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SOLUTIONS OF NAVIER-STOKES EQUATIONS WITH NON-DIRICHLET BOUNDARY CONDITIONS IN SPACE-PERIODIC DOMAINS

Petr Kučera, Jitka Píšová
Czech Technical University, Prague

In this contribution we model flow of an incompressible fluid in a domain Ω^* which consists of subdomains Ω_i which are disjoint and moved each other in one direction and we formulate steady, non-steady and time-periodic problem. We denote by Ω one of this subdomains ($\Omega = \Omega_0$). We describe the flow by the system of the non-steady Navier-Stokes equations on a domain Ω with boundary conditions which are "space-periodic in this direction".

At first we describe domains Ω and Ω^*. Suppose that

- $\Omega \subset \mathbf{R}^3$ is a bounded domain.
- $a \in \mathbf{R}$, $a > 0$, $\vec{\boldsymbol{\psi}} = (a, 0, 0)$ is a vector.
- $\partial\Omega = \overline{\Gamma_1} \cup \overline{\Gamma_2} \cup \overline{\Gamma_3}$, Γ_1, Γ_2 and Γ_3 are open disjoint subsets of $\partial\Omega$.
- $\overline{\Gamma_2} \cap \overline{\Gamma_3} \equiv \emptyset$.
- one-dimensional measures of $\overline{\Gamma_1} \cap \overline{\Gamma_2}$ and $\overline{\Gamma_2} \cap \overline{\Gamma_3}$ are zero.
- $(\Omega + \vec{\boldsymbol{\psi}}) \cap \Omega = \Gamma_3$ (By the symbol $A + \vec{\boldsymbol{\psi}}$ we mean set of all points $[x + a, y, y] \in \mathbf{R}^3$ where $[x, y, z] \in A$.)
- $\Gamma_2 + \vec{\boldsymbol{\psi}} = \Gamma_3$.

By the symbol Ω^* we denote $\cup_{j \in \{-\infty,\dots,\infty\}}(j \cdot \vec{\boldsymbol{\psi}} + \Omega)$.
Let $\tilde{\boldsymbol{f}}$, $\boldsymbol{u}_0$ be functions on $\tilde{\Omega}$ such that

$$\tilde{\boldsymbol{f}}(x + a, y, z) = \tilde{\boldsymbol{f}}(x, y, z)$$

and

$$\tilde{\boldsymbol{u}_0}(x + a, y, z) = \tilde{\boldsymbol{u}_0}(x, y, z).$$

For simplicity we denote $\boldsymbol{f} = \boldsymbol{f}|_{\Omega_0} = \boldsymbol{f}|_{\Omega}$, $\boldsymbol{u}_0 = \boldsymbol{u}_0|_{\Omega_0} = \boldsymbol{u}_0|_{\Omega}$, $Q = \Omega \times (0, T)$, where $(0, T)$ is a time interval, $0 < T < \infty$. We deal with the system

$$
\begin{aligned}
d\boldsymbol{u}/dt - \nu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla\mathcal{P} &= \boldsymbol{f} \qquad \text{on } Q, & (1)\\
\operatorname{div}\boldsymbol{u} &= 0 \qquad \text{on } Q, & (2)\\
\boldsymbol{u}(.,0) &= \boldsymbol{\gamma} \qquad \text{on } \Omega, & (3)\\
\boldsymbol{u}|_{\Gamma_1} &= \boldsymbol{0}, & (4)\\
\boldsymbol{u}|_{\Gamma_2} &= \boldsymbol{u}|_{\Gamma_3}, & (5)\\
(-\mathcal{P}\boldsymbol{n} + \nu\,\partial\boldsymbol{u}/\boldsymbol{n})|_{\Gamma_2} &= (-\mathcal{P}\boldsymbol{n} + \nu\,\partial\boldsymbol{u}/\boldsymbol{n})|_{\Gamma_3}. & (6)
\end{aligned}
$$

Here $\boldsymbol{u} = (u_1, \dots, u_m)$ denotes the velocity, $\mathcal{P}$ represents the pressure, ν denotes the kinematic viscosity, $\boldsymbol{g}$ is a body force, $\boldsymbol{\sigma}$ is a prescribed vector function on Γ_2, $\boldsymbol{n} = (n_1, \dots, n_m)$ is the outer normal vector on $\partial\Omega$ and $\boldsymbol{\gamma}$ is an initial velocity. We suppose for simplicity that $\nu = 1$ throughout the whole paper.

