NUMERICAL SOLUTION OF 2D INVISCID AND VISCOUS COMPRESSIBLE FLOW IN CHANNEL AND OVER THE PROFILE

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Abstract

The work deals with 2D numerical solution of the inviscid and viscous compressible flow in the channel and over the DCA profile in the blade cascade. Results are based on the solution of the system of Euler equations and Navier-Stokes equations.

Keywords: numerical solution, inviscid, viscous transonic flow, 2D channel, DCA profile

1 Introduction

Numerical calculations of the flow over the profile or blade cascade have been very useful in technical applications during the last years. With the significant development of modern fast computers during the last decades the computational time has been reduced and the basic numerical techniques become relatively fast as well the calculations of complicated numerical models.

Mathematical models used in this work are based on the solution of the inviscid compressible flow and viscous compressible flow over the profile. Computational domain represents GAMM channel with 10% DCA profile and section of blade cascade with 8% DCA profiles. Both solutions are compared to each other and with real measurements in the case of DCA blade cascade. The verified source code can be used for other industrial applications.

2 Governing equations

The viscous compressible flow is represented by the system of Navier-Stokes (NS) equations in following 2D conservation form

$$W_t + F(W)_x + G(W)_y = R(W)_x + S(W)_x \tag{1}$$

where

$$W = \begin{bmatrix} \rho & \rho u & \rho v & e \end{bmatrix}^T, \tag{2}$$

$$F(W) = \begin{bmatrix} \rho u & \rho u^2 + p & \rho uv & (e+p)u \end{bmatrix}^T,$$

$$G(W) = \begin{bmatrix} \rho & \rho uv & \rho v^2 + p & (e+p)v \end{bmatrix}^T,$$

$$R(W) = \begin{bmatrix} 0 & \tau_{xx} & \tau_{xy} & u\tau_{xx} + v\tau_{xy} + kT_x \end{bmatrix}^T,$$

$$S(W) = \begin{bmatrix} 0 & \tau_{xy} & \tau_{yy} & u\tau_{xy} + v\tau_{yy} + kT_y \end{bmatrix}^T,$$

$$\tau_{xx} = \frac{2}{3}\eta(2u_x - v_y), \ \tau_{xy} = \eta(2u_y + v_x), \ \tau_{yy} = \frac{2}{3}\eta(-u_x + 2v_y), \tag{3}$$

$$kT_x = \frac{\eta}{Pr}\frac{\kappa}{(\kappa-1)}\left(\frac{p}{\rho}\right)_x, \ kT_y = \frac{\eta}{Pr}\frac{\kappa}{(\kappa-1)}\left(\frac{p}{\rho}\right)_y, \tag{4}$$

$$p = (\kappa - 1)\left[e - \frac{1}{2}\rho\,(u^2 + v^2\,)\right],$$

(5)

$$\eta(T) = \left(\frac{T}{T_\infty}\right)^{3/4}, \quad Re = \rho_\infty c_\infty L_r / \eta_\infty.$$

(6)

Re represents Reynolds number, $Pr = 0.7$ is Prandtl number and $\kappa = 1.4$ is Poisson constant.

Inviscid flow represented by the system of Euler equations is simply reached after neglecting of viscosity [1].

3 Numerical schemes

Following numerical schemes were used for modelling of flows, Lax-Wendroff scheme - McCormack modification and Runge-Kutta scheme of 3rd order, both in variant for the finite volume method. Grid was used as a non-orthogonal with quadrilateral cells.

a) Lax-Wendroff scheme – MacCormack modification (MC)

Predictor step

$$W_{i,j}^{n+1/2} = W_{i,j}^n - \frac{\Delta t}{\mu_{i,j}}\sum_{k=1}^{4}\left[\left(\tilde{F}_k^n - \frac{1}{Re}R_k^n\right)\Delta y_k - \left(\tilde{G}_k^n - \frac{1}{Re}S_k^n\right)\Delta x_k\right]$$

(7)

