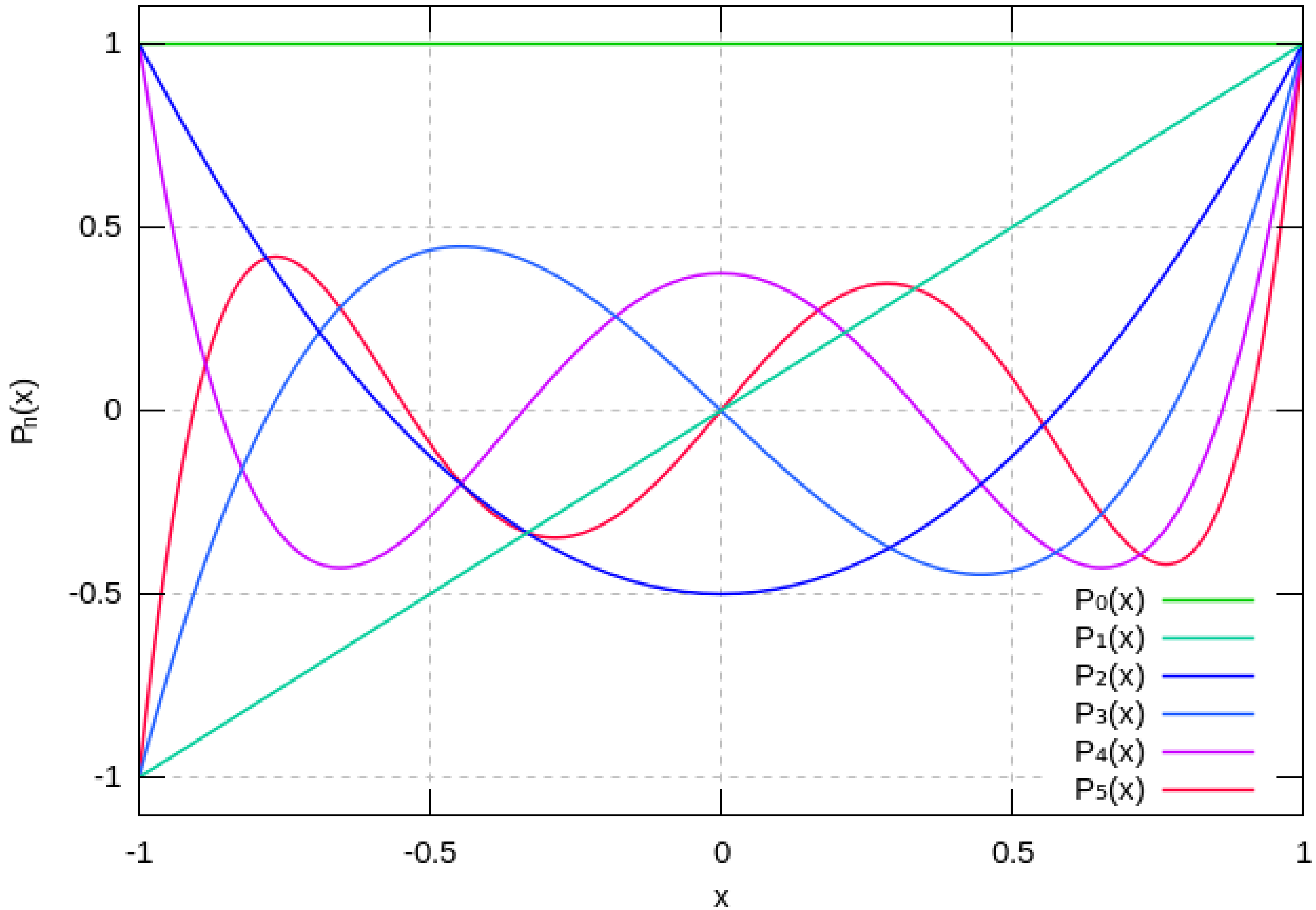
