**acta physica slovaca vol. 62 No. 6, 519 – 614** **December 2012**

# GALILEO, DESCARTES, AND NEWTON – FOUNDERS OF THE LANGUAGE OF PHYSICS

**Ladislav Kvasz***

*Institute of Philosophy, Academy of Sciences of Czech Republic*
*Jilská 1, 11 000 Prague, Czech Republic*
*Department of Mathematics, Pedagogical Faculty of Charles University*
*M. D. Rettigové 4, 116 39, Prague, Czech Republic*

Received 30 November 2013, accepted 2 December 2013, in final form 11 December 2013

Many of the outstanding discoveries in the history of physics were closely tied to fundamental linguistic innovations, which made them possible. There is an extensive literature discussing the scientific achievements of Galileo, Descartes, and Newton from various perspectives. The aim of the present paper is to contribute to these discussions with an analysis of the linguistic tools, by means of which these three authors made their scientific discoveries. We will focus on the main linguistic innovations contained in their works. Thus we will discuss not only the contributions of Galileo, Descartes, and Newton to the development of physics, but we will analyze also the linguistic tools by means of which they formulated these contributions. More specifically, in galileo we will focus on the innovations that he introduced into the experimental method and thus fundamentally changed the way how the expressions of the language of physics are related to reality. Since Galileo the majority of term that occur in physical formulas have an indirect, instrumentally mediated reference. Similarly in Descartes we will focus on the theoretical models, which he introduced into physics in order to explain various phenomena. For Descartes, and since Descartes for great part of physics, to understand a phenomenon means to construct its theoretical model, which by means of unobservable quantities and objects explains the observed phenomenon. In our exposition we will focus on the linguistic tools, by means of which these models are constructed. And finally in Newton we will focus on his description of interaction. For Newton, and since Newton for the entire physics, the description of reality consists in the representation of the temporal evolution of the state of the system. We will discuss the series of linguistic innovations that Newton had to introduce in order to create this way of representing reality.

DOI: 10.2478/v10155-012-0001-6

PACS: 01.65.+g, 01.70.+w

KEYWORDS: Galilean physics, Cartesian physics, Newtonian Physics, Absolute space, Idealization, Language of physics, Husserl

*E-mail address: kvasz@pedf.cuni.cz

## Contents

## Introduction

The Scientific Revolution of the 17th century was undoubtedly one of the most important events forming the Western civilization. About the Scientific Revolution and its principal actors we can find numerous books and papers written by historians of science, philosophers, scientists, as well as popular writers. The present paper tries to contribute to the discussions of the Scientific Revolution by a new interpretation describing this revolution as a *linguistic event*, as the creation of a formal language of a new kind. This new language is the language of modern physics that assigns to a physical system its *state* and then by means of a *differential equation* enables us to *determine the temporal development* of the system. We will try to show how the concept of state, the first differential equation, as well as the entire linguistic framework to which these two concepts belong, was created.

We will base our interpretation of the scientific revolution as a linguistic event on the analysis of idealization in Edmund Husserl's book *Die Krisis der Europäischen Wissenschaften und die Transzendentale Phänomenologie* (Husserl 1954; hereinafter referred to as *Krisis*). At first glance, such a choice may seem surprising, at least to the advocates of analytic philosophy. Analytic philosophy is the dominant current in the philosophy of science, and it also dominates the contemporary literature on the Scientific Revolution. Therefore, it might seem more natural to choose as the basis of our interpretation the methods of analytic philosophy instead of phenomenology. Analytic philosophy of science, however, considers science as a natural continuation of ordinary experience and common sense, and therefore it is blind to the radical discontinuity introduced by the emergence of modern science. It was Husserl who, thanks to the elaboration of the theme of lifeworld (*Lebenswelt*) in his *Krisis*, was able to see clearly the radical discontinuity that separates the lifeworld from the world of "*European sciences*". Even though contemporary phenomenology to a large extent abandoned Husserl's interest in science, there are still a number of philosophers who are dedicated to the analysis of science from the phenomenological perspective. Therefore, the choice of Husserl's phenomenology as a basis for the analysis of the process of creation of the language of science is not as strange as it may appear at first glance.

Husserl's interpretation of Galilean physics in the *Krisis* was highly original and attracted a stream of interest of philosophers (see Gurwitsch 1967, Garrison 1986, Heelan 1987, Soffer 1990, Mormann 1991, Drummond 1992). Nevertheless, Husserl's interpretation did not become the standard interpretation of Galileo's work. Thus it still holds what Gurwitsch said some forty years ago in his paper *Galilean physics in the light of Husserl's phenomenology*: "[...] *you must forgive me also for saying in conclusion, that notwithstanding the voluminous recent literature on the philosophy of science (whose value I do not in the least belittle), we do not yet possess a philosophy of science in a truly radical sense. Husserl's analysis of Galileo's physics indicates the direction in which a radical (i.e. a properly rooted) philosophy of science must develop*" (Gurwitsch 1967, p. 401). We believe that the source of problems with Husserl's interpretation of Galilean physics is that his interpretation is original but incomplete. It seems that, instead of an analysis of real physics, Husserl offered an analysis of the picture of physics as it was reflected in the German philosophical tradition, mainly under the influence of Kant and the neo-Kantians. For understanding physics it is not sufficient to analyze Galileo, as Husserl did, but it is necessary to analyze in a similar way also the works of Descartes and Newton. As Rupert Hall remarked: "*It is hardly too much to say that Newton had to write the Principia because Descartes had* *corrected Galileo's notion of inertia.*" (Hall 1967, p. 78).

Descartes and Newton corrected several of Galileo's errors and changed the direction which the program of mathematization of nature should take. Though Galileo's scientific ideas played a fundamental role in the rise of modern science, the form in which they were incorporated into the foundations of modern science differs in many respects from Galileo's original views. So, for instance, inertial motion is not the Galilean circular motion but the motion in a straight line. Modern science does not describe isolated natural phenomena, but it searches for universal laws. These laws are formulated not by means of triangles and circles, but by differential equations. Nevertheless, the idea of an inertial motion in a straight line, the idea of a universal natural law, as well as the notion of a differential equation, were absolutely alien to Galileo's views. They originated in the works of Descartes and Newton. Husserl's analysis of the birth of modern science is problematic because instead of three constitutive moments—the Galilean, the Cartesian, and the Newtonian—Husserl discussed only one. He then drew conclusions which simply do not hold. Husserl's analysis is not an analysis of physics, but only of a fragment thereof.

The question why Husserl confined his analysis of science to Galileo has an interesting answer. Even though Husserl's analysis was a criticism of positivist philosophy of science, he unwittingly remained in the framework in which positivism used to discuss science. According to positivism, the central issue in philosophy of science is to explain the relation of scientific theories to experience. According to positivism, scientific theories are based on accumulation and inductive generalization of empirical statements derived directly from neutral sense data. Husserl overthrew this picture, showing, that there is nothing like neutral sense data, and that from the very beginning we are dealing with an interpreted world, which he called lifeworld (*Lebenswelt*). Further, Husserl showed that science does not form its theories by accumulation and inductive generalization of empirical experience. On the contrary, the rise of modern science consisted in a very radical shift away from experience. Husserl called this radical shift *idealization*. Nevertheless, even though Husserl had overthrown the positivist philosophy of science, he still remained within the framework of positivist philosophy reducing the discussion of scientific theories to the question of their relation to experience.

A radical rejection of positivism requires rejecting not only what positivists say about science, but also the framework in which their theory of science is formulated. The positivist philosophy of science consists not only of all that that the positivists said about science, but also of all those aspects of science which they excluded from consideration. Modern science is based not only on Galilean empiricism which the positivists liked to contemplate. It is equally based on Cartesian and Newtonian metaphysics which the positivists passed over in silence and which, therefore, also Husserl did not analyze. Thus the difficulties with the phenomenological analysis of modern science lie in the fact that Husserl accepted the framework in which positivism discussed science. The foundations of modern physics lie besides the concept of *experiment*, a penetrating philosophical analysis of which was given by Husserl, also on the concept of the *state of a system* and the concept of the temporal evolution of the state, described by means of a *differential equation*. Therefore, we believe that Husserl's attempt to explain the relation of modern science to the lifeworld is interesting and original, but it is an analysis only of a fragment of physics.

The aim of the present paper is to supplement Husserl's analysis of Galileo by a similar analysis of Descartes and Newton. Husserl's achievement was the insight that the subject matter of physics is formed by *intentional objects* that are constituted in the process of *idealization*, in which some aspect of the lifeworld is *replaced by a mathematical ideality*. We do not believe, however, that the process of idealization in physics can be reduced to the analysis of Galileo. A fuller understanding of idealization requires complementing the analysis of Galileo's works by a similar analysis of the works of Descartes and Newton. Only these three layers of language—the Galilean, the Cartesian, and the Newtonian—taken together constitute the intentional objects of physics.

Although Galileo was the originator of the program of mathematization of nature, the tools by means of which he wanted to carry out this program (the triangles and circles, by means of which the book of nature is allegedly written) are too simple to be able to express universal laws of nature. In order to be able to express such laws, it was first necessary to create an entirely new mathematics. Descartes had to invent analytic geometry with the idea of a coordinate system, and Newton had to develop the differential and the integral calculus and to introduce the notion of a differential equation in order to be able to mathematically describe motion. Only on this level was it possible to accomplish Galileo's program of mathematization. Although Husserl correctly identified in Galileo the author of the intention of mathematization of nature, this intention could be fulfilled only by means of a radically new mathematics. So in the second and the third chapter we will complement Husserl's interpretation of Galilean physics by an analogous interpretation of the Cartesian and the Newtonian ones.

Idealization is not a single act, but rather a gradual process. Galileo, Descartes, and Newton brought three representations of reality that together laid the foundations of modern physics. The following theory of idealization is thus compatible with the concept of the fine structure of scientific revolutions, according to which every scientific revolution consists of a sequence of epistemic ruptures of smaller order (see Kvasz 1999, pp. 225-227). Thus an idealization consists of a sequence of three representations.[1] Koyré characterized the birth of modern science as a "*geometrization of space and the dissolution of the Cosmos*" (Koyré 1939, p. 3). This means that in terms of the classification of epistemic ruptures Koyré interpreted the emergence of modern science as a *representation*. We do not deny that this *representation* accompanied the rise of modern science, but we believe that the emergence of modern science was a more fundamental change, namely an idealization. In its course besides the ideal objects of ancient science (i.e. *geometric forms* and *numerical ratios*), ideal objects of an entirely new kind—*dynamic systems*—were created. Therefore Koyré's analysis, however interesting and stimulating, describes only a marginal aspect of the rise of modern science. At the heart of this process was not geometrization, but 'dynamization'. It was not about space or Cosmos, but about motion and interaction. In its course, the geometric principles used in the ancient description of the world were replaced by dynamic principles.

Koyré's view that Aristotelian physics is non-geometrical is based on the analysis of the manifest aspects of this theory. It is true that Aristotle did not use in his description of motion geometry (unlike Archimedes). On the other hand, Aristotle's theory of local motion is undeniably geometrical—it describes motion using *directions* like downwards and upwards, it speaks about the *centre* of the universe. Aristotle's physics is geometrical not apparently, because of its language, but inherently, because of the categories it uses. To characterize motions by means of the points towards which they are oriented is to characterize them geometrically, indicating the end-points of their trajectories. Therefore, the claim that geometrization of space occurred

[1] A short description of the classification of epistemic ruptures and a characterization of idealizations, representations, and objectivization can be found in (Kvasz 2008b, pp. 91-95).

only in modern times is incorrect. The term ‘geometrization’ can have different meanings. It is true that Aristotle did not use geometry as a tool for the *formulation* of his theory, but, nevertheless, on the level of *representation* and *idealization* his theory is a geometrical theory. Koyré’s mistake is rooted in the fact that he understands geometry in a narrow sense: he restricts it to the level of the explicit linguistic formulation of a theory. Aristotle, nevertheless, based his explanation of local motion on an ideal geometrical structure of the universe, towards particular points of which the motion is directed. The change of the structure of the universe during the scientific revolution is, in our view, more accurately described by Vladimir I. Arnold, one of the leading mathematicians of the twentieth century: “*Formerly, the universe was provided not with an affine, but with a linear structure (the geocentric system of the universe)*” (Arnold 1974, p. 5). According to Arnold, also the Aristotelian universe had a geometrical structure. Unlike the universe of modern physics, however, it was a linear structure, and not an affine one.[2]

While nobody would really question the value of Galileo’s contributions to the development of modern science, things are not nearly so simple with respect to Descartes. It is sufficient to quote the words of Stephen Gaukroger: “*With the exception of the work in optics, his contribution to the development of classical physics is minimal. Insofar as kinematics is concerned, Cartesian physics accomplishes considerably less than had been achieved by Galileo in his Two New Sciences, and insofar as Descartes’ physics can be considered a dynamical theory it is often hopelessly confused, particularly in comparison with Newtonian dynamics.*” (Gaukroger 1980b, p. 123). The views of Daniel Garber are similar: “*Descartes’ intellectual program failed, of course; while pieces of the program may have proved important inspirations to later thinkers, as an approach toward understanding the natural world Descartes’ program turned out to be a dead end. But while the design may have been faulty, and the edifice doomed from the start, it is fascinating to contemplate the entire structure as the architect planned it* . . . ” (Garber 1992a, p. 2). We will try to show that these judgments can be challenged. The fact that in kinematics “*Cartesian physics accomplishes considerably less than had been achieved by Galileo*” is its merit, because Galileo’s kinematics was misguided. Descartes realized the fundamental mistake in the orientation of Galileo’s research, something that reveals the depth of his insight. Similarly, we would like to show that the “*confusion of Descartes’ physics*” is not so hopeless. On the contrary, several of the ingredients of Newtonian physics have their origin in Descartes. Therefore, we think neither that “*Descartes’ intellectual program failed*” nor that it “*turned out to be a dead end*”, but that the Cartesian program was a bridge connecting Galileo and Newton.

A fragment of Cartesian physics is still included in the standard courses of mechanics. It is sufficient to open the classical textbook *Mechanics* (Landau and Lifshitz 1957). The first fifty pages of the book, devoted to an exposition of the Lagrangian formalism, are followed by a chapter on particle collisions, in which the Lagrangian function is not mentioned at all, no differential equations are solved, and the collisions are described entirely in the Cartesian spirit using conservation laws. Of course, besides the Cartesian law of conservation of momentum the law of the conservation of energy is used as well. Momentum, furthermore, is considered not as a scalar quantity, as the Cartesians would have it, but as a vector quantity. But these are technical details. The approach of the chapter differs so much from everything that precedes it as well as from what follows that we can in good conscience declare it to be a ***Cartesian relict***. Thus the

[2] A *linear space* has a special point—the origin of the coordinate system. In an affine space there is no such point—all its points are equivalent.

evaluation of Cartesian physics by historians of science is not in accordance with the practice of the scientists, who still include a part of it into their textbooks.

In our paper *Kuhn's Structure of Scientific Revolutions between sociology and epistemology* (Kvasz 2013), we propose to distinguish three types of scientific revolutions. The stimulus for the introduction of these distinctions was that the Einsteinian revolution was fundamentally different from the Scientific Revolution in the 17th century. After the triumph of the Einsteinian physics, the old Newtonian paradigm was not so radically rejected as the Aristotelian paradigm had been after the triumph of Newton (see Gillies 1992, p. 5). So the degree of incommensurability between the Aristotelian and the Newtonian physics must be greater than between the Newtonian and the Einsteinian physics. The fact that in Landau's textbook on mechanics we can find a fragment resembling the Cartesian way of describing collisions suggests that the transition from Descartes to Newton was similar to the transition from Newton to Einstein, where at least parts of the old paradigm are incorporated into the new one.

The tension between the evaluation of Descartes' scientific work by historians of science and its use by the scientific community shows the necessity of a reinterpretation of Descartes' scientific work. There are several possibilities how to reconstruct history of science. It can be reconstructed as a succession of *experimental discoveries* interpreted from the point of view of contemporary science. This approach is in the background of the above quoted citation from Stephen Gaukroger, who is willing to acknowledge only Descartes' contribution to the discovery of the law of refraction, denying at the same time the scientific value of the Cartesian system. Another possibility is to reconstruct the history of science on the *conceptual level* as changes of the fundamental categories and explanatory principles that scientists use to conceptualize their empirical data. This approach is in the background of the views of Daniel Garber, who interprets Descartes' physics as a metaphysical system. In his book *Descartes' metaphysical physics* (Garber 1992a) he offers a thorough reconstruction of the Cartesian system, describing it as a mistaken, but nevertheless outstanding, speculative achievement. A third possibility is to reconstruct the history of science on the *level of metaphors*, as a succession of visions and metaphors that form the basis of the conceptual schemes. This approach is in the background of the reconstructions by Koyré, who described the rise of modern science as the transition from the ordered cosmos to the infinite universe (Koyré 1957). But even for Koyré the transition from the Galilean mathematical physics to the purely verbal descriptions of the Cartesian system seemed to be a step in the wrong direction. Thus even if Descartes brought a new vision of the universe, Koyré did not take into account the connection between this vision and the universe of modern science. Husserl's reconstruction of Galilean physics, if we are prepared to interpret it radically enough, permits us to understand the positive side of Descartes' achievements. Husserl introduced the fourth level of reconstruction of the history of science, namely the *level of idealization*, by showing how physics systematically replaces various aspects of the lifeworld by mathematical idealities. This level of reconstruction opens the possibility of understanding Descartes' work and the role it played in the history of physics.

Adopting Husserl's approach we will try to interpret Descartes' contribution to physics as an idealization. Nevertheless, the Cartesian idealization was not an idealization of isolated phenomena, as was the case of Galileo, but it was an idealization of the ontological foundations of the lifeworld. The lifeworld has, beside its phenomenal level, also an ontological level. We understand the objects of our everyday experience, despite the great variety of phenomenal aspects we perceive in them, as possessing an ontological unity. We will interpret Descartes' contribution to the rise of modern science as the *replacement* of the objects of the lifeworld by their mathematical representation—extended bodies. Daniel Garber came close to this interpretation, when he remarked: "*The bodies Descartes shuffles out of his world in Meditation I, bodies which come up from time to time in the course of the first three Meditations, are the bodies of common sense, bodies known to me through my senses, and, like the piece of wax examined so carefully in Meditation II, endowed with scents, tastes, and tactile qualities. But when in Meditation VI the existence of bodies is proved, and the furniture removed by hyperbolic doubt in Meditation I is replaced, it has undergone a significant transformation. The sensual bodies we started with have been replaced by the lean, spare objects of geometry.*" (Garber 1992a, p. 75). Garber dedicated the next eighteen pages of his book to a reconstruction of Descartes' argument only to reach the conclusion that Descartes' arguments are insufficient to justify the replacement of sensory bodies by geometrical objects. In his reconstruction, however, Garber did not ask the question what objective does Descartes pursue by this replacement? If we interpret Garber's words in the light of Husserl's analysis of Galileo, we see that Descartes' replacement is a further step in the process of mathematization of nature. Just as Galileo had replaced the particular phenomena by mathematical quantities, Descartes replaced sensory bodies by geometrical objects.

When we, at variance with the practice of historians of science, integrate Descartes' physics into the mainstream of the history of physics, it creates a new perspective on the work of Newton. Many historians admit that Newton held during his youth a variant of Cartesian physics (see Cohen 1970 or Herivel 1970). Nevertheless, since Cartesian physics is not considered an integral part of the history of physics, its formal and conceptual structure has not been thoroughly studied. (An exception to this trend is the work of Alan Gabbey— see Gabbey 1980, 1985, and 2002.) In the third chapter of the present paper we will try to interpret Newton's physics against the foil of Cartesian physics as its wide-ranging and radical reconstruction. Similarly to Galileo, who brought idealization of the phenomena of the lifeworld, and to Descartes, who brought idealization of the ontology of the lifeworld, Newton's contribution can be interpreted as idealization of interaction. Further we will try to show that several aspects of the formal tools by means of which Newton achieved his idealization have their roots in Cartesian physics.

## 1 Galilean physics

There is a number of conflicting interpretations of the role of Galileo in the history of modern science (Mach 1883, Tannery 1901, Koyré 1939, Husserl 1954, Drake 1978, Naylor 1980, Wallace 1984, Wisan 1984, Hill 1988, Naylor 1990, De Caro 1993). Historians differ in their interpretations of the core of the Galilean project. Some of them see the main contribution of Galileo in his experimental method (Settle 1967, Drake 1978, Hill 1988), others in his mathematical Platonism (Koyré 1939, De Caro 1993), still others stress his use of the Aristotelian deductive method (Wallace 1984) or of a combination of experiment and deduction (Wisan 1984, Naylor 1990). An effort to present a more balanced picture of Galileo's achievements led the editors of the three collections: *Galileo, Man of Science* (McMullin, ed. 1967), *New Perspectives on Galileo* (Butts and Pitt, eds. 1978), and *The Cambridge Companion to Galileo* (Machamer, ed. 1998). Our aim is not to choose one of these interpretations, because we are convinced, that they do not exclude each other but rather represent different aspects of Galileo's work, which existed side by side and complemented each other, or perhaps they belong to different consecutive stages of the development of his thought.

The interpretation of Galileo as a Platonic is not convincing. Koyré bases his opinion on the argument that: "*No one could believe that there could be such a direct correlation between experiment and predictions! Indeed, despite the Galilean claim one has the temptation to doubt, for the simple reason: strict compliance as this is strictly impossible.*" (Koyré 1939, p. 107). In fact, Galileo's experiments were repeated and the obtained results are in good concordance with Galileo's own notes recording his observations (see Drake 1973, Naylor 1980, and Naylor 1990). In addition, anyone who has spent even a few hours in a laboratory knows that in each experiment there are measurement errors. This trivial fact was surely known to Galileo. To reject the experimental method for this trivial reason is ridiculous. Koyré is, of course, right in saying that the laws of physics cannot be derived directly from the experimental data, as some naive supporters of empiricism may think. But philosophical argument with empiricism should not be confused with history of science. We cannot understand Galileo if we try to interpret him as a pure empiricist, but similarly we cannot understand him if we interpret him as a Platonist. It cannot be denied that several Platonic themes permeate Galileo's work just like the themes of empiricism and Aristotelianism. We contend, however, that Platonism, as well as any other *–ism*, was just a marginal aspect of Galileo's achievements in physics. If we want to understand Galileo as one of the founders of modern science, we must first of all understand what was new in his work, i.e. how and why did he *transcend* Platonism, empiricism and Aristotelianism. In our view, the radically new element in Galileo's work was his ***understanding of motion as a geometric flow***, which is an idea foreign to Platonism, empiricism, or Aristotelianism. When we say that Galileo was a Platonic or an Aristotelian, we say nothing about what was new in his work, and therefore important for the rise of modern science, but we just describe what persisted in his work from the past. Neither Plato nor Aristotle created the theory of the free fall and Galileo differs from both of them precisely in creating such a theory. In history of science it is doubtlessly useful to distinguish between different traditions of thought—the Pythagorean, the Platonic, the Aristotelian, the Archimedean or the atomistic. But such distinctions themselves do not explain anything. We have to understand not only to what tradition someone belongs, but also to see in what directions he transcends each of these traditions. In our view, Galileo was first of all the founder of a new tradition, the tradition of modern physics, leading to Newton, Einstein, and Heisenberg. This tradition overcame Platonism. It replaced the ideas of the motionless Platonic ideal world by bodies in perpetual inertial motion.

Galileo Galilei (1564–1642) started his career as an adherent of Aristotelian philosophy at the University of Pisa, where between the years 1589 and 1592 he wrote the manuscript *De Motu*. In this work he tried to develop further the Aristotelian theory of motion by incorporating into it certain aspects of Archimedean hydrostatics as well as the scholastic conception of the impressed force. At that time Galileo accepted the division of motions into natural and non-natural ones. Nevertheless, in the case of the natural motions he replaced the Aristotelian division of elements into heavy (which naturally move downwards) and light (which naturally move upwards) by its Archimedean relativization, according to which an element is not light or heavy in an absolute sense, but only in relation to the surrounding medium. Thus for instance wood is heavy in the air and therefore it falls downwards, while in water it is light, and therefore it floats on the surface. Galileo explained the non-natural motions with the help of the scholastic concept of *virtus impressa* (impressed force). When one lifts a heavy body upwards, it receives lightness. If one drops it, the body starts to fall downwards and the inserted lightness is gradually diminishing, which manifests itself in acceleration of the motion. In the end, after all the impressed lightness is spent, the body acquires a uniform motion with a speed proportional to its specific weight.

In *De Motu* Galileo maintains that a body with a double weight would fall at a double speed. In that context he performed experiments with dropping bodies from a tower. The experiments did not confirm the proportionality of the speed to weight, but Galileo found an ingenious Aristotelian explanation of these negative results using the concept of impressed force. While one is holding a body on the top of the tower, his hand inserts some impressed lightness into the body. Now a body twice as heavy will receive twice as much of this lightness. Therefore if one drops two bodies having different weights, before the bodies reach their uniform motion, in which the twice-heavier body will move with double speed, there is a transitory period of accelerated motion, during which the bodies are losing their impressed lightness. Because the heavier body has more of the impressed lightness, it takes longer to get rid of it. The reason why we can not observe that a twice-heavier body falls with a double speed lies, according to Galileo, in that we do not have at our disposal a tower of a sufficient height so that the bodies could overcome the transitory phase of accelerated motion.[3] In 1604 Galileo in a letter to Paolo Sharpi held a totally different theory of motion. He was convinced that acceleration was a property of the free fall itself and not only of its transitory phase connected with losing the impressed lightness. Galileo turned to motion in the vacuum and formulated his law of the free fall.

In 1609 Galileo constructed a telescope through which he made a series of astronomical discoveries, which shook the Aristotelian theory of the universe. He published his astronomical discoveries in 1610 in his famous book *The Starry Messenger* (Galilei 1610). In 1613 he published his *Letters on Sunspots*, where he argued that sunspots appear and disappear directly on the surface of the Sun, which contradicted the Aristotelian doctrine of perfection and immutability of heaven. In 1623, Galileo published another astronomical treatise, *The Assayer* (Galilei 1623), in which he attacked peripatetic physics. While the book was being printed, Cardinal Barberini was elected as Pope Urban VIII. In 1624 Galileo came to Rome to ask the Pope for permission to publish a book dedicated to the discussion of the different theories of the structure of the universe. The Pope permitted him to discuss the theories only hypothetically. With

[3] An analysis of *De Motu* can be found in (Settle 1967).

this permission Galileo returned to Florence and started to write his *Dialogue Concerning the Two Chief World Systems—Ptolemaic and Copernican*, which appeared as (Galilei 1632). Presumably he was convinced that he fulfilled his promise given to the Pope, but many high rank representatives of the Church were of different opinion. Therefore Galileo was invited to Rome in 1633 to appear before the Holy Inquisition, where he was sentenced to home imprisonment (see McMullin 2009). In the solitude of his home imprisonment he wrote his most important work *Discorsi e Dimonstrazioni Matematiche*, *Intorno á Due Nuove Scienze* (Galilei 1638).

From this brief outline it can be seen, that the development of Galileo's thought followed basically the line of *objectivization*, *representation*, and *idealization* (see Kvasz, 1999, pp. 220-222).[4] Galileo started his career with an *objectivization* of the medium and an *objectivization* of the inertia. Thus at that time he was trying to preserve the general Aristotelian picture of the universe. He only wanted to introduce some new elements into this system and to relativize some of its distinctions in order to be able to explain the free fall and the projectile motion, which the Aristotelian theory could not explain in a satisfactory way. The failure of his attempts to develop a theory of these phenomena by introducing some new elements into the Aristotelian theory led Galileo to a radicalization of his views.

Around 1604 he abandoned the Aristotelian world picture and his effort to reform the Aristotelian theory through an objectivization. Now Galileo's aim was to replace the whole Aristotelian theory by a new *representation*. The series of astronomical discoveries opened the possibility that the Copernican astronomy could become the core of a new, non-Aristotelian world-view. Therefore the next more than 20 years Galileo devoted to the development of the Copernican theory. During this period his aim was not just to introduce some new objects or distinctions, which would fit into the global picture of the world, as he had wanted earlier. The change brought about by the acceptance of the Copernican astronomy was much more radical. It became absolutely impossible to explain motion of bodies on the Earth as a motion towards some natural places, because all places are in constant motion around the Sun. Therefore the Aristotelian theory of motion could not be saved by some new kind of impressed force. The whole Aristotelian idea of motion as a motion towards a place lost its meaning. The whole conceptual framework of the Aristotelian physics disintegrated.

For some time Galileo tried simply to replace the old crumbling Aristotelian representation of the world as an orderly system of natural places by the new Copernican representation of the world as a hierarchical system of circular motions. It is not the final rest in a natural place, but the eternal rotation in a circle, which is the principle of the construction of the universe. Nevertheless, there remained some tension in Galileo's work, because he conducted in fact two representations at the same time. The first of them was a *representation of the terrestrial motion*, according to which motion is not a transition to a given endpoint, but an inertial flow. In parallel to this radical change of the understanding of motion another representation took place, the *representation of the universe*, according to which the universe is formed not from some eternal changeless quintessence, as Aristotle claimed, but it is made of the same matter as the Earth. Earthly and celestial phenomena should therefore follow the same laws. Several aspects of these

[4] In our book on the development of mathematics (Kvasz 2008) we changed this terminology and instead of *objectivization* we used the term 'relativization' and instead of *representation* the term 're-coding'. Here it is perhaps not the appropriate place to introduce the technical terminology of (Kvasz 2008), and so we have decided to use the terms *objectivization* and *representation*.

two representations were in Galileo frequently in conflict.[5] In his theory of terrestrial motion Galileo *declared circular motion as inertial*. In his defense of the possibility of the motion of the Earth he said that when a ship is moving with inertial motion, a passenger locked in his cabin cannot determine experimentally whether the ship is moving or whether it is anchored in the harbor. On the other hand, in his theory of the tides he forgot about the inertial character of the circular motion and about his argument with the passenger locked in the cabin and he tried to explain the tidal wave as a consequence of the composition of two circular motions (the Earth's rotation and its revolution around the Sun), which does not make sense (see McMullin 1978).

If circular motion was really inertial then its existence should not manifest itself in any tidal wave. If we imagine a passenger locked in a cabin of a small ship, which is located inside a huge tanker half filled with water so that the small ship can float, the passenger locked in the cabin of the small ship cannot determine whether the ship or the tanker are moving. And this is exactly what happens to us on the Earth—at least according to Galileo: we are exposed to the composition of two inertial circular motions. Thus, the argument that Galileo considered a definitive proof of the motion of the Earth was at odds with his own theory of inertia. The error of Galileo's theory of the tides was noticed already by Descartes (see Shea 1978, p. 140). In his struggle with the tensions between his theory of motion and his theory of celestial phenomena Galileo probably realized that what is needed in order to build a consistent theory of motion is a more radical change. It is necessary to pass from the attempts to build a new representation of motion to its ***idealization***.

About two generations later Newton solved the conflict between the representation of the universe and our earthly experience with motion. He created a new picture of the universe as a centerless system of mutually interacting bodies. He replaced both the geocentric system of the Aristotelian physics and the heliocentric system of the Copernican astronomy by this picture. Galileo's views were far remote from this Newtonian picture and in several respects they were closer to Aristotle than to Newton. Galileo searched for a harmonic order of the universe, not for its dynamic laws; his universe had a fixed center, and there was no interaction between its bodies. On the other hand, it cannot be denied that Galileo opened many of the central issues of idealization of motion, as for instance the principle of inertia or the principle of relativity. Therefore we can say that with Galileo the process of idealization of motion started.

### 1.1 Galileo's instrumental idealization of motion

In a similar way, in which the notions *objectivization*, *representation*, and *idealization* characterize the different stages of Galileo's scientific development, we can use them to classify different interpretations of Galileo's work. Historians differ according to whether they interpret Galileo's contribution as an objectivization, a representation, or an idealization. In the following text we will concentrate upon the process of idealization. We will follow Husserl's interpretation from his *Krisis*. Husserl showed in his analysis of Galileo that there is an important epistemologi-

[5] The first of these representations could be characterized as the *transference of the inertial motion from the Heavens to the Earth* (in Aristotelian physics the heavenly bodies are in eternal, uniform, circular motion, and one could say that Galileo transferred this kind of motion onto the Earth by his principle of inertia), while the second one could be perhaps characterized as the *transference of matter from the Earth to the Heavens* (in Aristotelian physics the Earthly bodies consist of different matter to the Heavenly bodies, and one could say that Galileo, due to his astronomical discoveries transferred the Earthly matter onto the Moon and other heavenly bodies).

cal shift which separates the world of physics from the lifeworld. Therefore even though from the perspective of a science historian some of Husserl's statements are problematic, his drawing attention to the difference between the lifeworld and the world of science is of fundamental philosophical importance. Science historians usually ignore this difference, they do not question the scientific world-picture and present its creation simply as a process of further broadening and sharpening of our everyday life experience. Thus they generally ignore the process of idealization, and they direct our attention to epistemic ruptures of smaller magnitude.

Many confuse idealization with abstraction. This error is caused by the fact that we can obtain in our imagination a perfect geometric sphere from a real material ball by neglecting the unevenness and roughness of its surface, the elasticity of its matter, its weight, color, temperature, and even its taste and smell. What remains is a geometric sphere. Thus, at first glance, it seems that the perfect sphere is obtained by abstraction from the material ball. But in the process of abstraction something leads us; we are heading towards the perfect sphere. The perfect sphere must therefore exist before we start with the process of abstraction, so that we know what we have to neglect. This will become clear when we compare the geometrical idealization with the physical one. In the process of the physical idealization the ball retains its weight, hardness, and elasticity. An idealized physical sphere has weight, hardness, and elasticity, a perfect geometrical sphere lacks these properties. Thus, what has to be preserved in the process of abstraction is not arbitrary. The abstract object must fit into the linguistic framework, in our example into the linguistic framework of geometry or of physics. ***Abstraction is a linguistic reduction***; it is the replacing of reality by its linguistic representation. The syntax of the language leads us in the process of abstraction—it determines which properties can be neglected and which not.[6] The idealization which we study in this paper consisted in the construction of a new linguistic framework—the framework of physics. We can later choose this framework to lead us in the process of abstraction. Nevertheless, a*bstraction presupposes idealization, therefore it cannot explain it*.