Corrector step

$$W_{i,j}^{n+1} = \frac{1}{2}\left(W_{i,j}^n + W_{i,j}^{n+1/2}\right) - \frac{\Delta t}{2\mu_{i,j}}\sum_{k=1}^{4}\left[\left(\tilde{F}_k^{n+1/2} - \frac{1}{Re}R_k^{n+1/2}\right)\Delta y_k - \left(\tilde{G}_k^{n+1/2} - \frac{1}{Re}S_k^{n+1/2}\right)\Delta x_k\right] + AD\left(W_{i,j}^n\right)$$

(8)

b) Runge-Kutta scheme of 3rd order (RK)

$$ResW_{i,j}^{(r)} = \frac{1}{\mu_{i,j}}\sum_{k=1}^{4}\left[\left(\tilde{F}_k^n - \frac{1}{Re}R_k^n\right)\Delta y_k - \left(\tilde{G}_k^n - \frac{1}{Re}S_k^n\right)\Delta x_k\right]$$

(9)

$$W_{i,j}^{(0)} = W_{i,j}^n$$
$$W_{i,j}^{(r+1)} = W_{i,j}^{(0)} - \alpha_r\Delta t ResW_{i,j}^{(r)} + AD\left(W_{i,j}^n\right), \quad r = 0, 1, 2$$
$$W_{i,j}^{n+1} = W_{i,j}^{(3)}$$

(10)

$$\alpha_{0,1} = 1/2, \quad \alpha_2 = 1$$

Jameson artificial dissipation model was applied to damp the oscillations.

$$AD\left(W_{i,j}^n\right) = C_1\psi_1\left(W_{i+1,j}^n - 2W_{i,j}^n + W_{i-1,j}^n\right) + C_2\psi_2\left(W_{i,j-1}^n - 2W_{i,j}^n + W_{i,j-1}^n\right)$$

(11)

$$\psi_1 = \frac{\left|p_{i+1,j}^n - 2p_{i,j}^n + p_{i-1,j}^n\right|}{\left|p_{i+1,j}^n\right| + \left|2p_{i,j}^n\right| + \left|p_{i-1,j}^n\right|}, \quad \psi_2 = \frac{\left|p_{i,j+1}^n - 2p_{i,j}^n + p_{i,j-1}^n\right|}{\left|p_{i,j+1}^n\right| + \left|2p_{i,j}^n\right| + \left|p_{i,j-1}^n\right|}$$

4 Formulation of solved problems

In a case of GAMM channel it was used grid with 150x50 cells, in the case of DCA blade casdade the grid with 240x60 cells was used for inviscid flow and 160x60 cells for viscous flow with proper fine mesh close to the walls.

Inlet boundary conditions were realized for inviscid flow as follow: velocity M_∞ together with angle of attack α, density ρ_∞ and total energy per volume e_∞ were set; pressure p_∞ was extrapolated from the flow field. In a case of viscous flow the inlet pressure p_∞ was set.

Outlet boundary conditions were the same for the both type of flow. Outlet pressure was set and other variables extrapolated from the flow field.

Solid wall condition in a case of GAMM channel and combination of solid wall condition on the profile with periodic condition for the free stream were realized.

5 Numerical results

Transonic inviscid flow and viscous flow with inlet Mach number $M = 0.675$ in GAMM channel is shown at Fig. 1, Fig. 2 and Fig. 3 for the MacCormack and Runge-Kutta scheme.

