Non-linear contributions to interactions in climate networks:

sources, relevance, nonstationarity

Hlinka, J.; Hartman, D.; Vejmelka, M.; Paluš, M.

Institute of Computer Science, Academy of Sciences of the Czech Republic

EGU General Assembly
Vienna 2012

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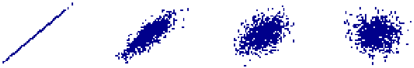

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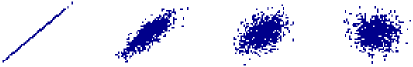

Measuring dependence:

Pearson's correlation $\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$

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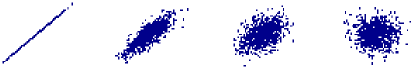

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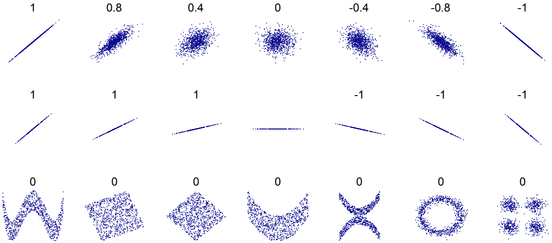

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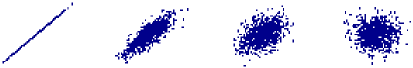

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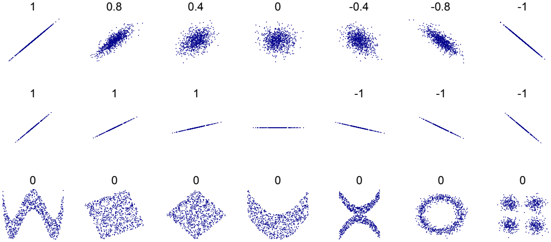

Mutual information:

$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log \left(\frac{p(x,y)}{p(x)\,p(y)} \right)$$

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  - relevance for higher-order analysis
  - treatment

Data and methods

Data: NCEP/NCAR reanalysis dataset

- surface air temperatures
- monthly data (years 1948 - 2007; 720 timepoints)
- global grid 73×144 points ($2.5\,\mathrm{deg} \times 2.5\,\mathrm{deg}$ sampling)
- yearly cycle removed (anomalies)

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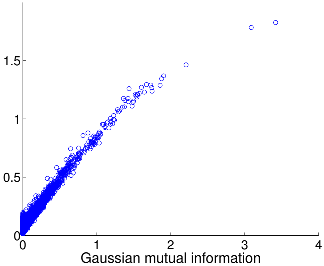
Methods: interaction/dependence quantification

- **nonlinear**: mutual information (pdf estimated using equiprobable binning; N=8)
- **linear**
  - Pearson's correlation
  - mutual information on linear surrogate data

Results: Existence



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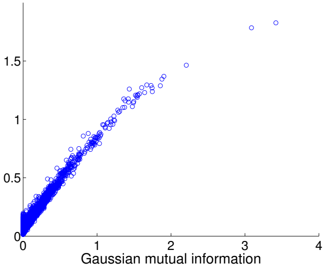

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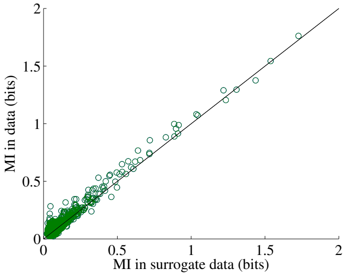

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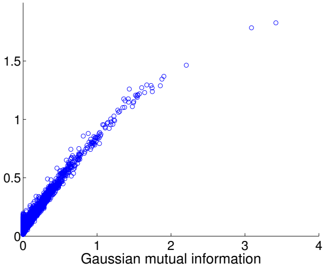

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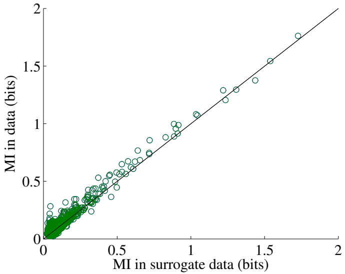

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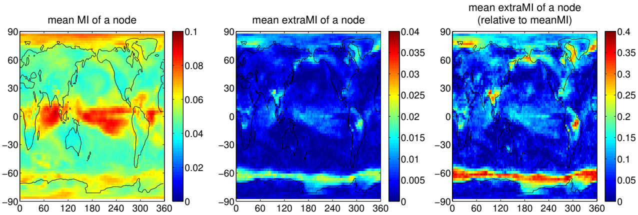