### 1.1.1 Mathematization of nature as Galileo's program

Edmund Husserl (1859–1938) described in his book *Die Krisis der europäischen Wissenschaften und transzendentale Phänomenologie* Galileo's main contribution to the development of European science as a mathematization of nature, as a turning the world of qualitative phenomena into a universe of mathematical quantities (Husserl 1954, pp. 43-49). In Aristotle's philosophy the world of celestial bodies was separated from the sublunary world. Aristotle attributed no change and permanent self-identity to the world of celestial bodies and therefore this world was the subject of mathematical representation. An example of such representation is Ptolemaic astronomy. On the other hand, the sublunary world with its characteristic irregularity and permanent change admits no mathematical description, and can be described, according to Aristotle, only approximately. Galileo set his concept of mathematization of nature against this officially adopted Aristotelian world-view. In Galileo's concept, every natural phenomenon is substantially mathematical. The mathematical essence of some phenomena such as number, length, or shape,

[6] People are unaware of the dependence of abstraction on the linguistic framework. They are usually concerned with abstraction in only one domain (in mechanics, or in geometry) and thus the linguistic framework can remain implicit. Only when we try to compare different idealizations of the same object, the dependence of idealization on the linguistic framework becomes obvious.

is immediately evident. In the case of such phenomena as pressure, heat, or motion, however, we can perceive no mathematical quantity immediately. But this is not significant. According to Galileo these phenomena, which Aristotle would never have considered as suitable for mathematical description, also have a mathematical essence. The only difference is that this essence remains hidden from our senses somewhere below the surface of phenomena. This means that Galileo attributed universal validity to mathematical description thus turning the world into a mathematical universe. Every phenomenon has an ideal essence, "*Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders in a dark labyrinth.*" (Galileo 1623, pp. 237-238).

### 1.1.2 The instrumentalization of observation and Galileo's astronomical discoveries

In the autumn of 1606 Dutch lens makers constructed an instrument which was able to enlarge distant objects. In January 1610 Galileo constructed a telescope, by the help of which he made a series of important astronomical discoveries. He discovered mountains on the Moon, the satellites of Jupiter, the phases of Venus, the sunspots, as well as a huge amount of new stars. Thus in one single month—January 1610—there occurred more changes in astronomy than during the whole preceding century. Galileo's discoveries played an important role in the defense of the Copernican theory (see Swerdlow 1998, Shea 1998). Our aim here is neither the exposition of these discoveries, nor the discussion of the arguments in favor of Copernicanism. We would rather like to draw attention to the difference between Galileo's notion of observation and the notion of observation used in the academic milieu of his time.[7]

Galileo published his astronomical discoveries in March 1610 in a small book *Sidereus nuncius* (Galileo 1610). The book created a real storm. The reason for the intense reactions was not only the novelty and deep significance of the discoveries themselves, but also the fact that he made them using a telescope. His critics accused Galileo of naivety. At those times the telescope was considered to be an illusionist toy, which shows the phenomena not as they really are, but altered. Therefore "observations" with a telescope are unreliable and cannot be a basis for a serious science. Science has to examine the phenomena as they really are. Galileo's grounding

[7] The Copernican revolution is not the subject of this paper. Therefore, we will not analyze in detail Galileo's astronomical discoveries. Very important among them was the *discovery of mountains on the Moon*, as it showed that the surface of the Moon resembles the surface of the Earth: it contains mountains and seas. It is thus likely that the Moon is composed of the same matter as the Earth. That the Moon is up in the sky is thus caused not by the substance from which it is composed, as Aristotle argued. The Moon is a huge stone, which, according to Aristotle's physics should fall to Earth. That it is not falling shows that Aristotle's theory is not valid. Equally important was the *discovery of a nova*, dating back to Galileo's youth in 1572. At first glance it was a trivial event—to the billions of stars a new one was added. But if in the translunar region changes can occur, so the possibility to describe it by means of mathematics is not the consequence of its changelessness, as Aristotle claimed. According to Aristotle the skies can be described mathematically, because they consist of a special substance, and therefore are changeless. However, when the skies may change, this means that the possibility to describe them by means of mathematics is unrelated to their changelessness. Therefore it may be possible to describe mathematically also the terrestrial phenomena. Great importance was also attached to the *discovery of the phases of Venus*. The phases of Venus indicate that this planet does not orbit in a circle around the Earth, as the Ptolemaic system prescribed, but it moves periodically away from and closer to the Earth, as the Copernican theory maintained.

scientific theories in „observations“ made with a telescope was considered to be similarly naive as trying to develop a theory from "observations" made by a curved mirror. Galileo wanted to persuade his opponents and therefore he sent them a telescope so that they could see with their own eyes what he was speaking about. "*The majority of the natural philosophers simply did not think it worthwhile even to look through his telescopes*." (Ronchi, 1967, p. 201). And it was not by mere reluctance. The book in which the new lenses were mentioned for the first time was published by Giovanni Battista Della Porta in 1589 with the title *Magia naturalis*. Its seventeenth chapter dealt with optical magic, among other things also with lenses. A lens creates images which are greater or smaller than the real object observed by the naked eye. The object seems to be nearer or more distant than it really is and sometimes it is even turned upside down. Thus the lenses do not show true images of the things we are looking at, but create illusions. In order to break the resistance of the academic community, Galileo persuaded the Duke of Tuscany to send a telescope as an official gift to other rulers. He knew that most of the rulers have in their courts mathematicians or astronomers and these will be assigned to review the new instrument regardless of whether they like it or not. So in 1610 the German emperor Rudolf II. received in Prague as a gift a telescope, which he let to be examined by his court mathematician Johannes Kepler. Kepler saw the satellites of Jupiter and he supported Galileo with his authority in his *Narratio de observatis a se quatuor Jovis satellitibus erronibus* ... (see Ronchi 1967, p. 202).

With his telescope Galileo brought a fundamental change in the notion of observation. The classical astronomical instruments like the sextant or the astrolabe were only put alongside the axis which connects the eye with the object in the sky. Thus they did not change the way in which a particular object is disclosed to our view in everyday experience. They modify only the conditions of its givenness (*Art der Gegebenheit*) in that with the help of an attached protractor it becomes possible to determine with a higher precision the position of the particular object in the sky. Therefore we can say that these classical instruments only sharpen our natural experience. Their precision has limits given by the resolving power of our eye. These limits were reached by the outstanding Danish astronomer Tycho Brahe. On the other hand, Galileo's telescope enters between us and the object we observe. It makes it possible to see things which, without its help, we are unable to see, as for instance the satellites of Jupiter or the countless new stars, whose magnitude is below the threshold of our eye's sensitivity. Thus the telescope expands in a fundamental way the scope of our experience; it shifts the visual horizon, and opens new unseen worlds to our gaze. By further improvements of the lenses of the telescope or changes in its design in principle it is possible to increase the precision of telescopic observations almost without limits. The design of increasingly larger and more sophisticated telescopes had a decisive role in the history of astronomy. In addition to the telescope, Galileo contributed significantly also to the discovery of the microscope. Altogether he created eight sorts of instruments, which are discussed in the article *The instruments of Galileo Galilei* (Bedini 1967).[8]

[8] The importance of instruments is in that they expand, stabilize, and homogenize our experience as well as increase its precision. Instruments make experience registrable, reproducible, standard, and intersubjective. These aspects, however, relate to the level of *representations*. We can say that each *representation* brings its particular instrumental practice. From the point of view of *idealization*, however, the mentioned aspects of the instrumental practice are of secondary importance, and so we will not analyze them here.

### 1.1.3 Experimental mathematization of phenomena and Galileo's law of free fall

Instruments like the telescope broaden the realm of our experience. Nevertheless, they do not intervene into the constitution of the observed phenomena. They only change the sharpness and the resolving power of our sight. There is, however, a whole range of phenomena for grasping of which the instrumentalization of observation is insufficient. If we take for instance the free fall, we are not able to see, whether it is uniform or not. The way the free fall is given to us in the immediate perception is too ambiguous for an exact mathematical description. We are not able to perceive it as something ideal and perfect. And no instrumentalization of perception can be of help here, because the problem lies not in the insufficient sharpness or resolving power of our sight, but in the ambiguity of the perception of motion. Free fall is a motion, and therefore time enters in a fundamental way into its constitution. Nevertheless, we have no specific organ for the perception of time, the resolving power of which could be eventually enhanced by some instrument. Here we need not sharpen our senses, with which we observe the phenomenon. We have to sharpen the phenomenon itself.

In spite of all this, according to Galileo's program of mathematization of nature, somewhere beneath the perceived surface of free fall there are hidden some ideal mathematical objects which determine this phenomenon in an absolutely precise way. The only problem is to reach them. Many phenomena, as they are present in nature, are too complex. This complexity is the reason why we cannot grasp directly the ideal forms which determine them. Therefore, according to Galileo, it is necessary to create simplified situations in which the phenomenon would disclose itself in its purity and would reveal its ideal essence. The creation of such simplified situations requires invention, and Galileo's analysis of free fall is a beautiful example of such an invention. For Aristotle free fall and horizontal motion were qualitatively distinct. Free fall was a natural motion, because the body moved towards its natural position. On the other hand horizontal motion (in the sublunary region, of course) was an unnatural motion, requiring an external mover. Galileo's idea was to consider these two motions from the point of view of the artificial motion on an inclined plane. Free fall is a motion on a totally inclined (i.e. vertical) plane, while horizontal motion is a motion on an inclined plane, the inclination of which is zero. So by continuously changing the inclination of the plane we can pass from free fall to horizontal motion and back. In this way Galileo's imagination, using the artificial device of an inclined plane, connected two phenomena which apparently have nothing in common.[9] This connection has considerable technical advantages, because the motion on an inclined plane is relatively slow and therefore it is more suitable for observation than free fall.

If we draw horizontal lines in constant distances from each other on "*an inclined plane, exquisitely polished and hard*, *upon which descends a perfectly round ball of some very hard substance*" (Galilei 1632, p. 23), we can observe where the ball will be after the first, second, third, etc. interval of time. What Galileo discovered was an impressive regularity. The distance passed by the ball grew as the square of the time. After the first pulse the ball reached the first line, after the second pulse the fourth, after the third pulse the ninth line. If we increase the inclination

[9] For connecting the seemingly unrelated phenomena of *free fall* and *horizontal motion*, the experience with Archimedean relativization of gravity could be of help. There Galileo had to do with two similarly unrelated phenomena of Aristotelian physics: *lightness* and *heaviness*. He connected them with the help of the environment (water) in which a piece of wood (a heavy object, i.e. an object falling downwards) became light (floating on the surface). The role of water in the Archimedean relativization resembles the role of the inclined plane.

of the plane, the motion will accelerate. Nevertheless, the regularity—distance proportional to the square of the time—will be preserved. From this we can derive the conclusion that in the limit case of the vertical plane the distance will be still proportional to the square of time. It is plausible, even if we have no possibility to observe it directly.[10]

Thus the experiment, ***by creating an artificial situation*** in which the ideal essence of the phenomenon is accessible to direct observation, sheds light on the natural situations, in which the ideal essence remains hidden. The motion down an inclined plane made it possible to discover the law of free fall. In this way we arrive at the concept of experiment, which is the central concept of the Galilean physics. An experiment consists in ***inventive disclosing of the ideal essence of phenomena using artificial situations***. It is based on realizing a certain relation, which enables us to create an artificial situation through which we disclose the ideal essence of the examined phenomenon. The ideal essence of a phenomenon expressed in the language of mathematics is an empirical law. Galileo's law of free fall was thus one of the first laws of modern physics.

### 1.1.4 Measurement as constitution of phenomena and the notion of atmospheric pressure

The purpose of an experiment is to create by the help of an artificial situation an access to the ideal essence of phenomena. After achieving that, its task is usually finished. When Torricelli created a vacuum in a glass tube (and so proved the existence of the empty space which contradicted the Aristotelian physics), it was not the end of the story. The reason was that the phenomenon of atmospheric pressure, the ideal essence of which he disclosed in this way, is not accessible in any other way. In ordinary experience we are not aware of atmospheric pressure, and many cultures did not even suspect that there existed something like this. In this respect there is a radical difference between heat and pressure. Heat is disclosed to ordinary perception and therefore the thermometer can be still interpreted as an instrument which only sharpens the perception of heat. With the atmospheric pressure the situation is rather different. Without a barometer we have no idea even of the existence of this phenomenon. That is why Torricelli's tube did not "end in a museum" (i.e. did not become of interest only to historians), but was transformed into the barometer, which is a device opening an access to atmospheric pressure.

This means that measurement is a standardization of experiment. Thus in order to understand what measurement is, one has to remember what an experiment is. An experiment is the disclosing of the ideal essence of a phenomenon through artificial situations. A measurement is based on the ***standardization of the artificial situation of the experiment*** i.e. of the objects, relations, and procedures that constitute it. For instance, in the case of the barometer we fix the diameter and the length of the tube, the amount of mercury. We may also determine the number scale which we fix to the tube, choose the suitable physical units and determine the scope of tem-

[10] The situation with the law of free fall is slightly more complicated. A sphere in its motion on an inclined plane not only slides but also rotates. Therefore, not only is its translatory motion accelerated, but also its spinning, thus it increases both its momentum and its angular momentum. If the ball moves without slip, the angular velocity of its rotation is directly proportional to the velocity of its translatory motion. (In the case of the sliding of a rigid body on an inclined plane without any rotation its velocity is $v = g.t.\sin(\alpha)$. In the case of rotation without slip, due to the losses on the angular momentum, the body will reach a smaller velocity $v = (5/7).g.t.\sin(\alpha)$, which is by almost 30% less.) Galileo, while watching the second case, believed that he is watching the first one. In the transition to the free fall ($\sin(\alpha)=1$) there is a problem, because the effect of the acceleration of the body's rotation is turned off gradually. Fortunately, both cases accelerate in the same way, so the law *path proportional to the square of time* is not altered by the presence of rotations. So the conclusion that Galileo drew from his observation holds, although it should be added that thanks to good luck.

peratures at which the barometer gives reliable results. In this way we secure the reproducibility and so the intersubjectivity of the measurement. Thus even if the ideal essence of phenomena such as pressure is not accessible to us directly, and we can disclose it only with the help of an artificial situation, by the means of standardization we can minimize the dependence of the phenomena upon the particular situation. The measurement device is thus a tool for the realization of Galileo's program of the mathematization of nature.

As long as physics operates in the area of phenomena, to which we have an immediate access through our senses, it is possible to understand measurement as a process of refinement of the picture of reality, offered by the senses. For instance in the case of *free fall* we are not able to decide by the use of our senses whether it is uniform or accelerated. Nevertheless, we know free fall from our experience and thus we are inclined to interpret the measurement as a device which only helps us to determine that free fall is accelerated. In the case of *temperature*, the interpretation of measurement as a process of refining the sensory image, which we get by the immediate contact with the body whose temperature we are measuring, becomes more problematic. The problem is that we are able to measure the temperature of bodies which are so hot that by touching them directly our hand would be carbonized immediately and so we scarcely can speak about some sensory image. In the case of the *atmospheric pressure* it is even worse. The gradual decrease of pressure manifests itself on the phenomenal level first by an unpleasant headache, but it ends with the explosion of our organism after the gases dissolved in the body reach their boiling point. To speak about the measurement of pressure as making our sensory impressions more precise is impossible. What sensory impression corresponds to the pressure of 0.01 atmospheres? This is absolutely beyond human imagination.

Thus measurement not only increases the precision of the phenomena of ordinary experience, but it also ***extends physical reality beyond the boundaries of phenomenal world***. The results of measurements, with which we are confronted, are sometimes quite different from ordinary experience. The physical world-picture is constantly adapting to the latest results obtained in the experimental practice, and so it is gradually moving away from the image of the world that we have created in our lifeworld. At first glance it may seem that sensory experience is in line with the instrumental reality and is only supplementing it with some subjective aspects. But this is not the case. Between the phenomenal and the instrumental reality there is a fundamental conflict. Husserl became aware of this conflict and he showed that the measurement device does not make the sensory perceptions more precise, but on the contrary, it replaces them by a number, i.e. by a mathematical ideality, which is something very different from a sensory perception.

A measuring instrument is according to Husserl a technical means which in a standard way transforms the changes of a phenomenon (such as temperature, atmospheric pressure, or color) into changes of length. However, length is of ideal nature, successive division of its unit leads in principle to absolute precision. In this way the measuring instrument ***replaces the phenomena***, presented to us by our senses as dim, vague, and uncertain perceptions by ideal mathematical objects. In this way nature, which for the Ancients was a realm of phenomenal contents, becomes a universe of physical quantities.[11] This universe of physical quantities is the basis for the mathematization of nature. Thus science does not mathematize the *physis* of Ancient philosophy. Phenomena like color or taste, as we perceive them in our ordinary experience, cannot be math-

[11] We understand here phenomenal contents as opposed to mathematical quantities (thus for instance the feeling of heat as opposed to the temperature measured with a thermometer; or the impression of color as opposed to the wavelength of the light, as measured in optics), and not as a member of the ancient opposition of form and content.

ematized. Science does not mathematize these phenomena, but only their pictures, obtained in measurement. The scientists believe, and the phenomenologists doubt, that this mathematization is faithful.[12]

### 1.1.5 Galileo's principle of inertia and idealization of motion

In section 1.1.3 we presented Galileo's theory of free fall, which originated from experimental investigation of the motion on an inclined plane (as the theory of motion on a totally inclined, i.e. vertical plane). Let us now turn to the second limit case of Galileo's experiment, namely to the horizontal plane. Imagine that the plane is inclined downwards from the left to the right and that we let a ball roll in the same direction. Obviously, the ball will accelerate. If we gradually diminish the slope of the plane, passing through the horizontal position to the opposite direction, while the ball is still moving in the same direction, we will find that by moving upwards its motion decelerates. That means that the motion downwards is accelerated, while the motion upwards is decelerated. That is why the horizontal motion should be neither accelerated nor decelerated. Similar considerations could have led Galileo to his fundamental *principle of inertia*: "*We may therefore suppose it to be true that in the ordinary course of nature a body with all external and accidental impediments removed travels along an inclined plane with greater and greater slowness according as the inclination is less, until finally the slowness comes to be infinite when the inclination ends by coincidence with the horizontal plane. . . . But motion in a horizontal line which is tilted neither up nor down is circular motion about the center; therefore circular motion is never acquired naturally without straight motion to precede it; but, being once acquired, it will* ***continue perpetually with uniform velocity***." (Galilei 1632, p. 28, stress L. K.)

According to Aristotle, every motion must have its motive cause. Aristotle's basic perception was thus a perception of rest, motion being conceived as its disturbance, a deviation from rest in consequence of some cause. Galileo comes forth with a new principle—the principle of inertia: if a body moving on an absolutely smooth horizontal surface would be left on its own, it would remain moving infinitely. This is something substantially new—not the motion itself, but only the change of motion is the point to be explained. We have to explain not why things are moving, but why they stop. We need not a theory of the "moving cause" (as Aristotle thought), but rather a theory of the "stopping cause" (i.e. the theory of friction). By his principle of inertia Galileo changed radically the concept of motion. But we have to remember that he did not discover inertial motion in ordinary experience, but on a perfectly smooth horizontal surface that was a component of an artificial experimental situation. We can say that Galileo adapted his understanding of motion to the experimental practice. The motion became for him something ideal. The Greeks were unable to imagine something ideal and at the same time subjected to change.

[12] According to Husserl, idealization is the solution of a *conflict* between the phenomenal reality of ordinary experience and the instrumental reality of the experimental practice of science. This conflict is perhaps most saliently manifested in the question of the motion of the Earth, where science teaches us not to believe our own eyes that the Earth is stationary. We must instead learn to believe that we are constantly driven by the Earth with a velocity of over 10,000 kilometers per hour. Scientists were led to this idea by astronomical observations. Science has a tendency to suppress this conflict. She tries to convince us that there is a continuous transition from the phenomena of everyday experience to the scientific world picture. Therefore, in books on popular science Aristotelian physics is portrayed as a collection of prejudices, while Galileo and Newton are portrayed as defenders of common sense. But let us not be fooled. It is the Aristotelian physics that is the physics of ordinary experience and common sense. Modern science has abandoned everyday experience and common sense in the name of instrumental observations, experiments, and measurements.

For them ideal meant changeless. This was why Aristotle denied a possibility of mathematical description of the sublunary world. Galileo was able to imagine *idealities subjected to change*—the free fall in a vacuum for instance is something ideal, and therefore subjected to mathematical description, but at the same time it is a motion, i.e. something subjected to change.

The principle of inertia reminds one in many aspects of the scholastic theory of the impetus. Nevertheless, there is a basic difference. The aim of the theory of impetus was only to incorporate the phenomenon of inertia into the Aristotelian world-view, i.e. to explain, why a stone preserves its direction of motion even after it leaves the hand that threw it.[13] Galileo (after years of futile attempts) came to the conclusion that it is not possible to incorporate this phenomenon into the Aristotelian system, and that for the sake of this phenomenon (as well as the acceleration of the free fall), it is necessary to abandon the Aristotelian *representation of motion* and to replace it by a new *representation*, in which inertia becomes one of the fundamental principles. Thus Galileo raised inertia to a principle. It is not just a marginal phenomenon which we can get rid of by introducing the impetus. On the contrary, inertia is the central property of motion, around which a new interpretation of motion should be built. Motion is not a transitory state, through which bodies reach final rest at their natural places. On the contrary, motion is a fundamental property of all bodies.

Nevertheless, every motion we encounter on Earth has a natural tendency to stop. Thus, after Galileo came to the conclusion that motion is inertial and its stopping is only a result of friction, physics left the realm of natural experience and came into a direct opposition to it. If a desk moved freely in the room, we would be probably surprised. But according to Galileo, we should not be surprised, because to move freely is the most natural thing that a desk can do. But it is surprising that the desk rests in its place. Thus not the motion of the desk, but rather its motionless rest is something unnatural which we have to explain. The explanation is that the surface of the room is not ideally smooth and ideally hard, and so it hinders the desk in manifesting its nature. Thus Galilean physics considers the real nature of bodies (i.e. their inertial motion) to be something which nobody ever experienced, and the way how bodies appear in our everyday experience, what happens with them each and every day (i.e. being at rest), is allegedly something absolutely unnatural. As Koyré said: "*Galileo's physics explains that which is by that which is not* [i.e. real motion by motion in a vacuum]." (Koyré 1939, p. 200).

### 1.1.6 The distinction between primary and secondary qualities

In his work *The Assayer* Galileo divided all characteristics we encounter in everyday experience into primary and secondary qualities: "*Hence I think that tastes, odors, colors, and so on are no more than mere names so far as the object in which we place them is concerned, and that they reside only in the consciousness. Hence if the living creature were removed, all these qualities would be wiped away and annihilated.*" (Galilei 1623, p. 274). This means that the image of reality that our senses present to us is not physically real, but only that part of it which we are

[13] The medieval theory of impetus was an *objectivization of inertia* in the framework of Aristotelian physics. Inertia was objectivized as the *impetus*, i.e. as a substance that is inserted by the mover into the moving body. Impetus is so just another element of the Aristotelian universe. The inertia of the projectile motion has, according to the theory of impetus, nothing to do with the inertia of the motion of the heavenly bodies and is thus limited to a narrow range of phenomena such as the motion of missiles or of thrown objects. In contrast, Galileo's principle of inertia is the basis of a new *representation*. The inertia of the motion of the earthly bodies and the inertia of heavenly motion is governed by the *same principle*.

able to grasp through measurement. Only that part can become the subject matter of scientific investigation. In the reduction of reality to the primary qualities we can see, at the one hand, a forerunner of the mechanistic worldview and, at the other hand, a remote predecessor of the notion of state of a physical system. Similarly, as in the case of Galileo's principle of inertia of circular motion there was a long way to go towards the principle of inertia of modern physics, also in the case of the reduction of reality to the primary qualities there is still a long way to go to the concept of state. Moreover, here, unlike in the case of the principle of inertia, Galileo's idea of reduction is not related to the idea of the temporal development, which is one of the most fundamental aspects of the notion of state. But despite these constraints one can not deny that we are here confronted with the ***idea of the reduction of phenomena to their mathematical description***, which was the key moment in the further development of physics.

### 1.1.7 Galileo's reduction of motion to a geometric flow

As we have already pointed out, in everyday experience we encounter motion as a process that sooner or later stops. Such motion is the subject of the Aristotelian physics. According to Aristotle, motion is simply a transition of a body from one place to another. Therefore, it is determined by two places. On the one hand it is the starting point, i.e. the place where the body rests before it starts moving. On the other hand motion is determined also by its final point, i.e. the point, towards which the motion is directed. The motion stops when the body reaches its final destination. Thus Aristotle's concept of motion can be characterized as a theory of ***geometric transition***. It is based on the geometric structure of the universe (the theory of natural places) and motion is a transition from one place to another.

Galileo's principle of inertia changes the concept of motion in a fundamental way. Galileo speaks about the motion which—strictly speaking—nobody ever saw, about motion as an eternal flow. According to Galileo, motion has no starting point and no destination; it is not a transition from one place to another. Of course, there are cases where a motion stops. For instance, in the case of a free fall the body stops when it reaches the surface of the Earth. But this stopping is only the consequence of hitting an obstacle. Therefore the terminal point does not belong to the motion itself. In contrast to the Aristotelian theory, according to Galileo, the terminal point of the motion appears only due to a violent external intervention. The free fall is not directed to any terminal point. It is only, due to the external circumstances, violently interrupted at the Earth's surface. Motion is, according to Galileo, neither a motion from a place nor a motion towards a place. Motion is a movement alongside a trajectory. Thus Galileo replaced the Aristotelian concept of motion that we have characterized as a geometric transition by a new concept of motion as a ***geometric flow***.

We can say that between the two points, by means of which Aristotle explained motion, Galileo inserted a curve connecting them. The motion passes along this curve, either uniformly or with acceleration. This concept makes it possible to describe motions that have neither a beginning nor an end, because the curve also may have no starting point and no endpoint. For Galileo, the universe is a geometrically ordered system of trajectories. This is an enormous change, because motion is no more a transitory disturbance of a fundamentally static order of the universe, as it was in the Aristotelian theory. Motion is an eternal flow, and so the order of the universe itself becomes a kinematic order. Nevertheless, Galileo's concept of motion is still a geometric one, because it represents motion by means of the geometric concept of a trajectory.

In the Aristotelian world-view each place had its fixed identity, determined by its relation to the order of the universe. The nature of a motion was determined by the place towards which it headed—it was a motion downwards, if it was directed towards the center of the universe, and it was a motion upwards, if it was directed towards the sphere of the stars. Thus Aristotle discerned different kinds of motion according to the final point towards which they moved. With Galileo, a motion can no more derive its identity from a place, because in his system there are no fixed places. Nevertheless, Galileo still discerned different kinds of motions as free fall, projectile motion, inertial motion etc. The identity of these different kinds of motion was not determined by the terminal point, but by the geometric form of the trajectory along which they moved. So for instance free fall was rectilinear, inertial motion was circular, and projectile motion was parabolic. Thus, even though Galileo changed the Aristotelian way of determining the nature of a motion, he did not abandon the very idea that ***there are motions of different kinds***. For Galileo the acceleration of the free fall was a property of that particular kind of motion, just as the parabolic shape of the trajectory was a property of projectile motions. Therefore Galileo saw no need to explain why free fall was accelerated. Acceleration was simply a property of this kind of motion.

Galileo was not yet in possession of sufficient mathematical resources that are necessary for the development of his new idea of motion. He did not posses analytic geometry that would allow him to describe trajectories of arbitrary shapes, and so in his theory of motion a dominant role was still given to circles and parabolas. In addition, he lacked the idea of infinity of space. Galileo's universe was still the finite universe of ancient science; his universe ended at the sphere of the stars. Galileo's theory resulted in a replacement of the Aristotelian idea of the universe as a hierarchically ordered system places by the idea of the universe as an ordered system of circular motions. The basic principle of ancient science that the order of the universe is given by geometric arrangement is still preserved. Galileo only replaced the static order of places by a kinematic order of trajectories. But the order of his universe is still a geometrical order, a harmonious order of trajectories of non-interacting bodies.

Galileo had no concept of gravity (see Koyré 1939, p. 199). He did not see acceleration as a result of the action of a force, but as a characteristic feature of the free fall, understood as a special kind of motion. Strictly speaking, at the phenomenal level such a view is understandable; we do not perceive any action of forces. Galileo described only what he was able to observe; he reduced physics to phenomenal reality. It was Descartes who expressed the idea that there is no natural acceleration and that there is only one kind of motion—uniform motion in a straight line. Free fall must be therefore the result of interaction. The fact that we do not see it is for Descartes irrelevant. If we do not see interaction, we must postulate it. Accelerated motion must be a result of interaction, whether we see it or not.

### 1.2 Shortcomings of Galilean physics

The importance of Galileo's thoughts for the development of modern science is generally accepted. But despite his fundamental contributions, Galileo's ideas had also some deep shortcomings which are the reason why the modern science is not a direct continuation of the Galilean project. We do not have in mind Galileo's mistakes (as was his conviction that inertial motion is circular or his ignorance of Kepler's discovery of the elliptic shape of the planetary orbits: he maintained that the orbits are circular). Such mistakes can be easily corrected. Neither have we in mind Galileo's opinion that the universe is finite. This can also be changed. What we have in mind are problems concerning Galileo's conceptual understanding of motion. These problems are the reason why the analysis of Galileo's work is not sufficient for the understanding of the structure of modern science.

The shortcomings of Galilean physics are most easily realized when we compare Galilean physics with the Cartesian one. Such a comparison allows us to understand not only *the limits of* Galileo's conceptual understanding of motion, but also *the relation between* Galilean and Cartesian physics. Even if this relation is not very important from the historical point of view (it seems that Descartes did not study Galileo's works in detail), it is important for the clarification of the place of Descartes in the history of physics. Therefore, we will list the deficiencies of Galilean physics in the order in which their corrections enter into the construction of the Cartesian system. In focusing on the shortcomings of Galilean physics, our aim is not to lessen Galileo's merits. We believe that shortcomings of scientific theories belong to the history of science in the same way as their successes. The advantage of the analysis of the shortcomings of scientific theories is that they allow us to see the specific features and the unique character of these theories, by pointing to their limits.

### 1.2.1 The circular character of inertial motion

*Galileo considered uniform circular motion as inertial.* This means that he was unaware of the centripetal acceleration of circular motion. The uniform circular motion is, according to Galileo, not the result of the action of a force, but it is inertial. It is obvious that the belief in the inertial character of circular motion is a remnant of the Aristotelian theory of celestial motion, even though, unlike Aristotle, Galileo attributed this kind of motion also to the earthly bodies.

### 1.2.2 Absence of the concept of a state

*Galilean physics lacked the concept of state*. Although Galileo formulated the idea of reduction of reality to primary qualities, he was unable to reduce the diversity of different kinds of motion. This fragmentation in the description of motion is a typical feature of Galilean physics. Aristotle separated the circular motion, which he considered the principle of motion of heavenly bodies, from the linear motion, which was the principle of motion of bodies in the sublunar realm. Galileo eliminated the Aristotelian separation of the heavenly and the sublunar realm when he discovered that the horizontal motion on the Earth's surface is inertial, and thus ascribed circular motion also to earthly bodies. We can say *that Galileo homogenized space*—he removed its division into the sublunar and celestial realms—but ***he did not homogenize motion***—he still distinguished different types of motion. The free fall is a rectilinear motion, inertial motion is circular, and projectile motion is parabolic. When a body falls down, it is governed by the law of free fall, when it moves horizontally, it is governed by the law of inertia, and when it is thrown obliquely upwards, its motion will be governed by the law of projectile motion. According to Galileo, there exist different kinds of motion (from the empirical point of view they do really exist), and the task of physics is to find their accurate mathematical description.

It seems that Galileo took the idea of distinguishing different kinds of motion from Aristotelian physics. This means that although Galileo rejected the Aristotelian way of classifying the particular kinds of motion (into natural and non-natural ones), he did not reject the very idea of the existence of different kinds of motions. Unlike Galileo, modern physics knows only one kind of motion—uniform rectilinear motion— everything else is the result of interaction.[14] It is easy to see that this shortcoming of Galilean physics is the result of its too close connection to experience. In ordinary experience we actually see different kinds of motion. To reduce all of them to only one, the uniform motion in a straight line, and to attribute all deviations from this uniform motion to the action of forces, requires a considerable degree of abstraction.

### 1.2.3 Absence of universal laws

*Galilean physics lacked universal laws*. Laws discovered by Galileo, such as the law of free fall, the law of isochrony of the pendulum, or the law of inertia, describe specific phenomena and so they cannot be applied to other phenomena. The law of free fall describes falling objects but it does not apply to a pendulum. The law of isochrony of the pendulum describes pendulums but not falling bodies. Galilean physics thus breaks nature into isolated phenomena, each of which is described by a special law written in the language of mathematics. Even though the Galilean mathematization of phenomena was a big step forward, the fragmentation of nature that it introduces indicates that Galileo did not yet found the optimal way of mathematization. Modern mathematical science is not a direct continuation of the Galilean project.

The source of this fragmentation of nature seems to have its origin in the experimental method. An experiment allows us to find for each phenomenon a corresponding mathematical law, but on the other hand, it isolates the studied phenomenon from all other phenomena. Galilean physics, due to its too narrow ties to experiment, lost sight of the unity of nature

### 1.2.4 Absence of the notion of interaction

*Galilean physics lacked any description of interactions between bodies*. All laws discovered by Galileo (the law of free fall, the law of inertia, the law isochrony of the pendulum), describe the motion of a single isolated body. In his investigation of nature Galileo isolated the bodies from their surroundings and studied their motion separately. Therefore he saw the acceleration of free fall not as the result of interaction, but is a characteristic feature of this kind of motion—just like the parabolic shape of the trajectory of projectile motion. Galileo did not see the necessity to explain how it is possible that free fall is accelerated. For him it was simply a fact that he wanted to describe as precisely as possible.

It is not difficult to see that the absence of interaction in Galilean physics is closely related to the geometrical language, by means of which he described motion. When he tried to grasp motion as a continuous passing along a trajectory, interactions were left out from the picture. An interaction impairs the given shape of the trajectory, and thus it interferes with the geometrical framework by means of which Galileo described motion. According to Galileo, the book of nature is written in the language of mathematics; "*its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it*" (Galilei 1623, p. 238). Thus Galileo choose the wrong mathematics. His geometrization of

[14] It is surprising that Galileo did not recognize uniform rectilinear motion: all motions that he described differ from the uniform rectilinear motion. Inertial motion is according to Galileo uniform, but it is circular (i.e. not straight); free fall is a motion along a straight line, but it is accelerated (i.e. not uniform).

motion was an important step forward on the road to modern science, but it was only the first step.[15]

### 1.2.5 Absence of the union of several bodies into a mechanical system

As already mentioned, Galilean physics studied the motion of isolated bodies. In addition to the impossibility of describing interaction among bodies, mentioned in the previous chapter, it also means the absence of bounds between them. In other words, Galilean physics lacked the theoretical tools for the description of a mechanical system consisting of several bodies. Of course, in the study of motion on an inclined plane or of a pendulum we are (from the Newtonian point of view) dealing with systems with bounds, but Galilean physics did not understand them this way. It simply neglected the arm of the pendulum or the inclined plane. It required the arm to be perfectly solid and have zero mass; just as it required the inclined plane to be perfectly hard and perfectly smooth. These requirements can be summarized into a single condition: the arm of the pendulum or the inclined plane should not interfere with the motion of the body under examination.

