Numerical instability in PIC simulations of weakly collisional magnetized plasmas

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Introduction

Theoretical description of plasma instabilities can be done with magnetohydrodynamics (MHD) theory, which is suitable for macroinstabilities, or the kinetic theory, which can be used for both micro- and macroinstabilities. Since instabilities are highly nonlinear phenomena, the fully analytical solution might be very complicated, and the problem must be linearized or solved by using some quasilinear theory.

Due to difficulties with analytical solutions, numerical simulations, which allow for detailed study of nonlinear phenomena such as instabilities or turbulence, became more important in last decades. However, when using numerical simulation one should be aware of particular issues that are important for numerical stability.

The most important parameters which affect numerical stability are spatial grid resolution Δx and size of the time step Δt. These arise due to approximations of derivatives by finite differences. The role of spatial and temporal spacing for numerical stability has been well studied already by 1980's [1]. For instance numerical instabilities due to spatial spacing will occur if $k\Delta x < 1$, or $\lambda_D/\Delta x < 1$.

In our recent paper, we studied the influence of collision type on the stability of weakly collisional plasma in $\mathbf{E}\times\mathbf{B}$ fields [2]. In the numerical study we encountered numerical instabilities even for the cases, which should otherwise be numerically stable according to the well-established stability conditions [1]. In this work we identify these instabilities.

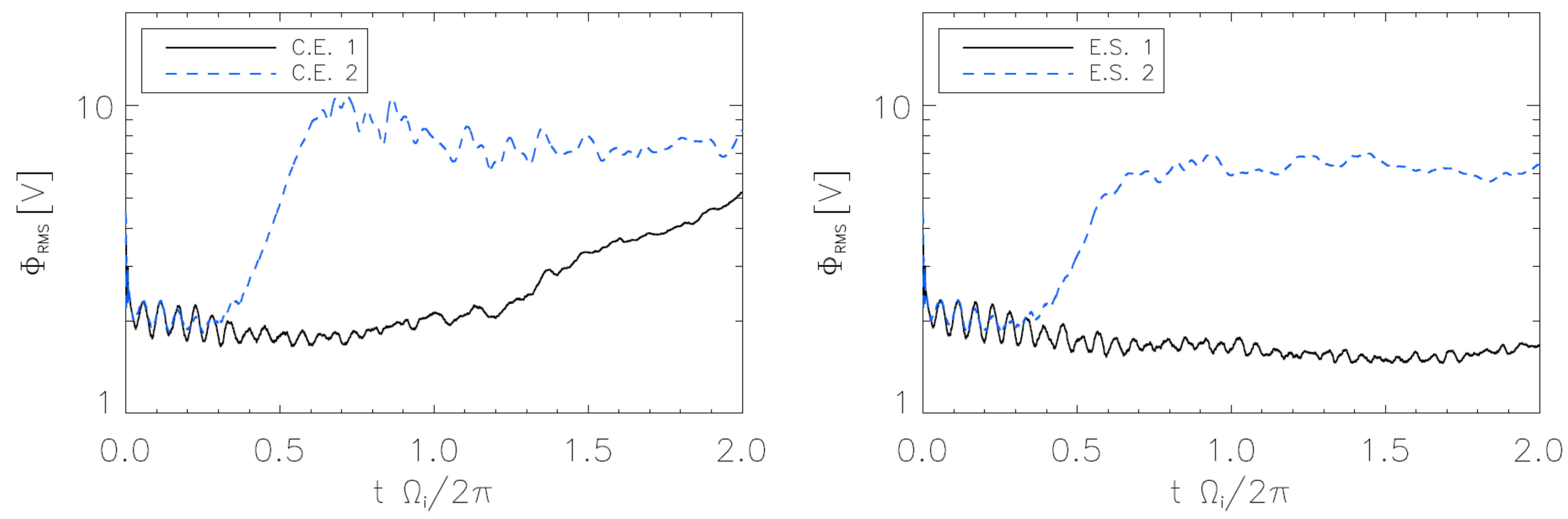
Parameters of the model

We use the self-consistent electrostatic 3D Particle-in-Cell (PIC) numerical simulations. Our code allows us to set external static magnetic and electric field in arbitrary direction and set collisions with neutrals using the Monte Carlo null collision method [3]. For the simulation we set the external magnetic field B magnitude to $\mathbf{B} = 0.005\,T$ in the $\hat{\mathbf{x}}$–direction, and the external electric field E magnitude to $\mathbf{E} = 550\,\mathrm{V}\cdot\mathrm{m}^{-1}$ in the $\hat{\mathbf{y}}$–direction. The plasma particle trajectories are calculated using the leap-frog method combined with the Boris algorithm [4]. In all simulations, the initial Maxwellian plasma is used as well as elastic (E.S. case) and charge exchange (C.E. case) collisions between charged particles and neutrals in weakly collisional regime ($\nu_\alpha/\Omega_\alpha < 1$). The stability condition $\lambda_D/\Delta x > 1$ was satisfied in all cases.

