APPLICATION OF A MODEL FOR SOLUTION OF SHOCK WAVE PARAMETERS IN STEAM TO EVALUATION OF VALUE OF SPEED OF SOUND

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Abstract

A model for solution of shock wave parameters in steam was developed, tested and verified. Physical principle of the model is based on balance equations and the IAPWS data on state of steam are applied. Numerical solution of thermodynamic and flow parameters is described. A special case, when the shock wave is extremely weak, offers to solve values of velocities representing speed of sound in steam. Achieved results are analysed and discussed.

1. Introduction

Speed of sound is a physical quantity closely connected with compressibility of matters. Physically, speed of sound expresses speed of advance of a disturbance in a respective matter. Generally, speed of sound is defined as square root of infinitesimal pressure disturbance ∂p related to infinitesimal change of density $\partial \rho$ at isentropic process:

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \tag{1}$$

In this paper, another principle of speed of sound will be treated. When an extremely weak shock wave is presumed, then the velocity of steam upstream of the shock wave is equal to velocity downstream of the shock wave and it has to be equal to speed of sound. The model for solution of thermodynamic and flow parameters of steam where normal shock wave appears will be described. Numerical solutions were performed. Results are compared with data of International Association for Properties of Water and Steam (IAPWS). Namely, the Industrial Formulation IAPWS-IF97 [1] is applied.

2. Speed of Sound in Steam

Fundamental definition of speed of sound is expressed by Eq.(1). There is no problem to evaluate speed of sound when state equation in form $f(p, \rho, s) = 0$ is known. It is possible to derive relation of speed of sound for an ideal gas as

$$a = \sqrt{\kappa \frac{p}{\rho}} = \sqrt{\kappa r T} \tag{2}$$

From Eq.(2) it follows that speed of sound in an ideal gas depends on temperature only because κ = const. is ratio of heat capacities (Poisson constant) and r is specific gas constant.

$$\mathrm{r} = \frac{\mathrm{R}}{\mathrm{M}} \tag{3}$$

R is universal gas constant R = 8314,41 $\mathrm{Jkmol^{-1}K^{-1}}$ and M is molar weight of the gas. But steam cannot be considered to be an ideal gas generally. Steam is a real gas and its state equation is more complex. The IAPWS investigated state equations of water substance and formulated two of them - one for general and scientific use IAPWS 95 [2] and for industrial use IAPWS-IF97 [1].

The formulation of IAPWS 95 is a fundamental equation for specific Helmholtz free energy $f(\rho, T)$. Its dimensionless form is separated into two parts:

$$\frac{f(\rho, T)}{rT} = \phi(\delta, \tau) = \phi^{o}(\delta, \tau) + \phi^{r}(\delta, \tau) \tag{4}$$

where $\delta = \rho/\rho_c$ and $\tau = T_c/T$. The IAPWS defined for water substance specific gas constant r = 461.51805 $\mathrm{J{\cdot}kg^{-1}K^{-1}}$, critical density ρ_c = 0.332 $\mathrm{kg{\cdot}m^{-3}}$ and critical temperature T_c = 647.096 K. Functions $\phi^{o}(\delta, \tau)$ and $\phi^{r}(\delta, \tau)$ are defined by IAPWS [2]. Speed of sound is then solved from:

$$a(\delta, \tau) = \sqrt{rT\left\{1 + 2\delta\frac{\partial\phi^{r}}{\partial\delta} + \delta^{2}\frac{\partial^{2}\phi^{r}}{\partial\delta^{2}} - \frac{\left(1 + \delta\frac{\partial\phi^{r}}{\partial\delta} - \delta\tau\frac{\partial^{2}\phi^{r}}{\partial\delta\partial\tau}\right)^{2}}{\tau^{2}\left(\frac{\partial^{2}\phi^{o}}{\partial\tau^{2}} + \frac{\partial^{2}\phi^{r}}{\partial\tau^{2}}\right)}\right\}} \tag{5}$$

The formulation of IAPWS IF-97 is divided to 5 regions where different fundamental equations are defined - Gibbs free energy $g(p,T)$ or Helmholtz free energy $f(\rho,T)$. Fundamental equation for Gibbs free energy is expressed in dimensionless form:

$$\frac{g(p, T)}{rT} = \gamma(\pi, \tau) \tag{6}$$

where π is dimensionless reduced pressure, and τ is dimensionless reduced temperature. Function $\gamma(\pi, \tau)$ is defined by IAPWS [1] speed of sound is solved from:

$$\mathrm{a}(\pi, \tau) = \sqrt{\frac{\mathrm{rT}\left(\frac{\partial\gamma}{\partial\pi}\right)^{2}}{\frac{\left(\frac{\partial\gamma}{\partial\pi} - \tau\frac{\partial^{2}\gamma}{\partial\pi\partial\tau}\right)^{2}}{\tau^{2}\frac{\partial^{2}\gamma}{\partial\tau^{2}}} - \frac{\partial^{2}\gamma}{\partial\pi^{2}}}} \tag{7}$$