### 1.2.6 The openness of the physical description of motion

*Galilean physics was not able to describe a closed dynamic system*. Galileo's law of free fall is one of the first laws of modern physics. But how strange a law it is! In free fall the body accelerates, thus its momentum and kinetic energy are increasing. Nevertheless, the question from where are these gains of momentum and energy coming is left by Galileo without any answer. The systems described by Galilean physics violate the conservation laws; their total energy and momentum spontaneously grows and falls. This shows that Galilean physics did not have the concept of a closed physical system.

### 1.2.7 Summary

Interestingly, the absence of the concept of state, the lack of universal laws, and the lack of description of interaction is not mentioned in the extensive literature on Galileo's work. Professional historians of science probably see comparisons of a theory with later stages in the development of the particular discipline as something inappropriate. They call it *Whig history* and see it as one of the fundamental methodological errors of their discipline. In our view, however, the comparison of a theory with the later stages of development can improve our understanding of the theory. The above listed shortcomings of Galilean physics do not diminish the importance of Galileo's contribution to science. Nevertheless, they enable us to understand more clearly the motives, the contents, and the context of the Cartesian project. If we realize that Galilean physics lacked the concept of state, the notion of a universal law, and the notion of interaction, we cannot ignore the role of Cartesian physics, which introduced these elements into the physical description of nature. We can neither omit Descartes from the history of physics, nor can we describe him as a strange marginal figure (the author of a metaphysical physics). We have to take

[15] The unsuitability of Galilean mathematization was noticed by Wisan: *it has been insufficiently noticed that to an important degree his* [Galileos] *'mathematicism' consists in the attempt to reduce natural science to the Greek mathematical model in order to achieve the logical certainty of mathematics*. (Wisan 1978, p. 3).

seriously Descartes' physics despite its many factual and conceptual errors, and integrate it into the history of physics as a bridge linking Galileo with Newton.

## 1.3 Husserl's analysis of Galilean physics

In chapters 1.1.1 – 1.1.6 we tried to show the relevance of Husserl's interpretation of Galileo. Nevertheless, the fact that we consider Husserl's interpretation of modern science to be a relevant one does not mean that we accept all aspects of his theory. In Husserl's text there are several technical faults which do not lessen the importance of his contribution, but their analysis can give us a better understanding of the limitations of his understanding of modern science. We hope that on the basis of this understanding we can overcome the limitations of the phenomenological theory of science.

### 1.3.1 Some technical comments concerning Husserl's interpretation of Galileo's physics

Our first comment concerns ***Husserl's understanding of mathematization***: "*But through Galileo's mathematization of nature, nature itself is idealized under the guidance of the new mathematics; nature itself becomes—to express it in a modern way—a mathematical manifold*" (Husserl 1954, p. 23). From the historical point of view, the mathematization of nature was not so straightforward. In the way in which Galileo attempted to mathematize nature (with the help of "*triangles, circles and other geometrical figures*") it is simply impossible to get beyond a mathematical description of simple phenomena. And apart from that, between Galileo's "mathematized nature" and the "new mathematics" which accompanied the role of its idealization, there was an important intermediate stage: the Cartesian physics.

Another problem concerns the ***algebraic language***: "*Here we must take into account the enormous effect—in some respects a blessing, in others portentous—of the algebraic terms and ways of thinking that have been widespread in the modern period since Viète (thus since even before Galileo's time)*" (Husserl 1954, p. 44). Husserl indicates here that alegraic formulas are somehow connected with Galileo's mathematization of nature. Even if he does not say explicitly that Galileo wrote some formulas, nevertheless he indicates the existence of such a connection. But Viète's symbols are rather too complicated and useless for physics. Therefore Galileo did not use any formulas and he expressed his laws in a purely verbal manner. The transcription of physical laws into algebraic symbolism was the achievement of the next generation of physicists, and the author of the algebraic symbolism, with the help of which this transcription was achieved, was Descartes.

The question of ***causality*** remains also unclear: "*The formulae obviously express general causal interrelations, "laws of nature", laws of real dependencies in the form of the "functional" dependencies of numbers*" (Husserl 1954, p. 41). If we rewrite Galileo's law of free fall with the help of algebraic formulas (of course, Galileo used no formulas), we obtain something like:

$$s = \frac{1}{2}gt^2.$$

It is obvious that this formula does not express any causal relation. It is simply an expression of a correlation between two aspects of the phenomenon of free fall without any recourse to causes.

Physics has achieved the level of the description of causal laws, but it was not Galilean physics but the Newtonian physics. Newton's law (expressed using modern notation)

$$F = m\frac{d^2x}{dt^2}$$

is an expression of the change of the momentum of a body due to the action of a force. Nevertheless, it is not an algebraic equation but a differential one. And this question leads us again to Descartes, who in his physics tried to overcome Galileo's reduction of physics to the purely phenomenal level and wanted also causality to be included into our description of nature. In this way Cartesian physics plays the role of an intermediate state between Galilean physics, which excluded causality from the description of nature, and the Newtonian physics, which describes causality through differential equations.

Husserl's view of 'algebraization' (or 'arithmetization') of geometry is also problematic: "*This arithmetization of geometry leads by itself in a sense to emptying of its meaning*." (Husserl 1954, p. 44). In our view, algebraic symbolism definitely does not empty the meaning of geometry, but on the contrary, it raises this meaning to a higher level of fullness. To see this, it is sufficient to realize why geometry can serve as a tool for (the Galilean) mathematization of nature in the first place. It is because the language of geometry was the first language that contained the idea of a variable in the implicit form of the *line segment of indefinite length*. The language of algebra brought this idea to an explicit form and thus brought the possibilities, which in geometry had been present only implicitly, to a much higher level of completion. Algebra brought a radical, structural deepening of the sense of geometry in the form of Descartes' analytic geometry.

### 1.3.2 The relation between Galileo's physics and Cartesian philosophy

The above mentioned details, in which Husserl's exposition contradicts the historical facts, lead us repeatedly to Descartes. At the same time anyone who read Husserl's *Krisis* might notice one remarkable feature of the book. Husserl analyzed thoroughly Galileo's physics (Husserl 1954, pp. 20-60) and Descartes' philosophy (Husserl 1954, pp. 60-85), but the relation between these theories he only vaguely indicated in three rather shorts notes: "*One can truly say that the idea of nature as a really self enclosed world of bodies* ***first emerges*** *with Galileo. A consequence of this, along with mathematization, which was too quickly taken for granted, is [the idea of] a self-enclosed natural causality in which every occurrence is determined unequivocally and in advance. Clearly the way is thus prepared for dualism, which* ***appears immediately afterward*** *in Descartes*" (Husserl 1954, p. 60, stress L. K.). "*After Galileo had carried out, slightly earlier, the primal establishment of the new natural science, it was Descartes, who conceived and at the same time* ***set in systematic motion*** *the new idea of universal philosophy: in the sense of mathematical or, better expressed, physicalistic, rationalism—philosophy as universal mathematics*" (Husserl 1954, p. 73, stress L. K.). "*Is Descartes here not* ***dominated*** *in advance by the Galilean certainty of a universal and absolutely pure world of physical bodies, with the distinction between the merely sensibly experienceable and the mathematical, which is a matter of pure thinking?*" (Husserl 1954, p. 79, stress L.K.).

We do not intend to question the importance of the relation between Galileo's physics and Descartes' philosophy which is indicated by Husserl. What is striking is the conceptual vagueness of the description of this relation, using the words like "*first emerges*", "***appears imme-***

***diately afterward***", "*set in systematic motion*", or "*dominated*". It reminds us of a description of the process of a geological folding, in which mountains *emerge*, continents *appear*, and the whole process is *set in motion* by tectonic forces. While Husserl offers a thorough intentional analysis of the works of Galileo and Descartes, the transition from one to the other is left unclear and has the form of a conceptual sedimentation.

In contrast to the above-mentioned technical comments, a deeper problem manifests itself in this ambiguity in the interpretation of the relation between Galileo and Descartes. We are convinced that Descartes' philosophy cannot be interpreted as an outcome of Galilean physics. In our view Descartes' philosophy is an outcome not of Galileo's but of Descartes' physics. Thus we come back to the point we mentioned at the beginning of our paper. Husserl successfully refuted the positivistic interpretation of the rise of modern science, but at the same time he took over the framework, in which positivism discussed this question. One characteristic feature of the positivistic interpretation of the rise of modern science was the omission of Descartes' physics (as a metaphysical theory) and the attempt to interpret Newton's physics as a direct continuation of Galileo's intentions.

We believe that the omission of Descartes' physics from the analysis of the rise of modern science is the reason why Husserl tries to connect the two incompatible theories. Galileo's physics cannot be connected directly with Descartes' philosophy, because they are separated by Descartes' physics. Descartes' physics brought a *radical alteration* of the entire Galilean project of the mathematization of nature, and Descartes' philosophy was a further radicalization of his physics.[16] Thus even if it is possible to give a clear conceptual explanation of the transition from Galilean physics to Descartes' physics, as well as of the transition from Descartes' physics to his philosophy, a direct transition from Galileo's physics to Descartes' philosophy cannot be described, because it just did not take place. Descartes went in his physics against the Galilean program, and so later, when he completed his physics by its philosophical reflection, the Cartesian philosophy which so emerged did not have with Galilean physics much in common. That is in our view the reason why Husserl connected the conceptually clear and precise exposition of the scientific works of Galileo with an equally clear and precise exposition of the philosophical works of Descartes in such a vague and obscure manner. An exposition of the rise of modern science requires first of all clarification of the relation between Galileo's project of mathematization of nature and Descartes' philosophy. This clarification has the form of an intentional interpretation of Cartesian physics, which is the content of the next chapter.

[16] Interpretation of Descartes' philosophy as a deepening of the project of Descartes' physics is not common. Historians of philosophy usually ignore Descartes' physics and try to explain Descartes' philosophy from purely philosophical motives (as the confrontation with skepticism). Nevertheless, Gaukroger's *Descartes, An Intellectual Biography* (Gaukroger 1995) is an exception from this trend.

## 2 Cartesian physics

Galileo made a series of important scientific discoveries by means of his experimental method. Nevertheless, Descartes realized the limited scope of the Galilean project of founding science solely on experiments. In a letter to Mersenne of October 11, 1638 Descartes wrote: "*without having considered the first causes of nature, he [Galileo] has merely looked for the explanations of a few particular effects, and he has thereby built without foundations*" (Clarke 1992, p. 271). In order to make a phenomenon accessible for experimental investigation, Galileo had to isolate it from the network of its relations with other phenomena. Therefore, the laws discovered by Galileo, for instance the law of the free fall, the law of the isochrony of the pendulum, the law of the parabolic trajectory of projectile motion, are all laws describing isolated bodies. Although Galileo succeeded in reducing these phenomena to mathematical relations between physical quantities, what he achieved was that for each phenomenon there was a specific law. Therefore even if the laws discovered by Galileo were true, which Descartes doubted, by Galileo's method nature would disintegrate into a set of unrelated processes. In opposition to Galileo, Descartes required that science must go beyond the phenomena and grasp the deeper ontological unity of nature. Each time we observe motion, it is the motion of a body, when we perceive a number, it is the number of certain bodies, when we see a shape, it is the shape of a particular body. Thus from the primary qualities, which were the core of Galilean science, it is necessary to proceed to their ontological foundations, i.e. to the extended body. Thus the method of science must be based on an ontological rather than on a merely phenomenal reduction.

Descartes moves from the description of isolated phenomena to the description of the state of a physical system. Only when we reach this deeper level of description, the unity of the world reveals itself. The task of physics is to find ***universal laws*** describing the ***changes of the state*** and not *particular regularities* occurring between the parameters characterizing *isolated phenomena*. Descartes thus in a fundamental way transgresses the level of generalization which can be justified by empirical experience. Strictly speaking, however, it is impossible to derive from any set of experiments that beyond particular phenomena there is an *ontological unity* (represented by the notion of state)— just as it is impossible to conclude from any correlation between experimental data that there are *universal laws* underlying these correlations. But on the other hand, we cannot deny that modern physics does precisely this. Instead of parameters characterizing the experimental behavior of a system, physics postulates its state described by the Lagrangian or Hamiltonian function. Similarly it tries to derive all observed correlations between the empirical parameters from some universal law, expressed in the form of Lagrangian or Hamiltonian equations. In other words, science follows Descartes' intentions of an ontological reduction and universal description of nature.

There is no doubt that this idea is a metaphysical one. In justifying his physical laws Descartes explicitly used metaphysical arguments. It is partially because of this close relation to metaphysics that positivistic historiography ignored Cartesian physics and that many modern historians tend to interpret it as a metaphysical system and not as a scientific theory. We would like to show that Cartesian physics is more than a *metaphysical system* and that it can be interpreted as a project of ***mathematical physics***. In our view the metaphysical underpinnings of the Cartesian system had only an auxiliary role because there was no mathematical language Descartes could use to work out his project. As soon as Newton created such a language, it became possible to replace Descartes' metaphysical principles by mathematical ones and to justify them by their success. We see Descartes' reliance on God in physics as a symptom of the fact that Descartes is transcending the way in which nature was described by his contemporaries, but he did not have the mathematical ground on which he could base his new way of describing nature. God has in Descartes' system a precise epistemological place.

Once Newton created the differential and integral calculus by means of which a mathematical description of nature could be successfully completed, large parts of the Cartesian metaphysics could be omitted and the new physics could be justified by its functioning. What Descartes grounded on metaphysics, Newton formalized. Metaphysics will be moved elsewhere—it will be used by Newton for a justification of the infinity of space and of the action at a distance—to gradually disappear almost completely. But if we want to explain the origins of modern science, we must not ignore the theological roots. When we look at the elements of Cartesian physics supported by a theological foundation, we can easily see that they are formal elements. Theological passages in scientific works can be seen as indicators of the birth of a new language (for more details see Kvasz 2008b). The formal aspects of a language consist in everything the language cannot express (or justify empirically) but only display. And there is no doubt that experience can not justify the conservation of the quantity of motion or the possibility to reduce all phenomena to extension and motion. It should be noted, however, that these principles are an essential part of Descartes' physics—they constitute the form of its language. The reluctance of philosophers to appreciate Descartes' fundamental contribution in the making of modern physics is understandable. To admit that Descartes could play an important role in the creation of physics would mean to admit that science is in a fundamental way metaphysical.

The recognition of the metaphysical foundations of modern science is hampered by the fact that in modern science the metaphysical foundations are operationalized. Scientists are not aware of the fact that in describing a physical system by its state and the temporal evolution of that state by a universal law they are using metaphysical principles going back to Descartes. Scientists do not ponder on such questions, they simply write down the equations and start to solve them. Thus it seems that Desmond Clarke is not right when he considers Descartes' requirement, according to which we have to construct metaphysics before formulating a physical theory, to be an obsolete, scholastic trait of Descartes' thought (see Clarke 1992, p. 272-273). On the contrary, this feature makes Descartes modern, as he proceeds in agreement with the practice of contemporary scientists. The only difference is that Descartes is fully conscious of the metaphysics and explicitly states it, while modern science has a formal metaphysics which is taken for granted and therefore perceived by nobody. But to write the Lagrangian function $L(q, q')$ describing the state of a system is a ***metaphysical move***. Strictly speaking, there is no reason why a system should have a state.[17]

Thus, we come back to Husserl and his questioning of the obviousness of modern science. Husserl showed that modern science replaces the phenomena of the life-world by mathematical quantities. A similar process takes place on the ontological level, where science replaces the ontology of the life-world by the description of states. The Cartesian rupture which separates the state of a system from the ontology of the life-world is thus similar to the Galilean rupture which separates mathematical quantities from the phenomena of the life-world. The similarity

[17] To ascribe a state to a system means to assume that its entire future can be determined from its mathematical description at the present moment of time. It means that we can ignore the entire history of the system, i.e. the trajectory, along which it reached its current state. All information essential for the future development is present in the mathematical representation at a single moment in time.

of these two ruptures can be expressed by the notion of idealization. Thus we would like to interpret the transition from the Galilean to the Cartesian physics as the ***transition from the phenomenal idealization to the ontological idealization***. Descartes moved from the Galilean world of mathematical quantities to the causally determined world of moving extended bodies.

Contemporary history of science has some difficulties in understanding the historical importance of Cartesian physics not only because of this physics' metaphysical foundations, but also because of its rather deep mistakes. Descartes is accordingly omitted in many expositions of the development of physics and Newton is seen as deriving directly from Galileo. Daniel Garber has accepted this account when he wrote: "*Numerous identified themselves as Cartesians, and numerous, like Spinoza, Leibniz, and Malebranche, were deeply influenced by the Cartesian idea of a mechanist system of metaphysics and natural philosophy, while significantly altering the details. But there was another important trend in seventeenth-century thought, a* ***nonmetaphysical*** *and problem-oriented conception of natural philosophy. This is found in Descartes' near contemporary, Galileo, and in* ***his successor, Newton***." (Garber 1992a, p. 307, stress L. K.). We believe that this interpretation of the rise of modern science is contentious. Excluding Descartes from the history of science prevents us from understanding the origins of two central features of modern physics, namely its ontological homogeneity and descriptive universality. Modern physics assumes that every system has a state and that there is a differential equation that describes the temporal evolution of this state. Nevertheless, these assumptions are empirically unjustifiable. The concept of state is not an empirical concept.

Another aspect of Cartesian physics discouraging its incorporation into the mainstream history of science is, besides its metaphysical roots, its verbal character. When Descartes arrived at the idea that physics should offer a universal description of nature, he did not have at his disposal the mathematics necessary to accomplish this task. Therefore he presented his idea of the universal description of nature only in a verbal form. Many historians of science might have been confused by this circumstance. When they compared the verbal and in many respects totally mistaken Cartesian physics with the Galilean mathematical description of motion, they came to the conclusion that Descartes was, when compared with Galileo, a step backwards to metaphysics. But such an interpretation of the relation between Descartes and Galileo is questionable. Descartes, if he had wanted to, could have worked out the Galilean project much further than Galileo was able to. Descartes was one of the creators of analytic geometry and he introduced the standard algebraic notation, which is still in use. He was well equipped to develop the ideas Galileo arrived at through a cumbersome symbolism and a rudimentary idea of a coordinate system in a much more elegant way. Nevertheless, Descartes was not interested in the motion of isolated bodies as Galileo was, but rather in the interactions among bodies, a phenomenon Galileo never understood. Despite Galileo's apparent similarity to Newton, due to his use of mathematical language, he was in fact closer to Aristotle. For all his use of mathematical language, Galileo was still developing only a geometric theory of motion. On the other hand, Descartes tried to grasp interactions, and thus despite his verbal formulations he was doing precisely the same thing that Newton was to do on a higher level, using his new mathematics; he tried to develop a dynamic concept of motion.

Thus Koyré's thesis that Descartes "*in identifying matter with extension substituted geometry for physics*" (Koyré 1939, p. 94) is problematic. Descartes did not substitute geometry for physics, because the Cartesian extended bodies are not geometrical objects—they interact. It is inconceivable that two geometric triangles could collide or that one circle would rebound from another in accordance with the Cartesian laws of impact. The figures of Euclidean geometry do not have a tendency to move. Euclid never formulated the law of inertia or the law of the conservation of the quantity of motion for his figures. Nevertheless, these are the two basic laws obeyed by all Cartesian bodies. What Descartes substituted for physics was definitely not geometry. Rather, Descartes, in contrast to Galileo, tried to incorporate the notion of interaction into the description of nature. Therefore if one of the above mentioned theories is a geometrical one, it is the Galilean. Similarly problematic is Koyré's view that Descartes "*substituted his concrete physics of motion in a plenum for the abstract physics of motion in a vacuum*" (Koyré 1939, p. 100). The transition from Galilean physics of motion in a vacuum to Cartesian physics of motion in a plenum is not a transition from the abstract to the concrete. Descartes was one of the leading mathematicians of his times and surely did not lack the capacity for abstraction. If he had wished to, he could have developed the physics of motion in a vacuum to a much higher level than Galileo could have dreamt of. Why Descartes did not embark on this project is something Koyré did not understand. The reason is simple—Galilean physics lacks the notion of interaction. Thus the transition from Galileo to Descartes was not a transition from the abstract to the concrete, but it was rather a transition from the representation of the world without interactions to a representation of the world in which interactions are incorporated.

We can not agree with the assessment expressed by William Shea, according to which "*Cartesian motion is neither dynamic (involving consideration of force) nor kinematic (involving only consideration of space and time), but merely diagrammatic (involving only consideration of space)*" (Shea 1991, p. 272).[18] Similarly, we cannot accept the interpretation of Descartes' scientific work as an attempt to reduce physics to kinematics. This is how Dijksterhuis sees Descartes' physics, and he goes further by claiming that Descartes reconstructs all physical phenomena in the language of matter and motion, eliminating in this way the concept of force (see Dijksterhuis 1961, pp. 403-418). As we will show, Descartes' physics is a dynamic, and not a kinematic theory, because it studies interactions between bodies, and it describes these interactions by means of forces. The only difference is that the forces in Cartesian physics are not forces of interactions, but forces of inertia. This difference, however, does not affect the overall character of Descartes' physics.

Already Gaukroger drew attention to the unacceptability of Dijksterhuis' interpretation, when he showed that there are forces in Descartes (Gaukroger 1995, p. 247). Soon, however, he himself gets astray when he writes: "*His [Descartes'] aim is not to reduce physics to kinematics, but rather to model it on hydrostatics... I believe it is this reliance on hydrostatics, rather than kinematics that explains Descartes' commitment to the notion of a plenum.*" (Gaukroger 1995, p. 247). Gaukroger's interpretation of Descartes' hydrostatic works is impressive and it shows a deep insight into the thinking of the young Descartes. Captured by his interpretation of Descartes' hydrostatic work (especially of Descartes' attempted to explain the hydrostatic paradox) Gaukroger began to see the entire physics of Descartes against this background. In our

[18] Shea is right that in comparison with Galileo's kinematic description of motion Descartes lacks the temporal dimension. We expressed this lack by calling Descartes' idea of motion a *transition*. Nevertheless, this transition is in our view a *dynamic* transition. Shea's term '***dynamic motion***' corresponds to Newton and thus to what we call a *dynamic flow*. Shea's term '***kinematic motion***' corresponds to Galileo and thus to our *geometric flow*. Finally his '***diagrammatic motion***' may correspond to Aristotle's theory of local motion and thus to our *geometric transition*. Thus we agree with Shea that Descartes' theory differs from the Galilean as well as from the Newtonian theory. His conceptualization of the different theories of motion has many similarities with ours. But we firmly believe that Descartes' theory of motion was a dynamic one, and thus we disagree with Shea in his last point.

view, however, Descartes' theory of motion is a dynamic theory of motion, and his theory of collisions is a (even though factually incorrect) dynamic theory of collisions. For Descartes' choice of plenum we need not invent any hydrostatic justification. The role of the plenum in Cartesian physics is to transfer action between bodies, and that is a purely dynamic affair, representing one of the main contributions of Descartes to physics. The plenum will become a marginal historical episode as soon as Newton replaces it by his forces acting at a distance. Thus the legacy of the positivist historiography is visible even in Gaukroger. This legacy refuses to give Descartes a place in mainstream history of physics, so that a historian like Gaukroger, who is well aware of the qualities of Descartes' physics, is forced to invent an interpretation, like that with the hydrostatic model, to find for Descartes at least some place in the history of physics. In the following paragraphs we will try to show that Descartes clearly belongs to the mainstream history of physics, and thus needs no such apology.

The development of Descartes' thought is remarkable for its internal coherence. It is from the beginning marked by the effort to find a new method of scientific research. His journey from mathematics through physics to metaphysics is an uncovering and clarification of this method. As Jean-Luc Marion writes: "*the starting point of the Meditations—the project of establishing science by means of hyperbolic doubt—is nothing else than the point reached by the end of the Regulae, namely science operating on the simple natures, both material and common*." (Marion 1992, p. 123).

Descartes was born in 1596 in La Haye. In **1618** he met the Dutch scholar Isaac Beeckman (1588–1637) who awakened in him the interest for mathematics and physics. Beeckman, who was eight years older than Descartes, presented him with various mathematical and physical problems and discussed with him their solutions. It is likely that Descartes adopted Beeckman's theory of atoms and of empty space, which he later in his philosophical works strongly rejected, as well as the idea of a mechanical reduction of natural phenomena, to which he adhered throughout his entire life. The influence of Beeckman on Descartes' thinking is described in Descartes' biography (Gaukroger 1995, pp. 68-103).

The decade **1618–1628** following his encounter with Beeckman was perhaps the most creative period in Descartes' life. During this period Descartes formulated the basic ideas of his method, created a new algebraic symbolism, laid the foundations of analytic geometry, and discovered the law of refraction. His attention was focused on mathematics, mainly on the possibilities opened up by the new algebraic language, which allows us to employ in our calculations an abstract quantity independently of whether it is an arithmetical, geometrical, or physical one.[19] Descartes soon realized that by reaching this level of abstraction it becomes possible to create a radically new method of pursuing the natural sciences. Science need not be bound to the descrip-

[19] A fine exposition of Descartes' mathematical works can be found in (Shea 1991, pp. 35-92). On page 43 Shea describes an alleged error of Descartes, consisting in the transition from the equation $\frac{x^3}{7} = x + 2$ to the equation $x^3 = x + 2$ of a simpler type, which he was able to solve graphically, while he *assumed that it* [the solution of the original equation] *could be reinstated by a simple multiplication after solving the equation $x^3 = x + 2$. There are two other instances in the Cogitationes Privatae where he makes the same mistake* .... Shea's comments this by saying: *These failings should not blind us to the magnitude of Descartes' discovery of a practical method of solving cubic equations.* It appears, however, that Descartes is right. If we take the equation $x^3 = x + \frac{2}{\sqrt{7}}$ (an equation of the simpler type with a modified last term) and find its solution $t$, then $x = \sqrt{7}t$ will be the solution of the original equation, which can be easily checked. So Descartes was right: from a solution of the equation of the simpler type it is possible by a simple multiplication obtain the solution of the original equation.

tion of concrete, isolated phenomena, but may seek abstract representation of reality and universal laws, just like algebra formulates general formulas expressing solutions of whole classes of algebraic equations, no matter what values their coefficients have and whether they represent arithmetical, geometrical, or physical quantities (see Schuster 1980). The universality of algebraic notation was probably the model for Descartes' idea that all physical properties can be reduced to extension and motion. This period culminates by writing the *Regulae ad directionem ingenii*, which, however, were published only in 1701. This work is interesting also because it contains an attempt to apply algebraic operations to geometric line segments. Descartes interpreted the product of the segments $a$ and $b$ as the area of the rectangle with the sides $a$ and $b$. The idea to interpret the product of the segments $a$ and $b$ as the segment $a.b$, which is the core of analytic geometry, was born only after Descartes had written the *Regulae*.

Descartes' mature natural philosophy started to crystallize in the years **1629–1633** when he abandoned the mathematical study of physical phenomena and turned to the development of a general world-view. Letters from the years 1629 and 1630 suggest that he intensively worked on the theory of motion, on optics, meteorology, and anatomy. One could say that he was applying his algebraic method to the world as a whole. This period culminated in 1633 in writing *Le Monde* (Descartes 1662), in which he *rejected the existence of a vacuum*, proposed a *mechanical interpretation of gravity* and *subscribed to the Copernican theory of Earth's motion*. In *Le Monde* Descartes described a model of the world. This model is purely hypothetical, its construction surpassing the horizon of our experience. Scientific theory thus becomes a hypothetical model of reality, and it is no longer just its true mathematical description, as it was for Galileo. Using the words of Desmond Clarke we can say that for Descartes "*to explain any natural phenomenon is equivalent to constructing a model*" (Clarke 1992, p. 266). When in 1633 Descartes heard about the condemnation of Galileo, he decided not to publish his *Le Monde*. He wrote: "*this has so astonished me that I almost resolved to burn all my papers, or at least not to let anyone see them. For I cannot imagine that Galileo, who is Italian and even well-loved by the Pope, as I understand, could have been made a criminal for anything other than having wanted to establish the motion of the earth.*" (Ariew 1992, p. 77).

In the years **1634–1637**, after realizing that *Le Monde* may not be published, Descartes returned to the special problems of mathematics and physics and developed several themes of the *Regulae*. In 1637 he published three essays—*Dioptrics*, *Meteors*, and *Geometry*—, united into one volume, and completed by a preface entitled *Discourse on Method* (Descartes 1637). The *Dioptrics* and the *Meteors* contained a number of important scientific results, such as the law of refraction, the theory of visual perception, and the theory of the rainbow, which had been, at least partially, contained already in *Le Monde*. The *Geometry* is the birth place of analytic geometry and it also contained significant advances in algebraic symbolism (see Grosholz 1980, Mancosu 1992). Descartes introduced the convention to use letters from the end of the alphabet ($x$, $y$, $z$...) for the unknowns, while letters from the beginning of the alphabet ($a$, $b$, $c$ ...) for the parameters; he proposed to use the top right index to indicate powers ($x^3$, $x^4$, $x^5$ ...). The *Discourse on Method* includes an analysis of several problems of methodology, metaphysics, and physics. The book, however, lacks even a mention of the problem of the Earth's motion. The book received positive acceptance, as it contained a number of scientific discoveries, a new algebraic notation, and analytic geometry. Seeing the success of the *Discourse,* Descartes decided to publish his philosophical system.

In the years ***1638–1650*** Descartes embarked on intensive work. He published his *Meditations* *on first philosophy*, containing his metaphysical views. He developed his physics on metaphysical principles, especially on the principle of immutability of God. Soon after the *Meditations* he published the *Principles of Philosophy* (Descartes 1644), a work containing his physical ideas in a systematic form. This book can be regarded as a Cartesian physics textbook. Descartes wrote in a letter to Constantijn Huygens, that the *Principles* are only a translation of *Le Monde* into Latin. Indeed, the contents of these two works are in many ways overlapping. *Le Monde* just like the *Principles* contains the theory of collisions, based on three laws of nature; they both base the description of the solar system on the vortex theory of gravity. However, if we compare these two works in detail, we see that Descartes changed in the meantime some of his views and clarified others. His theory of collisions received a more detailed formulation and the Copernican doctrine receded into the background.

Despite the undeniable differences in detail, however, there is a fundamental similarity between the *Principles of Philosophy* (1644) and the *Le Monde* (1633), just like between the *Discourse on Method* (1637) and the *Regulae ad directionem ingenii* (1628). This similarity is a testimony of the internal integrity and coherence of Descartes' thought. His works were not written in response to external stimuli, as were several works of Galileo, but in the process of gradual clarification and refinement of a vision that emerged sometime between 1618–1619 when Descartes was only twenty years old.

### 2.1 Descartes' ontological idealization of state

In section 1.1 we presented an interpretation of Galilean physics as an *idealization of motion*. We believe that Descartes made a further important step in the process of idealization by turning to the ***idealization of state***. Husserl did not study this phase of the process of idealization and tried to reduce the ideality of modern science to the Galilean idealization. Nevertheless, Galilean physics was able to idealize only isolated phenomena. If science was based only on observation, experiments, and measurement, it could never introduce quantities such as force, energy, or action. It would have no reason to transcend the horizon of phenomenal reality. Science, however, does surpass this horizon and we believe that this is due to Cartesian physics. Descartes gave the program of mathematization of nature a much more radical form than Galileo. According to Descartes, mathematization should not just replace isolated phenomena by mathematical quantities but it should also reduce their ontological substratum to the mathematical notions of extension and motion, or—using the words of modern science—to the notion of state. Therefore we will call Descartes' idealization the idealization of state.

Many historians are not aware of this process. For instance Stephen Gaukroger writes: "*Even if we could establish the essentialist thesis that extension is the only property that we cannot conceive of matter lacking without its ceasing to be matter, what relevance does this have for mathematical physics? More specifically, first, why should physics be based on this conception of matter and not another; second, why must physical concepts be dependent upon an abstraction argument; and third, why should we want an essentialist physics in the first place?*" (Gaukroger 1980b, p. 132). Obviously Gaukroger did not notice that here Descartes was not concerned with a metaphysical question, he was not developing any essentialist notion of matter. Descartes' question was an epistemological one; it was the question of *how the quantities determining the state of a system can be distinguished from the remaining physical quantities*. The importance of this question for mathematical physics is obvious. Only after solving this question can we start to develop mathematical physics. Thus the problem here does not concern the idea of matter but the idea of its description. Descartes' argument from abstraction, according to which every quantity we can abstract from a body without destruction of its ontological integrity as a body, does not belong to the quantities that determine the state, is a reasonable argument. A body's state is something it cannot be deprived of. Descartes' strategy for obtaining the notion of state by a systematic elimination of all attributes the body can be deprived of at least in principle is not so misguided. In our imagination, for instance, we can deprive bodies of their colors and therefore colors do not belong to the quantities determining state. We do not mean to say that Descartes' solution was correct. Newton showed that although we are able to imagine a body without mass, the mass must nevertheless participate in the determination of state. Thus Newton changed the criteria determining the state parameters. But if we want to understand Descartes, it is not sufficient to say that he embarked on some strange essentialist enterprise completely unrelated to physics. We must understand the reason why he was pursuing it, namely, in order to introduce the concept of state. Even if Descartes' notion of state was abandoned and *extension and motion* are no more used as state variables, we should not forget that it was he who introduced the idea of state into physics for the first time. Therefore, we see Gaukroger's criticism of Cartesian physics as unjustified.

In our interpretation of Galileo we mentioned that he did not have analytic geometry at his disposal, and so his mechanics was only fragmentary, limited to a few isolated phenomena. Galileo lacked the mathematical language which would enable him to systematically develop his physical theories. Descartes created such a language in the form of analytic geometry. Therefore, we might expect that in his interpretation of motion he will utilize this mathematical discovery and present a theory of motion which would use a much better mathematical apparatus than the mechanics of Galileo. On the background of such expectations Descartes' *Principles of Philosophy* is a disappointment. It seems that Descartes gave up mathematization and returned to the Aristotelian verbal style of describing motion. But this first impression is misleading. Descartes gave up only the ***Galilean way of mathematization*** of motion as geometric flow, because he realized that it is not possible to incorporate into it the description of interactions among bodies. This very fact reveals the depth of Descartes' mathematical insight. He refused to walk along the Galilean path, because he saw that it does not lead where he wanted to go. Thus, although he created analytic geometry that would allow bringing Galilean kinematics in unprecedented perfection, he decided not to use it in his physics. He understood well that analytic geometry cannot be used to describe interactions. In this he was absolutely right. As it turned out, it was necessary to create an entirely new mathematics for a mathematical description of interaction, the differential and integral calculus. Thus Descartes' resignation to continue in the development of the *Galilean way of mathematization* was not a manifestation of some metaphysical inclinations. On the contrary, it shows an understanding of the problems associated with the mathematical description of motion as well as a deep insight into the possibilities of mathematics of his days. Descartes' refusal to continue the Galilean mathematization of motion is therefore, in our view, an expression of his philosophical grandeur, his ability to sense the limits of mathematics. The idealization of interaction had to wait until Newton and Leibniz created the calculus, a tool strong enough to accomplish this goal.

