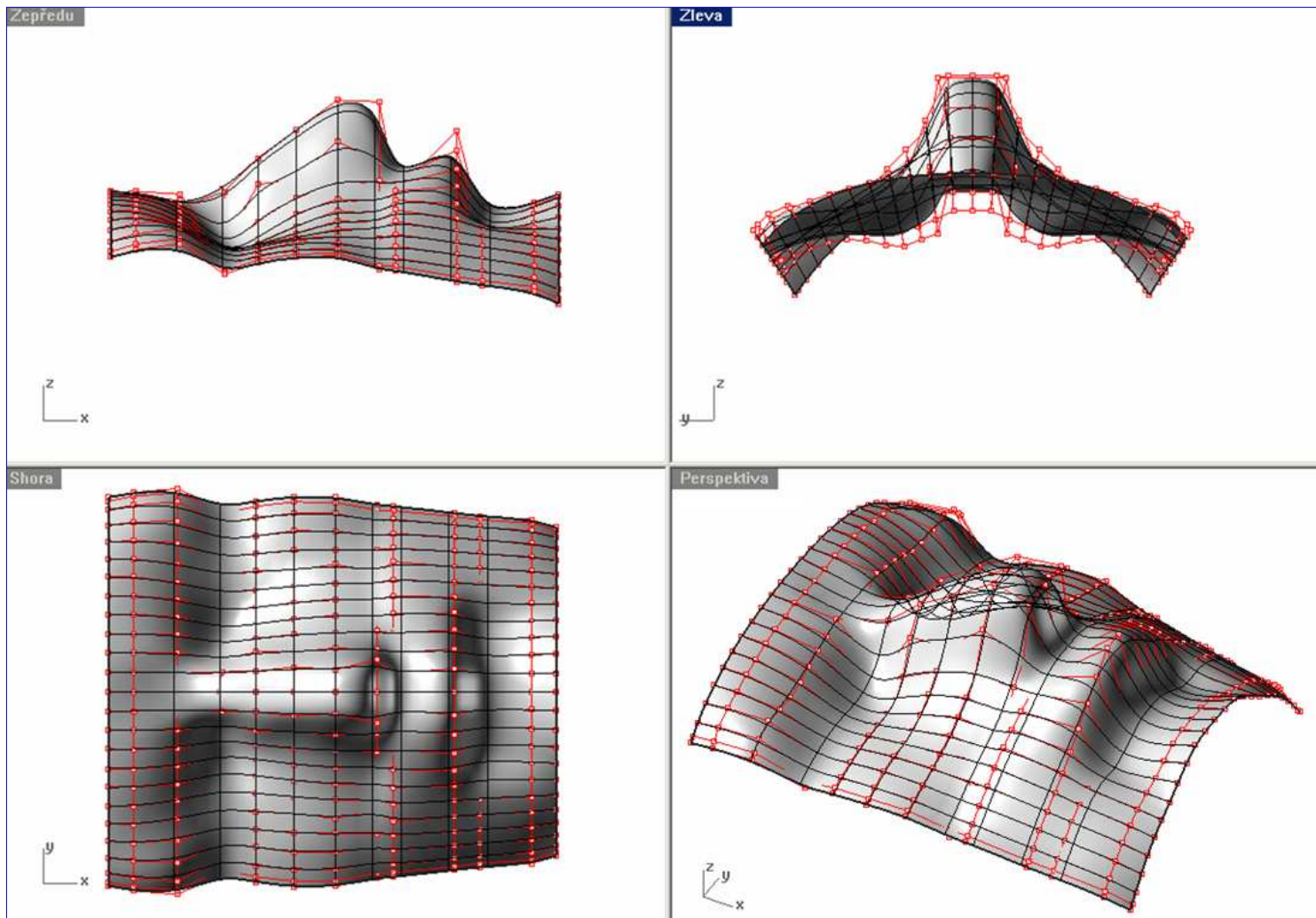
ŘÍDICÍ SÍŤ, BÁZOVÉ FUNKCE, PLOCHA



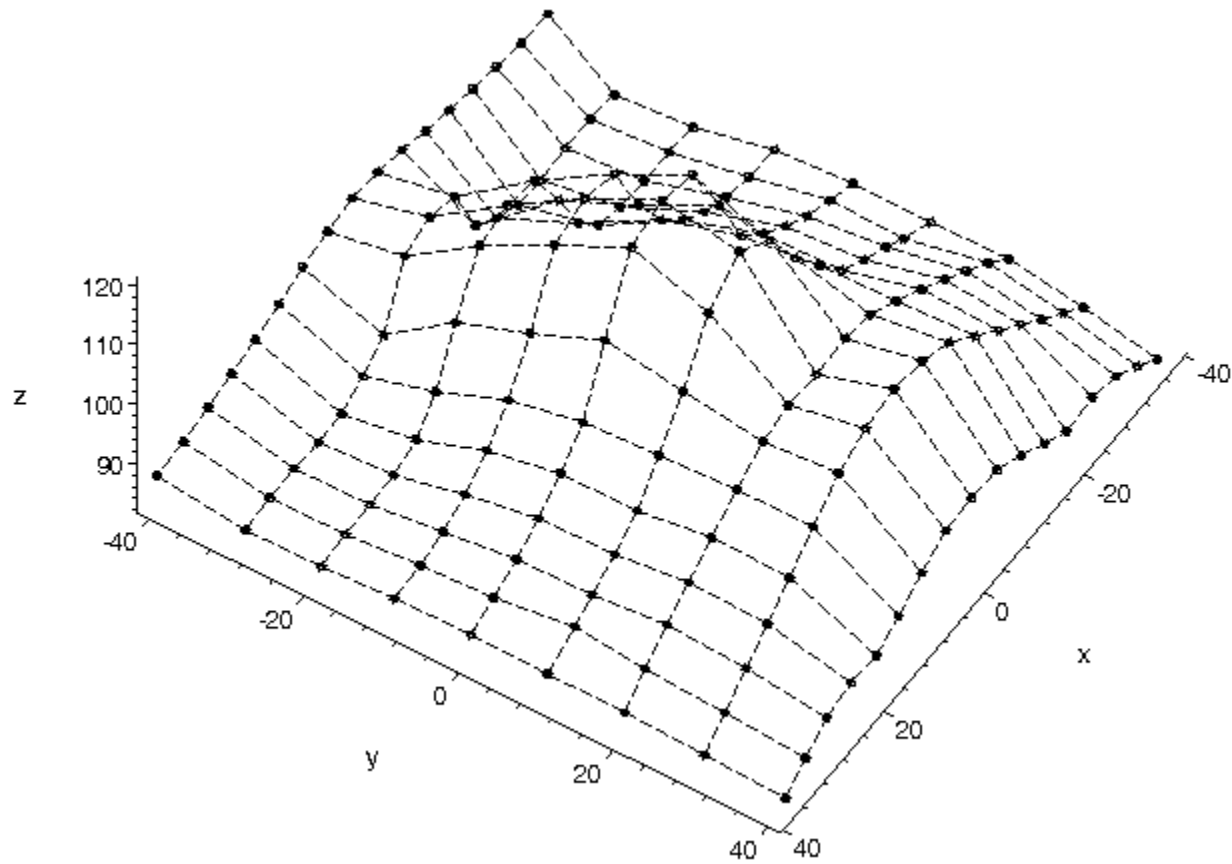
APROXIMAČNÍ PLOCHA