In a case of blade cascade different inlet Mach number were tested together with the different attack angle of the inlet stream. It was compared different behaviour by using the different numerical schemes as well the differences between calculation of the inviscid and viscous flow due to viscosity. Results were also compared with the experimental data.
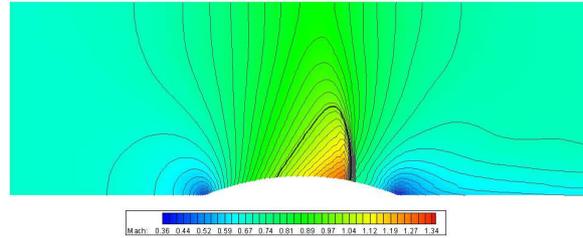
Figure 1: Inviscid flow over 10% profile in GAMM channel $M = 0.675$ - MC scheme
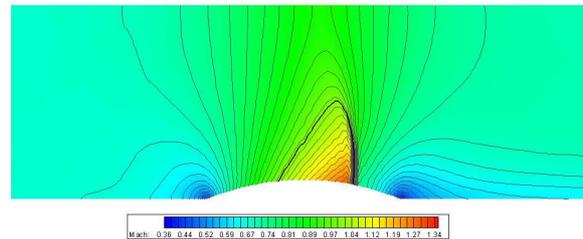
Figure 2: Inviscid flow over 10% profile in GAMM channel $M = 0.675$ - RK scheme
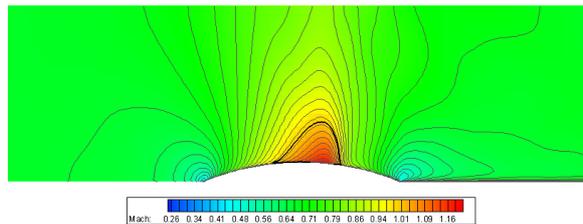
Figure 3: Viscous laminar flow $Re = 1.27 \cdot 10^7$ in GAMM channel $M = 0.675$ - MC scheme
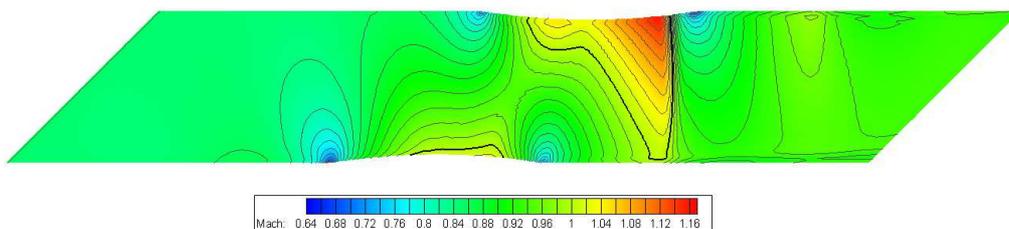
Figure 4: Inviscid flow in 8% DCA blade cascade $M = 0.88$, $\alpha = 0.6°$ - MC scheme
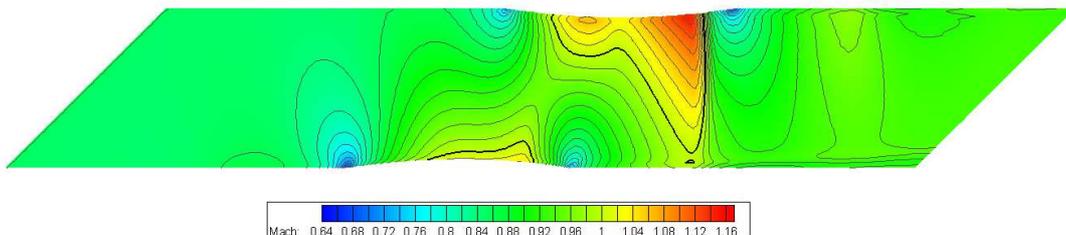
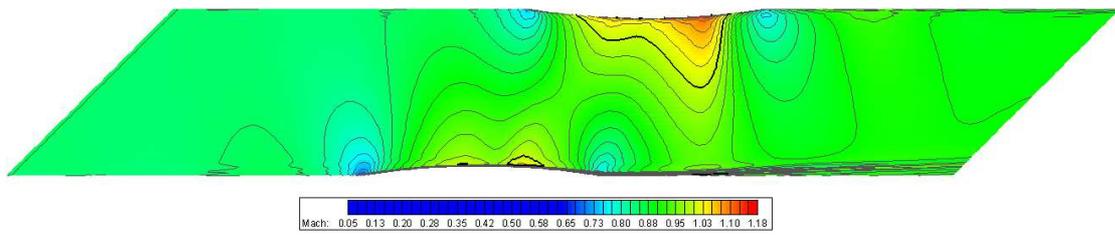*Figure 5: Inviscid flow in 8% DCA blade cascade $M = 0.88$, $\alpha = 0.6°$ - RK scheme*