Values of speed of sound were evaluated according to Eq.(7) and presented in phase diagram p-t (pressure-temperature) [3] and are shown in Fig.1. It is evident that theory of ideal gas cannot be applied in the case of water and steam.

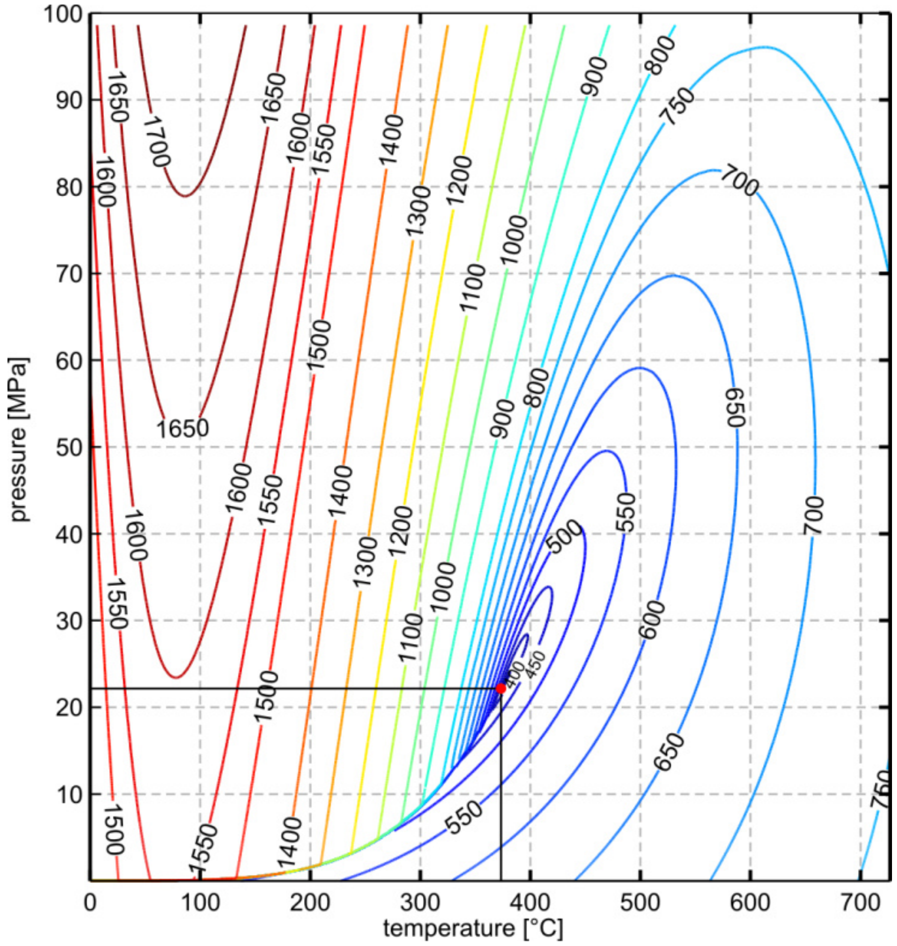

Fig.1 Values of speed of sound in water and steam

3. The model for solution of thermodynamic parameters of steam downstream of a normal shock wave

An equilibrium model of shock wave in steam is formulated. Theoretical approach for solution steam parameters is based on balance equations for steam passing the infinitesimally thin control volume on the shock wave (Fig.2). The modified balance equations are:

Balance of mass $$\frac{\dot{m}}{A} = \rho_1 v_1 = \rho_2 v_2 \tag{8}$$

Balance of momentum $$p_1 - p_2 = \frac{\dot{m}}{A}\left(v_2 - v_1\right) \Rightarrow p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 \tag{9}$$

Balance of energy $$h_0 = h_{01} = h_{02} = h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} \tag{10}$$

Equation of state for steam $$\frac{1}{\rho} = f\left(p,\ h\right) \tag{11}$$

where $\dot{m}$ is mass flow, A is cross section of the control volume, ρ is density, v is velocity, p is pressure, h is enthalpy, s is entropy, x is dryness fraction; indices $_1$ is upstream of the shock wave, $_2$ is downstream of the shock wave, $_0$ is total value.