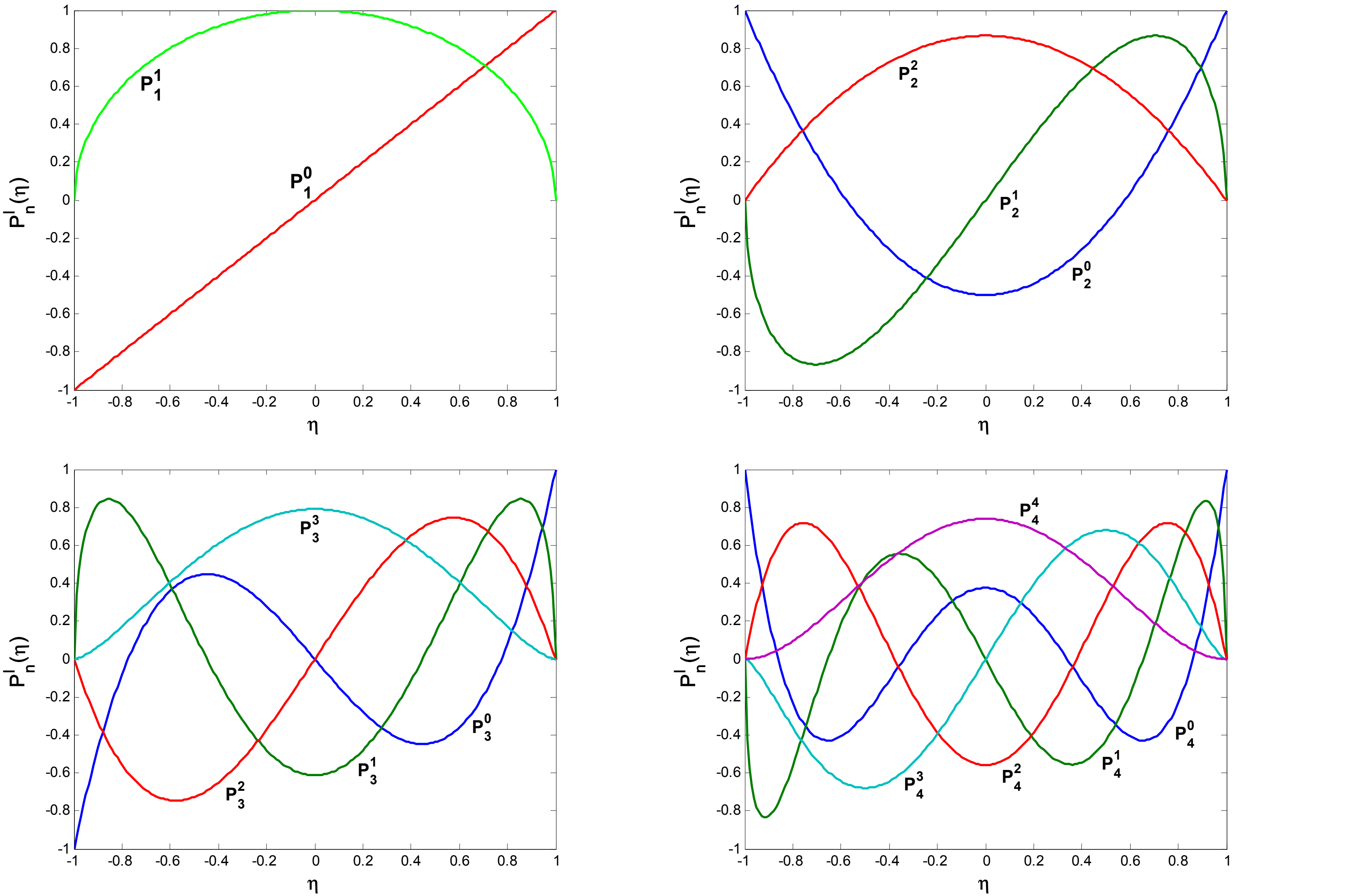
Legendrovy polynomy