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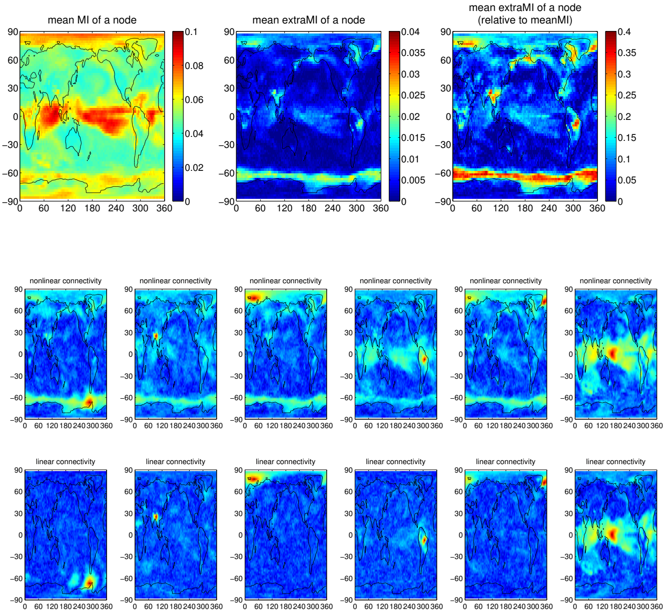
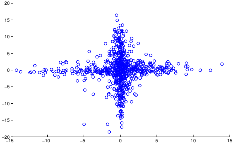


Form/origin



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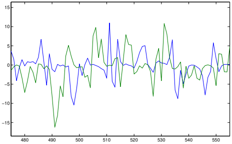

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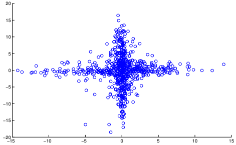

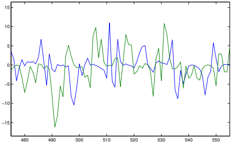

- introduce conservative preprocessing: month-wise variance equalization

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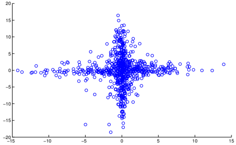

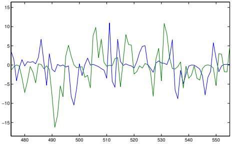

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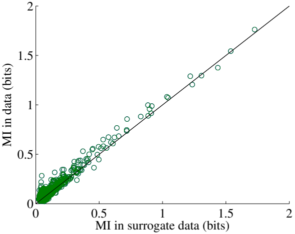

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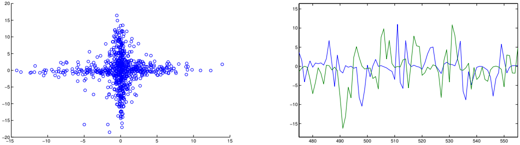

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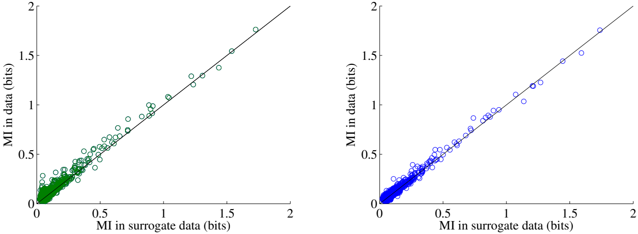
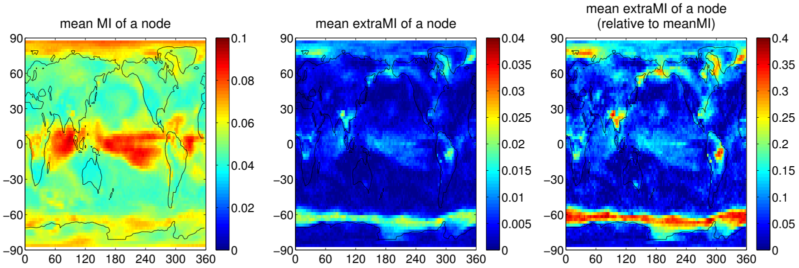