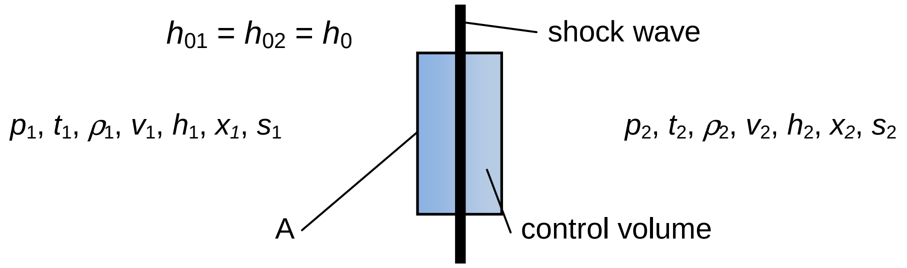

Fig.2 The scheme of a shock wave, control volume, and parameters on a normal shock wave in steam

All thermodynamic parameters upstream of the shock wave are given (except total thermodynamic parameters). In the paper [3], the calculation procedure for given pressure downstream of the shock wave p_2 (when $p_2 > p_1$) was derived. The iterative procedure is based on the modified balance Eqs. (8) to (11) and on the chosen value of density of superheated steam downstream of the shock wave $\rho_2^{(1)}$ in first iterative step :

$$h_2^{(n+1)} = h_1 + \frac{1}{2}\left(\frac{1}{\rho_1} + \frac{1}{\rho_2^{(n)}}\right)\left(p_2 - p_1\right), \tag{12}$$

where index $^{(n)}$ denotes n-th iteration step. After application of equation of state (11), value of density superheated steam downstream of the shock wave is solved:

$$\rho_2^{(n+1)} = \frac{1}{f\left(p_2, h_2^{(n+1)}\right)}, \tag{13}$$

The solved value of density downstream of a shock wave $\rho_2^{(n+1)}$ is used in next $(n + 1)$ -th iteration step into the Eq. (12). It was proved that the iterative procedure is relatively fast.

In the paper [4] the solved thermodynamic parameters on a normal shock wave in all region of saturated steam were solved according to iterative process Eqs.(12), (13). In the Fig.3, phase diagram (pressure p - temperature T) shows dependences of parameters of supersaturated steam downstream of normal shock waves in saturated steam for p_2/p_1 = const. Thermodynamic parameters are also presented in diagram in Fig.4 (enthalpy h - entropy s) as dependences for p_2/p_1 = const. The calculation offers solutions of another parameters.