Descartes' contribution to physics can be characterized as the idealization of the notion of state, which entails an ***ontological homogenization*** and ***nomological unification*** of the world. Husserl pointed out to a break that separates the phenomena of the life world from the mathematical variables by means of which science represents them. Nevertheless, to explain the origin of the ontological homogeneity and the nomological unity of the world the idealization of isolated phenomena is not enough. The basis of Cartesian physics is the notion of state. Husserl characterized Galileo's intention as the program of mathematization of nature. But Galileo's mathematization of isolated phenomena is not the project of modern mathematical physics. Mathematics was for Galileo only *a language suitable for the description of phenomena*. It was Descartes who first arrived at the idea of a mathematical physics. As we have already explained, it is not important that Descartes presented his theory only in a verbal form. His theory is a mathematical theory in a deeper sense, not just in the sense of the language employed. When Descartes says that everything can be reduced to extension and motion, it means that mathematics is the ontological foundation of reality. So geometry is not just a language suitable for the description of phenomena, as it was for Galileo. Reality itself is nothing else but mathematical bodies in motion. Descartes had probably something like this in mind, when in his letter to Mersenne from 27. July 1637 he wrote: "*My entire physics is nothing else than mathematics.*"

### 2.1.1 Descartes' correction of Galileo's principle of inertia

The law of inertia is in the literature usually attributed to Galileo. This tradition dates back to Newton, who attributed the merit of his discovery entirely to Galileo while he did not even mention Descartes (see Koyré 1939, p. 129). Galileo, however, considered inertial motion as circular and not rectilinear. In his experiments with the inclined plane he discovered that motion on a perfectly smooth horizontal plane is inertial. But motion on a horizontal plane is a motion at a constant height above the Earth's surface, i.e. a motion in a circle. In this form, the principle of inertia is of course incorrect. In addition, the principle of inertia applies in Galilean physics only to a special kind of motion—the motion on a perfectly smooth horizontal surfaces. Other kinds of motion do not obey the principle of inertia.[20]

One of the first thinkers who realized that ***inertial motion is rectilinear*** and that the ***principle of inertia applies to all motions*** was Descartes. In the *Principles of Philosophy* he writes: "*The first law of nature: that each thing, as far as it is in its power, always remains in the same state;*

[20] Descartes may have realized that it were the instrumental techniques of experimentation and measurement that divided Galileo's world into isolated phenomena. Descartes' method of *clara et disctincta perceptione* can be seen as an alternative to Galileo's method of experimentation and measurement. While in a *measurement* we isolate the phenomena from their natural surroundings, in a ***perception*** the phenomenon is left as it is, so that we can understand it together with its ties to its surroundings. However, it is important that this perception was ***clear*** (Galileo achieved clarity by means of the *artificial experimental situation* that allowed for instance to clarify the character of free fall) and ***distinct*** (Galileo reached distinctness by *precise measurement*). Descartes seems to be, as concerns the method of clear and distinct perceptions, inspired by mathematics, and so his abandoning of the experimental method turned out to be a step backwards. Newton corrected this and returned to the instrumental techniques of experimentation and measurement as the foundation of physics. Nevertheless, Newton was able to compensate the Galilean isolation of phenomena by his mathematics. Phenomena, which the instrument isolates, he connected by their mathematical description.

Despite of this it is interesting to notice that in the field of optics Descartes experimented and his theory of the rainbow is based on experimental investigation of the refraction of light on a spherical glass container filled with water. However, here Descartes did not go beyond the concept of Galilean experiment. He created an artificial situation, which allowed him to mathematically describe the rainbow, i.e. the phenomenon that defies direct mathematization—just like Galileo in his experimental study of the free fall. Shea noted that Descartes in his experiments with the container filled with water simplified the situation by assuming that the refraction of light on the walls of the container can be neglected (see Shea 1991, p. 206). Neither here, however, did Descartes differ from Galileo, who in his experiments on the inclined plane neglected the rotational motion of the balls.

*and that consequently, when it is once moved, it always continues to move. ... The second law of nature: that all movement is, of itself, along straight lines; and consequently, bodies which are moving in a circle always tend to move away from the center of the circle which they are describing.*" (Descartes 1644, p. 59-60). These two laws can be considered the forerunners of Newton's law of inertia: "*Every body preserves in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.*" (Newton 1687, p. 416-417). Newton's law is a combination of refinement and two Descartes' laws.

In the question of the relation between Descartes' first two laws of and Newton's law of inertia there is no consensus. Some historians point to the contextual and conceptual differences between them (e.g. Gabbey 1980, p. 286-297). It appears that the way how we see the relation between two versions of a law depends on the perspective from which we look at it. We can examine this relation from three perspectives: the perspective of idealizations, of representations and of objectivizations. From the perspective of *idealizations*, Descartes' and Newton's formulations of the law of inertia are equivalent because they fix in the particular system the ideal nature of the uniform rectilinear motion. From the perspective of *representations* some differences emerge. Descartes described interactions as singular events in which a body can change its direction of motion without changing its velocity. For example, if we bounce a ball against the wall, according to Descartes, after hitting the wall the ball instantaneously changes the direction of its motion, without changing its velocity. Therefore Descartes formulated the conservation of the state (i.e. of the quantity of motion) in the first law and the conservation of the direction of motion in the second. In the Cartesian system, the conservation of the velocity and the conservation of the direction are unrelated. When Newton inserted between the Cartesian states before and after the collision a continuous process which he described by means of a differential equation, he found out that after touching the wall the motion of the ball gradually slows down, until it stops for an instant, and then the elastic forces begin to accelerate it in the opposite direction. The change of direction of motion (which Descartes understood as an instantaneous turn) is thus in fact a gradual process of change of velocity. Therefore there is no reason to describe changes of state and changes of direction separately. From the perspective of *objectivizations* some contextual differences came to the fore. In Descartes' system, the separation of the velocity from the direction in the formulation of the principle of inertia is important in several contexts. Most important is the context of the relation between the soul and the body. In this context, the separation of the velocity and the direction of motion makes it possible for the soul to influence the body.[21] Since our aim is to analyze idealization, we will not discriminate between Descartes' and Newton's formulation of the principle of inertia. We will thus consider Descartes the author of this principle. In Descartes we can find both the rectilinear character, as well as the universality of the inertial motion which are missing in Galileo.

Whenever Galileo studied a moving body, he isolated it from its surroundings. According to Descartes, such an idea of motion is totally misleading. For one thing, it can lead us to

[21] According to Descartes, the body is a hydromechanical machine. Nerve fibers are tubes in which flows a fine fluid that affects muscle contraction and blood circulation in the body. If we replace the fluid in the nervous fiber by an electric current, Descartes' idea is not at all naive. From anatomical studies he learned that a large number of nerve fibers end in the pineal gland, and therefore he situated there the place of the contact between the soul and the body. The soul cannot change the ***quantity of motion*** (i.e. the velocity of the flux of the fluid), because this quantity is constant, but Descartes attributed to the soul the ability to influence the ***direction of the flux*** of the fluid in the pineal gland, thus to determine where the blood will flow, and so to interfere with the body's movements. In Newton's system such influence of the soul on the body is not possible any more.

erroneous conclusions, as for instance Galileo's belief that inertial motion is circular. This error is a consequence of Galileo's tendency to isolate the moving body, to exempt it from the influences of its surroundings and to describe how it would move if there were no other bodies and no friction. In his study of motion on a horizontal surface Galileo thus abstracted from friction, air resistance and from the influence of all other bodies. But in this process of abstraction he happened not to abstract from weight. Only on account of that did inertial motion remain circular. If he had really abstracted from everything that surrounds the moving body, the circular motion would become rectilinear. Galileo did not realize that removing the surrounding medium would also destroy the agent causing the curvature of the body's trajectory. He mistakenly assumed that, after the elimination of all surrounding bodies interacting with the studied body, it would preserve the circular form of its trajectory. Thus Galileo believed that, if we eliminate the influence of the surrounding medium, each motion will preserve its particular character. Moreover, in a vacuum the motion will manifest its character all the more clearly.

Descartes had the idea that there is only one kind of motion, uniform motion in a straight line, and that everything else is the consequence of interaction. Thus, according to Descartes, Galileo's theory of motion was erroneous in a fundamental way, because it abstracted from the surrounding medium as well as from the neighboring bodies. In reality we can eliminate neither and therefore we cannot study what would happen after such elimination. Therefore it is very probable that Galileo's theory of free fall was just as mistaken as was his claim that inertial motion is circular. Galileo claimed that his law of free fall described the falling of a body in a vacuum. But free fall is an accelerated motion, and in the vacuum there is no agent to accelerate the motion, so a body falling in the vacuum would have to move with a constant velocity.[22] It cannot accelerate itself. Acceleration is a consequence of interaction. In order to accelerate the motion of the body there must be something acting on the body, some other body which causes the acceleration. For this reason Descartes rejected Galileo's theory of the free fall. According to Descartes, if a vacuum was possible at all, all bodies placed in it would move with constant velocities. Accelerated motion is possible only as a consequence of an action. Accelerated motion in a vacuum is nonsense.[23]

Imagine Galileo studying the flight of a bird. He would probably, as he did in the case of the free fall, turn to the study of the flight of the bird in a vacuum. Galileo's method was to get rid of the environment that complicates the motion. Once we get rid of the air, the bird would fall to the ground. Just like in the case of the free fall, once you strip away the environment, the body stops to move with acceleration. In order to keep the bird afloat and the free fall accelerated we need a causal agent that causes these kinds of motion. When Galileo abstracted from the medium, in

[22] Galileo's law of free fall contradicts the law of conservation of momentum. Galileo did not have the notion of a closed physical system; he did not find it strange that in a free fall the total momentum is increasing. Only Descartes, who abandoned the description of motion in a vacuum and returned to motion in a medium, was able to identify the source from which the falling body obtains its increasing momentum. In the Galilean system the source of the increasing momentum of the falling body remained a mystery.

[23] Newton introduced forces acting at a distance and we have a tendency to see Galileo's law of free fall against the background of the Newtonian gravitational force. But Galileo had no forces acting at a distance, and he would reject them as occultism. Thus Descartes' criticism of Galilean physics is fully justified and fatal. Newton accepted Descartes' argument that a body moving with acceleration must be subject of action of a force. The only difference is that Newton did not require for this action a material carrier. According to Newton, the acceleration of free fall is the result of action, just like according to Descartes. The only difference is that Newton described this action by means of forces that spread through empty space without any physical mediation, which Descartes did not think possible.

his imagination he could envision a bird flapping its wings in a vacuum or a body falling there with acceleration. But according to Descartes, such motions are absurd; they are 'phenomenon without any causal substratum'. We are able to imagine them, but they cannot exist. Thus the entire Galilean project is mistaken. Descartes realized that the scientific description of a particular phenomenon must take into account the ontological basis that determines that phenomenon. It is not possible to restrict science to the phenomena, as Galileo was trying to do. Now we understand what Descartes meant when he criticized Galileo by saying that "*he has thereby built without foundations*".

Galileo's mistake is systematic. He tried to abstract from the influences that disturb the motion, while still supposing that the circular shape of the inertial motion or the acceleration of the free fall would remain. But both these effects are consequences of interaction. As soon as we eliminate interaction, the circular character of the inertial motion as well as the accelerated character of the free fall vanishes. What remains is uniform motion in a straight line.

### 2.1.2 Mathematization of the ontological basis of the phenomena

From the Galilean notion of motion as a geometric flow it is necessary to move to a ***dynamic notion of motion as a state***.[24] According to Descartes, motion is not a process but a state; it is not an activity but passivity. Descartes thus introduces a radically new kind of ontology when he declares that the essence of the world is extension and motion. The importance of this change cannot be overestimated. Aristotle in the *Posterior Analytics* asserted that mathematics cannot be used in scientific explanation of natural phenomena. He based his view on the argument that a scientific explanation must be a causal one, i.e. it must be based on the causes which actually determine the phenomenon explained. According to Aristotle mathematics is unable to provide causal explanations. In their descriptions of nature mathematicians use a system of abstract constructions, as for instance the epicycles and deferents in the Ptolemaic system. These abstract constructions do not exist in reality, and therefore cannot be the causes of the studied phenomena. It would be absurd to maintain that the epicycles and deferents are the causes of the retrograde motion of the planets. They only describe that motion, but they cannot cause it. In other words, mathematics is suitable only for the description of phenomena, but it is unable to deal with the real causes that determine them. According to Aristotle, the material cause, i.e. the matter, out of which the particular bodies are made, is one of the causes of each phenomenon. Only an explanation taking into account the material substance can be a causal one, i.e. a scientific

[24] Modern logic does not discriminate between *states* and *attributes*. ***State*** is a physical analogy of the geometric concept of ***position***. Modern logic originated from Frege's analysis of arithmetic and numbers have no relative positions. A geometric figure is characterized by the fact that it can be placed in different places. Thus position in geometry serves individualization, an aspect which the language of arithmetic does not have. In geometry an object is not determined by its attributes—besides of them we must specify its position, which is not a characteristic of the object itself. Further equivocalities of the objects of the language of geometry are ***size*** and ***orientation***. These three aspects refer to three groups—the group of congruencies, the group of scalings and the group of mirror symmetries. The choice of position is the choice of a representative with respect to the group of congruencies, the choice of size is the choice of a representative with respect to the group of scalings and the choice of orientation is a choice of representative with respect to the group of mirror symmetries. To speak means to introduce differences. To speak the language of geometry means to interfere with the symmetries of this language and into the homogeneous, isotropic and scaling invariant background introduce signs that destroy its symmetry and create the ***here***. The language of physics goes further when it introduces the notion of state. State resembles position, but it is not restricted to the group of geometrical transformations, but to the Galileo group. Besides here it can say also ***now***.

explanation of the particular phenomenon. Mathematical abstractions are unable to offer causal explanations.

Galileo yielded to this Aristotelian argument. What he aimed at in his physics was a purely mathematical description of phenomena and he completely gave up the ambition of offering explanations of their causes. In this way he accepted the role Aristotle had allotted to mathematics. He was probably convinced that science can do no more than offer a precise mathematical description of the studied phenomena. Descartes did not shrink from the Aristotelian challenge. On the contrary, he welcomed it. According to Descartes a mathematical explanation of phenomena is possible, because the mathematical form, i.e. extension, is the ontological basis of nature. Therefore a mathematical description of the phenomena is the description of the causal basis of the world and a mathematical explanation is a causal explanation. In other words, Descartes raised the geometric form to the ontological level; he ***converted mathematical form into physical substance***. Mathematics does not abstract anything, as Aristotle believed. It grasps the ontological essence of things, because extension and motion form the ontological essence of bodies. Thus, according to Descartes, not only the particular physical quantities are mathematical. The ontological basis of the physical world is mathematical as well. Descartes moved from the Galilean idealization of the particular physical quantities to the idealization of the ontological foundation of the world.

We have thus reached a deeper understanding of the sense in which Cartesian physics is mathematical. It is not mathematical in the superficial sense in which Galilean physics can be called mathematical. For Descartes, mathematics is not just a language we can use to describe nature. According to Descartes all that exists is extension and motion and thus the mathematical description of extension and motion is a causal description of the world. The fact that Descartes formulated it with the help of ordinary language shows his deep understanding of the possibilities of contemporary mathematics. In comparison to Descartes' system, the Galilean theory of the universe with its search for geometric harmony of circular trajectories is a naive overestimation of the possibilities of geometry. Descartes understood clearly that we have to give up the ancients' preference for geometry. We have to give up the search for order in the universe, both in the form of a system of natural places as in Aristotle, and in the form of a system of circular motions as in Galileo. Galileo's views considering triangles and circles to be the letters in which the book of nature is written are naive. It is not only that the book of nature is written in the language of mathematics, but nature itself is embodied mathematics. Therefore the question of the applicability of mathematics in the description of natural phenomena is according to Descartes meaningless. ***We do not apply mathematics to nature; nature itself is mathematical***.

The transition from the epistemological to the ontological use of mathematics is closely connected with the rise of modern algebraic symbolism. It is the algebraic symbolism that makes the creation of a universal description of nature possible. From the algebraic point of view it is unimportant what kind of quantity is represented by a variable. A variable $x$ can stand for the length of a geometric line, for the temperature of water, or for the speed of a stone. Descartes was probably the first thinker who clearly realized the possibilities of this new symbolic language. Algebraic language enables us to move from the description of appearances to the description of the universal relations that constitute them. On this deeper level, where the objects are stripped of their accidental qualities (where an object is just an $X$, i.e. something capable of entering into relations with other objects), the world can be mathematized. To this deeper level, disclosed by the language of algebra, Descartes ascribed an ontological status.

Thus Cartesian physics is more algebraic than geometrical. It is the Galilean theory that is geometrical. Galileo conceptualized motion as a geometric flow along a trajectory; and the trajectories of all bodies are ordered to form the geometrical harmony of the heliocentric system. Thus it can be said that Galileo only replaced the Aristotelian geometrical hierarchy of places by a similarly geometrical hierarchy of trajectories. The Sun is in the center of the universe, around which the planets move in a neat geometrical order. On the one hand it is an important step forward, because in the Galilean system motion ceases to be a mere disturbance of order. In contrast to Aristotle, Galileo sees motion as a constitutive element of the order of the universe. But on the other hand the order of the universe is still conceived as a geometrical order. Descartes was one of the first scientists for whom the universe was not a geometrically ordered system of trajectories, but a dynamic system of interacting bodies.

### 2.1.3 Descartes' law of conservation of momentum as the first universal law

Descartes formulated a law, according to which the total quantity of motion in the universe is constant.[25] In *Principia philosophiae* he gave a theological justification of this principle: "*it is most in harmony with the reason for us to think that merely from the fact that God moved the parts of matter in different ways when he first created them, and now conserves the totality of that matter in the same way and with the same laws with which he created them earlier, he always conserves the same amount of motion in it.*" (Descartes 1644, part II sec. 36). Descartes' notion of the quantity of motion is in many respects close to the modern concept of momentum. Of course, Descartes did not have the concept of mass which enters into our definition of the momentum, and used the notion of the size of the body instead. Nevertheless, it might be argued that in his system the size of the body is equivalent to its mass, because his geometrical substance has "constant density". Another peculiarity of Descartes' quantity of motion is its scalar character. In spite of this some historians tend to substitute our expression *mv* for the Cartesian term *quantity of motion*. Thus for instance Martial Gueroult writes: "*The characteristic of these forces... is that they... can be calculated at each instant for each body, according to the formula mv*" (Gueroult 1980, p. 198). This may seem questionable, because Descartes did not have the notion of mass; therefore the use of the symbol $m$ is unjustified. But the transcription of Descartes' views into modern formalism may help us to understand more clearly what Descartes was actually doing.

Other historians object to such interpretations. For instance Daniel Garber writes: "*It is important here not to read into Descartes' conservation principle the modern notion of momentum, mass times velocity. First of all, Descartes and his contemporaries did not have a notion of mass independent of size... What is conserved is size times speed simpliciter, so that when a body reflects, and changes its direction, then as long as there is no change in its speed, there is no change in the quantity of motion. Descartes' conservation principle was extremely influential on later physicists... Unfortunately, the law turned out to be radically wrong.*" (Garber 1992b, p. 313-314). Nevertheless, this view can be challenged, as well. Descartes' theory can be reconstructed

[25] An interesting interpretation of the origins of Descartes' law of conservation of the quantity of motion can be found in a paper of Alan Gabbey (Gabbey 1985, pp. 38-41). Beeckman has noticed that in the collisions of bodies their motion is slowing down—the faster body loses some of its velocity. Therefore he asked why we do not observe in the universe a universal rigidity. This problem of the mechanical universe resembles the problem of the thermal death of the universe, which emerged in the 19th century. It is not excluded that Descartes came to the discovery of the law of conservation of the quantity of motion just when he ruminated about this issue.

on different levels of complexity: the level of idealization, the level of representation, or the level of objectivization. The replacement of the size of the body by its mass in the definition of the quantity of motion, accomplished by Newton, is an objectivization—the objectivization of density of matter. Another correction of the Cartesian concept of the quantity of motion, introducing its vector character, which was accomplished by Huyghens, is similarly only an objectivization. The fact that these corrections can be achieved at the level of objectivizations indicates that at the level of idealization, which is the topic of this paper, Garber's objections can be omitted. At this level Descartes' principle of conservation of quantity of motion is the first conservation law in the history of science. This principle is the precursor of a series of similar laws of conservation. But even more important is the fact that it was the first example of a ***universal law***. It is not a law describing only a particular phenomenon, as were the Galilean laws. Descartes' law does not split the universe into an infinite number of isolated regularities. On the contrary, this law grasps the unity of the world, describing an aspect of the world which unites the world into a whole—the whole having an invariant quantity of motion. This law cannot be derived from experience, because it is impossible to measure the quantity of motion of the whole universe. Despite this, Descartes asserted that it was invariant.

### 2.1.4 Descartes' description of interaction as a collision

Descartes replaced Galileo's notion of motion as an ideal flow along a trajectory by the notion of motion as a state. This enabled him to pose the fundamental question of how to describe the changes of this state. The changes of the state result from interactions. The law of the conservation of the quantity of motion required that these interactions consist in transmissions of momentum from one body to another. Thus Descartes radically changed the picture of the world presented by Galileo. The Galilean universe was a kinematic universe: it was an ordered system of inertial circular motions. Galileo lacked any notion of interaction between bodies. In contrast to this, the Cartesian is a dynamic universe, a universe of bodies in perpetual interaction. According to Descartes, the interactions between bodies have the character of ***collisions***. The first and the second laws of Cartesian physics say that bodies remain in the state of rest or uniform rectilinear motion as long as possible. For a body it can become impossible to preserve its state in two following ways: either two bodies are heading for the same place, or one body is already at rest at a place towards which another body is heading. Then a collision is inevitable. For Descartes collision is the paradigmatic kind of interaction and he describes it by his third law: "*When a moving body comes upon another, if it has less force for proceeding in a straight line than the other has to resist it, then it is deflected in another direction, and retaining its motion, changes only its determination. But if it has more, then it moves the other body with it, and gives the other as much of its motion as it itself loses.*" (Descartes 1644, part II, sec. 40).

In his theory of collision Descartes introduced the ***notion of force***: "*Here we must carefully note that the force each body has to act on another or to resist the action of another consists in this one thing, that each and every thing tends, insofar as it can to remain in the same state in which it is, in accordance with the law posited in the first place.. . . That which is at rest has some force for remaining at rest, and as a consequence has some force for resisting all those things which can change that; that which moves has some force for preserving in its motion, that is, in a motion with the same speed and toward the same direction.*" (Descartes 1644, part II, sec. 43). Descartes' notion of force is remarkable because his forces are entirely passive; their purpose is only to preserve the state. Thus in contrast to the Newtonian system, the Cartesian forces are not forces of interaction; they are not forces by means of which one body would act upon the other. The Cartesian force is an inertial force, preserving the state of the body. From the metaphysical foundations of the Cartesian system it follows that it is God who, because of his immutability, preserves a constant quantity of motion in the universe. And it is the immutability of God that is the reason why the Cartesian forces cannot be active. God does not interfere with the world; he only preserves the world as it was at the moment of creation. Even though God is the source of the inertial forces, he is not affected by them. Therefore, in Descartes' philosophy, the forces have a very complex ontological status. With respect to God they are the consequences of his immutability, with respect to the world they are its modes. In a letter to More from 1649 Descartes wrote: "*Moving force is the force of God Himself conserving as much displacement in matter as He put in it at the first moment of creation ... And this force in created substance is its mode, but it is not a mode in God; but this being somewhat above the understanding of the common run of mind, I have not wanted to deal with the question in my writings so as not to seem to support the opinion of those who consider God as a world-soul united to matter.*" (Gueroult 1980, p. 199). Thus Descartes' universe is opened to the action of God. God can act upon the world without being affected by it. Therefore, in the Cartesian system, the law of action and reaction does not hold, because the forces act only in one direction, from God towards the world. In contrast to this, in the Newtonian system God is not the origin of the forces, he only assures their passage through empty space. Newtonian forces themselves belong to the world, they are forces of interactions between bodies. Thus, while in the Cartesian system forces originate in God and act in the world, the Newtonian forces both originate in and act on bodies.

**2.1.4.a A formal reconstruction of Descartes' theory of collisions** In the literature we can find several proposals for a formal reconstruction of the Cartesian collision rules (see e.g. Gabbey 1980, Garber 1992a, or Coehlo 2002). The aim of these reconstructions is to help us to understand what Descartes was actually doing when he formulated his rules. They differ by the degree to which they adhere to the precise wording of the Cartesian system and in the extent they rely on modern formalism. The outcome of the reconstructions is a more detailed and differentiated assessment of the relation of the Cartesian rules to the actual behavior of solid bodies in collision. In presenting a new reconstruction, we do not mean to question the main results of the previous ones. Our aim is rather to pose a new question. We intend to reconstruct not the empirical content of the Cartesian laws (i.e. how far they agree with the facts about collision) but rather their formal relation to the Newtonian system (i.e. how far they agree with the Newtonian description of the collisions). Thus we ***reconstruct not the content of one system, but the relation between two systems***. In this way we hope to be able to give some meaning also to laws that are empirically incorrect, and could not, therefore, be properly dealt with by the usual methods of reconstruction. The techniques used in our reconstructions are known as *perturbation theory*. They were developed in the 19th century in astronomy and played an important role in the development of various areas of quantum mechanics. Their original purpose was to transfer knowledge about the behavior of the solutions of a dynamic system, which can be handled by analytic means, to a system that is not analytically solvable, but which is in some respects close to the original system. Our aim is to use perturbation theory in the reconstruction of the relation between scientific theories. We have used perturbation theory in (Kvasz 1999), and here we would like to use it in the reconstruction of the Cartesian theory of collision.

Descartes described the collisions of bodies by his seven rules. From the point of view of contemporary physics these rules look rather strange. Descartes' fourth rule, for instance, says: "*If body C were entirely at rest, and were just a bit larger than B, then whatever the speed with which B moved toward C, it would never move C, but would be repelled by it in the opposite direction, since a resting body resists a greater speed more than it does a smaller one, and this in proportion to the excess of the one over the other. And therefore there would always be a greater force in C to resist, than there would be in B to impel.*" (Descartes 1644, part II, sec. 49). We will try to give an epistemological reconstruction of two of the Cartesian collision rules. Our approach differs from the approaches of many renowned historians. Martial Gueroult, for instance, remarked with respect to the above-mentioned quotation: "*This law is false, but we are concerned here not with the scientific truth of Cartesian physics but with the coherence of this physics with the metaphysics which should provide the foundations for it*" (Gueroult 1980, p. 224). In our view an epistemological reconstruction of a theory should not be restricted to the description of its historical context and its internal consistency. A reconstruction should offer more. It should not just show that at the time they were formulated the views of Descartes were meaningful and that they are to some extent internally consistent. We believe that in order to be able to play such an important role in the history of science, the theories of Descartes must have had a ***factually correct core***. That means that there must be a class of phenomena for which a great deal of what Descartes said about motion was correct. Thus we do not defend Descartes as a philosopher, as a creator of internally consistent systems of categories. Our aim is to defend him as a scientist, to show that his views are factually valid for at least a segment of reality.

Therefore we must first of all find a situation, in which the Cartesian collision rules (at least some of them) would be meaningful not just internally, i.e. from the point of view of the Cartesian system itself, but also from the point of view of the Newtonian mechanics. Then we must find a parameter of the Newtonian theory which, when decreased to the limit zero, yields a system behaving in accordance with the Cartesian rules. It seems that for the Cartesian theory, such a parameter could be the ratio of the masses of the colliding bodies. Thus we would like to show that Descartes' theory is a factually correct theory of collision of bodies with enormously different masses.

Let us first take the collision of a ***light body B moving towards a heavy body C***, which is at rest. We will describe this collision using the formulas of the Newtonian mechanics, expressing the laws of conservation of momentum and of energy:

$$m_B.v_B = m_B.V_B + m_C.V_C\,,$$

$$\frac{1}{2}m_B.v_B^2 = \frac{1}{2}m_B.V_B^2 + \frac{1}{2}m_C.V_C^2\,.$$

In these equations we consider the masses $m_B$ and $m_C$ of the colliding bodies, as well as the velocity $v_B$ of the body $B$ before the collision to be known, and our task is to determine the velocities $V_B$ and$V_C$ after the collision. After elementary transformations we obtain the formulas:

$$V_B = v_B\frac{m_B - m_C}{m_B + m_C} \qquad\qquad V_C = v_B\frac{2m_B}{m_B + m_C}\,. \tag{2.1}$$

This result contradicts the Cartesian assertion, according to which the body $C$ will preserve its state of rest after the collision. The velocity $V_C$ is a positive quantity and so Descartes' assertion is wrong. Nevertheless, let us divide the numerators as well as the denominators in the formulas (2.1) by $m_C$ and let us then expand the resulting expressions into a series according to $\frac{m_B}{m_C}$. We obtain:

$$V_B = v_B \left[ -1 + 2\frac{m_B}{m_C} - + \cdots \right] \qquad V_C = v_B \left[ 2\frac{m_B}{m_C} - 2\left(\frac{m_B}{m_C}\right)^2 + - \cdots \right].$$

If we now take the limit $\frac{m_B}{m_C} \to 0$, we will obtain ***precisely what Descartes asserted***. The body $C$ will remain at rest, because in the limit case $V_C$ will equal zero. On the other hand, the body B will rebound, because there is the factor $-1$ in the formula for $V_B$, and this factor indicates that the velocity $V_B$ of the body $B$ after the collision will be equal to $-v_B$. That means that the body $B$ will rebound with the same velocity $v_B$ and will move in the opposite direction. This shows that even if in the general case the Cartesian assertion is wrong, (for finite values of the ratio $\frac{m_B}{m_C}$ the resulting speed of the body $C$ will be positive), in the limit case (the ratio $\frac{m_B}{m_C}$ approaching zero) the system behaves in accordance with the Cartesian theory. Thus the Cartesian theory of motion is more than just a consistent philosophical system. It is a scientific theory, because at least for a small segment of reality it really holds.

Let us now consider the situation, in which the ***heavy body B is moving towards the light body C***, which is at rest. The solutions (2.1) are valid, because in the Newtonian system the equations are the same regardless of the masses of the bodies. Nevertheless, what changes is the parameter according to which we expand our solutions into infinite series. It cannot be the ratio $\frac{m_B}{m_C}$, because now the body $B$ is heavier, and so this ratio is greater that 1. Instead, we have to choose $\frac{m_C}{m_B}$, and by transformations similar to the above ones we obtain:

$$V_B = v_B \left[ 1 - 2\frac{m_C}{m_B} + - \cdots \right] \qquad V_C = v_B \left[ 2 - 2\frac{m_C}{m_B} + - \cdots \right].$$

We can see that there is a fundamental change in the behavior of the body $C$ in comparison to the previous case. While in the previous case the body $C$ stayed at rest, now, in the limit $\frac{m_C}{m_B} \to 0$ we obtain a nonzero velocity for both bodies. Thus both bodies would move in the same direction, precisely as Descartes asserted. Nevertheless, there is a difference between our result and Descartes' prediction. Descartes thought that after the collision both bodies would move with the same velocity, while our result indicates that the lighter body $C$ will move with double velocity compared to the heavy body $B$. This difference draws our attention to another peculiarity of the Cartesian theory of impact. While in the first case which we have reconstructed, Descartes described the collision as elastic, in the second analyzed case he described the collision as totally inelastic. Thus in order to obtain precisely what Descartes said, it would be necessary to introduce an additional term, expressing the energy loss into the second case. This would make our formulas a little bit more cumbersome, but our general result would not change. Thus we can consider the ***asymptotic validity of the Cartesian theory*** to be established.

Our reconstruction of Cartesian physics is in a sense a middle position between its assessment by positivist historiography and its philosophical reconstruction. According to the positivist historians, if Cartesian physics was a scientific theory, it must have been an inductive generalization of the empirical data. But as Descartes' collision laws are obviously wrong, they cannot be obtained in that way, and therefore Cartesian theory cannot be a scientific theory. That is why most positivist historians of science do not even discuss it. On the other hand the historians of philosophy tend to consider Descartes' physics to be a purely metaphysical system, and so they restrict their own task to showing its internal coherence. They usually do not even formulate the question of its empirical validity. Thus both these interpretations agree in ignoring Descartes' scientific aspirations.

But Descartes' physics is more than just a coherent conceptual system. It is related to reality, but this relation is not as direct as positivists would like it to be. Thus in our reconstruction we are not giving up the question of the empirical validity of the Cartesian system. Nevertheless, we analyze this question not via a direct confrontation of the theory with the experimental data. Rather, we confront Descartes' theory with reality in an indirect way, using its formal reconstruction in the framework of Newtonian physics. Thus we accept Newtonian physics as a true representation of reality and confront the Cartesian physics only with this Newtonian representation. While the positivists use the correspondence theory of truth, and while philosophers like Daniel Garber use the coherence theory of truth, our approach is based on a combination of the two. In the case of the Newtonian theory, which we used in our reconstructions, we adhere to the classical correspondence theory of truth. But in the case of the Cartesian theory we test only its coherence with the Newtonian system. Therefore our result is that ***Cartesian physics is coherent with a theory that corresponds to reality***. Thus the Cartesian theory is true in a stronger sense than the philosophical reconstructions based on the coherence theory of truth can provide. On the other hand our approach protects Descartes against the strict verdict of the positivist historiography, because it does not require a direct correspondence to reality.

### 2.1.4.b A conceptual comparison of Descartes' theory of collisions with Newton's theory

Our interpretation of Descartes' theory of collisions will be based on the paper *Force and momentum in the Seventeenth Century: Descartes and Newton*, in which Alan Gabbey used in the interpretation of Descartes' physics the formalism of Newtonian mechanics. Such an approach may seem unjustified, and Gabbey himself felt the need to justify it by saying, "*I do not read Descartes through Newtonian glasses, how someone may believe ... I use rather Newtonian mirror to properly display the essential aspects of Descartes' theory of collision, and thus clarify our understanding of this theory.*" (Gabbey 1980, p. 314, note 169). We will go beyond such a use of Newton's theory as a means for 'mirroring' Descartes' theory. In our view, in the epistemological reconstruction of the development of pre-Newtonian physics Newton's theory has to play a much more fundamental role. We see the birth of Newtonian physics first of all as the birth of a new language having a new syntax. Therefore, our aim is *not* to compare ***what Descartes said*** about motion with what Newton said about the same topic. We are interested in comparing the way ***how Descartes speaks*** about motion with the 'Newtonian syntax'. We consider Descartes a linguistic innovator. The fact that some of his formulations turned out to be false is from our perspective of secondary importance. The important point is that in formulating these views he fundamentally changed the language by means of which we describe nature. Thus we are interested here in a ***comparison of the language of Cartesian physics with the Newtonian language***. We are not going to judge Descartes' views from the viewpoint of the "*Newtonian truth*". We rather analyze Descartes' language from the viewpoint of the "*Newtonian syntax*".