$$P_n(\eta) = \frac{(-1)^n}{2^2 n!}\frac{d^n}{d\eta^n}\left(1-\eta^2\right)^n, \quad n = 0,1,2,\ldots \qquad \int_{-1}^{1} P_n(\eta)P_m(\eta)\,d\eta = \int_0^{\pi} P_n(\cos\theta)P_m(\cos\theta)\sin\theta\,d\theta = \frac{2}{2n+1}\delta_{mn}$$



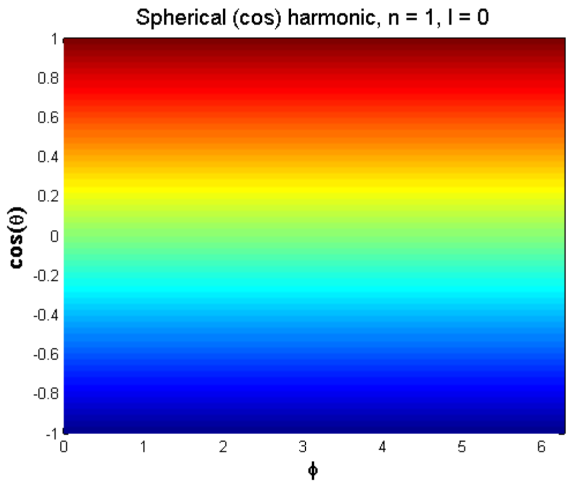
Přidružené Legendrovy funkce

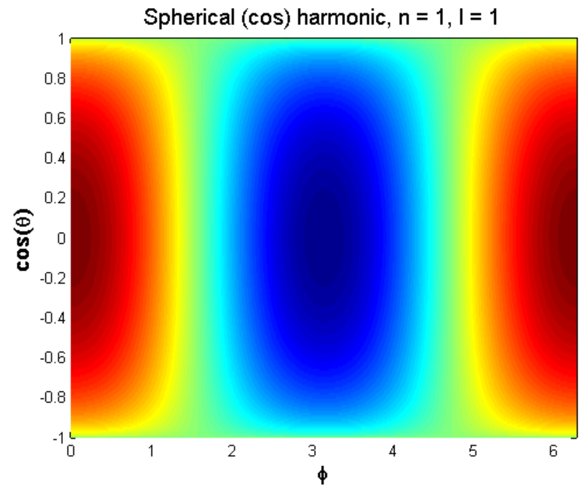
$$P_n^l(\eta) = \frac{(-1)^{n+l}}{2^n n!}(1-\eta^2)^{l/2}\frac{d^{n+l}}{d\eta^{n+l}}(1-\eta^2)^n,\ l = 0,1,\ldots,n,\ \int_{-1}^{1} P_n^l(\eta)P_{n'}^l(\eta)d\eta = \frac{2(n+l)!}{(2n+1)(n-l)!}\delta_{nn'},\quad \int_{-1}^{1} P_n^l(\eta)P_n^{l'}(\eta)\frac{d\eta}{1-\eta^2} = \frac{(n+l)!}{l(n-l)!}\delta_{ll'}.$$