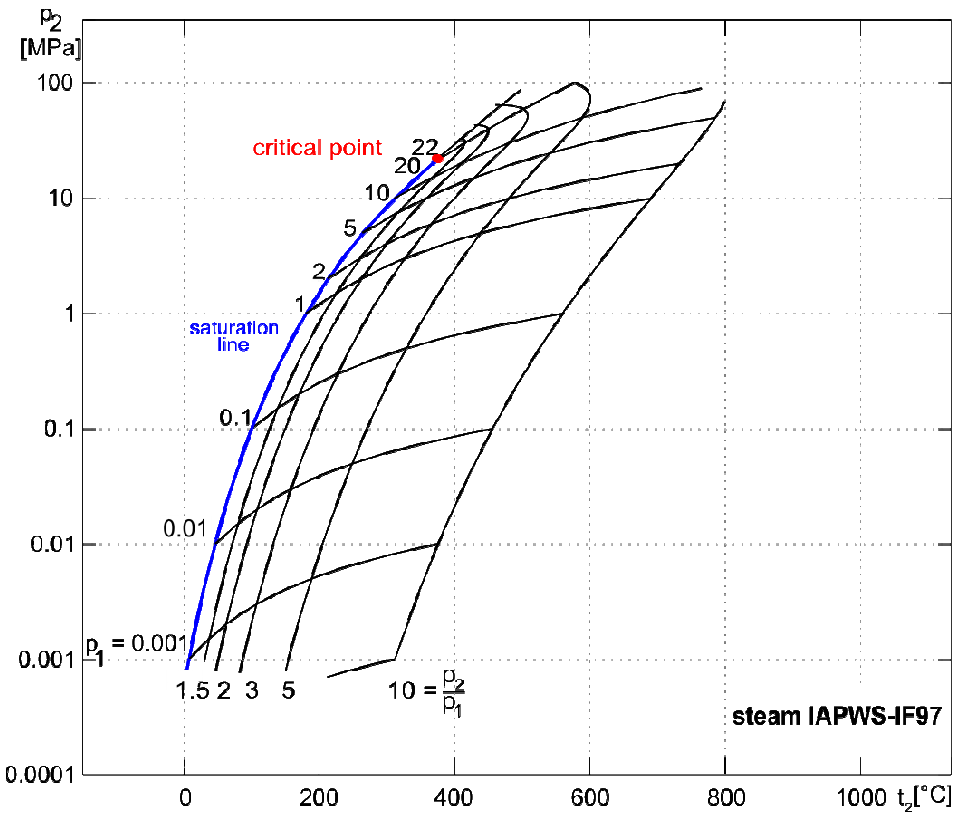

Fig.3 State parameters downstream of normal shock waves in saturated steam in phase diagram, for p_2/p_1= const.

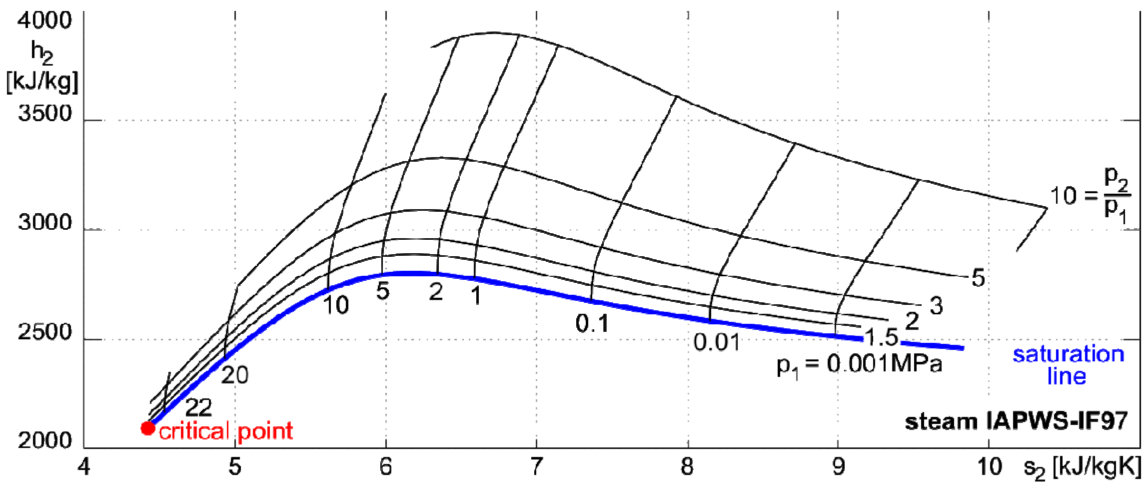

Fig.4 State parameters downstream of normal shock waves in saturated steam in the diagram enthalpy-entropy, p_2/p_1= const.

An equilibrium model of shock wave in wet steam was formulated in [4]. The model is based on isobaric separation of wet steam into saturated steam and saturated liquid water. Parameters of superheated steam downstream of the normal shock for the saturated steam component are solved by means of the iterative procedure, Eqs.(12), (13), while for the saturated liquid water are solved at assumption of isentropic compression. Isobaric mixing of both components is finally solved. Figure 5 shows diagram enthalpy-entropy with results of calculations for dryness x_1 = 0,879 and for pressures in all region of wet steam upstream of the shock.