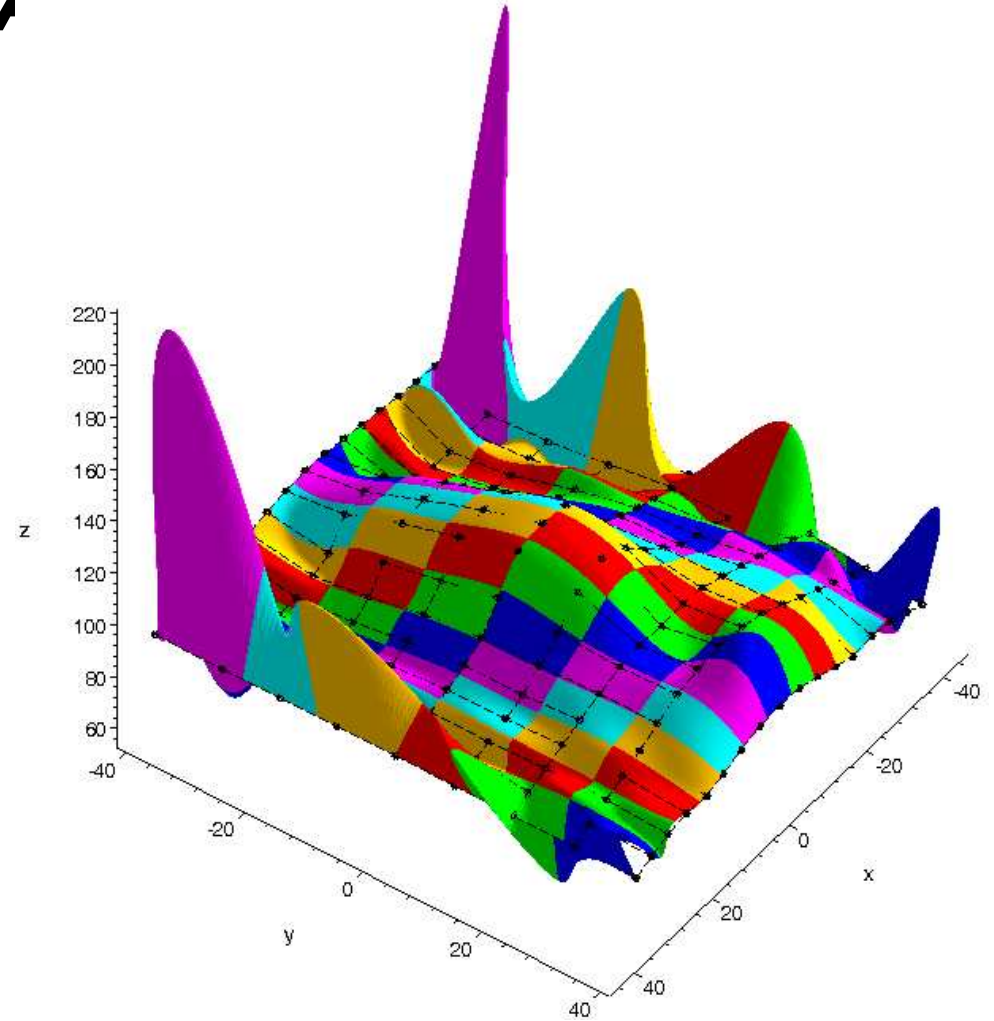
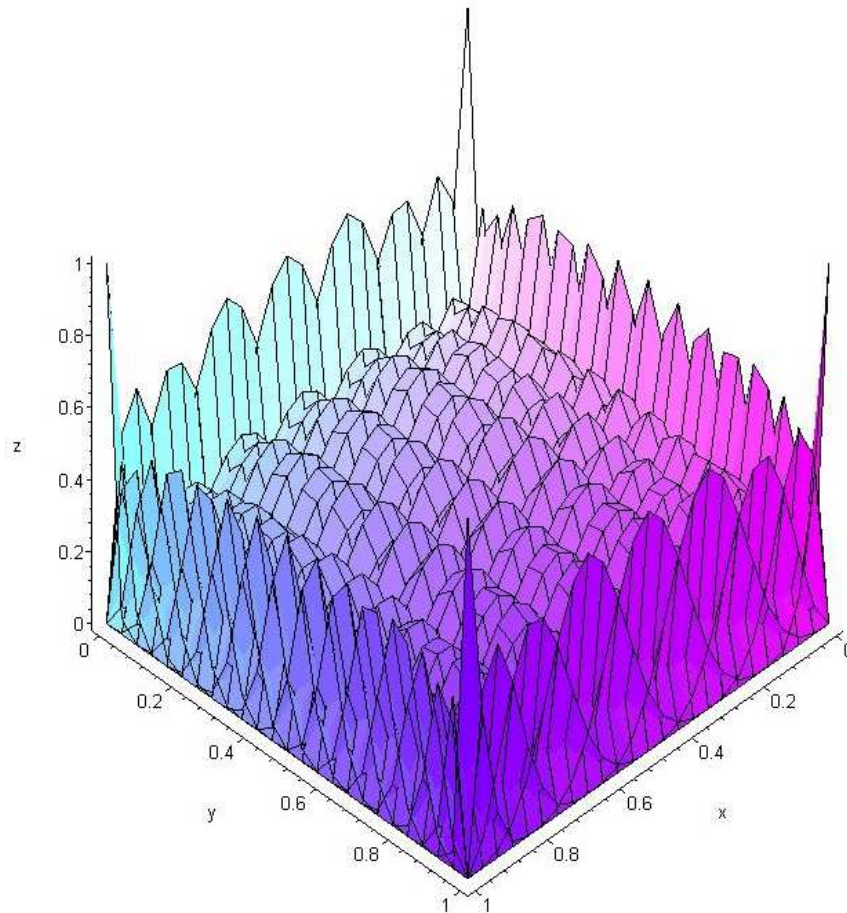
APROXIMAČNÍ PLOCHA



DEFINIČNÍ SÍŤ



BÁZOVÉ FUNKCE, PROSTÁ INTERPOLAČNÍ BLOČKA



LITERATURA

Linkeová, I.: NURBS křivky, ČVUT v Praze, 2007.

Piegl, L., Tiller, W.: The NURBS Book, Springer, London, 1997.