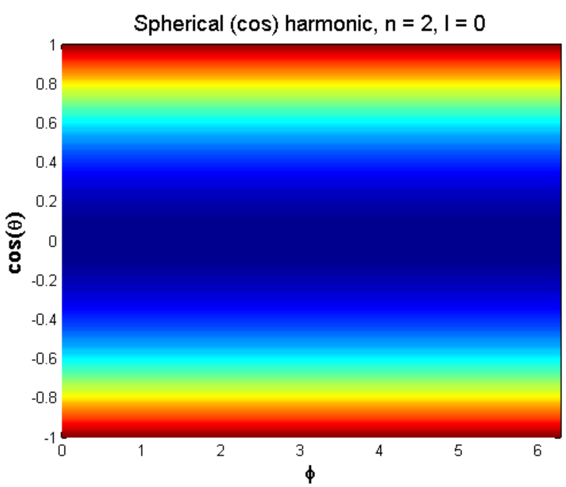


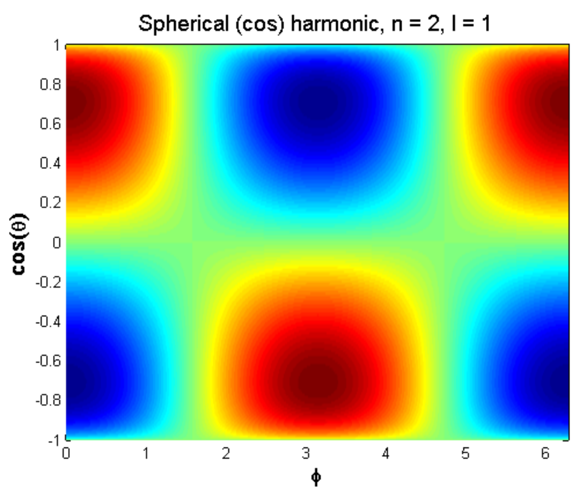
Sférické harmonické

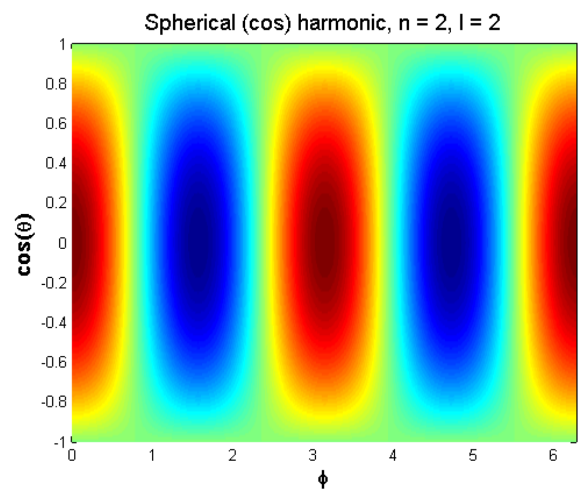
$$Y_n^l(\theta,\varphi) = \sqrt{\frac{(2n+1)(n-l)!}{4\pi(n+l)!}} P_n^l(\cos\theta) e^{il\varphi}, \quad \mathrm{Re}\left\{Y_n^l(\theta,\varphi)\right\} = \sqrt{\frac{(2n+1)(n-l)!}{4\pi(n+l)!}} P_n^l(\cos\theta)\cos l\varphi, \quad \int_0^{2\pi}\int_0^{\pi} Y_n^l(\theta,\varphi) Y_{n'}^{l'*}(\theta,\varphi)\sin\theta d\theta d\varphi = \delta_{nn'}\delta_{ll'}$$