Figure 6: Viscous laminar flow $Re = 10^6$ in 8% DCA blade cascade $M = 0.88$, $\alpha = 0.6°$ - MC scheme

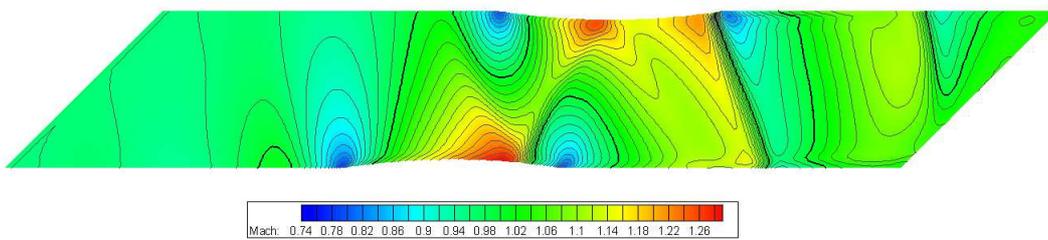*Figure 7: Experimental data $M = 0.832$, $\alpha = 0°$*

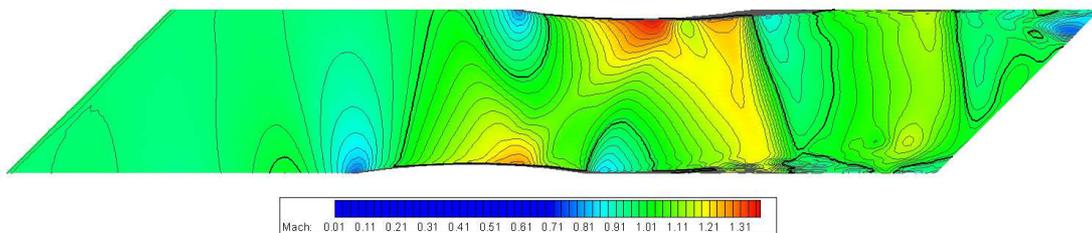*Figure 8: Inviscid flow in 8% DCA blade cascade $M = 1.02$, $\alpha = 0°$ - MC scheme*

Figure 9: Viscous laminar flow $Re = 10^6$ in 8% DCA blade cascade $M = 1.02$, $\alpha = 0.5°$ - MC scheme

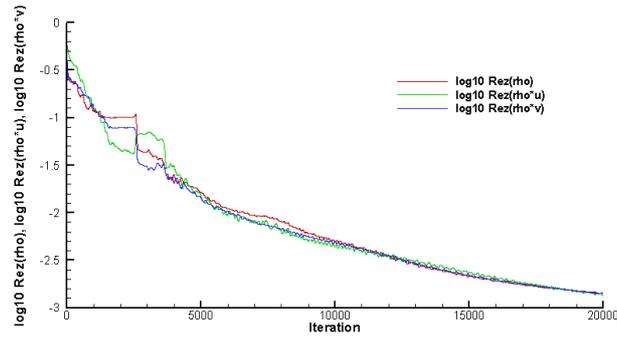

Figure 10: Experimental data $M = 0.946$, $\alpha = 0°$

Figure 11: History of convergence process to the Fig.8

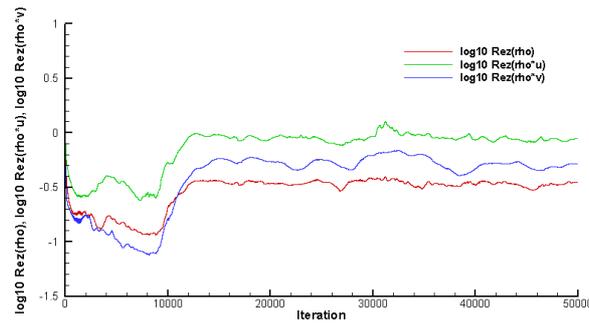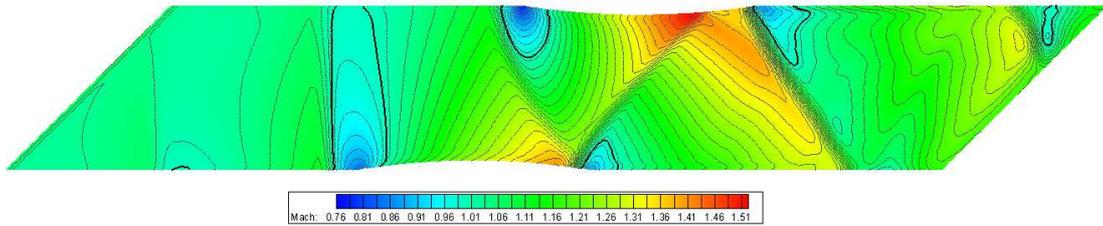*Figure 12: History of convergence process to the Fig.9*