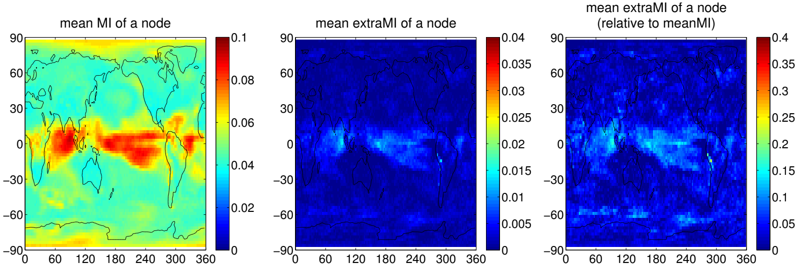
Statistical testing against surrogates: 8% links above 95th percentile

Form/origin II

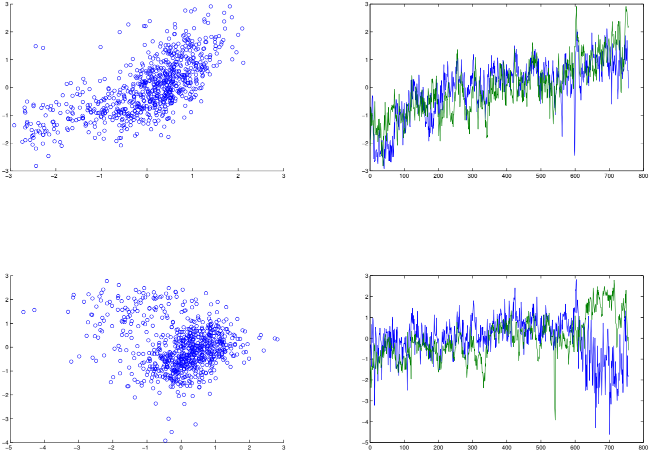
Temperature anomalies:



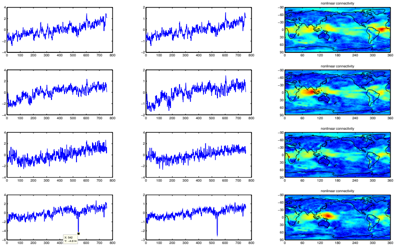
After additional normalization of variance:

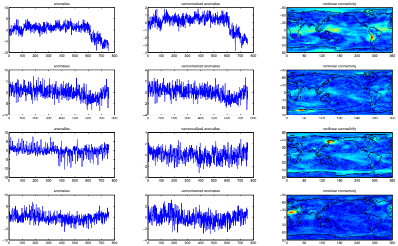


What about remaining ‘non-linearities’?



More examples





Form/origin III

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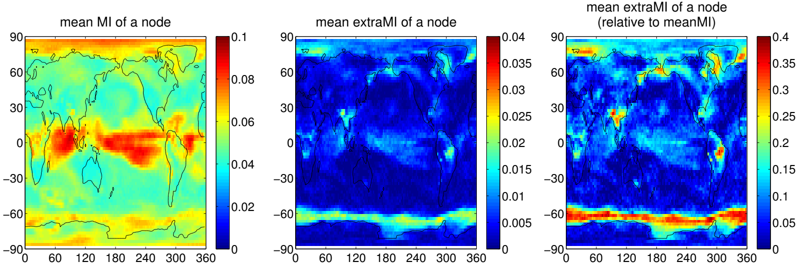

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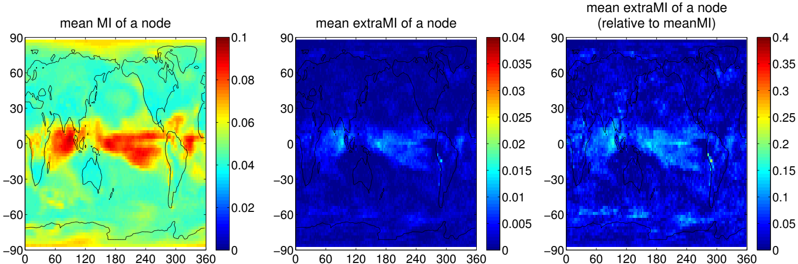