If we want to understand Descartes' theory of collisions, we must turn to the letter of 17

February 1645 to Clerselier, where he wrote: "*My reason for saying that a body without motion can never be moved by another which is smaller, with whatever speed it might move, is that it is a law of nature that a body which moves another must have more force to move it than the other has to resist it. But this excess can depend only on its size; for the one without motion has as many degrees of resistance as the other, which is moving, has of speed. The reason being that if it is moved by a body moving twice as fast as another, it must receive from it twice as much motion; but it resists twice as much this double quantity of motion. For example, B cannot push C except it move it as fast as it itself would move after having pushed it: that is, if B is to C as 5 to 4, of 9 degrees of motion in B it will have to transfer 4 of them to C to make it go as fast as itself: which for it is easy, for it has the force to transfer up to 4 1/2 (that is the half of all it has), rather than be reflected in the opposite direction. But if B is to C as 4 to 5, B cannot move C, except it transfer 5 of these nine degrees, which is more than half of what it has, and against which C consequently resists more than B has the force to act; that is why B must be reflected in the opposite direction rather than not move C*." (Gabbey 1980, p. 269).

Descartes describes here a collision as ***an event happening in a single moment of time***. Bodies collide and at the moment of their collision it is decided which force will prevail: whether the motive force of the body $B$, or the resisting force of the body $C$. As a result of the understanding of motion as state, every body strives to preserve its state as long as possible. The state is preserved by means of the forces of inertia—the motive force in the moving body or the resisting force in the resting body.

Descartes introduced the *force of inertia* that preserves the motion of the body $B$ that moves with the velocity $V_B$ (i.e. the ***motive force***) simply as the product of the size of the body and its velocity. In the case of a body at rest this definition is useless, because it would always give the value zero. Therefore, for a body at rest Descartes gave a different definition of the force of inertia, according to which the *force of inertia* of a resting body to resist an attempt to set it into motion (i.e. the ***resisting force***) is equal to the product of the size of the body and the speed at which it would move after the collision. Thus according to the first definition, the force of inertia is equal to the ***total momentum*** of the body, while according to the second it is equal to the ***change of momentum***.[26] These definitions describe the situation where the bodies are 'isolated'. When they collide, the situation becomes more complex. Gabbey writes: "*The supporting arguments Descartes provides in this passage, are of greater significance than assessments, based on the comparatively mundane grounds that the rule is empirically absurd*." (Gabbey 1980, p. 269).

Gabbey's interpretation of Descartes' theory can be summarized as follows: If the body $B$ moving at the velocity $V_B$ manages to move the resting body $C$, so after the collision they will move together with the velocity $\frac{B \times V_B}{B+C}$.[27] This result is a consequence of the conservation of momentum and it is valid also in the Newtonian theory. The quantities of motion of the bodies

[26] One of Newton's fundamental innovations was that he unified these two definitions and put force ***proportional to the change of momentum***. It resembles Newton's unification of Descartes' first two laws into a single law of inertia by introducing velocity as a vector. Moreover, Newton describes interaction not as a singular event but as a process filling an interval of time, which allowed him to define force as the ***velocity of the change of momentum***.

[27] In the previous chapter we used Newtonian physics to reconstruct some aspects of Descartes' theory of collisions. Therefore, instead of the size of a body, which Descartes refers to by the symbol $B$, we used Newton's symbol for mass $m_B$. After we have clarified, in the language of Newtonian physics, the content of Descartes' theory, we can return to the use of Descartes' symbolism. The reader used to Newton's symbolism can always replace the symbol $B$ by its Newtonian translation $m_B$.

after the collision will be $\frac{B^2 \times V_B}{B+C}$ and $\frac{C \times B \times V_B}{B+C}$ respectively, because the quantity of motion is the product of velocity and the size of the body. Until now we have been in agreement also with Newton. But now a peculiarity of Descartes' theory enters the picture. The body $C$, which was initially at rest, resists the change of its state and the adoption of the above specified quantity of motion. Its resisting force is $\frac{C \times B \times V_B}{B+C}$. This force of inertia of the resting body $C$ must be overcome by the force of inertia of the moving body $B$, which keeps it in motion. The total quantity of motion of the moving body $B$ before the collision was $B \times V_B$. But a portion of this quantity, equal to $\frac{C \times B \times V_B}{B+C}$, must the body $B$ transfer to the body $C$ and it can retain only the rest, namely $\frac{B^2 \times V_B}{B+C}$. At the moment of the collision, it will be decided whether the body $B$ can move the resting body $C$ or it just rebounds from it. The outcome of the collision depends on whether the force of inertia of the moving body $B$will exceed the resisting force of the body $C$ against the adoption of the particular quantity of motion. Descartes thus describes collision as a process taking place on two levels. On one level occurs the transfer of the quantity of motion $\frac{C \times B \times V_B}{B+C}$ from the moving body $B$ to the resting body of $C$.[28] (This transfer occurs also according to Newton, even if according to him it is mediated by forces.) But in addition to this transfer of momentum, there is the process of deciding whether the transfer of momentum will take place at all. In the process of decision, the motive force of the moving body $B$ stands against the resisting force of the resting body $C$. The body $C$ will be moved if its resisting force $\frac{C \times B \times V_B}{B+C}$ is smaller than the motive force $\frac{B^2 \times V_B}{B+C}$ of the moving body $B$. After simple cancellations we get Descartes' condition $C < B$.

This derivation is interesting because Descartes views collision as a ***transfer of momentum*** and describes it ***by means of forces***, i.e. just like Newton. However, the two levels of description—the transfer of momentum and the action of forces—are separated. The transfer of momentum happens somehow spontaneously, the momentum simply passes from the body $B$ to the body $C$. The forces do not enter into the transfer of momentum; they act only in deciding whether the transfer will happen at all. The reason is that Descartes' forces are not forces of interaction, but they are forces of inertia by means of which bodies preserve their states. Gabbey characterized the Cartesian theory of collision as the ***contestant view of force***. He writes: "*Descartes' assumption that the key to solving a collision problem lies in setting the motive forces of one body against the resisting forces of the other, and calculating the excess, on which depends the retardation of the acting body, or acceleration of the recipient. Accordingly, there will be an exchange of motion when the motive force exceeds the resisting force*." (Gabbey 1980, p. 246). Forces are not associated with the change of state (as in Newton), but with preserving it. Comparing the forces we can find out which of them will prevail and determine the outcome of the collision. If the motive force prevails, the bodies will move together. If the resisting force prevails, the body that was at rest will remain at rest, and the moving body will bounce.

On Descartes' theory of interaction it is fascinating to see how close he got to Newton. We can say that Descartes had all the ingredients from which Newton would later build his equation of motion. These ingredients were, however, in Descartes' theory put together in a rather different

[28] God preserves in the universe a constant quantity of motion, and it is irrelevant, whether he preserves it in the body $B$, or in the body $C$. From the point of view of the law of conservation of the quantity of motion, the transference of a certain quantity of motion from the body $B$ to the body $C$ is unproblematic. The only problem is that the body $C$ resists the acceptance of the particular quantity of motion.

way. Descartes had the ***concept of interaction***, but he described interaction as a singular event and not as a process. He had the ***concept of force***, but his forces were only preserving the states of the bodies and not changing them. He understood interaction as a ***transfer of momentum***, but this transfer was separated from the action of forces. Thus all major components of the Newtonian theory of interaction can be found already in Descartes, although they did not fit together in the way they were later combined by Newton. This is the reason why, if we want to understand Newton, we must first analyze Descartes. Newton's contribution to physics was neither the *concept of interaction*, nor the *concept of force*, or the interpretation of *interaction as a transfer of momentum*. All these ideas were already in Descartes. Newton's most important contribution was the ***linguistic framework*** that combined these Cartesian notions (after their corrections and refinement) into the formal syntax mechanics.

### 2.1.5 Descartes' theory of gravity and the vortex of fine matter

One of the main achievements of Galilean physics was its description of the free fall. According to the law of free fall discovered by Galileo, all bodies on the surface of the Earth fall by uniformly accelerated motion with acceleration constant for all bodies. Galileo did not ask the question where the acceleration of the falling bodies comes from. The falling bodies clearly violate the law of the conservation of momentum. According to Descartes the motion of the falling body must be accelerated by something; the body must get its momentum from somewhere. But for Descartes the only mechanism of interaction was an interaction by contact (i.e. push or pull). The agent that confers the additional quantity of motion onto the falling body must itself be in motion (in order to be able to transfer motion it must possess some), and it must be in constant contact with the falling body (so that the acceleration could be the same during the entire motion). Finally, since all bodies on the surface of the Earth are falling, the agent must be omnipresent.

Due to these attributes of free fall, it is natural to conclude that the Earth is in the center of a huge vortex of invisible matter which presses all bodies down to the surface of the Earth. This invisible matter must be itself in motion (so that it could pass portions of its motion to the falling object), and this motion must have a consistent and steady character. So we come to Descartes' vortex of fine matter as an explanatory model by means of which he explained gravity. According to Descartes, the universe is filled with matter; around every celestial body is a vortex similar to that which surrounds the Earth, and thus on every celestial body we would feel gravity just like on the Earth. Descartes' vortex theory is thus a *natural explanation of gravity* as soon as one realizes that gravity needs an explanation, and assumes that this explanation must be based on a contact theory of interaction. Descartes' theory of gravity has the advantage that it integrates the universe into a system of causally interacting bodies.

In the Galilean universe, bodies were isolated by empty space; they were individual objects moving through space without mutually influencing each other. The unity of the universe was, according to Galileo, a kind of harmony of its elements that can be compared to the unity of tones in a chord. It is a mathematical unity rather than a causal one. Galileo would probably dismiss the idea that the planets, for instance Venus, affect the objects on the Earth (including people), as an astrological superstition, as an irrational belief in occult powers. Against the background of the Galilean universe we see the predominance of the Cartesian understanding of the universe as a system of causally interacting bodies. Thus fine matter unites the entire universe into a single, causally connected system of mutually interacting bodies. While this was a purely speculative idea, it has a correct core. Descartes realized that, in addition to the description of interaction of individual bodies by means of collision, he needs a mechanism to unite the parts of the world-system into an integrated whole. As in the previous cases, also here Descartes lacked the mathematical tools by means of which this unification would be accomplished by Newton. Instead of a formal unification Descartes introduced a material one (in the form of vortices of fine matter). But whatever form his causal unification of the universe takes, it was an important step forward.

#### 2.1.6 Descartes' concept of motion as dynamic transition

From the point of view of Cartesian physics we could say that Galileo's theory of motion described only that which was trivial—the motion of a body on which no other body acted. What Galileo understood as motion is actually the temporal evolution of the state of a body in the case when we neglect all other bodies. On the one hand, this is uninteresting, because in reality there is only one kind of such motion—the uniform motion in a straight line. All other examples of Galileo, as they contain no description of interaction, must be mistaken. On the other hand, what Galileo proposed is not physics but an exercise in geometry. According to Descartes, physics must turn from the Galilean concept of motion as a geometric flow to the ***dynamic concept of motion as a change of state***.

Aristotle described motion as a ***geometric transition***, i.e. a transition of the body from an initial place to a terminal one. Between the initial and the terminal places Galileo inserted a trajectory connecting them. In this way he transformed motion into a ***geometric flow***, into a continuous "sliding" along a trajectory. Descartes described interaction as a collision, i.e. as a transition. In contrast to Aristotle's *geometric transition* from the initial to the terminal position, Descartes described motion as a ***dynamic transition***, as a transition from an initial state (state before the collision) to a terminal state (state after the collision). As he did not have the calculus at his disposal, he described this transition using ordinary language. Descartes was among the first thinkers who viewed the universe not as a geometric system of harmonically ordered circular motions, but as a dynamic system of interacting bodies.

### 2.2 Shortcomings of Cartesian Physics

Descartes' physics brought, with respect to Galilean physics, major conceptual advances. It introduced the concept of state, the description of interaction and a new conception of natural law as a universal law describing motion of bodies. Besides these advantages, however, Descartes' physics had a number of serious shortcomings. The shortcomings of the Cartesian system most frequently mentioned in the literature are its verbal formulation and its empirical falsity. We, however, do not consider these two shortcomings as too important. As we already mentioned, the verbal formulation of the Cartesian system can be interpreted as a manifestation of Descartes' insight that mathematics of the first half of the 17th century (to the development of which Descartes substantially contributed) was not suitable for the description of interaction. From this perspective the ***verbal formulation of the Cartesian system is its merit rather than its shortcoming***. It seems that it was the choice of an inappropriate mathematics that prevented Galileo from incorporating interaction into his representation of motion. Triangles and circles are changeless and it is not clear how we can by their means describe interaction. Descartes refused the use of triangles and circles and opted for ordinary language, and it is probable that this decision enabled him to introduce interaction into physics.

As to the empirical falsity of Cartesian physics, our reconstruction of Descartes' theory of collisions showed that ***Descartes' theory is not completely false, it has an empirically correct core***. The fact that positivist philosophy of science was not able to identify this core is a consequence of the insensitivity of the correspondence theory of truth by means of which it approached Descartes. In section 2.1.4.a we have shown that Descartes' theory of collisions—despite its empirical inadequacy—can be put into correlation with reality. Nevertheless, for this we need Newton's theory as a mediator. Despite the fact that according to the correspondence theory of truth Descartes' theory of collisions is false, by combining the correspondence and the coherence theories of truth we showed that ***Cartesian theory is coherent with a theory that corresponds to reality***.

When we refuse the shortcomings that are most often attributed to Descartes, this does not mean that we consider Descartes' theory being perfect. In our opinion, Descartes' theory has a number of conceptual shortcomings, but in order to identify them, we must enter deeper into the Cartesian system.

### 2.2.1 Inability to define rectilinear motion

The first problem of Cartesian physics has to do with this definition of motion. Although Descartes was convinced of the correctness of the Copernican theory, after the condemnation of Galileo he was reluctant to take a stance on this issue. He avoided this problem by his definition motion: motion is "*the transference of one part of matter or of one body, from the vicinity of those bodies immediately contiguous to it and considered as at rest, into the vicinity of others*" (Desecrates 1644, p. 51). Thus we can speak about motion of a body only in relation to bodies that surround it, and the Earth is, of course, relatively to its immediate environment (i.e. the atmosphere) motionless. Descartes writes: "*However, in common usage, all action by which any body travels from one place to another is often also called movement; and in this sense of the term it can be said that the same thing is simultaneously moved and not moved, according to the way we diversely determine its location. From this it follows that no movement is found in the Earth or even in the other Planets; because they are not transported from the vicinity of the heaven immediately contiguous to them, inasmuch as we consider these parts of the heaven to be at rest.*" (Descartes 1644, p. 94). So Descartes used his definition of motion to reject Copernicanism. Earth is motionless because it does not move in relation to the matter that surrounds it.

But Descartes' definition of motion cannot be explained only as a concession to the pressure of the Church, because it is deeply connected with the whole Cartesian system. Among other things, the definition of motion is of fundamental importance for the theory of collisions. A strange aspect of Descartes' description of collisions is that if a lighter body moves towards a heavier one, it bounces and the two bodies will remain separate; on the other hand, when a heavier body moves towards a lighter one, it will set it in motion and both bodies will continue to move together. From the point of view of Newtonian physics this is false, because these two cases differ only in the reference system in which we describe them. First we describe them in the system coupled with the heavier body, then in the system coupled with the lighter one. The choice of the reference system, however, cannot affect the outcome of a collision. Imagine that the Earth is colliding with a small comet, and on the Earth, as well as on the comet there are Cartesians. The Earthly Cartesian sees a small comet moving towards the enormous Earth, so he believes that the comet will bounce. On the other hand, the Cartesian living on the comet sees the huge Earth moving towards him, so he believes that after the collision both bodies will move together. What will actually happen cannot be decided by Cartesian physics.

In the manuscript *De gravitatione* from 1673 Newton criticized Descartes' definition of motion. His criticism was that if motion is defined as "*the transference of one part of matter or of one body, from the vicinity of those bodies immediately contiguous to it*", then it is not possible to define rectilinear motion. If the bodies surrounding the given body move in different directions, some are accelerating, others decelerating, it is not clear what it means that "*all movement is, of itself, along straight lines*", as the second law of Cartesian physics requires. With respect to some bodies, the given body may move in a straight line, while with respect to others it may move along a curve.

### 2.2.2 Separation of the velocity of motion from its direction

Descartes understood velocity as a scalar quantity, and so he separated it from the direction of motion. Therefore, he had to formulate one principle for the conservation of velocity and another principle for the conservation of the direction of motion. The separation of the velocity from the direction, however, had a number of other consequences which could not be solved so easily. For instance, motions in which only the direction changes (as in the bounce of a ball from the wall, or in the uniform circular motion) Descartes did not perceive as changes of state. He described them simply as changes of direction. This is false, because when the ball bounces from the wall, it passes through "*all degrees of slowness*", as Newton will formulate it, thus its velocity is changing as well.

### 2.2.3 Scalar character of Descartes' quantity of motion

Although Descartes formulated the predecessor of the first conservation law—the law of the conservation of momentum—we cannot attribute to him the full credit for this discovery, because his formulation was incorrect. What is conserved is not the scalar quantity of motion, as Descartes believed, but the vector of momentum. An example, in which we can realize the difference between these two formulations, is the system of two bodies that attract each other by gravitational force, and consequently move towards each other with acceleration. In this system the (scalar) quantity of motion increases, because the bodies are moving faster and faster, but the (vector) momentum remains constant, because their motions are in opposite directions so that the increases of momentum of the two bodies cancel each other. This system, of course, is not a counterexample to Cartesian physics, because according to Descartes the increases of the velocity of the bodies are at the expense of the vortex of fine matter, which is the cause of the acceleration. The vortex loses exactly the same quantity of motion which the bodies acquire, so that the law of conservation of the quantity of motion (at least according to Descartes) is not violated. Although this example does not show the 'internal inconsistency' of the Cartesian system, it definitely shows its 'empirical inadequacy'.

### 2.2.4 Speculative character of the Cartesian explanatory models

After introducing the ontological level of description of nature, Cartesian physics split the description of nature into two components. On the one hand, there is the description at the phenomenal level where the goal is to acquire exact, quantitative data about the phenomenon. For example, in the case of refraction of light Descartes created a detailed table that contained the results of thorough measurements of the angle of refraction for various angles of incidence. On the other hand, there is the description on the ontological level where Descartes introduces his explanatory models, the aims of which is to offer a causal explanation of the studied phenomenon. Typical Cartesian explanatory models were, for example, the vortex of fine matter explaining gravity, or the tennis balls explaining refraction of light. These models represent progress in comparison with Galilean physics, which was restricted to a quantitative description of phenomena and did not attempt to explain them causally. On the other hand, many Cartesian models were pure speculations. Descartes, the main advocate of scientific method, lacked any methodical guidance in the construction of his explanatory models. Therefore it is no wonder that many of these models—the model of the vortex of fine matter, or the model of the tennis balls—have turned out to be misguided.[29] Sporadic errors can be tolerated, but such an accumulation of errors indicates a serious problem of the conceptual structure of Cartesian physics.

### 2.2.5 Disconnectedness of the phenomenal and the ontological levels of description

The explanation of gravity by means of the vortex of fine matter is a paradigmatic example of the Cartesian method of explanation by means of a reduction of a phenomenon to its ontological basis. This method, however, has a very loose connection between the phenomenal and the ontological levels of description. In the case of gravity, although we know that it is caused by the vortex, there is no link between the properties of gravity (its magnitude, homogeneity, and direction) and the properties of the vortex (its orientation, direction, and velocity). Descartes postulated his models without any possibility of empirically testing their properties. Probably against this aspect of the Cartesian system were directed Newton's famous words "*Hypotheses non Fingo*".

### 2.2.6 Impossibility to incorporate friction into the description of interaction

Another problem of Cartesian physics is that it is not possible to include friction into its description of motion. Descartes understood interactions as collisions, and he described them by comparing the states of the system before and after the collision using the law of conservation of the quantity of motion. This means that Descartes cannot describe those interactions in the course of which the total quantity of motion of the system is changed—and these are all interactions in which friction plays a role. When a body moves with friction, we can not equate the quantity of motion before and the quantity of motion after a particular period of time (as the Cartesian description requires), because some quantity of motion is lost due to friction. A Cartesian could argue that motion is not lost, but only transferred to the tiny particles of matter. In principle this explanation is correct: friction only changes mechanical motion into molecular motion that we

[29] Even if many of Descartes' explanatory models were wrong, his explanation of the rainbow turned out to be correct.

perceive as heat. Nevertheless, this does not change the fact that motion of a mechanical system with friction cannot be described by means of Cartesian physics.

This deficiency is not a marginal problem—friction touches the very heart of Cartesian physics. In order to explain the circular form of planetary orbits, Descartes filled the universe with fine matter. For the development of physics it was important because Descartes thus achieved the causal interconnectedness of all phenomena. Any body of the Cartesian universe could in principle act on any other body through the mediation of fine matter. Nevertheless, the vortex of fine matter has one major drawback. If the form of the Earth's orbit was really caused by the vortex, the vortex would have to interact strongly with the Earth. In six months, during which the Earth traverses half of its orbit and its velocity takes the opposite direction, the fine matter would have to transfer to it a momentum equal to twice the product of the mass of the Earth and its velocity (that is 2 times $5.97 \times 10^{24}$ kg times $2.98 \times 10^{4}$ m.s$^{-1}$, which gives the unimaginable $3,56 \times 10^{29}$ kg.m.s$^{-1}$). If fine matter interacted with the Earth so intensively, friction would appear which would stop the motion of the Earth.[30]

The same substance, by means of which Descartes explained gravity, had such a disastrous consequence for the kinematics of his universe. In order to cause gravity, fine matter must strongly interact with the macroscopic bodies. But once it starts such interaction, friction appears, due to which after a short period of time all motion ceases. Newton dedicated the second book of the *Principia* to the calculations of various scenarios of motion in an environment. The failure to include friction into his physics was not an oversight of Descartes. Friction cannot be included into the Cartesian system because it violates the law of conservation of the quantity of motion. Newton had to leave the Cartesian world filled with fine matter and return to the universe of bodies moving in a vacuum. Nevertheless, this will be not the Galilean universe of isolated bodies. Newton took over into his system one of Descartes' main achievement—the causal interconnectedness of all phenomena.

### 2.2.7 Contact theory of interaction as collision

Descartes' conceptualized interaction between bodies as a conflict understood as a clash of their tendencies to preserve their state (of motion or of rest). The basic model of interaction was the model of collision, i.e. of an immediate contact. We do not want to say that there are no collisions. Collisions are, however, not the only kind of interaction. Descartes' vortex of fine matter was an attempt to fit also gravity into the framework of his theory of interaction. We saw that Descartes was led to his vortex theory by the assumption that interaction happens through contact. Even if the vortex theory had worked, it was too complicated for a mathematical description. That was why Descartes did not try to mathematize gravity. He believed that Galileo's law of free fall was false, because he could not imagine that a vortex could lead to such a simple law: "*path proportional to the square of time*". But Descartes' theory of gravity was so complicated only thanks to his contact theory of interaction. When Newton admitted forces acting at a distance, gravity became accessible to mathematical description.

---

[30] Since Descartes considered the quantity of motion a scalar quantity, according to Cartesian physics during the Earth's motion around the Sun, the quantity of motion does not change—the Earth changes only the direction of its motion, while its quantity of motion is unchanged. Thus Descartes did not realize this fundamental problem—the necessity to transfer huge momentums between the Earth and the vortex of fine matter.

### 2.2.8 Understanding of interaction as a singular event happening instantaneously

According to Descartes, the motion of a body is composed of sections of uniform rectilinear motion when the body preserves its state of motion, which are separated by collisions, i.e. by *singular events*, during which the body undergoes changes of state. Singular event here means an event that happens at an instant i.e. at a 'point of time'. This has the consequence that the rates of change of momentum as well as the rates of change of all other temporal characteristics of the body are infinite. It is not difficult to see that it was Descartes mathematical apparatus that forced him to understand collisions as singular events. The law of conservation of the quantity of motion is an algebraic equation and it was the language of algebra that did not allow Descartes to describe interaction as a continuous process.

### 2.2.9 Inability to describe a bounded closed mechanical system

A further problem is that the Cartesian law of conservation of the quantity of motion applies only to the entire universe. Cartesian physics is therefore unable to describe not only systems with friction, but actually any physical system that is less than the entire universe. In Cartesian physics, it is not possible to describe the motion of a closed bounded system of bodies, because all bodies are submerged in the vortex of fine matter which constantly interacts with them. Into every bounded system of bodies, the vortex brings or takes away certain quantities of motion. Cartesian physics was therefore unable to define a closed bounded system—the only system that it was able to describe was the entire universe. But the universe as a whole is not accessible to empirical investigation, but only to speculation.

Descartes' system is thus open. First of all, it is open to the work of God who maintains a constant quantity of motion in the universe. Secondly, it is open to the actions of the soul. Descartes' separation of the direction of motion from its quantity, and his limitation of the conservation law solely to the quantity of motion, forms the basis of the Cartesian explanation of the action of the soul on the body. According to Descartes, the body is a hydraulic machine driven by the circulating blood. In addition to the blood vessels, in which the blood flows, the body contains also nerve fibers, which are fine tubes, in which, according to Descartes, a spiritual fluid circulates. Its circulation can affect the muscles by closing or opening the flaps that direct the flow of blood, and so cause muscles to contract or expand. The center of the circulation of the spiritual fluid is, according to Descartes, the pineal gland, into which enter a great number of different nerve fibers from the whole body, including the nerves from the two eye balls. The pineal gland is the place where the contact between the body and the soul occurs. On the one hand, the flow of the spiritual fluid can cause motions of the pineal gland which the soul perceives as sensory perceptions. On the other hand, the soul can interfere with the flow of the spiritual fluid by redirecting it from one nerve fiber to another one. Descartes' law of conservation of the quantity of motion is not violated because the redirecting of the flow of the spiritual fluid does not change the quantity of motion. The erroneous notion of the (scalar) momentum thus allowed him to describe the interaction between the soul and the body. As soon as Newton corrected Descartes' concept of momentum (by turning it into a vector quantity), the physical world became causally closed, and in the physical universe there remained no place for the interference of the soul.

### 2.2.10 Summary

We introduced the shortcomings of Cartesian physics in the order in which Newton gradually eliminated them. The first three of them—the impossibility to define rectilinear motion, the separation of velocity of motion from its direction, and the scalar nature of the quantity of motion—Newton ***explicitly rejected at the very beginning*** of his scientific career, and replaced them with his theory based on the concept of absolute space and time. The next three shortcomings—the speculative character of the explanatory models, the disconnectedness of the phenomenal and the ontological level, and the failure to include friction into the description of interaction—Newton ***overcame gradually*** as he replaced the Cartesian speculative explanatory models by the mathematical description of forces. The last three shortcomings—the contact model of interactions, the interpretation of interaction as an instantaneous event, and the openness of the Cartesian description of nature—Newton ***abandoned only tacitly*** as he developed his system and the particular aspects of the Cartesian description of interaction became redundant. Against the last three aspects of the Cartesian theory Newton never openly objected.

## 3 Newtonian physics

Husserl interpreted idealization as a process in which an aspect of the lifeworld is replaced by a mathematical ideality. Galilean physics replaced phenomena such as speed or heat by mathematical quantities of velocity or temperature. Cartesian physics replaced the objects of lifeworld by extended bodies of the Cartesian universe. In the present chapter we would like to interpret Newtonian physics as a further step in the process of idealization, a step in which action that we encounter in our lifeworld (like pulling, pushing, dragging, etc.) was replaced by action of forces. The Newtonian replacement of the ordinary action by the action of forces can be seen as a continuation of the Cartesian reduction. Even though on the ontological level Descartes abandoned the lifeworld and created his mathematical universe of extended bodies, his understanding of action (as pushing and pulling) remained close to the ordinary notion of action. Pushing and pulling is precisely what we do in our everyday lives. When we write, we push the pen against the paper, and when we want to undo the shoelaces, we simply pull them. Thus Descartes transferred our ordinary notion of action into his mathematical universe of extended bodies. In what follows we would like to show that many aspects of Newtonian physics can be understood as a consequence of the replacement of the Cartesian notion of action based on everyday experience by a new, mathematical notion of action that is absolutely alien to any experience; the action of forces at a distance. In other words ***we will try to interpret Newtonian physics as idealization of action***.[31] It was the mathematical description of action that enabled Newton to complete the process of mathematization of nature, started by Galileo.

The idealization, on which modern physics is based, has thus three layers. The first layer is the Galilean *instrumental idealization of phenomena*. It consists in the replacement of the phenomena of the lifeworld by mathematical quantities, obtained by techniques of measurement. The second layer is the Cartesian *ontological idealization of objects*. It consists in the replacement of the objects of the lifeworld by extended bodies obtained in the process of the ontological reduction of reality. The third layer is the Newtonian ***analytical idealization of action***. It consists in the replacement of the action between objects of the lifeworld by forces acting at a distance. By identifying these three levels of idealization we actually claim that *physical quantities* are not true pictures of the phenomena of the lifeworld. Physical quantities are unambiguous, intersubjective, and reproducible, while the phenomena of the lifeworld contain a subjective dimension, they are often ambiguous and non-reproducible. Similarly, we claim that *physical bodies* are not identical with the objects of the lifeworld. Physical bodies have unambiguous quantitative characteristics that can be combined into the “finitely representable” state,[32] which contains all information about their future. Objects of the lifeworld have qualitative properties, and they are characterized by their purpose rather than state. And finally we claim that *physical interactions*

[31] In the chapter devoted to Galilean physics, we pointed out that Husserl took over the horizon in which the positivists formulated their philosophy of science. The positivists tried to reduce the discussion of science to the analysis of its empirical methods, while ignoring the ontological and causal aspect of scientific theories. In the controversy with positivism, Husserl refuted the positivist interpretation of science, pointing at a discontinuity between the lifeworld phenomena and their scientific description. But because the other two aspects of scientific theories were avoided by positivism, the ontological and the causal aspect of science was avoided also by Husserl. The concept of idealization, which we use here, is thus broader than that introduced by Husserl in the *Krisis*.

[32] In quantum mechanics, the states are given by vectors of an infinite-dimensional Hilbert space. Thus ‘final representability’ is not meant in the sense of final dimension of the representation. But the Hilbert space, despite its infinite dimensions, is given by a finite number of axioms, and so it is manageable by a ‘final, human mind’.

are not identical with the causal relations as we encounter them in the lifeworld. Physical systems are mono-temporal and causally closed. The objects of the lifeworld are changing simultaneously on several time scales and are in a fundamental sense open.

Our aim is to show that by joining these three layers of idealization, that is, by putting together the mathematical description of quantities, of states and of action, Newton created ***an idealized world, by which modern science replaces the world of our ordinary experience***. This replacement is so successful because, besides its own empirical basis, the world of science has its own ontology and its own causality. Thus the world of science is closed not only on the empirical level of facts, but it is closed also on the ontological level of objects and on the causal level of action. Modern science is able to predict not only the outcomes of experiments, but also the existence of new objects.[33]

The universe of physics is separated from the lifeworld by three layers of idealization which together constitute the language modern physics. Modern physics replaces the lifeworld by descriptions of its language. This replacement is so suggestive that many of us believe that we actually live in the world that is described by the scientific theories. The world of physics is operationally, ontologically, and causally closed. *The world of physics is operationally closed*—that means that only those phenomena are physically real which can be reproduced by instrumental procedures; *it is ontologically closed*—that means that physical phenomena are the manifestations of physical objects; and *it is causally closed*—that means that physical objects can interact only with other physical objects. Husserl argued that we must not equate the descriptions of the language of physics with reality, in order not to lose the possibility to understand the process of the creation of that language and to appreciate the difficulties that its creators had to overcome. The aim of the phenomenological criticism of physics is not to challenge the physical picture of the world, but to understand its roots.

A remarkable aspect of Cartesian physics is that Descartes in a sense failed to incorporate interaction into his mathematical description of nature. Cartesian physics contains a tension. In contrast to Galileo, Descartes realized that it is necessary to incorporate interaction between bodies into the physical description of nature. On the other hand, he was not able to create a mathematical description of interaction. Thus in Cartesian physics, when we describe a collision of two bodies, we describe the state of the system before the collision as well as the state of the system after the collision. Nevertheless, what we do not describe is what happens in the very moment of the collision. Therefore, instead of what is changing, that is, instead of the process of the gradual change of state during the interaction, Cartesian physics describes only what is preserved, that is, the quantity of motion. One of the fundamental contributions of Newton to the development of science was the creation of a mathematical description of interaction. Into the gap, which in Cartesian physics separated the state of the system before the collision from the state after the collision, Newton inserted the process of continuous change of state caused by the action of a force. Descartes described a ball's bouncing from the wall as an immediate change of the direction (the technical term was directedness) of the motion of the ball, without a change of

[33] One of the first such prediction was the prediction of the planet Neptune. Physicists understood the perturbations of the motion of Uranus not as mere phenomena requiring an exact description (as a Galilean scientist would proceed), but postulated the existence of a ***causal*** agent, which by means of its gravitational attraction causes these perturbations, and interpreted this agent ***ontologically***, as a celestial body. When astronomers focused their telescopes on the place where, according to the calculations of the physicists, this celestial body should be, they found a new planet there. Since the discovery of Neptune, this scenario has been repeated many times.

its velocity. Newton inserted between the two Cartesian states (representing the ball before and after the bounce) a process of gradual deceleration, until the ball stopped for a moment. Then the forces of resilience started to accelerate its motion as long as the ball reached the same velocity as that with which it hit the wall, but of the opposite direction. Thus, according to Newton, the rebounding ball passes all degrees of velocity reaching zero, while according to Descartes it only turns around without any change of velocity at all. This example illustrates clearly that Descartes excluded the process of gradual deceleration followed by gradual acceleration from the description of collision and he simply connected the initial and the final state by his conservation law. In this way Descartes reduced the bouncing of the ball to a change of its direction.[34]

The creation of a mathematical description of interaction enabled Newton to close the causal network of the physical picture of the world. He replaced the hypothetical fine matter, by means of which the Cartesians tried to embed the physical phenomena into a network of causal relations, by a mathematical description of interaction. This step is in agreement with Husserl's interpretation of idealization as a replacement of an aspect of our lifeworld by a mathematical representation of it. Therefore we suggest the main contribution of Newton to physics was not in his discovery of the law of universal gravitation or of the law of action and reaction, but in the creation of a mathematical language that makes it possible to predict the future behavior of a mechanical system in an analytic way. As long as the discussions of Newton's contribution to physics are restricted to the discussions of his particular discoveries, we are stuck in the positivist framework. Then Newton's fame represents a mystery, because in the case of each particular discovery, which we can ascribe to him, a whole range of predecessors and contemporaries emerge, who require their share in the discovery. And so Newton's exceptional place in the history of science remains unexplained. Nevertheless, as the author of a mathematical description of interaction, Newton stands alone and ahead of his time. In order to appreciate the importance of Newton's work we have to accept that ***Newton's main contribution to science was not an empirical one but a linguistic one***. Newton is the author of the mathematical language which enables us to describe interaction.