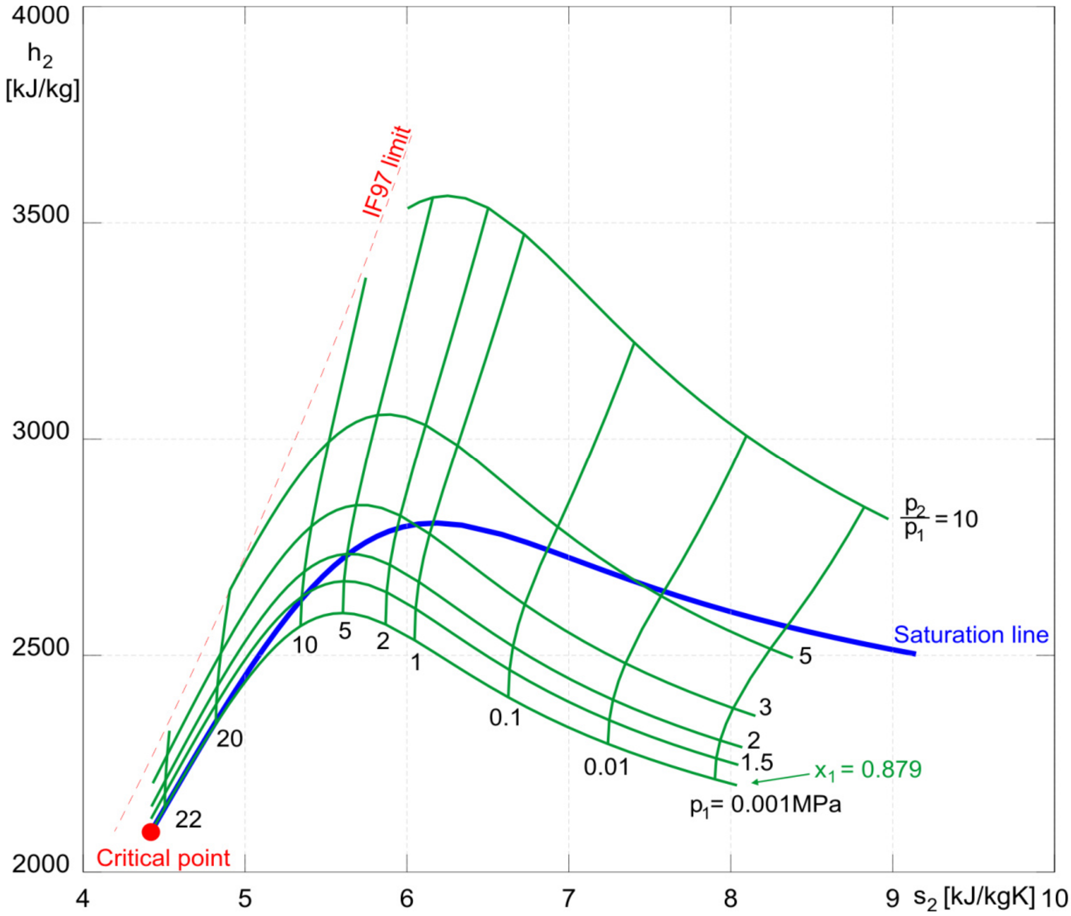

Fig.5 State parameters downstream of normal shock waves in wet steam in the diagram enthalpy-entropy, $p2/p1$ = const.

4. Solution of flow parameters of steam on the normal shock wave

From the balance equations Eqs.(8), (9), (10) relations for velocities from solved thermodynamic parameters can be derived. Velocity v_1 of steam upstream of the normal shock wave can be solved according to relation

$$v_1 = \frac{\dfrac{p_2 - p_1}{\rho_1}}{\sqrt{2\dfrac{p_2 - p_1}{\rho_1} - 2(h_2 - h_1)}} \ . \tag{14}$$

Velocity v_2 downstream of the normal shock wave is derived from equations (8) and (9) so can be solved according to relation

$$v_2 = v_1 - \frac{p_2 - p_1}{\rho_1 v_1} \ . \tag{15}$$

Achieved results from calculations of flow velocities on a normal shock wave in saturated steam [9] are presented for all region of pressures of saturated steam in all region of parameters of saturated steam are introduced in Fig.6 as dependencies for p_2/p_1 = const.
Velocities on a normal shock wave in wet steam for x_1 = 0,879 in all region pressures p_1 for given p_2/p_1 = const. are presented in diagram in Fig.7 as dependencies for p_2/p_1 = const.