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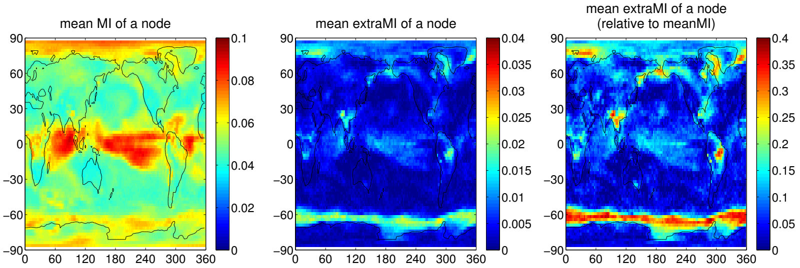
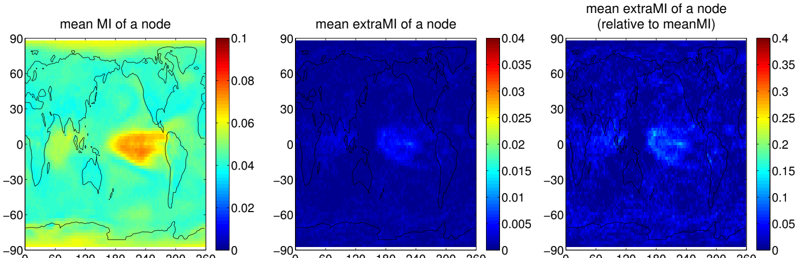


After additional detrending:



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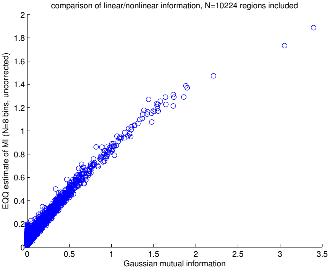

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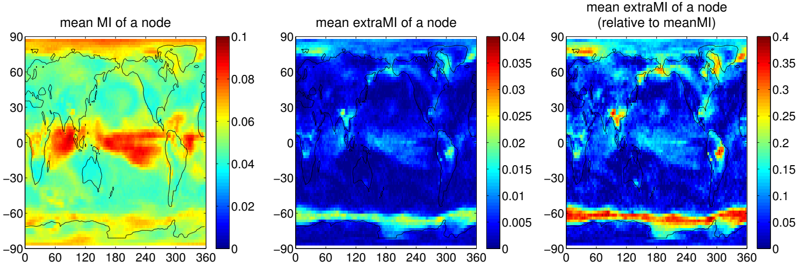

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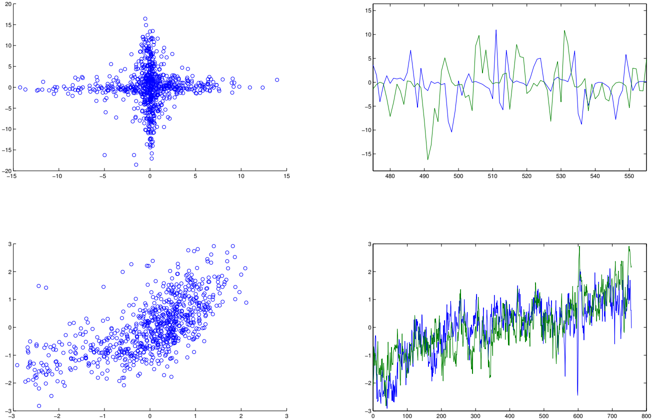

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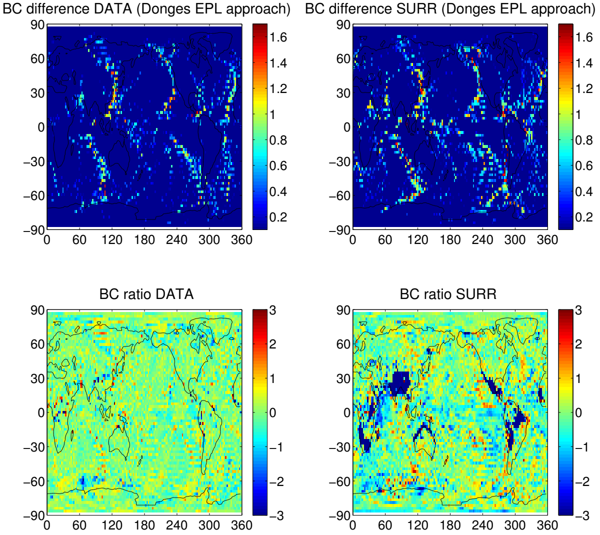
Thank you for your attention!

This study was supported by the Czech Science Foundation project No. P103/11/J068.

Relevance for graph topology

Donges et al., 2009: nonlinearity key for global topology



Other datasets: ERA