### 3.1 Newton's analytical idealization of interaction [35]

Descartes biographer Stephen Gaukroger writes: "*Newton, the success of whose work was largely responsible for the demise of Cartesianism later in the century, was himself a Cartesian in the early 1660s, before he developed his own distinctive natural philosophy.*" (Gaukroger 1995, p. 4). Newton read from Descartes everything that was available, and certainly read *Dioptrics*, *Geometry* and *Principles of Philosophy*, whose topic will appear again and again in his mature work (see Whiteside 1970, p. 72). His main scientific work *Philosophiae naturalis principia*

[34] Cartesian physics has a mathematical ontology but it lacks a mathematical description of interaction. Its verbal formulation is the consequence of its inability to mathematically describe the interaction of mathematical objects.

[35] To call the Newtonian idealization *analytical* may cause some doubts. The term analytical is usually associated with Lagrange, who used it in the title of his treatise on mechanics. Newton called his theory *rational* mechanics. When we use, in connection with Newton, the adjective analytical, we would like to call attention to a remarkable aspect of Newton's physics. At the surface, Newton's *Principia* are synthetic, because they are written in the language of synthetic geometry. But if we examine their structure in more detail, it turns out that *Newton, using the language of synthetic geometry, presented a concept of interaction that is fully analytic*. Newton's mechanic was the first theoretical system, in which the causes and the effects of a mechanical interaction were connected in such a way, that from the effects (e.g. the elliptic form of the planetary trajectories) it was possible to deduce the causes (the inverse square law). The transition from the effects to the causes is typical for the analytic method.

*mathematica* is in many respects a result of critical coming to terms with Descartes' *Principia philosophiae*. Newton took from Descartes several incentives, among the most important of which were understanding motion as a state; describing interaction as a change of state; and the notion of a natural law. Newton based his system on three laws, the same number as Descartes:[36]

**I.** Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.

**II.** A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.

**III.** To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction. (Newton 1687, p. 416-417).

At the beginning of the *Principia* we can find the following definitions:

**I.** *Quantity of matter* is a measure of matter that arises from its density and volume jointly.

**II.** *Quantity of motion* is a measure of motion that arises from the velocity and the quantity of matter jointly.

**III.** *Inherent force* of matter is the power of resisting by which every body, so far as it is able, perseveres in its present state either of resting or of moving uniformly straight forward.

**IV.** *Impressed force* is the action exerted on a body to change its state either of resting or of moving uniformly straight forward. (Newton 1687, p. 403-405).

In these definitions, Newton used Cartesian terminology (quantity of motion, inherent force). Nevertheless, he gives these terms a new meaning that is absolutely alien to Cartesian physics. At the beginning of his career, between the years 1665–1673, Newton held a kind of Cartesian mechanical philosophy, which he gradually abandoned. The process of abandoning the Cartesian philosophy is visible on the development of Newton's notion of inertial force (see Gabbey 1980, p. 273-274).

Newton's reaction to Cartesian physics can be divided into three areas. The first area concerned motion and the concepts of space and time. In this area, Newton rejected Descartes' views as early as in the 1670s. Already in his manuscript *De gravitatione* from 1673 Newton subjected Descartes' identification of matter with extension to severe criticism, as well as his definition of motion, and showed the inconsistency of the Cartesian system. Perhaps the most important argument of Newton was that if we define motion as the replacement of a body with respect to the bodies of its immediate neighborhood, then it is impossible to introduce the notion of rectilinear motion. The manuscript *De gravitatione* is remarkable also because Newton used here for the first time the term *absolute motion.* Thus Newton's idea of the absolute space has its roots in a confrontation with Descartes (see Böhme 1989, and Steinle 1991). In the *Principia* we can find

[36] Newton united the first two laws of Descartes (the law of inertia of motion and the law of conservation of the direction of motion) into his law of inertia. He replaced Descartes' third law by his law of action and reaction. Between these two laws he, finally, inserted his law of force.

the notion of absolute space in an explicit form, so that when he was writing the *Principia*, the Cartesian theory of motion was already rejected.

The second area of Newton's reaction to Cartesian physics concerned methodology. Newton criticized the Cartesians for the speculative nature of their explanatory models. Overcoming this deficiency meant abandoning the Cartesian models and replacing them with theoretical explanations, in which the phenomena are tied to the causes by means of an experimental method so that the causes are no longer speculative. Newton used this method already in *Lectiones Opticae*, a work written at the end of the 1670s (Newton 1729). In the sphere of methodology, however, the surpassing of Cartesianism was not as easy as in the question of motion, because for each Cartesian explanatory model it was necessary to develop a mathematical theory based on experiments.

Newton's third area of reaction to Cartesian physics concerned interaction. Unlike the Cartesian notion of motion, which he rejected right at the beginning of his career, in the description of interaction Newton remained for a long time faithful to the Cartesian contact theory of interaction and emancipated himself from it only gradually. At first, he translated the Cartesian theory of interaction into the language of forces, which made it more precise, but the forces by means of which he described collisions were still the Cartesian contact forces acting in a singular moment of time. As it was pointed out by I. Bernard Cohen, Newton's second law presented in the *Principia* reads as follows: "*A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.*" Force is thus proportional to the change of (the quantity of) motion, thus

$$\boldsymbol{F} = d(m\boldsymbol{v})\,. \tag{3.1}$$

This is something fundamentally different from what we understand as Newton's second law today when we put force equal to the rate of change of momentum:

$$\boldsymbol{F}dt = d(m\boldsymbol{v}) \qquad \text{or} \qquad \boldsymbol{F} = \frac{d(m\boldsymbol{v})}{dt}\,. \tag{3.2}$$

In Newton's formulation of his second law, force is directly proportional to the change of momentum, and *not to the rate of change of momentum*, and it is an *instantaneous force* (Cohen 1970, pp. 144-159). It is the Cartesian conception of action as collision, described in the language of forces.[37]

In addition to the *instantaneous forces*, which act at the moment of collision, Newton has already around 1665 introduced the description of continuous action of forces. It was the action of the centrifugal force in rotational motion, which he described by a sequence of evenly spaced impulses of instantaneous forces, and he approximated the smooth trajectory by an inscribed polygon. In a limit the polygon approaches the smooth trajectory and the impulses of the instantaneous force approach the action of a continuous force (Herivel 1970, p. 125). Later he used also "continuous forces" operating in a time interval (the gravitational force). But for these continuous forces he did not formulate any special law of motion, and kept on describing their action as the limit of a large number of impulses of instantaneous forces. He spread out these instantaneous forces evenly over an interval of time, so that in the limit he obtained from

[37] Newton speaks about proportionality between force and change of motion, so that his formulation does neither contradict equation (3,2), nor does it explicitly assert it.

formulas of the type (3.1) a description of action equivalent to (3.2). Therefore, as Cohen writes: "*Surely there can be no doubt that Newton thus knew and stated explicitly the Second Law of Motion in a form equivalent to* [(3.2)]." (Cohen 1970, p. 157).

It seems that Newton considered for a long time the introduction of a (continuous) force of gravity only ***an effective way of describing*** the gravitational action, the mechanism of which is unknown, but which has the form of collisions with some kind of eather. This eather cannot be the Cartesian fine matter, because Newton knew that the Cartesian model does not work. He believed, however, that we will find a mechanistic explanation of gravitational interaction. So everything that we have said until now (the translation of interaction into the language of instantaneous forces and an exact mathematical formulation of the law describing their action), still belongs to the second area of Newton's reaction to Cartesian physics. Instead of an arbitrary speculative explanation of gravity using a hypothetical fine matter, Newton substituted its quantitative and experimentally controllable description by means of instantaneous forces. Newton passed to the third area when he realized that he will probably never find anything better than the continuous forces, and he turned from viewing the continuous forces only as an effective way of description of some unknown contact interaction to accepting the continuous forces as forces acting at a distance (i.e. forces without any further mechanical explanation).[38]

It is not clear how far towards the adoption of forces acting at a distance Newton actually went. It is obvious that these forces appear in the *Principia* in the description of the planetary motion. Newton, however, did not introduce them into the conceptual foundations of his theory. Newton's *Principia* is built on the concept of instantaneous forces and the continuous forces acting at a distance appear on the scene only incidentally. It may be that he thought that the theory of gravity built on forces acting at a distance would incite opposition, and therefore he introduced in the first pages of his work, where he formulated the laws of motion, only instantaneous forces. However, it is equally possible that he still believed in the possibility of finding a mechanical model of gravitational interaction, which would replace the forces acting at a distance, and so he built the *Principia* solely on instantaneous forces. Fortunately, we can leave this question unanswered and include Newton's description of interaction by means of continuous forces among his achievements.

### 3.1.1 Mathematization of nature as Newton's program

Galilean and Cartesian physics represent two fundamentally different approaches to mathematization of nature. According to Galileo, mathematics is a ***language*** in which the book of nature is written. This is perhaps the most widespread understanding of the role of mathematics in physical sciences. When we open a book on modern physics, we find mathematical language used in the description of phenomena and the derivation of laws. This fact may create the impression that contemporary physics is the continuation of the Galilean mathematization. In the previous chap-

[38] In this respect the fate of Newtonian *gravitational force* is not different from the fate of the *electromagnetic field* (introduced by Faraday as a convenient description of the action of charges and currents), or *Planck's action quantum* (introduced by Planck as a formal trick in the course of his derivation of the law of black body radiation). Just as after Faraday came Maxwell, and after Planck came Einstein, who ascribed the electromagnetic field or the action quantum respectively an ontological status, so after Newton came Euler, who formulated for forces acting at a distance the equation (3.2), which we call Newton's second law. Thus the birth of field theory or of quantum mechanics was in many respects similar to the birth of Newtonian mechanics. An entity, introduced as a convenient tool of description acquired the status of physical reality.

ter we argued that this is not the case. The Galilean program was a blind alley and it soon petered out. Its fundamental fault was that it accepted the role which Aristotle ascribed to mathematics: not to meddle in the ontological interpretation of reality and to restrict itself to the description of the phenomena. The expansion of the program of mathematization to the ontological level was the merit of the Cartesians.

Descartes did not understand mathematics as a mere language useful for the description of reality. For Descartes, the reality itself is mathematical because things are extended bodies. Mathematics thus becomes the ***ontological foundation*** of reality. It does not describe physical phenomena (i.e. phenomena that according to Aristotle have a material, thus for mathematics inaccessible foundation). Mathematics, according to Descartes, represents the states of the physical system and so it comprises all the information needed for its understanding. In this way Descartes terminated a dualism stemming from antiquity, according to which reality is the union of form and matter. According to Descartes a thoroughgoing mathematization of nature is possible, because there is no substance different from form. Mathematical objects—Descartes' extended bodies—represent not only the form but also the ontological foundation of reality. They are form and matter at the same time.

Nevertheless, despite its radical character, the Cartesian program did not succeed. Let us turn to the difficulties which terminated it. The most fundamental among them was Descartes' failure to create a mathematical representation of interaction. The program of mathematization was accomplished by Newton who carried mathematization even further. According to him, mathematics is not only a language suitable for the description of phenomena, as the role of mathematics was understood by Galileo. Neither is it only an ontological foundation of reality, as its role was understood by Descartes. Newton introduced a third approach, according to which mathematics is the ***form of analytical representation*** of reality; the form of representation of time, space and interaction.[39]

### 3.1.2 Transformation of the instrumental practice – instrument as a physical object

Instrumentalization of observation was one of the main achievements of Galilean physics. Instruments such as the telescope or the barometer allow mathematization of various phenomena; the instruments themselves, however, were not subject to mathematical description. At the time of his astronomical discoveries, Galileo did not know the law of refraction. And even if he had known it, he probably could not have built a theory of the telescope, because Galilean physics was not suitable to explain the operation of complex instrumental devices. A comprehensive mathematization of the world, including the instruments and the human body, was introduced only by Cartesian physics. There every object, be it an organ of the human body such as the eye, or a scientific instrument, such as the telescope, was subject to mathematization. This made it possible to interpret observation as an interaction of the observed object with the sensory organ of the human body. So the very act of observation became part of the physical description of nature. Nevertheless, Cartesian physics remained in many respects qualitative and was focused on the construction of qualitative explanatory models of the organs of human body rather than on the development of a quantitative theory of physical instruments. It was only Newton who made the instruments subject of theoretical description. We can fully realize the importance of

[39]For more details concerning the meaning of the phrase 'form of analytical representation of reality' see section 3.2.

this shift on the problem of the analysis of errors of measurement.

In Galilean physics, the errors caused by friction, surface imperfections, or deformations of physical instruments are something that *has to be eliminated, and not studied*. According to Galileo, the smaller the friction, the smoother the surfaces, and the stronger the materials, the closer we get in our observations to the perfect law of nature that describe the ideal processes occurring in a vacuum. Descartes believed that vacuum is not possible, and thus the ideal world described by Galilean physics does not exist. Disturbances can not be eliminated. Bodies are constantly exposed to disturbances from the vortex of fine matter, and these disturbances are so complex, that any mathematical analysis of the errors of measurement is beyond the point.

Only in Newtonian physics, which introduced the concept of a closed physical system, it became possible to develop for different instruments a theory of their operation, and thus to obtain a clearer picture of their reliability. An instrument is a physical system like any other, so it can be studied by standard methods of physics. Newton's incorporation of instruments, and thus also of the process of observation, experiment and measurement, into the physical picture was essential for connecting science and technology. This connection is a feature that distinguishes modern science from its predecessors in antiquity and the early modern period.[40]

### 3.1.3 Newton's analytical notion of experiment and the inductive proofs from phenomena

Galileo's approach to the experimental method was motivated by his effort to find the ideal form of phenomena by means of artificial experimental situations. The paradigmatic example of this approach is the experiment on the inclined plane which led to the discovery of the law of free fall. The acceleration of free fall is a phenomenon known from everyday experience and the task of science is to make this phenomenon accessible to mathematical description. Galileo's notion of experiment can be interpreted as ***synthetic*** in the sense that the mathematical quantity or the mathematical law appear in the final part of the experiment, just as the constructed object appears in the final step of its construction in synthetic geometry. Descartes showed that Galileo's understanding of physics was too narrow. It is necessary to complement the mathematical description of phenomena by a mathematical description of the bodies which cause the phenomena. Nevertheless, in order to be able to offer such causal explanation of the phenomena, Descartes had to introduce several purely speculative entities. For instance, in order to explain gravity he introduced a vortex of fine matter. But he left unanswered the question how we can study this vortex. Thus even if Descartes introduced explanatory models into physics, he did it in a speculative way. The contribution of Newton to the experimental method consisted in finding a way ***of studying empirically the ontological basis of reality***.

Newton developed his new approach to experimental method during his study of colors in 1665–1667 (Hakfoort 1992, p. 115-121). Hooke and Huyghens rejected Newton's theory of colors, because they viewed it against the background of the style of experimental work that was typical for the Cartesian mechanical philosophy. When Cartesian physics is criticized for being

[40] Newton's incorporation of observation into the physical representation of reality changed the nature of physical knowledge—post-Newtonian physics reflects its own process of acquiring knowledge in itself. Philosophy ignored this fact for a long time, and interpreted knowledge as a kind of sensory perception. For example, Kant tried to put Newton's physics into the Cartesian epistemological framework of sensory perception. It is with naturalized epistemology that philosophy begins to discover something that in physics has been valid for three hundred years.

purely speculative and not caring for empirical data, this criticism is not justified. For instance, Hooke was one of the most prominent experimental scientists of his time. Nevertheless, his style of experimental work was Cartesian. The Cartesian mechanical philosophy grew out, at least partially, from a criticism of the Galilean experimental method. According to Descartes, it is not enough to study different aspects of a physical problem, but it is necessary to create an overall mechanical picture about its functioning. The core of Cartesian science is the endeavor to discover the mechanisms at the core of the experimental data. Thus the problem is not that the mechanical philosophy would not use experiments, but rather that ***the experiments are separated from the theoretical work*** on the explanatory models which remained speculative. Descartes made experiments, but he used them only in order to activate his imagination. Theoretical work started after experimental work finished.

Newton realized that for the further development of mechanical philosophy an experimental control of its theoretical models was necessary. The difficulty was that in experiments only the phenomena are accessible, while the theoretical models postulate ontological entities we cannot experience directly (for instance the vortex of fine matter, which is, according to Descartes, the cause of gravity). Newton's answer to this dilemma was his method of ***inductive proof from phenomena***. Questions about Newton's inductive method led to many misunderstandings. After Mill interpreted induction as a logical method, Popper presented its fundamental criticism. Nevertheless, we believe that neither Mill's interpretation nor Popper's criticism concern the method Newton had in mind when he said that he had proven the inverse square law by induction from the phenomena. In other words, despite Popper's criticism of the inductive method we still believe that Newton's method works and that it lies at the foundations of modern physics.

Every kind of induction is based on some linguistic framework. Just as *mathematical induction* is based on the well ordering of the set on which the inductive proof is performed, Newton's *analytic induction* is based on the language of the calculus. In the process of empirical data interpretation (Newton considered Kepler's laws empirical data and called them phenomena) he was searching for a function which would describe the dependence of the force from parameters such as distances or masses. Nevertheless, he was searching for this function not among the *logically possible functions*, because here Popper is right: for any finite empirical data there is an infinite number of logically possible functions fitting the data, and so it is impossible to determine one of these functions by induction. But if we do not jump at once to the realm of the logically possible functions (in Newton's times this realm was unknown), but we restrict ourselves to "*analytically suitable functions*", we obtain a framework analogous to the framework in which mathematical induction operates. For instance, if we assume that the attractive force is proportional to some power of the distance $f(r) = r^k$, then from the fact that the planetary orbits are closed curves we can prove that $k$ is either –2 or +1 (see Arnold 1974, p. 38). From the logical point of view this proof is not satisfactory, because the fact that the trajectories form a closed curve is not a simple spatio-temporal event, and our derivation is based on a series of hidden assumptions (that motion is described by a second order differential equation, that space is three-dimensional, that all the functions are differentiable as many times as needed, etc.). From the logical point of view these assumptions are doubtful. Thus if we interpret Newton's inductive proof as a logical derivation, it is obvious that it does not work. But it is Popper's decision to read Newton's words using the framework of logic. Newton did not work in the framework of formal logic because in his times formal logic did not exist. What we wanted to show is that if we reconstruct Newton's inductive method using not formal logic, as Popper did, but the framework of the calculus, which Newton himself invented, then we can give Newton's words a clear meaning.

We suggest calling Newton's method of *inductive proof from phenomena* interpreted in the above mentioned manner ***analytical approach to the experimental method***.[41] The analytical approach in the contemporary sense was born in algebra in 1591, when Viète published his *In artem analyticam isagoge*. In 1637 Descartes transferred it to geometry. We would like to interpret Newton's contribution to the experimental method as a further step in this expansion of the analytic approach. According to Viète, the core of the analytical method consisted in three steps. In the first step we *mark by letters* the known, as well as the unknown, quantities. The purpose of this step is to cancel the epistemic difference between the known and the unknown. In the second step we *write down* the relations which would hold between these quantities if the problem was already solved. In the third step we *solve* the equations, and find the values of the unknown quantities. Newton made important contributions to algebra as well as to analytical geometry, which shows that he mastered the analytical method on the creative level. This increases the plausibility of the interpretation of Newton's experimental method as a further expansion of the analytical approach. In contrast to the analytical method as we know it from algebra or geometry, where the basic difference is an *epistemic difference* between the known and the unknown quantities, in the analytical approach to the experimental method the fundamental difference is a *methodological difference* between the measurable quantity (position, velocity) and the non-measurable quantities (forces). Thus Newton first *marks by letters* the measurable quantities as well as the non-measurable quantities. In this way he cancels their methodological difference. Then he writes down the equations that hold between these quantities. Finally, he derives from these equations some relation containing only measurable quantities, which relation can be therefore checked experimentally.

In order to see the novelty of this method, let us compare it with the methods of Galilean and Cartesian physics. Galileo simply refused to speak about non-measurable quantities and thus he considered unscientific all theories which supposed for instance an influence of the Moon on earthly phenomena. For Galileo, the world of science was restricted to phenomenal reality. In this respect, Cartesian physics was a step forward. It was able to conceive an influence of the Moon onto earthly phenomena. The vortex of the fine matter could in principle transfer such an influence. Nevertheless, Descartes was not able to say anything specific about the physical characteristics of this vortex and so, in the end, he was not able to say anything specific about this influence itself. This is so because in Cartesian physics the world-picture is split into two parts. One part is formed by ordinary bodies accessible to experimental investigation, the other by hypothetical substances, through which the results of the experiments are explained. The hypothetical substances are not accessible to experiments; they are accessible only to speculation. Thus we can say that *Cartesian physics has its phenomenal and ontological levels of description unconnected*.

Newton realized that the relation between the phenomenal and the ontological levels of de-

[41] The term 'analytical approach to the experimental method' can provoke some objections. In philosophy the term '*analytic*' is usually understood as the opposite to 'synthetic'. A proposition describing the outcome of an experiment is usually considered as a synthetic proposition, so it seems that the experimental method cannot be analytic. In our view, however, the opposition between analytic and synthetic is a consequence only of ignoring the role of formal languages in physical knowledge. The reality, which we want to examine experimentally, can be put in correlation with some calculus, by means of which the experiment is incorporated into a network of analytical relations which can provide the experiment with an analytical dimension. How this can work will be shown in more detail below.

scription in Cartesian physics is analogous to the relation of the known and the unknown quantities in algebra or analytic geometry. The non-measurable quantity (e.g. the force by which the Earth attracts the Moon) has to be marked by a letter and this letter has to be inserted into the equations which hold for such forces. Then some consequence of these equations should be derived in which only measurable quantities occur. Finally, this empirical prediction should be experimentally tested. In this way a measurement becomes a test of the analytic relations from which the prediction was derived. The greatest advantage of Newton's method is that we are not obliged to measure directly the quantity we are interested in. The network of analytic relations, into which this quantity is embedded, makes it possible to apply the experimental techniques at that particular place of the network which is most suitable for measurement. Thus, for instance, using the empirical data about the elliptic form of the planetary trajectories it is possible to test the form of the dependence of the gravitational force from distance. Precisely this was done by Newton and consequently he was angry when Hooke, who only guessed the correct formula, made priority claims to its discovery. In the framework of Cartesian physics in which Hooke was working, it was not possible to "*prove by induction*" the law of universal gravitation because in that framework the description of phenomena (data about the planetary trajectories) and the description of mechanical action (the law of gravitation) are not connected. Thus, even if from the logical point of view all Newton's assertions are hypotheses, just as the whole Cartesian vortex theory is a hypothesis, there is a fundamental difference between them. The hypotheses used by Newton are analytically bound to the rest of his theory. The hypotheses of the Cartesians, on the other hand, are only qualitative and their relation to the empirical phenomena is loose. This gives a deeper sense to Newton's assertion "*Hypothesis non fingo*". Newton does not make up his hypotheses but derives them from the data.

### 3.1.4 Measurement techniques and the "weighing" of the Earth

The characterization of measurement as a standardization of an experiment makes it possible to derive, from our interpretation of Newton's experimentation as an analytical approach to the experimental method, a new understanding of Newton's innovations in the field of measurement. As an illustration of the possibilities disclosed for physics by Newton's approach to measurement we would like to take the "weighing" of the Earth. In 1798, that is more than hundred years after the publication of the *Principia*, Henry Cavendish (1731–1810) successfully measured the force of attraction between two heavy spheres using very fine torsion weights. When he compared this tiny force with the weight of the heavy spheres, that is, with the force with which they are attracted to the Earth, he was able to calculate the mass of the Earth. Cavendish's measurement was therefore often called „weighing of the Earth". In order to understand the novelty of the Newtonian approach to measurement employed by Cavendish, let us compare it with the Galilean approach. In his measurement, Cavendish used fine torsion weights, i.e. an *instrument* by means of which Coulomb measured the attraction between two electric charges some ten years earlier. In his laboratory, Cavendish created an *artificial situation* that was thoroughly designed to eliminate all disturbing effects which could distort the result of the measurement. So far everything is in accordance with the Galilean approach. Nevertheless, the Galilean scientist would stop here and he would add the new phenomenon of attraction between the spheres to the known phenomena in a similar fashion as he added more than a century earlier the atmospheric pressure or temperature to them. The novelty of the Newtonian approach lies in the network of analytical relations which makes it possible to relate the measured force of attraction between the spheres to their weights and from this relation to determine the mass of the Earth. That presupposes the existence of a universal law, i.e. something entirely alien to Galilean physics. Of course, *after* the measurement of the attractive force, the Galilean scientist would also probably come up with the idea of an analogy of this force with gravity, and maybe after some time he would arrive at something analogous to Newton's inverse square law. But Newton formulated this law a century before Cavendish's measurement.

If we realize how tiny the force between the two spheres is, it becomes clear that a Galilean scientist had no chance to discover it by some lucky coincidence. He had no reason to construct such ingenious experimental equipment as Cavendish created in order to make the force measurable. In ordinary experience there is no clue which could lead him to the discovery of the attraction of bodies, like the failure of pumping of water from deep shafts that led Torricelli to the discovery of atmospheric pressure. In everyday experience there is no phenomenon which would reveal that macroscopic bodies attract each other. Therefore for a Galilean scientist (and for his positivist followers) the gravitational force would probably forever remain undiscovered. Newtonian science differs from the Galilean in that it embeds the phenomena into a framework of analytic relations. It can then use this framework to search for artificial phenomena suitable for testing its predictions. Cavendish used the law of universal gravitation when he planned the experimental situation in which the forces predicted by this law would become measurable. The law gave him an estimate of the magnitude of the force, and thus also an estimate of the precision that his instruments must reach. Thus while Galilean science uses the technical equipment in an experiment only to alter the *phenomena which already exist in our ordinary experience*, Newtonian science goes much further than that. It ***constructs new phenomena which have no parallel in ordinary experience***. Of course, by this we do not mean that for instance the forces of attraction between macroscopic bodies did not exist before Cavendish measured them. These forces existed, but they were not phenomena, they were not accessible to human experience.

### 3.1.5 Newton's critique of Descartes' definition of motion and the principle of inertia

In paragraphs 3.1.2, 3.1.3 and 3.1.4 we described the changes that Newton brought to the empirical basis of physics. Generally we can say that *Newton planted experience into a firm linguistic framework* and inserted the phenomena into a network of analytical relations. This made it possible to deploy the experimental techniques at the best accessible point of the entire network, and so to obtain answers to questions whose the direct experimental testing would be difficult. In the following paragraphs we will analyze the principles that Newton formulated to make the experimental results intelligible.

As already mentioned, Newton subjected Descartes' definition of motion to criticism. Descartes wrote that motion is "*the transference of one part of matter or of one body from the neighborhood of those bodies that immediately touch it and are regarded as being at rest, and into the neighborhood of others*" (Desecrates 1644, part II, sec. 25). Newton realized that when we accept this definition, it becomes impossible to define rectilinear motion. If all bodies in the neighborhood of a given body move in different directions, then with respect to some of them the motion of the given body may be rectilinear, while with respect to others it may not be so. So it is not clear what Descartes' second law of nature is about when it says that "*Each and every part of matter, regarded by itself, never tends to continue moving in any curved lines, but only in* *accordance with straight lines*" (Desecrates 1644, part II, sec. 39). Newton rejected the Cartesian definition of motion and introduced his new concept based on the idea of absolute space (see Jammer 1954, p. 93-124)

In defense of Descartes' theory of motion it can be said that he did not attach great importance to the definition of rectilinear motion, because according to him the universe is filled with matter, so that rectilinear motion is not possible. The purpose of the concept of rectilinear motion in the Cartesian system is not to describe some actually existing motion. Rectilinear motion does not belong to the level of phenomena; it belongs to the level of explanatory models. The concept of rectilinear motion is a theoretical notion which makes the phenomenal reality intelligible. It is against the background of this concept that circular motion appears to be a motion that at every point deviates from rectilinear motion, and so there is a need to give a causal explanation of these deviations.[42] Descartes constructed several models, by means of which he explained the observed phenomena. The hypotheses on which these models were based, however, did not have to be precise. It was enough that they showed that in principle the studied phenomenon can be explained using the particular mechanical model. Descartes felt that he is at the beginning of a long journey and so he left the details to future generations.

Newton rejected such approach to theoretical models. Unlike in Descartes' physics, in Newton's the phenomenal and the ontological layers were analytically connected. Hypotheses were no longer a matter of speculation. They had analytically derivable implications which were empirically testable. Therefore it was impermissible to use in the models so hazily defined concepts as Descartes' concept of rectilinear motion. In Newton's system, all concepts were embedded into a single linguistic framework, and so it was necessary to define the terms of the ontological level with the same care as those of the phenomenal level. That is why the *Principia* open with definitions of the basic terms.

Although we tried to play down the severity of the problems associated with the definition of rectilinear motion in Cartesian physics, nevertheless, these problems are not the result of Descartes' lack of care or patience. The problems with the notion of motion are a systematic feature of the whole Cartesian system and are connected with Descartes' *identification of matter with extension*. It is because Descartes identified the extended body with space that he lost any reference system in which he could define motion. The extended substance is at the same time that which is moving (i.e. 'matter') as well as that with respect to which motion is defined (i.e. 'space'). Therefore, if Newton wanted to create a coherent description of motion, he had to divide the Cartesian extension again into space and matter. In this way space became a *framework* on the background of which Newton could *define uniform motion in a straight line* and formulate the law of inertia. Thus even though Newton's wording of his first law resembled the formulations of Descartes, the absolute space, against the background of which this law was formulated, is fundamentally anti-Cartesian. Newton first planted the description of motion into the framework of absolute space and only with its background did he formulate the principle of inertia. In addition to an unambiguous definition of uniform rectilinear motion, Newton changed

[42] Here we see the pre-eminance of Cartesian physics over the Galilean, according to which circular motion is natural. Descartes saw circular motion against the background of the concept of uniform rectilinear motion and so, regardless of whether he had the term 'uniform rectilinear motion' correctly defined or not, he clearly understood that circular motion is the result of interaction. This example illustrates the relation between the phenomenal and the ontological level of Cartesian physics. The role of the ontological model was to shed light on the phenomenon, to make the phenomenon intelligible. So it was not necessary for the model to be accurate in every detail.

the formulation of the law of inertia as well, by combining Descartes two laws into a single one. Newton reached the understanding of velocity as a vector quantity, i.e. a quantity that is characterized by magnitude and direction which had been separated in Cartesian physics. Newton's law of inertia is thus in many respects fundamentally anti-Cartesian.[43]

One of Newton's important discoveries was the discovery of the absolute nature of circular motion. Descartes, in order to avoid the dangerous question of the Earth's motion, declared that motion is relative. The Earth is at rest with respect to the air surrounding it. According to Descartes, it is more natural to describe the motion of a body from the perspective of the bodies immediately contiguous with it. Therefore he declared the Earth to be motionless. Newton realized that the rotation of the Earth can be determined in an absolute sense. In a letter to Hook from 1679 he suggested an experiment to prove this rotation. If we let a body fall from a high tower (Newton threw stones from the dome of St Paul's Cathedral in London), it will not fall down along the perpendicular to the surface. Earth's rotation will deter its motion from the perpendicular direction. This effect is, for a fall from the height of 50 meters (the altitude of the dome), only 0.5 centimeters, which is smaller than the effects of draft in the cathedral. So Newton's experiments were, despite the fact that he had all windows closed, inconclusive. However, regardless of the fact that Newton could not measure this effect, the effect as such really exists, and thus Newton was on the right track.[44]

As a further argument in favor of the absolute nature of the Earth's rotation, Newton invented his experiment with the bucket. If we hang a bucket full of water onto the end of a twisted rope, the rope will start to unroll and it will bring the bucket into rotation with respect to the water in the bucket. At the beginning the bucket will quickly rotate, but the water will be still. Gradually the friction of water with the sides of the rotating bucket will bring the water into motion, and the water will rotate together with the bucket. According to the Cartesian physics, which defines motion with respect to the immediate surroundings of the body, the water was at the beginning of the experiment in motion, because it constantly changed its position with respect to the bucket. At the end of the experiment, on the other hand, the water is at rest, because it is motionless with respect to the bucket. Nevertheless, if we look at the surface of the water, we will find out that at the beginning of the experiment the surface of the water had the form of a horizontal plane, while at the final stages of the experiment it acquired a parabolic shape. The parabolic shape of the surface of water reveals the presence of centrifugal forces, and thus it proves its rotation in an absolute sense. Whether the water rotates or not we can thus determine from the shape of its surface in an absolute sense, regardless of what is happening with the bucket. Thus this thought-experiment is another argument against the Cartesian definition of motion.

Newton introduced his concept of absolute space and absolute time in confrontation with Descartes' definition of motion. Mach subjected these concepts to fundamental criticism. Al-

---

[43] Newton's second component of the Cartesian extended body, namely *matter*, is also interesting. Descartes, by raising geometric form to physical substance, eliminated the ancient amorphous *hylé*, which was considered the ontological substrate of the world. When Newton split the Cartesian extended body into its spatial and material components, he did not return to the amorphous *hylé* of ancient philosophy. Newton's matter took from the Cartesian substance its mathematical definiteness. All attributes of the Newtonian matter are clear and distinct. So, even if Newton rejected Descartes' identification of matter with space and returned to the separate categories of matter and space, he retained the main achievement of Cartesian physics, namely the mathematization of the ontological substrate of the world. Newton's matter is a purely mathematical substance, just as Descartes' extended body was. Thus the ancient *hylé* was definitively destroyed by Descartes.

[44] Later Foucault devised a better experiment for the demonstration of the motion of the Earth.

though Mach's criticism is justified and it can be seen as a first step towards the theory of relativity, we cannot agree with Mach's view that: "*This absolute time can be measured by comparison with no motion; it has therefore neither a practical nor a scientific value... It is an idle metaphysical conception.*" (Mach 1883, p. 224). As we tried to show in (Kvasz 2011), the concepts of absolute space and absolute time form a formal structure.[45] Mach was able to criticize these concepts because he had at his disposal a richer language through which it was possible to analyze the language of Newton's theory and discover its weaknesses. However, the language of Mach's theory has an analogous formal structure which Mach cannot criticize. He has to accept it uncritically, just like Newton accepted the concepts of absolute space and absolute time.