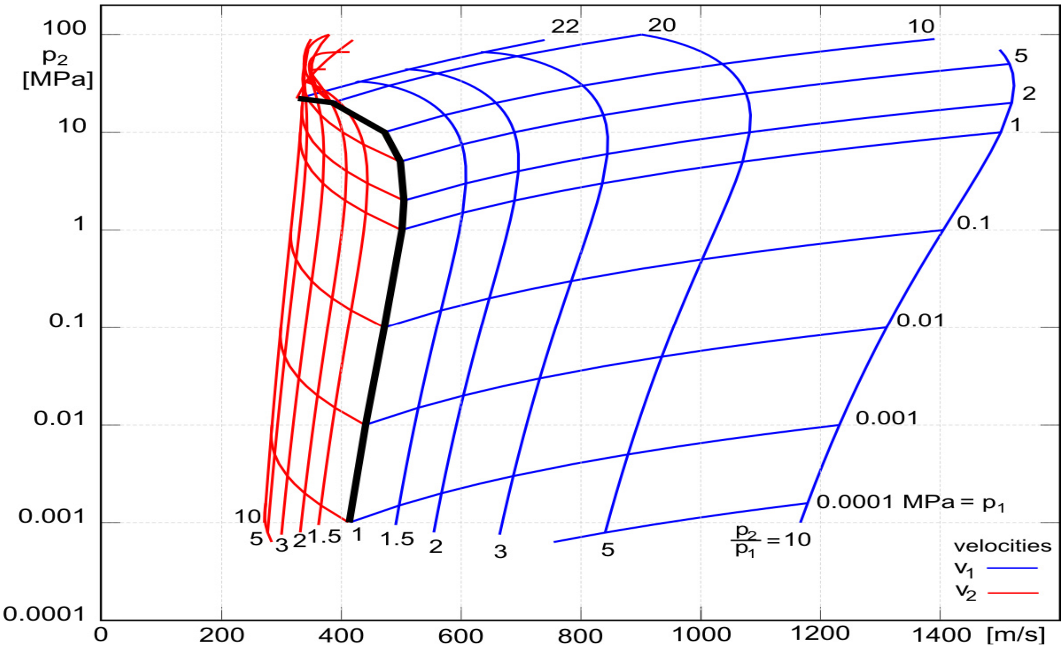

Fig. 6 Velocities of steam upstream of (blue) and downstream of (red) a normal shock wave in saturated steam, p_2/p_1 = const.

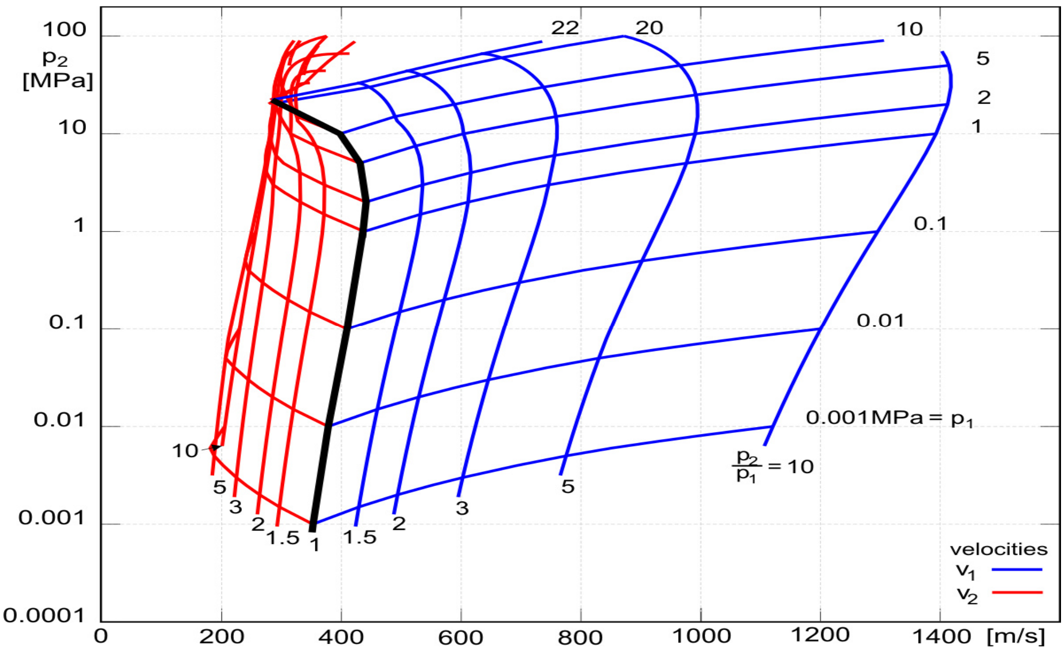

Fig. 7 Velocities of steam upstream of (blue) and downstream of (red) a normal shock wave in wet steam, x_1 = 0,879, p_2/p_1 = const.

5. Solution of speed of sound of steam

The model for solution of shock wave parameters was applied for superheated, saturated and wet steam for different ratios of pressures p_2/p_1. Achieved velocities v_1 and v_2 are shown in Figs.6 and 7. Special case is for $p_2 = p_1$, then $v_1 = v_2 = a$, (a is speed of sound). Numerical procedure was performed for $p_2/p_1 = 1.01$ and 1.2. From those points (4 points of the velocity-pressure diagram) the intersection of velocities v_1 and v_2 is calculated. Examples of velocities a are shown in Figs.6 and 7 as black lines. Figure 8 compares the solved values of speed of sound for superheated steam with the IAPWS-IF97 solved according to description in Sect.2. There is a good agreement with speed of sound calculated according to the IAPWS-IF97. The same comparison for saturated steam is on Fig. 9. The results for the saturated steam are also in good agreement with IAPWS-IF97. The error depends on chosen pressure ratios for intersection calculation. This problem will be further investigated.