Suppose that there exists a strong solution of problem (1)–(11) for given data. To prove result of existence of a strong solution for a data which are small perturbation of the previous one we use methods which is motiveted by the technique described in [1]–[7].

Corresponding steady problem is the following:

$$
\begin{aligned}
-\nu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla\mathcal{P} &= \boldsymbol{g} \qquad \text{on } \Omega, & (7)\\
\operatorname{div}\boldsymbol{u} &= 0 \qquad \text{on } \Omega, & (8)\\
\boldsymbol{u}|_{\Gamma_1} &= \boldsymbol{0}, & (9)\\
\boldsymbol{u}|_{\Gamma_2} &= \boldsymbol{u}|_{\Gamma_3}, & (10)\\
(-\mathcal{P}\boldsymbol{n} + \nu\,\partial\boldsymbol{u}/\boldsymbol{n})|_{\Gamma_2} &= (-\mathcal{P}\boldsymbol{n} + \nu\,\partial\boldsymbol{u}/\boldsymbol{n})|_{\Gamma_3}. & (11)
\end{aligned}
$$

Here, we want to prove local solvability in the neighbourhood of famous solution also. Moreover, we want to prove regularity of corresponding Stokes solution.

We formulate also time-periodic problem on time interval $(0, T)$. .

$$
\begin{aligned}
d\boldsymbol{u}/dt - \nu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nabla\mathcal{P} &= \boldsymbol{f} \qquad \text{on } Q, & (12)\\
\operatorname{div}\boldsymbol{u} &= 0 \qquad \text{on } Q, & (13)\\
\boldsymbol{u}(.,0) &= \boldsymbol{u}(.,T) \quad \text{on } \Omega, & (14)\\
\boldsymbol{u}|_{\Gamma_1} &= \boldsymbol{0}, & (15)\\
\boldsymbol{u}|_{\Gamma_2} &= \boldsymbol{u}|_{\Gamma_3}, & (16)\\
(-\mathcal{P}\boldsymbol{n} + \nu\,\partial\boldsymbol{u}/\boldsymbol{n})|_{\Gamma_2} &= (-\mathcal{P}\boldsymbol{n} + \nu\,\partial\boldsymbol{u}/\boldsymbol{n})|_{\Gamma_3}. & (17)
\end{aligned}
$$

Here, we want to characterize the set of solution such that the problem is local solvable in their neighbourhood.

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Reference

[1] M. BENEŠ, Mixed Initial-Boundary Value Problem for the Three-dimensional Navier–Stokes Equations in Polyhedral Domains. DCDS suppl. **1**, 135–144, (2011).

[2] M. BENEŠ, P. KUČERA, Solutions of the Navier–Stokes equations with various types of boundary conditions. Arch. Math. **98**, 487-497, (2012).

[3] S. KRAČMAR, J. NEUSTUPA, Modelling of flows of a viscous incompressible fluid through a channel by means of variational inequalities. ZAMM **74**, 637-639, (1994).

[4] S. KRAČMAR, J. NEUSTUPA, A weak solvability of a steady variational inequality of the Navier-Stokes type with mixed boundary conditions, Proceedings of the Third World Congress of Nonlinear Analysis, Nonlinear Anal. **47**, 4169–4180, (2001).

[5] P. KUČERA, Z. SKALÁK, Solutions to the Navier-Stokes Equations with Mixed Boundary Conditions, Acta Applicandae Mathematicae, **54**, Kluwer Academic Publishers, 275-288, (1998).

[6] P. KUČERA, Basic properties of solution of the non-steady Navier-Stokes equations with mixed boundary conditions in a bounded domain. Ann. Univ. Ferrara **55**, 289–308, (2009).

[7] P. KUČERA, The time periodic solutions of the Navier-Stokes Equations with mixed boundary conditions. Discrete and continuous dznamical systems series S. **3**, 325–337, (2010).