#### 3.1.6 Newton's correction of the Cartesian notion of state

Despite the fact that Descartes introduced the concept of interaction into physics and for its sake he created the concept of state, he was not able to describe the process of change of state. According to Descartes, bodies change their state abruptly in moments of collisions. Cartesian physics is able to describe the state before the collision and the state after the collision, but it fails to describe what happens at the moment of collision. The changes of state are singular events that defy physical description. As we will see below, Newton radically changed the Cartesian picture of interaction when he turned interaction into a continuous process, during which infinitely small impulses of forces cause infinitely small changes of states. Absolute space and absolute time are elements of the framework in which *the infinitely small impulses of forces can be tied to infinitely small changes of states and to infinitely small changes of position*. It is the framework that makes possible the integration of the phenomenal and the ontological level of description of a physical system into a single structure. In Cartesian physics, these two levels were separated and their separation was probably the greatest weakness of the Cartesian system. When Newton tried to unite the phenomenal and the ontological levels of description of a physical system, he needed for this a stable linguistic framework within which it would be possible to represent these two levels. Absolute space and absolute time are elements of such a unifying framework in which the differentials of state variables are continually integrated into the process of generating the trajectories. In order to accomplish this integration, Newton had to introduce a fundamental change in the description of the state. Descartes described the state of a physical body by means of its quantity of motion. Newton transformed Descartes' quantity of motion into momentum (which he, however, continued to call by its Cartesian name 'quantity of motion').

The first difference between the Cartesian description of state by means of quantity of motion and its Newtonian description by momentum is that the Newtonian momentum is a *vector quantity*. Newton realized that the changes of direction of motion in elastic bounces from an obstacles or in uniform circular motion involve also changes of velocity, and hence, (despite Descartes' denial) changes of state. This transition is of great importance for the understanding of planetary motion. When Descartes claimed that the Earth is with respect to her surroundings at rest, it meant that it is not exposed to any interaction, and orbits plunge into the vortex of fine matter. It is a natural idea which everyone, who has ever swum in a river, knows well. The water of the river carries us with a relatively high velocity, but locally we do not feel this drift, because we are moving with the same velocity as the surrounding water. This situation is in conformity with the

[45] Howard Stein called this structure in (Stein 1970) a kinematic connection.

Cartesian thesis that a body (e.g. Earth) in a uniform circular motion does not change its state, because it moves in concordance with the bodies that surround it. Newton realized that this idea is false. In a uniform circular motion the body constantly changes the direction of its motion, thus changes also the momentum, which means that it changes its state. For the description of this change of direction in circular motion Newton introduced the concept of centripetal force which is the cause of this change.

Clarification of the vector character of momentum is important not only for the understanding of the uniform circular motion, but also for the clarification of the relation between the soul and the body, and it led finally to the causal closure of the physical description of nature. Descartes described the action of the soul on the body, hence of a non-physical system onto a physical one, as changing the direction of the flow of the animal spirits. Since changes of the direction of motion do not violate the law of conservation of the quantity of motion, such mechanism of action of the soul on the body was in principle possible. When Newton proclaimed the changes of direction to be changes of state, the action of the soul on the body was no longer possible. Newton thus closed the causal description of nature. The price for this was that the interaction between body and soul became a philosophical problem.

The second change Newton introduced into the description of state concerns the fact that the Cartesian quantity of motion was the product of the (scalar) velocity and the *size* of the body. For Descartes, that definition was natural because the geometric substance was the ontological basis of the world; therefore its 'quantity' in the form of the size of the body entered the definition of the quantity of motion. When Newton divided the Cartesian extension into matter and space, he defined quantity of matter as the product of *volume* and *density*. The Cartesian concept of size is thus divided into two components: one *geometrical*, namely volume, and one *dynamical*, namely density.

The third change Newton introduced into the description of state has to do with the fact that, in addition to momentum, state also involves the position of the body. In Descartes, there was a lack of clarity on this issue because, as a consequence of the identification of matter with space, bodies lost position in the ordinary sense of the word. According to Descartes, the body's position is determined solely by the bodies in its vicinity, and as such position is hardly distinguishable from motion which is defined almost by the same words. More specifically, the concept of rest and the concept of position coincide in the Cartesian system. Only when Newton separated matter and space could these two terms be clearly distinguished. *Rest* is a motion with zero velocity (it is a particular value of the vector of momentum), while *position* is a value of the vector of position. The introduction of absolute space and time was an important step bringing clarity into the description of motion. Only when Newton included position into the concept of state, it became possible to interpreted state as a characteristic of the system allowing to predict its complete future temporal evolution. The Cartesian concept of state was incomplete, and as such it did not allow any such prediction.

### 3.1.7 Replacement of the law of conservation of the quantity of motion by the law of action and reaction

In the previous two paragraphs we described the changes introduced by Newton into the spatio-temporal framework on which the description of motion is based. Only after the articulation of this framework was it possible to turn to the formulation of the laws of dynamics. We will start interpretation of the laws of Newtonian dynamics with the law of action and reaction because this law has a direct predecessor in Descartes' law of conservation of the quantity of motion. Nevertheless, just like in the case of the state, also in this case Newton introduced several changes.

The first change was that Newton did not refer by his law to the entire universe but only to a closed physical system. For Descartes, the quantity of motion included the motions of all bodies in the universe and therefore it was impossible to calculate its actual value. In contrast to this, Newton's momentum was defined for relatively small systems and so it was possible to determine its value. It is true that Descartes used specification of the value of the quantity of motion in his description of collision of bodies. For instance, he asserted that the quantity of motion before and after the collision is the same. But such descriptions were counterfactual because in reality, according to Descartes, all bodies are submerged in the vortex of fine matter which can take away a portion of the quantity of motion. Therefore, strictly speaking, in the Cartesian system the law of conservation of the quantity of motion holds only for the entire universe. The quantity of motion of any restricted system of bodies cannot be constant. Only after Newton turned to the empty space as the background of mechanics, the conservation of momentum in smaller systems became possible. By eliminating the Cartesian vortex of fine matter, Newton opened the possibility to ***describe closed mechanical systems***. Galileo did not have any mechanical systems at all; he described only isolated bodies (as his bodies did not interact). On the other hand, Descartes included into his system the whole universe. Only Newton was able to describe something between these two extremes.

The second change concerns the role of the law of conservation of the quantity of motion. For Descartes, it was a fundamental principle on which the description of interactions rested. In Newtonian physics, the fundamental level of description of interaction is that of forces. The law of conservation of momentum in the Newtonian system is only one of the consequences of the law of action and reaction. When every action is accompanied with a reaction of the same magnitude and opposite direction then the momentum a body acquires during a period of time from the acting force will be precisely equal to the momentum another body or bodies lose as an effect of the force of reaction.[46] Therefore, the total momentum of a closed system remains constant. Thus the fundamental principle of the Cartesian physics became *Corollary 3* to Newton's third law of motion.

### 3.1.8 Newton's representation of interaction as an action of forces

In the interpretation of Cartesian physics we have mentioned as its main advantage over Galilean physics the notion of interaction that Descartes brought into physics. Descartes understood interactions as collisions and described them by means of the law of conservation of the quantity of motion. Despite its ground-breaking novelty, this way of describing interaction had a number of conceptual limitations. In section 2.2 we described the most important of them (the inability to incorporate friction into the description of interaction, and the inability to mathematically describe gravity). In addition to these conceptual limitations, the Cartesian theory of collisions has a further serious drawback: in does not work. If we translate the rules by means of which

[46] In this derivation, it is essential to understand the momentum as a vector. In Cartesian physics, the quantity of motion is a scalar, and therefore from the Cartesian point of view the force of action and the force of reaction would produce the same positive quantity of motion. So the action of Newtonian forces violates the Cartesian law of conservation of the quantity of motion.

Descartes described collisions into the language of Newtonian physics, we find out that most of them are false. By means of a formal reconstruction in section 2.1.4.a we have tried to show that these rules have, in spite of their falsity, a correct core. Nevertheless, this does not change the fact that they contradict experience. Newton was aware of the limitations of the Cartesian description of interaction as well as of the falsity of the Cartesian theory of collisions. Therefore, he abandoned the Cartesian description of interaction by means of the law of conservation of the quantity of motion and started to ***describe interactions as an action of forces***. Initially, he perhaps did not want more than to increase the precision of Descartes' theory, and so he left the basis of the Cartesian model of interaction (i.e. collisions) unchanged. He only introduced into it the instantaneous forces. However, in order to be able to formulate the theory of interaction mathematically, he had to make a number of changes which we will discuss briefly.

**3.1.8.a The replacement of inertial forces by forces of interaction** The role of forces in the Cartesian system was to *preserve* the state of a body. They did not act between the bodies, but they bound each body to its present state of rest or uniform rectilinear motion. Therefore, we can represent them using arrows oriented downwards. They play a role only during the moments of collision when they decide the direction and velocity of the next interval of uniform rectilinear motion of the body. Let us consider a moving body $B$ colliding with a resting body $C$. Depending on which force is greater—whether the *force for proceeding* $F_B$ in the body $B$, or the *force of resisting* $F_C$ in the body $C$—the outcome of the collision will be either that $B$ imposes its motion on $C$, or that $B$ will simply rebound, while $C$ preserves its rest.[47] Thus, according to Descartes, the force of a body acts upon the body itself and it simply preserves its state of motion or rest.


Fig. 1 Descartes' notion of force.

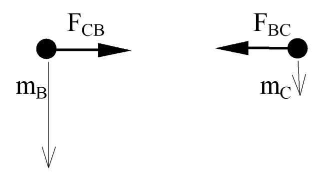


Fig. 2 Newton's notion of force.

According to Newton, a force is something by the virtue of which one body acts on another body and causes a change of its state. The role of the preservation of the state, which Descartes ascribed to forces, is in the Newtonian system played by masses.[48] By introducing the notion of mass, Newton liberated the forces from the role of binding bodies to their own states and thus opened the possibility to ascribe forces a new role—the role of changing the states of other bodies. Newtonian forces are forces of interaction; they act along the line connecting the two bodies (see Jammer 1957)

[47] The derivation of the formulas for the force of proceeding and the force of resistance is in (Gabbey 1980).

[48] Newton retained the Cartesian terminology and, in the context of inertia, he spoke about the 'inherent force of matter'. The fact that it is a residuum of the Cartesian terminology is evident from the fact that this force is the only force that is subject neither to the second law nor to the law of action and reaction. Newton formulated his second law only for impressed forces, but it is still strange to introduce a force that is without any effect.

**3.1.8.b The replacement of the notion of interaction understood as a singular event by the notion of interaction understood as a continuous process (of change of state)** The Cartesian notion of interaction is based on the idea of a *contest*, understood as the collision of two tendencies to preserve the previous state (of rest and motion respectively). The result of the contest is the victory of one tendency at the expense of the other. The paradigmatic model of interaction is the collision of two bodies. According to Descartes, the greater body determines the outcome of the collision and thus the further motion of the bodies. If the moving body is greater, then after the collision both bodies will move together in the direction of the original motion. If, on the contrary, the resting body is greater, then it will remain in its state of rest also after the collision and the moving body will bounce. The description of interaction is ***separated*** from the description of motion. As long as a body can, it moves uniformly in a straight line. When such motion becomes impossible, a collision occurs causing the transition into a new dynamic state. Thus the motion of a body consists of periods of uniform rectilinear motion separated by ***singular events***—collisions during which the state of the body is changed.

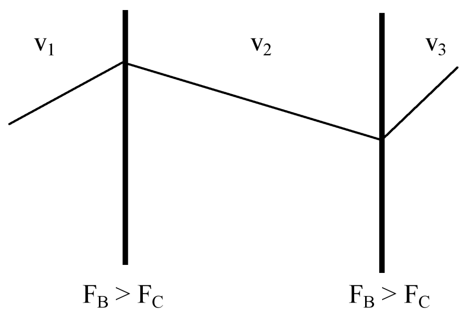


Fig. 3 Descartes' notion of interaction.

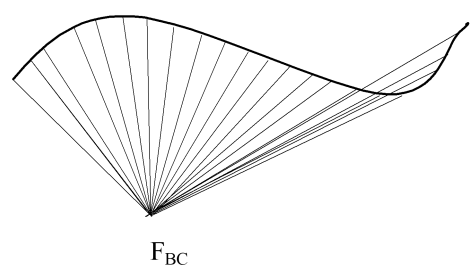


Fig. 4 Newton's notion of interaction.

In contrast to Descartes, Newton understands interaction as cooperation. In the process of interaction the faster body accelerates the slower body and at the same time the slower body decelerates the faster one. The resulting motion is a compromise; it is the result of the action of both bodies. Thus the result of a collision is neither a simple re-bouncing from the obstacle, nor is it a coupling of the two bodies into one, but something in-between. A second, maybe even more important, change is that motion and interaction happen ***simultaneously***. They are not separated from each other as in Descartes. According to Newton, the forces act all the time and their action accompanies the whole motion. A third change is that interaction is not a singular event, it happens not during isolated moments in time as in Descartes. According to Newton, a body acts on the other body during an ***infinitesimal time interval dt*** (or ***o***). It is true that Newton still speaks about impulses of forces, but in all concrete calculations he makes a limit transition. In the limiting case the impulses are becoming infinitesimally dense, and the magnitude of each separate impulse becomes infinitesimally small, thus at the end we are getting a continuous picture. And it is this continuous picture, which is important, because all the relations, which Newton uses in his calculations, hold only for this limiting case.

**3.1.8.c The connection of the action of forces with the transition of momentum** As we stated in our description of the Cartesian theory of collisions, Descartes described collisions on two levels. On one level the forces decide the outcome of the collision—whether the moving body will simply bounce or whether it will subdue the resting body to its motion. After the forces decided the next course of motion, they do not enter into further description of the interaction and the bodies exchange the particular quantity of motion independently of the forces, just on the basis of the law of conservation of the quantity of motion.

Newton radically changed this scenario. He excluded the decision about the outcome of the collision from the description of the collision and asserted that the moving body always transmits some portion of its momentum to the body with which it interacts. Thus a bounce without a change of state is not possible. The forces that he liberated from their Cartesian role of contest he engaged in the transfer of momentum. This is the content of his second law: "*A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed*." (Newton 1687, p. 416). We see that the impressed force has the form of an impulse and causes directly a change of (the quantity of) motion, that is, of momentum. As a consequence of the law of action and reaction, every action causes a reaction that changes the state of the acting body.

Newton's *Principia* are built on the idea of the description of interaction by means of impulses of forces. According to Howard Stein (Stein 1970), one of the central arguments of Newton's book was the derivation of the law of universal gravitation. This derivation consisted of three steps. In the first step, using a purely mathematical argument, Newton proved that Kepler's laws are equivalent to the statement that the planets are attracted to the Sun by a centripetal force inversely proportional to the square of the distance. In this argument, the action of the centripetal force is represented by a set of discrete impulses. In the second step, by comparing the acceleration of the motion of the Moon with the acceleration of free fall on the Earth's surface, Newton shows that the gravity on the Earth's surface obeys the same law as the centripetal forces that control the motion of the planets. That means that all planets have around them a gravitational force just like the Earth. On the basis of the law of action and reaction, however, we further get that also the bodies on the surface of the Earth attract the Earth itself, from where Newton concluded that (as the Earth is just a large piece of matter) the bodies also mutually attract each other, even though with a force that is too small to be directly measured. The postulation of the gravitational attraction among all bodies was a great success of Newtonian physics. Before Newton nobody knew that the macroscopic bodies attract each other, and this prediction could be experimentally confirmed only more than hundred years later.

In the derivation of the law of universal gravitation we can see an illustration of the prevalence of the Newtonian description of interaction by means of forces over its Cartesian description by means of collisions. It was the precise *quantitative formulation* of the gravitational *force* which made it possible to discover the *universal character* of gravity. Newton perhaps initially believed that the gravitational force is a result of an unknown mechanism of contact action, and he perhaps considered the quantitative law that describes this force only as an *effective mathematical description* which will be later derived mechanistically. This belief explains the fact that Newton in his description of the gravitational force in the *Principia* represented this force still as a result of a large number of discrete impulses, i.e., as if it was the result of collisions with some aether. But despite this Cartesian representation of the gravitational force, the advantage of Newton's effective mathematical description of gravitational interaction in comparison with its Cartesian description is the ***possibility of precise quantification of the size of impulses of the gravitational force***.

As in the case of gravity, Newton's way of representing interaction by means of forces has an advantage also in the description of friction. As we have mentioned, Cartesian physics was unable to incorporate friction into the description of motion, because friction violates the law of conservation of the quantity of motion. For Newtonian physics friction presents no problem; it is a force like any other.

### 3.1.9 Newton's analytical idealization of interaction by means of continuous forces

We are coming to one of the central moments of Newtonian physics—to the idealization of action. We interpreted Galileo's contribution to the development of physics as the *idealization of the phenomena of the lifeworld* (i.e. as the replacement of these phenomena by physical quantities), and Descartes' contribution as the *idealization of the ontology of the lifeworld* (i.e. as the replacement of ordinary things by extended bodies). Similarly, we propose to interpret Newton's contribution as the *idealization of action* (i.e. as the replacement of action understood as a contact—as action is encountered in the lifeworld—by action of continuous forces).

In the description of the gravitational interaction, a force entered the Newtonian theory which does not have the character of discrete impulses and which could not be reduced to contact interaction. It was a force that is ***acting at a distance***. Thus we put the Newtonian *forces acting at a distance* besides the Galilean *physical quantities* and the Cartesian *states of physical systems* as the third kind of ideal objects by means of which physics replaces aspects of the lifeworld. The introduction of these forces is an example of idealization, since such force is a (vector) function, i.e. a mathematical object; just as mathematical objects are the physical quantities or the states of physical systems.

Cartesian physics describes the world on two levels: an ontological and a phenomenal. The description of the collisions between bodies and particles of the fine matter belongs to the ontological level. These collisions give rise to gravity that we perceive on the phenomenal level. Nevertheless, these two levels of description are unconnected. There is no way how to determine from the phenomena the properties of the fine matter, or conversely how to deduce from the properties of fine matter the features of the phenomena. One of Newton's methodological insights was the idea that the relation between the phenomenal and ontological levels of description of interaction among bodies was analogous to the relation between the known and the unknown quantities in algebra or analytical geometry. In the case of forces, which belong to the ontological level of description, and are thus inaccessible to direct measurement, this insight means that we can mark them with letters like $F$ and then work with them as if they were measurable quantities: with the letter $F$ we can perform all formal manipulations as if it denoted a measurable quantity. We can *represent it as a function of other measurable quantities*, for instance, we can represent the gravitational force as a function of distances and masses of the other bodies. This incorporates the forces, i.e. ontological quantities, into the network of analytical relations and thanks to Newton's *analytical notion of experiment* its empirical examination can start. Due to the fact that Newton considered forces as functions of other quantities, he was able to deduce from Kepler's laws his famous law of gravity

$$F = \kappa \frac{m_1.m_2}{r^2}\,. \tag{3.3}$$

Here $F$ is the gravitational force by means of which a body with a mass $m_2$ acts on the body with the mass $m_1$, which is at a distance $r$. Neither Leibniz nor Huyghens were willing to accept

Newton's derivation of this law, because they were not able to imagine a mechanism that would be so incredibly regularly structured that it could ensure that every body would be attracted by every other body in the universe with a force precisely equal to (3.3). Leibniz and Huyghens apparently remained in the captivity of the image of interaction by contact; that is in the captivity of how we encounter interaction in the lifeworld. They were not willing to replace this idea by a mathematical ideality—a functional relation.

Newton did not list the law of universal gravitation among his basic laws of nature, because he understood the laws in the list as fundamental principles of the description of nature. The law of gravitation, on the other hand, is an empirical law which can be derived from the astronomical data. Nevertheless, if we abstract from the specific form of this law (the determination of which is an empirical problem), then there still remains a principle. We suggest calling it the principle of analytic representability of forces and adding it to the three laws of Newtonian physics. This principle is a rather general assumption that ***forces are functions of measurable quantities***. Thus even if forces are not measurable directly, the quantities from which they depend are measurable. The task of physics is then to experimentally determine the particular functions that express the different kinds of forces. This is a further point in which Newtonian mathematization transgresses the boundaries of the Galilean approach. Galileo understood mathematics as a language suitable to express the *phenomena*. According to Newton, not only the phenomena but also the forces can be expressed using mathematics.

Newton's basic premise is that the forces (i.e. hypothetically postulated causes) can be expressed as a function of measurable quantities, and thus ***incorporated into the language of physics***. McGuire notes that [Newton's] "*earliest program in dynamics was probably the hope of extending the scope of the concept of force essential for analyzing collisions, to all forces whatsoever*." (McGuire 1970, p. 187). Newton, however, gradually abandoned the perspective of collisions and developed a mathematical description of forces independent of their mechanistic explanation. This transition was gradual, but it had dramatic consequences for the entire physics.

### 3.1.10 Newton's syntactic closure of the description of motion

Idealization of interaction allowed Newton to initiate perhaps the most radical transformation of the language by means of which we describe nature. This transformation was, in our opinion, more radical than the one introduced by the theory of relativity or by quantum mechanics. Newton ***syntactically closed the language of physics***; he created a formal language of a new kind that allows calculating the temporal evolution of the state of a physical system in an analytical way. The temporal evolution of the state is described by Newton's second law. If we want to understand the meaning of this law, is not sufficient to analyze its wording, and to show that it is based on the idea of discrete impulses of a force (i.e. to show that Newton's second law is a translation of the Cartesian understanding of interaction into the language forces). We also have to analyze the examples in which this law is used. These examples show that in the background of Newton's notion of interaction there is the continuous process of change of state under the action of forces. By syntactically closing the language of physics, Newton created an ideal object of a new kind, ideal object into the constitution of which time enters in a fundamental way. We get thus to the reconstruction of the most important aspect of the birth of physics. Newton developed an ***analytical representation of a dynamic process***, and thus completed the mathematization of nature. He turned motion that we encounter in the lifeworld into an ideal object. This change has several aspects which we will describe briefly.

**3.1.10.a Unification of the phenomenal and the ontological levels of description** Cartesian physics describes motion on two levels: on the *kinematic* (or phenomenal) level of the description of changes of the measurable quantities, such as position or velocity, and on the *dynamic* (or ontological) level of the description of interaction by means of forces. To create an analytical representation of a dynamic process means to *create a mathematical language, that makes it possible to merge the kinematic and the dynamic levels of description of motion into a single process.* Newton created such a linguistic framework by immersing the kinematic and the dynamic changes into the continuous flux of absolute time. Let us see how this merging works.

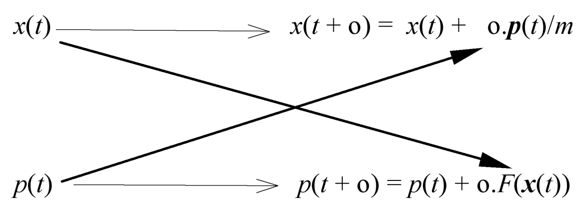


Fig. 5 Newton's merging of the phenomenal and the ontological levels of description.

On the one hand, the actual state of the system represented by the momentum $p(t)$ (i.e. a dynamic quantity), gives after the division by the mass of the body its velocity, and manifests itself in the changes of the position of the body (i.e. on the kinematic level). Thus the (phenomenal) changes of position ***are determined by*** the dynamic state of the body—this relation is in Fig. 5 represented by the arrow pointing upwards. On the other hand, the actual position of the body $x(t)$ appears in the formula expressing the force (for instance in (3.3)), and so, thanks to the equation (3.2) it determines the changes of state. So the changes of state ***are determined by*** the position of the body—this relation is in Fig. 5 represented by the arrow pointing downwards. These two relations connect the kinematic and the dynamic levels of description, and they are indicated by bold arrows in Fig. 5.

The kinematic (or phenomenal) as well as dynamic (or ontological) levels are further embedded into the flux of time. On the kinematic level, the change of the coordinate during a short moment of time is given by a kinematic equation known already by Galileo: we obtain the position $x(t + dt)$ in the moment $t + dt$, when we add to the position $x(t)$ in the moment $t$ the distance $dt.p(t)/m$ that the body traversed during the infinitesimal period $dt$ with the velocity $p(t)/m$ that it had in the moment $t$.[49] This is indicated by the thin horizontal arrow in the first line of the diagram. Similarly, on the dynamic (or ontological) level the temporal change of momentum is described by Newton's second law: we obtain the momentum $p(t + dt)$ in the moment $t + dt$,

[49] The infinitesimal time interval is here represented by the Leibnizian symbol *dt*. Perhaps it would be more appropriate to use the Newtonian symbol *o*, but our aim is to make the text as readable as possible. Also in the formulation of Newton's second law we will use the symbolism of the Leibnizian calculus, because it is more convenient than the symbolism of the theory of fluxions. In his *Principia*, Newton suppressed the language of his calculus and made all derivations by means of synthetic geometry. We are thus faced with the choice to use Newton's geometric language, the symbolism of the theory of fluxions or the Leibnizian differentials. We have chosen the third option, but there is in principle no problem rewriting the entire paper in any of the other two.

when we add to the momentum $p(t)$ at the moment $t$ the impulse $dt.F(x(t))$ of the force $F$ that acted at the place $x(t)$ where the body was at the moment $t$. This is indicated in Fig. 5 by the thin horizontal arrow in the second row of the diagram.

The diagram in Fig. 5 represents how the infinitesimal changes on the phenomenal and the ontological levels are interconnected to create a single ***dynamic system***.[50] The force $F(x)$ transfers the changes of position of the bodies onto the changes of their states. The momentum $p(t)$ acts in the opposite direction and transfers the actual state of the bodies into changes of their positions.

**3.1.10.b The embedding of interaction into the flux of time** An important aspect of Newton's new way of describing action is the continuity of the action of force. A body is exposed to the action of forces from the other bodies during ***an infinitesimal time interval dt***, and not during a singular moment as was the case in the Cartesian physics. Even if Newton still speaks about impulses of forces, in most examples he passes to the limit where these impulses are becoming infinitely dense and indefinitely weak, until finally he obtains the continuous image. It is this continuous image that corresponds to physical reality, because Newton uses geometrical relations that are valid only in the limit. Therefore, we will focus on the continuous image. When we compare it with the Cartesian description of collisions, we will find a number of important differences.

According to Descartes, the result of a collision depends on the question which force is greater, whether the force for proceeding $F_B$ in the body $B$, or the force of resisting $F_C$ in the body $C$. The collision is thus determined by a relation of the form[51]

$$\frac{B^2 \times V_B}{B+C} > \frac{C \times B \times V_B}{B+C}\,. \tag{3.4}$$

The quantity on the left-hand side is the force for proceeding in the body $B$, the quantity on the right-hand side is the force of resisting in the body $C$. These quantities have the same denominators, and actually the only difference between them is the magnitude of the bodies. The other quantities cancel each other. Thus we arrived at the Cartesian result that the moving body $B$ "wins the contest" if and only if $B > C$, i.e. if the magnitude of the body $B$ is greater than the magnitude of the body $C$.

Now we see why Descartes maintained (in spite of contrary evidence) that a moving body can bring a resting body into motion only if it is greater. It is a necessary consequence of the formula (3.4). When this happens, the moving body $B$ must pass a portion of its quantity of motion to the resting body $C$ in order to start its motion. We see that Descartes saw correctly that interaction consists in the transference of a particular quantity of motion from one body to

[50] Here it is important that the second law has the form (3.2) and not (3.1), so that the evolution of the position $x(t)$ and the evolution of the momentum $p(t)$ are embedded into a common continuous flux of time.

[51] In the presentation of Descartes' theory we will use his symbolism. We will talk about size $B$ of the body instead of its mass $m_B$. It may seem that we contradict the spirit of the footnote 46, where we in the presentation of Newton's theory decided to use the Leibnizian notation. However, there is a fundamental difference here. Newton's and Leibniz's versions of the infinitesimal calculus are to a large extent equivalent, so the symbolism of one theory can be used in the interpretation of the other without distortion of its content. Newton's and Descartes' theory, however, are not equivalent. Therefore, if we want to understand the differences between them, it is better to present Descartes' system by its own means. Similarly, we retained Descartes' symbolism in section 2.1.4.b (unlike in section 2.1.7.a, which was a reconstruction Descartes' system in the framework of Newton's theory.)

the other. Nevertheless, in the Cartesian system, the passing of motion from one body to another was ***separated*** from the action of forces. The forces decided the outcome of the contest, they decided whether the result will be the re-bouncing of the moving body or a common motion of the two bodies. The forces did not play any role in the following transference of motion from one body to the other. This transference was governed only by the law of conservation of the quantity of motion. Descartes thus represented interaction on two levels. The first level consisted in the contest of the forces and it was governed by the formula (3.4). The second level consisted in the transference of motion between the bodies, and was governed by the conservation law. But these two levels were separated from each other.

The unfolding of the interaction from a singular moment into the time interval *dt* and the new notion of forces as forces of interaction enabled Newton to join the action of forces with the transference of momentum. Descartes defined the force for proceeding as $F_B = \frac{B^2 \times V_B}{B+C}$, that is as the product of the magnitude of the moving body $B$ and the common velocity after the collision $V = \frac{B \times V_B}{B+C}$. The force of resisting $F_C = \frac{C \times B \times V_B}{B+C}$ he defined as the product of the magnitude of the resting body $C$ and the common velocity after the collision. Even if these two definitions seem similar, there is a remarkable conceptual conflict hidden in them. Descartes defines the force for proceeding equal to the *residual momentum*, which is ***left*** to the body $B$ after the collision, while he defines the force of resisting equal to the *gain of momentum*, which the body $C$ ***acquires*** in the collision. Thus it seems as if Descartes hesitated between two ways of connecting forces with momentum.

According to Newton, force is equal neither to momentum nor to change of momentum, but to the ***velocity of the change of momentum***. Descartes could not understand this connection, because he described interaction as a singular event in time. Therefore, in the Cartesian system, there is no way how to introduce the notion of velocity of change of momentum. We see the fundamental importance of the fact that Newton embedded the transference of momentum into the flux of time. This made it possible to connect force with the velocity of the change of momentum in his second law

$$\boldsymbol{F}.\boldsymbol{dt} = \boldsymbol{dp}\,. \tag{3.5}$$

Thus one of the fundamental achievements of Newton was that he ***connected the change of momentum with the action of a force***. The importance of this fact is often misunderstood, and the law (3.5) is considered as a mere definition of the concept of force. The fundamental conceptual work which lies behind it is thus veiled. Newton had to make profound changes in both the concept of force and the concept of momentum, and above all to embed the whole interaction into a continuous flux of time, to be able to connect the action of a force with the change of momentum. To describe Newton's second law as a definition is from an epistemological point of view a deep mistake.

**3.1.10.c The closing of the description of a physical system** Newton closed the description of a physical system in many respects. First of all, he ***closed it causally***. In section 3.1.5 we stated that Newton replaced the Cartesian scalar quantity of motion by the vector of momentum, and thus causally closed the physical systems to the action of non-physical causes, such as the soul. Form Newton onwards, only physical causes can causally act on a physical system.

The second respect in which Newton closed the description of a physical system is related to the fact that he replaced the Cartesian vortex of fine matter by forces acting at a distance. As these forces decrease with the square of the distance, Newton was able to ***close the description of a physical system extensionally***, to the action of very distant objects. The possibility of describing a limited dynamic system, consisting of a small number of bodies, dates back to Newton.

As a consequence of Newton's causally closing the physical description of nature, the relation between mind and body became a philosophical problem, in the solutions of which one side of the problem is usually deprived of significance. On the one hand, there are the proposals that take seriously the physical description of nature and try to construe the freedom of the will as an epiphenomenon. On the other hand, there are the approaches that try to restrict the validity of the physical description in order to exclude from it the description of human behavior. However, we have to realize that this problem arises only if we accept Newton's *causal closure* of the description of nature, but ignore his *extensional closure* thereof, i.e. if we make a strange mixture of Newtonian deterministic causality and Cartesian all encompassing extensionality of the physical description of nature. In Descartes' system, there is no mind-body problem, because its causality is not physically closed, and thus the soul can influence the body. But neither is there any problem in Newton's system, because its extensionality is limited and thus the human body is not (in all its aspects) a physical system. When we are interested in the details of human behavior (and not just for example in the velocity at which a human body hits the surface of the Earth after a free fall from a particular altitude), we simply have no idea how to ascribe to the human body a state that would give sufficiently detailed information about its behavior.

**3.1.10.d The analyticity of the rules of dynamic** The merging of the phenomenal and the ontological levels of description creates a single process from motion so that ***the temporal evolution of this process can be analytically calculated***. This means that using fully explicit and purely formal rules of language we can pass from one state of a physical system to its subsequent states. This situation is the following scheme:

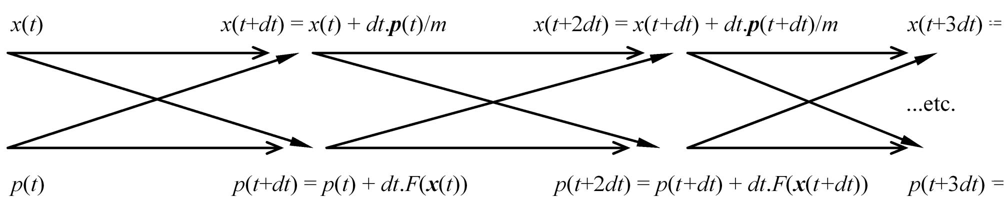


Fig. 6 Newton's merging of the infinitesimal steps into a continuous motion.

Figure 6 shows that Newton found a set of rules that fully determine the temporal evolution of a physical system. He thus changed a system of physical objects into a dynamical system, i.e. an ***ideal object*** that is analogous to a number or a mathematical structure. We consider the dynamic system an ideal object, because just like in the case of numbers or mathematical structures it is constituted by language. Dynamic systems differ from the two other kinds of ideal objects in that time enters into the language that constitutes them. Numbers and mathematical structures are atemporal idealities; dynamic systems are temporal idealities.

### 3.1.11 Newton's notion of motion as dynamic flow

The diagram in Fig. 7 represents the relationships between Aristotle's, Galileo's, Descartes' and Newton's conception of motion. Aristotle and Galileo developed *geometric theories of motion* (first row), while Descartes and Newton were building *dynamic theories* (second row). Aristotle's and Descartes' theories were formulated using *ordinary language* (first column), while Galileo's and Newton's theories were formulated using the *language of mathematics* (second column).