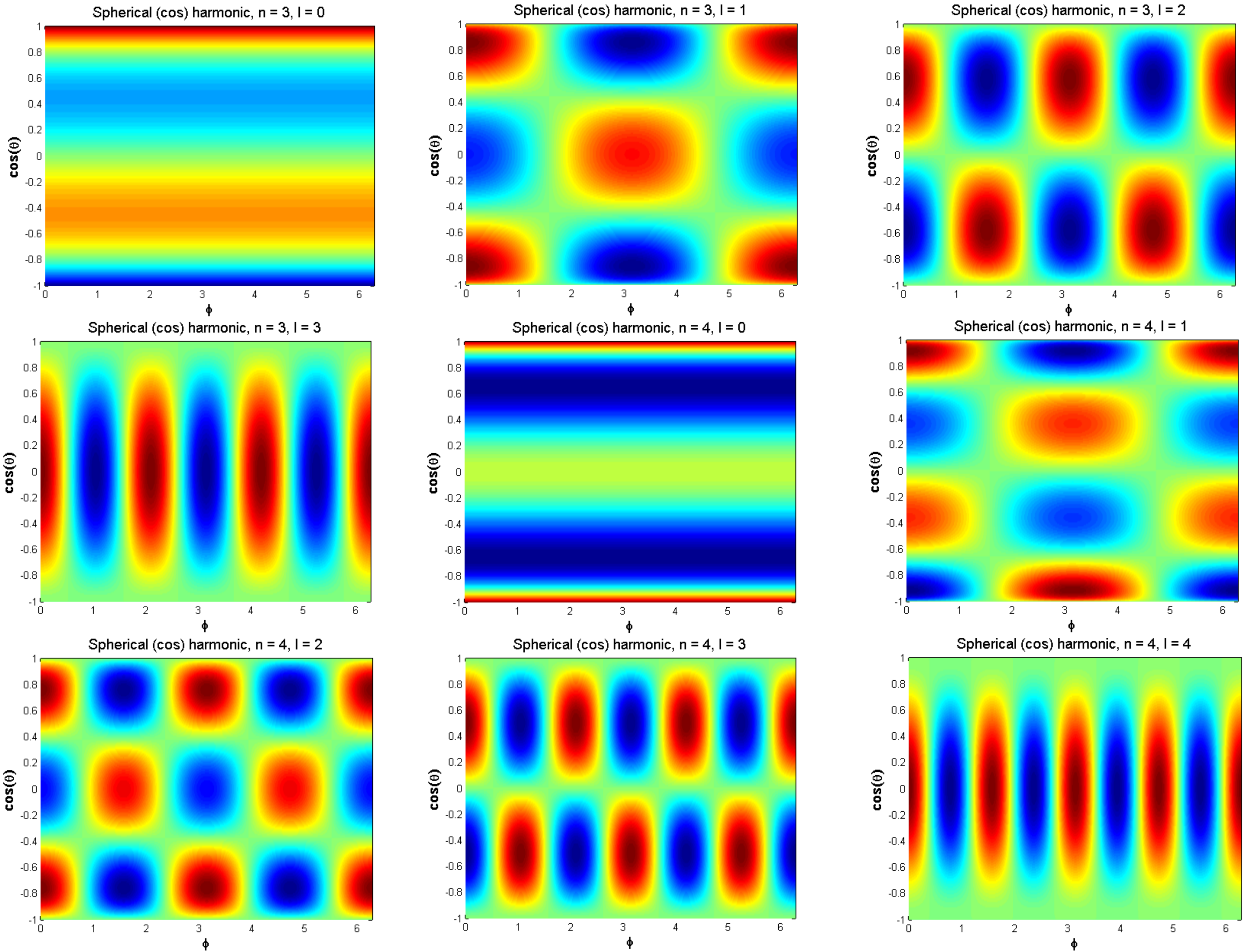








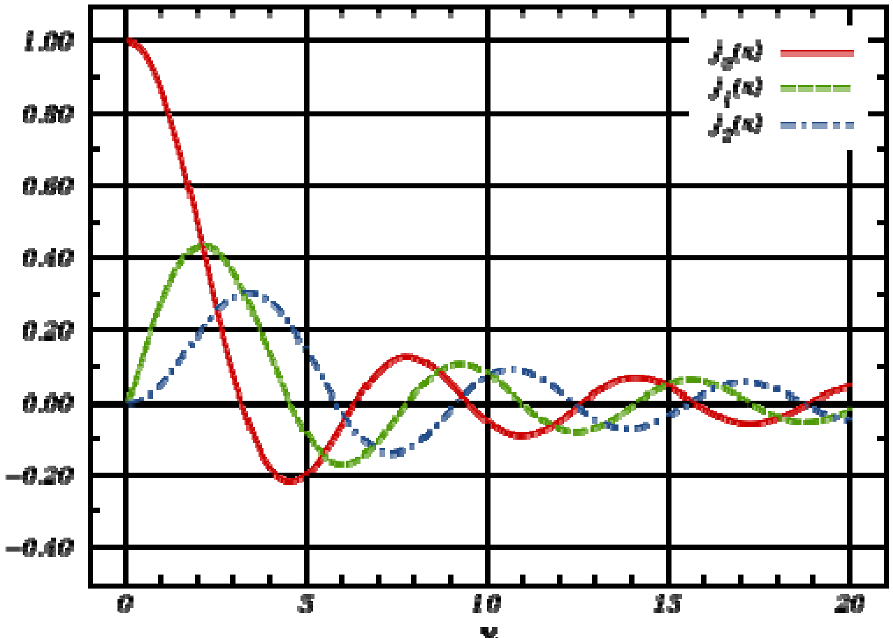
Sférické harmonické

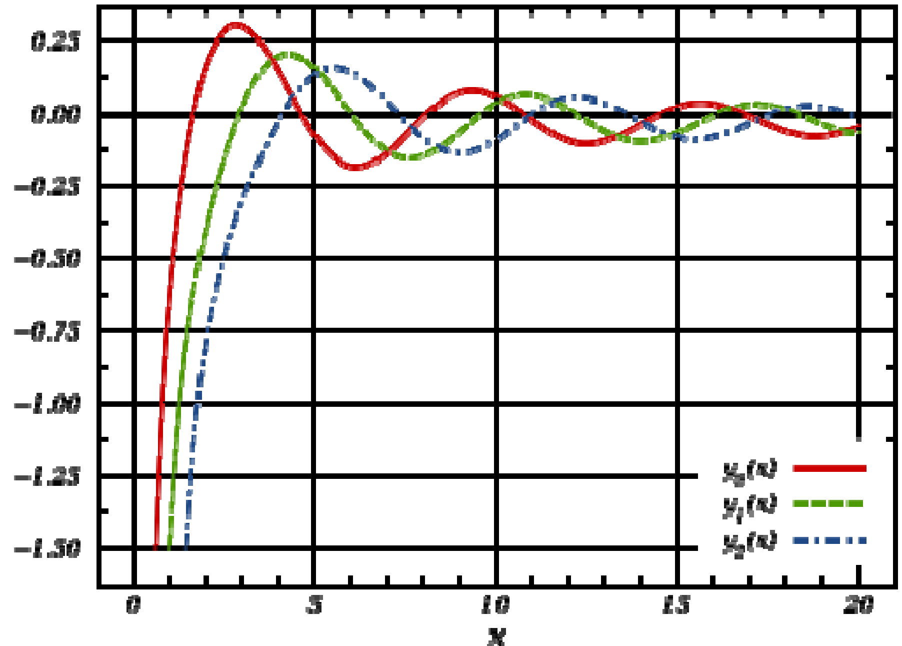


Sférické Besselovy funkce

$$j_n(\rho) = \sqrt{\frac{\pi}{2\rho}} J_{n+\frac{1}{2}}(\rho), \quad y_n(\rho) = \sqrt{\frac{\pi}{2\rho}} Y_{n+\frac{1}{2}}(\rho), \quad h_n^{(1,2)} = j_n(\rho) \pm y_n(\rho)$$

$$j_n(\rho) = (-\rho)^n \left(\frac{1}{\rho}\frac{d}{d\rho}\right)^n \frac{\sin\rho}{\rho}, \quad y_n(\rho) = -(-\rho)^n \left(\frac{1}{\rho}\frac{d}{d\rho}\right)^n \frac{\cos\rho}{\rho}$$





$$j_0(\rho) = \frac{\sin\rho}{\rho}, \qquad y_0(\rho) = -\frac{\cos\rho}{\rho},$$

$$j_1(\rho) = \frac{\sin\rho}{\rho^2} - \frac{\cos\rho}{\rho}, \qquad y_1(\rho) = -\frac{\cos\rho}{\rho^2} - \frac{\sin\rho}{\rho},$$

$$j_2(\rho) = \left(\frac{3}{\rho^2} - 1\right)\frac{\sin\rho}{\rho} - \frac{3\cos\rho}{\rho}, \qquad y_2(\rho) = \left(-\frac{3}{\rho^2} + 1\right)\frac{\cos\rho}{\rho} - \frac{3\sin\rho}{\rho},$$