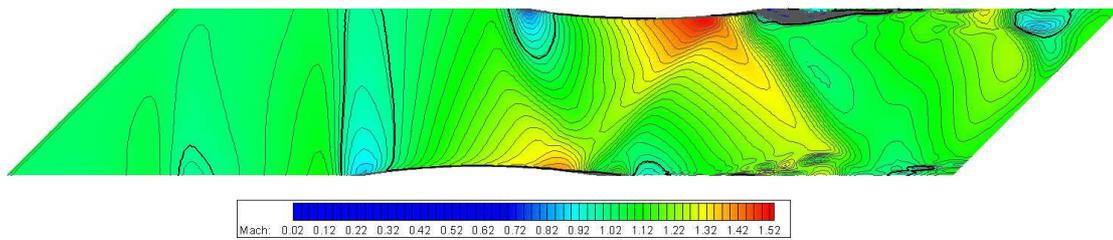*Figure 13: Inviscid flow in 8% DCA blade cascade M = 1.12 , $\alpha = 0°$ - MC scheme*

Figure 14: Viscous laminar flow Re = 10^6 in 8% DCA blade cascade M = 1.12 , $\alpha = 0.5°$ - MC scheme
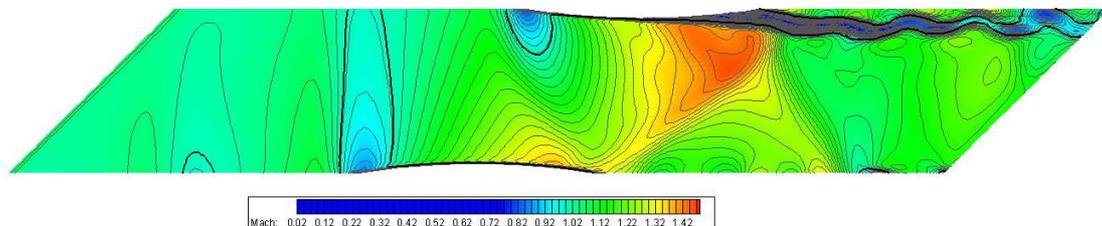

Figure 15: Viscous laminar flow Re = 10^6 in 8% DCA blade cascade M = 1.12 , $\alpha = 0.5°$ - RK scheme
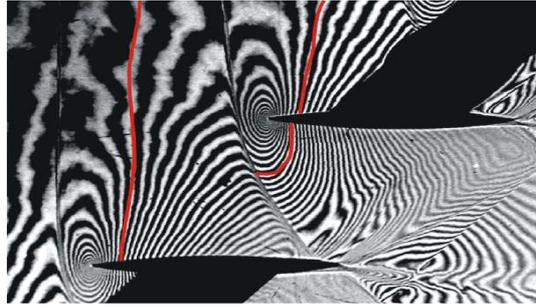
Figure 14: Experimental data $M = 1.013$, $\alpha = 0°$

6 Conclusions

The possibilities of source code use are shown in this contribution. Achieved results are comparable with other authors. Setting of artificial dissipation parameters together with inlet Mach number and angle of attack is quite time consuming process. Slightly higher inlet Mach number has to be used to eliminate the influence of numerical viscosity. The numerical results give us quite good correlation with the experimental data. The source code can be used for the calculations of transonic flow for variable profiles. The next step will be extension to viscous turbulent model.

Acknowledgements

References

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