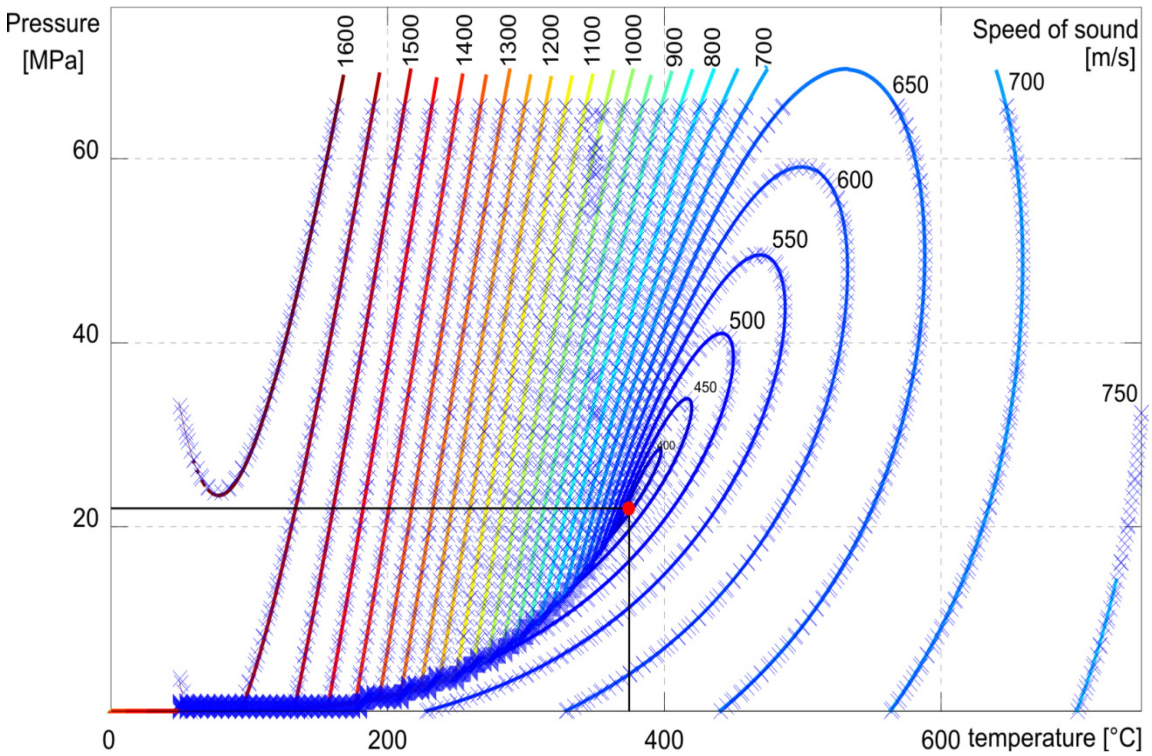

Fig. 8 Comparison of calculated values (×) against IAPWS-IF97 isolines in superheated steam