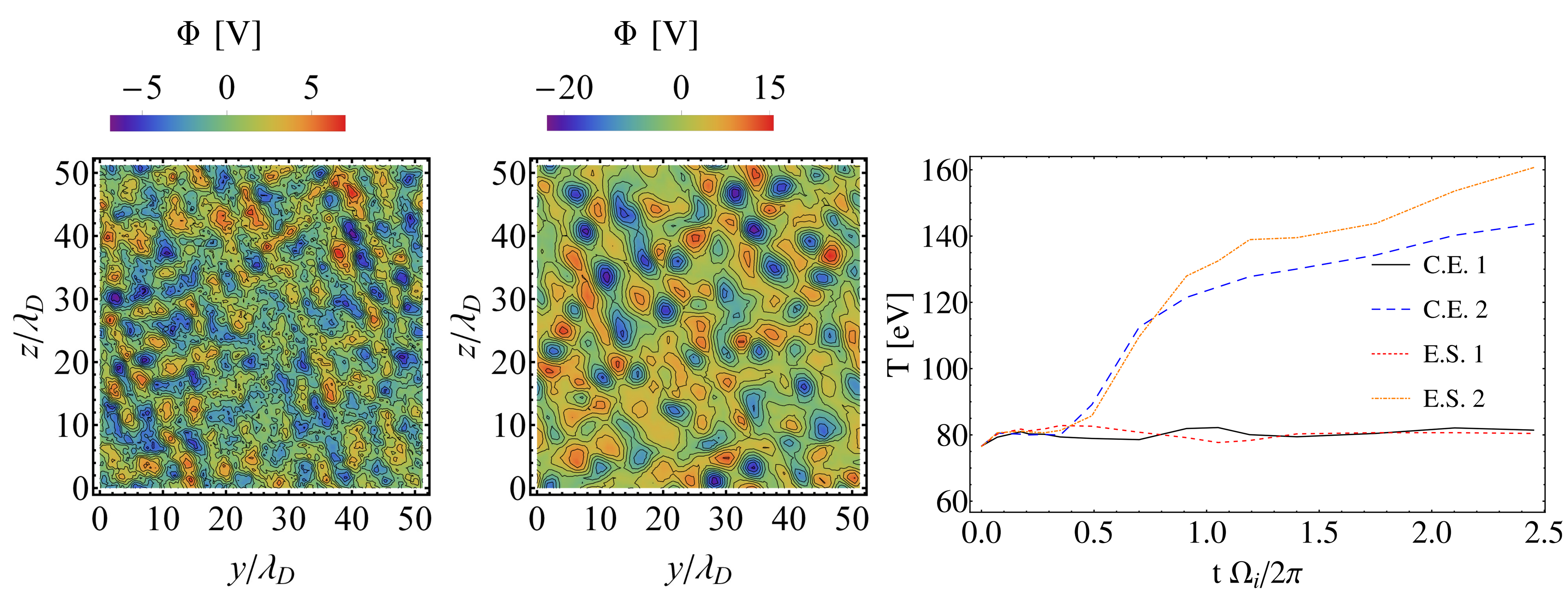
The setup of all four simulations is sumarized in the table below.

Parameter	C.E. 1	C.E. 2	E.S. 1	E.S. 2
n_g	128	65	128	65
L	0.5 m	0.5 m	0.5 m	0.5 m
Δx	3.9 mm	7.8 mm	3.9 mm	7.8 mm
N_i, N_e	$4\cdot 10^7$	$4\cdot 10^7$	$4\cdot 10^7$	$4\cdot 10^7$
$R_L/\Delta x$	1.05	0.52	1.05	0.52
$\lambda_D/\Delta x$	2.5	1.25	2.5	1.25

Numerical instability



In our simulations, we have monitored the RMS values of electrostatic potential fluctuations. It is clearly visible that the spatial resolution of electron gyro-radius significantly affects the stability and electron heating.



We have also monitored potential density in the plane perpendicular to the magnetic field. For numerically unstable case with elastic collisions the creation of filament structures along the magnetic field was identified. Artificial heating for numerically unstable cases is depicted on the right panel.

Kinetic equation and challenges

To analytically identify the numerical instability is useful to know the analytical physical solution. The initial Boltzmann equation (using the BGK approximation) is:

$$\frac{\partial}{\partial t} f_\alpha + (\mathbf{v}_\alpha\cdot\nabla_\mathbf{x}) f_\alpha + \left(\frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}_\alpha\times\mathbf{B}_0)\cdot\nabla_\mathbf{v}\right) f_\alpha = -\nu_\alpha(f_\alpha - f_{0\alpha}).$$

Initial distribution is Maxwellian including $\mathbf{E}\times\mathbf{B}$ drift velocity. External fields and subsequent drift velocity were chosen as following:

$$\mathbf{E}_0 = (0, E_0, 0),\ \mathbf{B}_0 = (B_0, 0, 0),\ \mathbf{v}_d = \frac{E_0}{B_0}(-\mathbf{e}_z),\ \text{and } \nu_\alpha < \Omega_\alpha.$$

Using integration over unperturbed orbits [5] we obtain this expression for particle density perturbation:

$$n_{1\alpha}(t,\mathbf{x}) = \int_{-\infty}^{\infty} \mathrm{d}^3\mathbf{v}' \left[C\exp(-\nu_\alpha t) - \frac{q_\alpha}{m_\alpha}\int_{-\infty}^{t} \mathrm{d}t' \left(\mathbf{E}_1(t',\mathbf{x}')\cdot\frac{\partial f_{0\alpha}(t',\mathbf{x}',\mathbf{v}'_\alpha)}{\partial \mathbf{v}'_\alpha}\exp(\nu_\alpha t')\right)\right],$$

where primed denotes variables along unperturber orbits. This result yields several questions which should be addressed before other calculations. These questions are: i) Is this problem analytically solvable or we will have to use some simplification? ii) Can be the transition process term neglected? iii) If not, should we use initial condition $f_{1\alpha(t=-\infty)} = 0$ or $f_{1\alpha(t=0)} = 0$ for finding the constant C?

These questions are also subject to our current work as answering them will help us finding the dispersion relation of the studied problem.

References

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