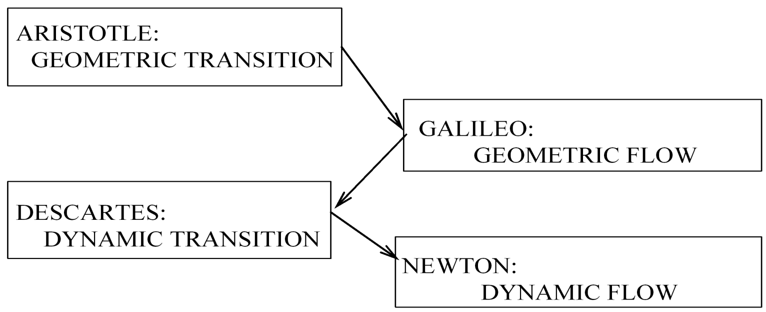


Fig. 7 Scheme representing the basic notions of motion.

***Aristotle*** represented motion as ***geometric transition***, as a transition of the body from one place to another. According to Aristotle, the universe is a *geometrically* ordered whole in which each body has a natural place. When something disrupts this geometric order and a body finds itself outside of its natural place, a motion starts, in the course of which the body reaches its natural place and the order gets restored. The terminal place is the carrier of the identity of motion. Aristotle's theory represents each motion by a pair of "photographs"—a photograph of its initial place and a photograph of its terminal place. Nevertheless, this theory does not describe what happens between these two places.

***Galileo*** inserted between the initial and the terminal place of an Aristotelian motion a trajectory in the form of a *geometric* curve. Thus he converted motion into a ***geometric flow***, a continuous sliding of the body along a trajectory. The geometric shape of the trajectory became the carrier of the identity of motion. For Galileo, motion is thus *a series of* ***static*** *photographs* ***put on*** *the thread of time*. This understanding of motion is still a geometric and not dynamic one, which we express by saying that Galileo's pictures are ***static***, i.e. they capture only the geometric position and not dynamic state of the body. The velocity of the body is on the Galilean photographs invisible. In order to determine the velocity, we need to take two photographs: one at the time $t$, the other at the time $t + dt$, and find out how much the position of the body in the second photograph has changed compared to its position in the first one. When this change of position is divided by the time interval $dt$, we get the velocity of the body. The motion is a *geometric flow*, because it is a set of positions that are inserted into the continuous flow of time (*the photographs are* ***put*** *on the thread of time*), and thus time is added to the photographs, so to say, from the outside. If we stretched the thread of time so that the distance between the photographs would double, the Galilean formalism would not notice it. A deceleration of time is from the Galilean point of view unobservable. That is what we want to express by the metaphor of photographs put on the thread of time.

***Descartes*** has enriched the concept of motion by the description of the interaction between bodies. He, nevertheless, interpreted this interaction as a collision, which he described as a transition. In contrast to the Aristotelian geometric transition from the initial position to the terminal position, Descartes described motion as a ***dynamic transition*** from the initial state (the state before the collision) to the final state (the state after the collision). In the language of our metaphor of photographs put on the thread of time, Descartes turned to ***dynamic*** *photographs*: on each photograph is captured, together with the position of the body, also its instantaneous velocity.[52] But Descartes, just like Aristotle, was satisfied with taking only two of them, one photograph of the state before the collision, and one photograph of the state after the collision. The difference is, of course, that Descartes' photographs were not geometric, but dynamic.

***Newton*** inserted, between the initial state and the final state of an interaction, a continuous process of the action of forces which he described by means of a differential equation. For Newton, motion is *a series of* ***dynamic*** *photographs* ***integrated*** *by the flux of time*. His notion of motion was a dynamic one, which means that his *photographs* captured the dynamic state of the body. We can see directly in the photograph what the velocity of the body is. If we take two photographs, one at the time $t$, the other at the time $t + dt$, and look at how much the position of the body in the second photograph has changed in comparison to its position in the first one, the change of position of the body read off from the two photographs must correspond with the value that we get when we multiply by $dt$ the velocity of the body taken from the first photograph. Newton's motion is a dynamic flow, it develops from itself (*the photographs are* ***integrated*** *by the flux of time*), i.e. time is not added to the photographs from the outside, but on the contrary, from the *inside*, it is present in the photographs (in the form of velocity). If we stretched Newton's series of dynamic photographs so that the distance between the photographs doubled, the equation of motion would cease to function. The changes of position would not be in accordance with the data in the photographs.[53]

We call Newton's notion of motion a ***dynamic flow***, because his law ***F.dt = dp*** tells not only that a certain amount of momentum $dp$ was added to a body (as Descartes' conservation law would tell), but the differential of time $dt$ indicates the time interval during which this increase of momentum occurred. ***The embedding of interaction into the flux of time*** is one of the most

[52] Instead of a *dynamic photograph*, it would be perhaps more appropriate to speak about an *infinitely short video*. An infinitely short video contains in addition to information about the positions also information about the instantaneous velocity of all bodies. But not quite, because the real video is a sequence of static images and the illusion of motion arises only in our head. In contrast, a dynamic photograph (i.e. the state of a dynamic system) contains only a single image, but that image itself is dynamic.

[53] In the language of infinitely short videos Newton's contribution can be expressed as follows: If we take two infinitely close videos, we let *the first video* run for the infinitely short period of time that separates these videos; it will end with a picture that is identical to the first picture of the second video. Time is thus integrated into the syntax of the language. The transition from one video to another through the period of time that separates the infinitely close videos is equivalent to an infinitely short run of the video. This is something that Galileo's pictures lacked. When we stretch the thread of time on which the pictures are hanging, Galileo cannot notice it. When we do the same thing with Newton's videos, he will notice it, because if we take two videos separated by an infinitely short period of time $dt$, the first frame of the second video will not be identical with the picture that we obtain by running the first video for the infinitely short time $dt$ (because the video was removed from its appropriate place). Here we see that in Galileo, time is only an external parameter on which static pictures are hung, while Newton's time is a parameter entering into the syntax of the language (i.e. the equation of motion).

important contributions of Newton. The notion of a dynamic flow is fundamental for the entire physics. Whether we take hydrodynamics, thermodynamics, electrodynamics, and quantum mechanics, everywhere we find the change of state of the physical system described as a dynamic flow. The motion of fluids, heat, charge, or quanta is represented as a continuous process described by a differential equation.

It is interesting to compare Newton's notion of motion also with that of Galileo. Galileo understood motion as a continuous process. Nevertheless, he introduced regularity into this process using geometry. According to Galileo, a specific kind of trajectory corresponds to each kind of motion and the regularity of a motion is given as the regular sliding of the body along this trajectory. Unfortunately, such a notion of motion is not able to represent interactions. This is so because an interaction disturbs the regular sliding of the body along the trajectory, and often it even deters the body from the trajectory itself. Newton replaced the geometric curve as the basis of the regularity of motion by the regularity of the flux of time. The flux of time became a *form*, common to all motions. Thus, whereas in Galileo *each body followed its own trajectory* and ignored the motion of the other bodies, Newton integrated the motions of all bodies in a *common flux of time*. In this flux, the trajectories of all bodies are generated simultaneously under the influence of mutual interactions. Newton called this regular flux of time, which is the common form unifying the motions of all interacting bodies into a dynamic system, ***absolute time***.

Diagram in Fig. 7 shows that Descartes moved in many respects against Galileo and Newton. First of all, he went against Galileo's and Newton's use of mathematics back to ordinary language. At this point, Descartes seemingly returned to Aristotle. But we must not let us be deceived, as it happened to many historians of science. Newton's physics is a continuation of Descartes', and not of Galileo's project. Only positivist historiography, which excluded Aristotle and Descartes from the history of physics, had no other choice than to connect Newton's project directly with that of Galileo.

The four conceptions of motion differ in how time enters into the description of motion. In the Aristotelian theory of geometric transition, time is not present; the theory says nothing about when the motion started, how long it took and when it ended. It conceptualizes motion in terms of position rather than time. In the Galilean theory of geometric flow, time is present as a parameter $t$ that parameterizes the trajectory of a given motion, and for ***each particular moment of time*** it indicates the point of the curve where the moving body is situated (in case of the free fall $x = \frac{1}{2}g.t^2$). In the Cartesian theory of dynamical transition, we are comparing states at ***two consecutive moments***, but the distance between them is not yet part of the description. Finally, in the Newtonian theory of dynamic flow, time is present in the equation of motion as variable according to which we differentiate.

### 3.2 Accomplishment of the mathematization of nature: Mathematics as a form of representation

Husserl's notion of idealization can be characterized as a ***substitutive idealization***; as a replacement of an aspect (a phenomenon or an object) of the lifeworld by an ideal object, i.e. the intentional object, constituted by means of a formal language. In this context, we propose to distinguish between a substitutive idealization and a constitutive idealization. In the case of Galileo and Descartes, the ideal objects that they needed for their *substitutive idealizations* were already present—they were supplied by mathematics. Nevertheless, in the case of Newtonian idealization the ideal objects of mathematics were not sufficient and thus Newton had to create a new kind of formal languages (the language of physics), the intentional objects of which he then used in the process of idealization. We suggest calling ***constitutive idealization*** the process of creating a new kind of formal language, the intentional objects of which can be used later in the process of substitutive idealization.

In the history of civilization, we can identify three constitutive idealizations. The first of them was the ***constitutive idealization of number***, which consisted in the creation of the formal language of arithmetic. In the ordinary understanding, the number sequence ends at a more or less fixed horizon where all counting stops. In ancient agricultural civilizations, symbolic number systems were created which simply prolonged the number sequence beyond any given horizon. By means of these number systems, i.e. formal linguistic systems, these civilizations constituted *numbers as ideal objects*, i.e. as intentional objects that exist independently of whether in reality there is something we can count by means of them. The language of arithmetic led to the *replacement* of the phenomenon of number that we use in the lifeworld by *numbers constituted in the formal language of arithmetic*.

The second constitutive idealization was the ***constitutive idealization of shape*** consisting in the creation of the language of classical geometry, by means of which the natural phenomenon of shape, i.e. the shape nestled within the natural horizon of similarities was replaced by geometric form. In the natural understanding, shape is an individual form, something like the face of the object. According to this understanding, each shape is unique, different from all others. In the ordinary understanding of shape, there is no perfect geometric cube but only individual cubes, each of which is slightly different from the others. The constitutive idealization of shape took place in ancient Greece between Thales and Euclid. It consisted in the creation of a language by means of which all imperfections, typical for shapes of the bodies of the lifeworld, were deleted. Geometric idealities, i.e. intentional objects constituted by the language of geometry, were born. The introduction of these ideal forms led to the *replacement* of the ordinary phenomenon of shape as an aspect of our experience, by geometric forms. This replacement was so successful that natural shapes with all their irregularities and imperfections started to be seen as imitations of these ideal geometric forms.

We propose to include, into this sequence, as the third constitutive idealizations the ***constitutive idealization of motion***, i.e. the process that took place in the period between Galileo and Newton. As in the previous two cases, also here the constitutive idealization consisted in the creation of a new language, the language of physics. We have seen that the process of constitutive idealization of motion consisted of three consecutive steps of substitutive idealizations. The first of these was the ***substitutive idealization of phenomena***, i.e. the Galilean replacement of the phenomena of the lifeworld by mathematical quantities obtained in measurement. This step consisted in the introduction of the notion of motion conceived as a geometric flow and was accomplished by means of the language of classical mathematics: the curves, by means of which Galileo described motion, were the curves of ancient geometry—the circle and the parabola. The second step was the ***substitutive idealization of state***, the Cartesian replacement of the objects of the lifeworld by extended bodies in motion. Descartes' extended bodies are objects of classical geometry elevated onto the ontological level of *real entities* and endowed with *motion*. The third step was the ***substitutive idealization of action***, Newton's replacement of action, understood as pushing or pulling, by the mathematical description of action using forces. During this third step, however, Newton had to turn from substitutive idealization of action to a more radical change, to the *constitutive idealization*. He had to create a new formal language—the language of dynamical systems—in order to mathematize interaction, and so to finish the process of idealization started by Galileo. By means of this new language we are able to calculate the temporal evolution of a dynamic system analytically, i.e. by formal manipulations with symbols.

The objects of physics are thus ideal objects; they are intentional objects constituted by the language of physics. Probably the strangest among them are forces acting at a distance. In this case, the replacement of the natural phenomenon by an ideal object is perhaps most visible. Newtonian forces act at a distance—a body does not need to touch another body in order to act on it. Nevertheless, there exists no explanatory model of this strange action. Instead of an explanatory model, Newton offers a mathematical description. Our suggestion is to put the Newtonian forces acting at a distance alongside Galilean quantities and Cartesian states simply as a further sort of mathematical objects by which science replaces different aspects of the lifeworld. Thus we maintain that in this case, just like in the other two, there is nothing to be explained or justified.

Galileo understood mathematics as a language that enables us to describe nature. According to Descartes, mathematics does not describe nature; nature itself is mathematical. Newton introduced an even more radical conception of the relationship between mathematics and nature. Mathematics is a ***form of representation of nature, a form of representation of time, space, and action***. This form is something more than just a language to describe phenomena. It captures not only the phenomenal side of reality, it describes not only ***what appears***—it captures also the ontological basis that causes the phenomena, thus it describes ***what there really is***. Mathematical description is not limited to what is apparent or measurable, but passes beyond the apparent towards the ontologically real, as Descartes wanted. But Newton did not stop even at ontology. Mathematics describes, according to Newton, not only the ontological basis of the world, it says not only what is real. Mathematics describes also how this ontological basis changes with time, it describes the process of the temporal development of state, and so it says also ***what is possible*** and what, from that which is possible, ***becomes real***.

Since antiquity, there has been a tradition that connects mathematics with the eternal. This was the core of Aristotle's arguments against the applicability of mathematics to describe nature. Mathematics embodies the eternal, while nature is the realm of the variable. Descartes refuted Aristotle's argument when he declared mathematics the ontological basis of reality, but in the description of nature even he could not overcome the connection of mathematics with the eternal. When Descartes formulated his three laws, he expressed by these laws only what is not changing—the principle of inertia expresses the constancy of the dynamic state, while the law of conservation of the quantity of motion expresses the immutability of the quantity of motion in the universe. Only now are we ready to appreciate Newton's radicalism. His laws are not *laws of conservation* (of the velocity, the direction, or the quantity) of motion, but they are ***laws of generation*** of motion. Nature is lawful not in its being, but in its becoming. Mathematics, according to Newton, is the form allowing description of these changes. ***Mathematics is the form of description of the change of state***.

The three layers of mathematization of nature—the Galilean, the Cartesian and the Newtonian—can be nicely illustrated by the Newtonian equations of motion

$$F dt = dp \,. \tag{3.5}$$

Here *dt* is an element of the Galilean layer of mathematization, grasping time as a physical quantity. Galilean time is read off from a watch, i.e. from an instrument which allows us to grasp its passing in a reproducible way. When we measure time, we have it in front of us; we hold it away from our body. Galilean time is "external" with respect to us; it is a regular repetition of a periodic process that "does not concern us". $F$ and *dp* are elements of the Cartesian layer of mathematization. They represent the fundamental idea of Descartes that we must move from the description of phenomena to the description of the changes of state. Descartes also understood that a change of state can be described as a transfer of a quantity of motion $p$, and that the direction, in which this transfer takes place, is decided by forces $F$. Thus we see that all the ingredients that occur in Newton's equation were known already before him.[54] However, these ingredients were isolated. In Descartes the forces were separated from the exchange of momentum. The inertial forces decided the outcome of the collision, i.e. the direction in which the momentum was transferred, but they did not enter into the transfer as such. In Descartes, forces were separated also from the flux of time. He described interaction as a singular event comparing states in two isolated moments. The time that elapsed between these moments was from the Cartesian viewpoint irrelevant.

The equation (3.5) can be seen as Newton's contribution to the mathematization of nature. It is a differential equation representing a form that ***integrates the Galilean flow with the Cartesian change of state***. The change of state is not a singular event as in Descartes, but it takes place in an interval of time. The time in which this change occurs, however, cannot be kept away from the body by means of an instrument. The time in the equation of motion is not the Galilean time measured on a watch. It is the mathematical (absolute) time in which the new state of the system is born as a result of the action of forces. Newton's time is a form that integrates all systems and all phenomena.

**Acknowledgement**

I would like to thank to Vladimír Balek, Pavel Bóna, Vladimír Bužek, Petr Dvořák, Donald Gillies, Juan Javier del Granado, Filip Grygar, Eberhard Knobloch, Paolo Mancosu, Miroslav Marcelli, Andrej Pázman, Ján Pišút, Paul Rusnock, Dimitri Sokoloff, Friedrich Stadler, Friedrich Steinle, Michael Stöltzner, Jan Sebestik, Marek Tomeček, Elena Várossová, and Pavol Zlatoš for their comments and criticism of the previous versions of this paper. The paper uses material from my previous publications, namely Galilean physics in light of Husserlian phenomenology. *Philosophia Naturalis* **39**, pp. 209-233, The Mathematization of Nature and Cartesian Physics. *Philosophia Naturalis* **40**, pp. 157-182, and The Mathematization of Nature and Newtonian Physics. *Philosophia Naturalis* **42**, pp. 183–211. I extend thanks to Vittorio Klostermann GmbH for permission to use this material. The paper originated in the *Fellowship of Jan Evangelista Purkyně* in the Institute of Philosophy of the Academy of Sciences of Czech Republic.

[54] Of course, Newton had to replace quantity of motion by momentum; inertial forces by forces of interaction, and instead of finite quantities he had to take infinitesimal ones.

## References


Ariew, R. (1992): Descartes and scholasticism: The intellectual background to Descartes' thought. In: Cottingham 1992, pp. 58-90.

Arnold, V. I. (1974): *Mathematical Methods of Classical Mechanics*. Springer, New York 1989.

Bedini, S. (1967): The instruments of Galileo Galilei. In: McMullin 1967, pp. 256-292.

Bechler, Z. (ed. 1982): *Contemporary Newtonian Research.* Kluwer, Dordrecht.

Bechler, Z. (1991): *Newton's Physics and the Conceptual Structure of the Scientific Revolution.* Kluwer, Dordrecht.

Bedini, S. (1967): The instruments of Galileo Galilei. In: McMullin 1967, pp. 256-292.

Böhme, G. (1989): Philosophische Grundlagen der Newtonschen Mechanik. In: Hutter 1989, pp. 5-20.

Butts, R. (1978): Some Tactics in Galileo's Propaganda for the Mathematization of Scientific Experience. In: Butts and Pitt 1978, pp. 59-86.

Butts, R. and Pitt, J. (eds. 1978): *New Perspectives on Galileo*. Reidel, Dordrecht.

Clarke, D. M. (1992): Descartes' philosophy of science and the scientific revolution. In: Cottingham 1992, pp. 258-285.

Clavelin, M. (1983): Conceptual and Technical Aspects of the Galilean Geometrization of the Motion of Heavy Bodies. In: Shea 1983, pp. 23-50.

Coehlo, R. L. (2001): *Zur Konzeption der Kraft in der Mechanik.* Waxmann Verlag, Münster.

Coelho, R. L. (2002): Zur Physik von Descartes: Naturgesetze und Stossregeln. *Philosophia Naturalis* **39**, pp. 45-60.

Cohen, I. B. (1960): *The Birth of New Physics.* W. W. Norton, New York 1985.

Cohen, I. B. (1970): Newton's Second Law and the Concept of Force in the *Principia*. In: Palter 1970, pp. 143-186.

Cohen, I. B. (1971): *Introduction to Newton's Principia.* Cambridge University Press, Cambridge.

Cohen, I. B. (1980): *The Newtonian Revolution*. Cambridge University Press, Cambridge.

Cohen, I. B. (1982): The *Principia*, Universal Gravitation, and the „Newtonian Style". In: Bechler 1982, pp. 21-108.

Cohen, I. B. (2002): Newton's concepts of force and mass, with notes on the Laws of Motion. In: Cohen and Smith 2002, pp. 57-84.

Cohen, I. B. and Smith, G. E. (eds. 2002): *The Cambridge Companion to Newton.* Cambridge University Press, Cambridge.

Cohen, I. B. and Westfall, R. S. (eds. 1995): *Newton, Text, Backgrounds, Commentaries. A Norton Critical Edition*. W. W. Norton, New York.

Cottingham, J. (ed. 1992): *The Cambridge Companion to Descartes*, Cambridge University Press, New York.

Czermak, J. (ed. 1993): *Philosophie der Mathematik*. Holder-Pichler-Tempsky, Wien.

De Caro, M. (1993): Galileo's Mathematical Platonism. In: Czermak 1993, pp. 13-22.

Descartes, R. (1637): *Discourse on Method, Optics, Geometry, and Meteorology*, translated by Paul J. Olscamp. Boobs-Merrill, Indianapolis 1965.

Descartes, R. (1644): *Principles of Philosophy*, translated by V. R. Miller and R. P. Miller. Reidel, Dordrecht 1983.

Descartes, R. (1662): *Le Monde, ou Traité de la lumiére*, edited and translated by Michael Sean Mahoney. Abaris books, New York 1979.

Dijksterhuis, E. J. (1961): *The Mechanization of the World Picture.* Clarendon Press, Oxford.

Disalle, R. (2002): Newton's philosophical analysis of space and time. In: Cohen and Smith 2002, pp. 33-56.

Drake, S. (1957): *Discoveries and opinions of Galileo*, New York.

Drake, S. (1973): Galileo's Experimental Confirmation of Horizontal Inertia. *Isis* **64**, pp. 291-305.

Drake, S. (1978): *Galileo at Work: His Scientific Biography*, New York.

Drummond J. J. (1992): Indirect Mathematization in the Physical Sciences. In: Hardy and Embree 1992, pp. 71-92.

Fauvel, J. (ed. 1993): *Newtons Werk. Die Begründung der modernen Naturwissenschaft*. Birkhäuser, Basel.

Gabbey, A. (1980): Force and Inertia in the Seventeenth Century: Descartes and Newton. In: Gaukroger 1980, pp. 230-320.

Gabbey, A. (1985): The mechanical philosophy and its problems: Mechanical explanations, impenetrability, and perpetual motion. In: Pitt 1985, pp. 9-84.

Gabbey, A. (2002): Newton, active powers, and the mechanical philosophy. In: Cohen and Smith 2002, pp. 329-357.

Galilei, G. (1610): *The Starry Messenger*. In: Drake 1957, pp. 21-58.

Galilei, G. (1623): *The Assayer*. In: Drake 1957, pp. 229-280.

Galilei, G. (1632): *Dialogue Concerning the Two Chief World Systems—Ptolemaic & Copernica.* Translated by Stillman Drake, Berkeley and Los Angeles 1967.

Galilei, G. (1638): *Dialogues Concerning Two New Sciences*. Translated by H. Crew and A. de Salvio, New York 1914.

Garber, D. (1992a): *Descartes' metaphysical physics*, The University of Chicago Press, Chicago.

Garber, D. (1992b): Descartes' physics. In: Cottingham 1992, pp. 286-334.

Garrison, J. W. (1986): Husserl, Galileo, and the processes of idealization. *Synthese* **66**, pp. 329-338.

Gaukroger, S. (ed. 1980a): *Descartes, Philosophy, Mathematics and Physics*. The Harvester Press, Sussex.

Gaukroger, S. (1980b): Descartes' Project for a Mathematical Physics. In: Gaukroger 1980, pp. 97-140.

Gaukroger, S. (1992): The nature of abstract reasoning: philosophical aspects of Descartes' work in algebra. In: Cottingham 1992, pp. 91-114.

Gaukroger, S. (1995): *Descartes, An Intellectual Biography*. Clarendon Press, Oxford.

Gillies, D. (ed. 1992): *Revolutions in Mathematics*. Clarendon Press, Oxford.

Grosholz, E. (1980): Descartes' unification of algebra and geometry. In: Gaukroger 1980, pp. 156-168.

Gueroult, M. (1980): The Metaphysics and Physics of Force in Descartes. In: Gaukroger 1980, pp. 196-230.

Guicciardini, N. (2002a): *Reading the Principia. The Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736.* Cambridge University Press, Cambridge.

Guicciardini, N. (2002b): Analysis and synthesis in Newton's mathematical work. In: Cohen and Smith 2002, pp. 308-328.

Guicciardini, N. (2009): *Isaac Newton on Mathematical Certainty and Method*. The M.I.T. Press, Cambridge, Massachusetts.

Gurwitsch, A. (1967): Galilean physics in the light of Husserl's phenomenology. In: McMullin 1967, pp. 388-401.

Hakfoort, C. (1992): Newtons Optik, Wandel im spektrum der Wissenschaft. In: Fauvel 1993, pp. 109-133.

Hall, R. (1967): The significance of Galileo's thought for the history of science. In: McMullin 1967, pp. 67-81.

Hardy, L. and Embree, L. (eds. 1992): *Phenomenology of Natural Science*, Kluwer, Dordrecht.

Harper, W. (2002): Newton's argument for universal gravitation. In: Cohen and Smith 2002, pp. 174-201.

Hatfield, G. (1992): Descartes' physiology and its relation to his psychology. In: Cottingham 1992, pp. 335-370.

Heelan, P. A. (1987): Husserl's later philosophy of natural science. *Philosophy of Science* **54**, pp. 368-390.

Herivel, J. (1965): *The Background to Newton's Principia*. The Clarendon Press, Oxford.

Herivel, J. (1970): Newton's Achievement in Dynamics. In: Palter, ed. 1970, s. 120-135.

Hill, D. K. (1988): Dissecting trajectories: Galileo's Early Experiments on Projectile Motion and the Law of Fall. *Isis* **79**, pp. 646-668.

Husserl, E. (1954): *Die Krisis der Europäischen Wissenschaften und die Transzendentale Phänomenologie*. English translation: *The Crisis of European Sciences and Transcendental Phenomenology* by David Carr, Evanston, Northwestern University Press 1970.

Hutter, K. (ed. 1989): *Die Anfänge der Mechanik, Newtons Principia gedeutet aus ihrer Zeit und ihrer Wirkung auf die Physik*, Springer Verlag, Berlin.

Hyder, D. and Rheinberger H.-J. (eds. 2010): *Science and the Life-World.* Stanford University Press, Stanford.

Jammer, M. (1954): *Concepts of space.* Harvard University Press, Cambridge.

Jammer, M. (1957): *Concepts of force.* Harvard University Press, Cambridge.

Jammer, M. (1960): *Concepts of mass.* Harvard University Press, Cambridge.

Kockelmans, J. J., and Kisiel, T. J. (eds. 1970): *Phenomenology and the Natural Sciences*, Evanston.

Koyré, A. (1939): *Galileo Studies*, translated by J. Mepham, The Harvest Press, Hassocks 1978.

Koyré, A. (1944): *Entretiens sur Descartes*. New York.

Koyré, A. (1957): *From the Closed World to the Infinite Universe*. The John Hopkins University Press, Baltimore.

Koyré, A. (1959): Giembattista Benedetti, critic of Aristotle. In: McMullin 1967, pp. 98-117.

Koyré A. and Cohen I. B. (1972): *Isaac Newton's Philosophiae naturalis Principia mathematica*. Cambridge University Press, Cambridge.

Kvasz, L. (1999): On Classification of Scientific Revolutions. *Journal for General Philosophy of Science* **30**, pp. 201-232.

Kvasz, L. (2002): Galilean physics in light of Husserlian phenomenology. *Philosophia Naturalis* **39**, pp. 209-233.

Kvasz, L. (2003): The mathematization of nature and Cartesian physics. *Philosophia Naturalis* **40**, pp. 157-182.

Kvasz, L. (2005): The Mathematization of Nature and Newtonian Physics. *Philosophia Naturalis* **42**, pp. 183–211.

Kvasz, L. (2008a): *Patterns of Change, Linguistic Innovations in the Development of Classical Mathematics.* Birkhäuser Verlag, Basel.

Kvasz, L. (2008b): Forms of Trascendence in Science and in Religion. *Theology and Science* **6**, pp. 89-106.

Kvasz, L. (2011): Classical Mechanics between History and Philosophy. In: Máté, Rédei and Stadler 2011, pp. 129-154.

Kvasz, L. (2013): Kuhn's *Structure of Scientific Revolutions* between sociology and epistemology. To appear in *Studies in History and Philosophy of Science*.

Landau, L. D. and Lifshitz E. M. (1957): *Mechanics*, Pergamon Press, Oxford 1969.

Le Grand, H. E. (1978): Galileo's Matter Theory. In: Butts and Pitt 1978, pp. 197-208.

Mach, E. (1883): *Die Mechanik in ihrer Entwicklung*. Brockhaus, Leipzig 1897. English translation by Thomas McCormack *The Science of Mechanics, A Critical and Historical Exposition of Its Principles.* Cambridge University Press, Cambridge 2013.

Machamer, P. (1978): Galileo and the Causes. In: Butts and Pitt 1978, pp. 161-180.

Machamer, P. (ed. 1998): *The Cambridge Companion to Galileo* New York.

Mancosu, P. (1992): Descartes's *Géometrie* and revolutions in mathematics. In: Gillies 1992, pp. 83-116.

Marion, J. L. (1992): Cartesian metaphysics and the role of the simple natures. In: Cottingham 1992, pp. 115-139.

Máté, A., Rédei, M., and Stadler, F. (eds. 2011): *The Vienna Circle in Hungary.* Springer, Vienna.

McGuire, J. E. (1970): Comment. In: Palter 1970, pp. 186-191.

McMullin, E. (ed. 1967): *Galileo, Man of Science*, New York.

McMullin, E. (1967a): Introduction: Galileo, man of science. In: McMullin 1967, pp. 3-52.

McMullin, E. (1978): The Conception of Science in Galileo's Work. In: Butts and Pitt 1978, pp. 209-258.

McMullin, E. (1985): Galilean idealization. *Studies in the History and Philosophy of Science* **16**, pp. 247-273.

McMullin, E. (2009): The Galileo affair: two decisions. *Journal for the History of Astronomy* **xl**, pp. 192-212.

Mormann, T. (1991): Husserl's philosophy of science and the semantic approach. *Philosophy of Science* **58**, pp. 61-83.

McTighe, T. (1967): Galileo's Platonism: A Reconsideration. In: McMullin 1967, pp. 365-387.

Naylor, R. H. (1974): Galileo and the Problem of Free Fall. *The British Journal for the History of Science* **7**, pp. 105-134.

Naylor, R. H. (1980): Galileo's Theory of Projectile Motion. *Isis* **71**, pp. 550-570.

Naylor, R. H. (1990): Galileo's Method of Analysis and Synthesis. *Isis* **81**, pp. 695-707.

Newton, I. (1673): De gravitatione. In: Newton 1962, pp. 89-121.

Newton, I. (1684): De Motu Corporum in Gyrum. In: Herivel 1965, pp. 257-303.

Newton, I. (1687): *The Principia*, A New Translation by I. Bernard Cohen and Anne Whitman, Proceeded by *A Guide to Newton's Principia*. University of California Press, Berkeley 1999.

Newton, I. (1729): *Lectiones opticae.* London. Russian translation by S. I. Vavilov: *Lekcii po optike*. Izdatel'stvo AN SSSR, Moskva 1946.

Newton, I. (1962): *Unpublished scientific papers of Isaac Newton*, ed. A. R. Hall and M. B. Hall, Cambridge.

Newton, I. (1972): *Philosophiae naturalis Principia mathematica*, ed. A. Koyré and I. B. Cohen, Cambridge.

Palter, R. (ed. 1970): *The Annus Mirabilis of Sir Isaac Newton 1666-1966.* The M.I.T. Press, Cambridge, Massachusetts.

Palter, R. (1970): Newton and the Inductive Method. In: Palter 1970, pp. 244-258.

Pitt, J. C. (1978): Galileo: causation and the Use of Geometry. In: Butts and Pitt 1978, pp. 181-196.

Pitt, J. C. (ed. 1985): *Change and Progress in Modern Science.* D. Reidel, Dordrecht.

Ronchi, V. (1967): The influence of the early development of optics on science and philosophy. In: McMullin 1967, pp. 195-206.

Schuster, J. (1980): Descartes' *Mathesis universalis*: 1619-28. In: Gaukroger 1980, pp. 41-96.

Settle, T. (1967): Galileo's use of experiment as a tool of investigation. In: McMullin 1967, pp. 315-337.

Shapiro, A. E. (2002): Newton's optics and atomism. In: Cohen and Smith 2002, pp. 227-255.

Shea, W. R. (1978): Descartes as Critic of Galileo. In: Butts and Pitt 1978, pp. 139-160.

Shea, W. R. (ed. 1983): *Nature Mathematized, Historical and Philosophical Case Studies in Classical Modern Natural Philosophy*. D. Reidel, Dordrecht.

Shea, W. R. (1983a): Do Historians and Philosophers of Science Share the Same Heritage? In: Shea 1983, pp. 3-20.

Shea, W. R. (1991): *The Magic of Numbers and Motion, The Scientific Career of René Descartes* Science History Publications.

Shea, W. (1998): Galileo's Copernicanism: The science and the rhetoric. In: Machamer 1998, pp. 211-243.

Simonyi, K. (1978): *A fizika kultúrtörténete*. Gondolat, Budapest 1986.

Smith, G. E. (2002): The methodology of the *Principia*. In: Cohen and Smith 2002, pp. 138-173.

Soffer, G. (1990): Phenomenology and scientific realism: Husserl's critique of Galileo. *Review of Metaphysics* **44**, pp. 67-94.

Stein, H. (1970): Newtonian Space-Time. In: Palter 1970, pp. 258-284.

Stein, H. (2002): Newton's metaphysics. In: Cohen and Smith 2002, pp. 256-307.

Steinle, F. (1991): *Newtons Entwurf „Über die Gravitation ..“* Frank Steiner Verlag, Stuttgart.

Swerdlow, N. (1998): Galileo's discoveries with the telescope and their evidence for the Copernican theory. In: Machamer 1998, pp. 244-270.

Tannery, P. (1901): Galileo and the Principles of Dynamics. In: McMullin 1967, pp. 163-177.

Wallace, W. A. (1984): *Galileo and His Sources: The Heritage of the Collegio Romano in Galileo's Science*. Princeton.

Westfall, R. S. (1971): *Force in Newton's Physics*. Neale Watson, New York.

Westfall, R. S. (1980): *Never at Rest*. Cambridge University Press, Cambridge.

Whiteside, D. T. (1970): Sources and Strengths of Newton's Early Mathematical Thought. In: Palter 1970, pp. 69-87.

Wilson, C. (2002): Newton and celestial mechanics. In: Cohen and Smith 2002, pp. 202-226.

Wisan, W. L. (1974): The New Science of Motion: A Study of Galileos *De motu locali*. *Archive for the History of Exact Sciences* **13**, pp. 103-306.

Wisan, W. L. (1978): Galileo's Scientific Method: a Reexamination. In: Butts and Pitt 1978, pp. 1-58.

Wisan, W. L. (1984): Galileo and the Process of Scientific Creation. *Isis* **75**, pp. 269-286.