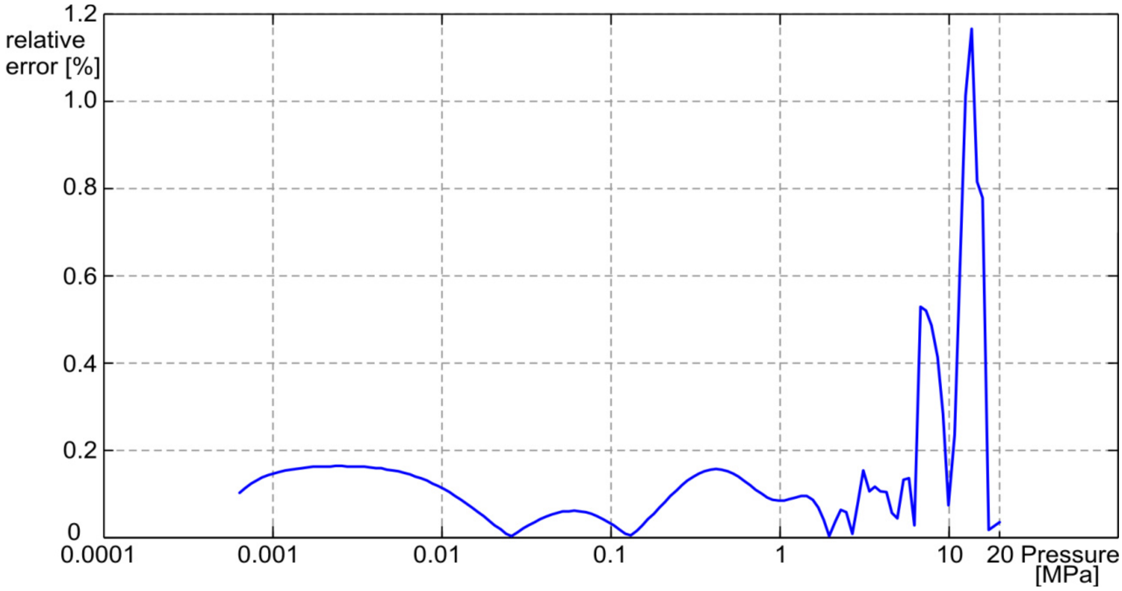

Fig. 9 Relative error of calculated values against IAPWS-IF97 in saturated steam

The most interesting results are regarding the wet steam. Using the isobaric separation model for the wet steam as described in [4], the speed of sound resulting from the theory of shock wave was calculated. As there is no IAPWS-IF97 equation for speed of sound in the wet steam, it was compared to the calculated speed of sound directly using the equation (1) and using the wet steam state parameters from IAPWS-IF97. The pressure difference *dp* deviation was considered to be 1 Pascal. There was a very good agreement of the results presented on Fig.10.

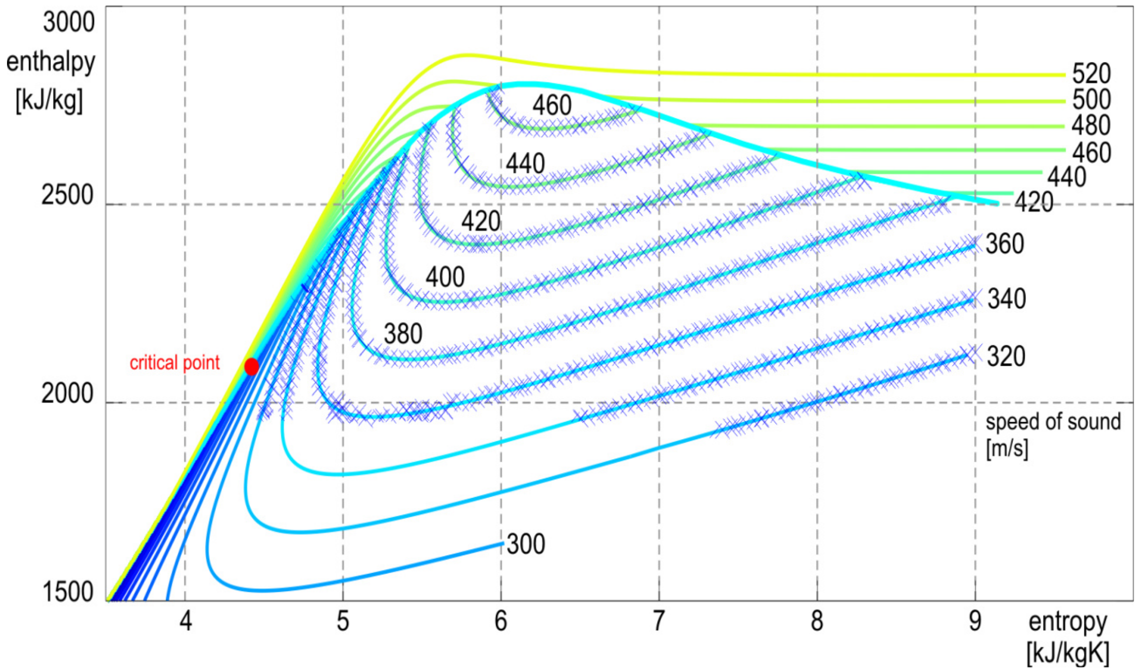

Fig. 10 Comparison of calculated values (×) and the values according to Eq. (1) in wet steam

6. Conclusion

Theoretical and numerical model for solution of thermodynamics and flow parameters of steam on normal shock wave is developed and verified. The model is applied for special case of very weak shock wave to solve values of speed of sound. Obtained results are compared with data of IAPWS for superheated and saturated steam. Achieved results in superheated and saturated steam are in agreement with IAPWS-IF97 calculation. This theory gives us a very good tool for further development and evaluation of the wet steam separation model and also other applications.

References

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Acknowledgement

This research was supported by the Technology Agency of the Czech Republic under project No.TE01020036.

The Doosan Škoda Power enables to materialize this research.