Quadrupole radiation diagram

$$\bar{Q} = \begin{pmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{pmatrix}, \quad \mathrm{Tr}\{\bar{Q}\} = 0 \quad \Rightarrow \quad Q_1 + Q_2 + Q_3 = 1;$$

$$\vec{Q} = \bar{Q} \cdot \vec{r} = \vec{x}^0 Q_1 \sin\vartheta\cos\varphi + \vec{y}^0 Q_2 \sin\vartheta\sin\varphi - \vec{z}^0(Q_1 + Q_2)\cos\vartheta;$$

$$\vec{r}^0 \times \vec{Q} = -\vec{x}^0\left[\left(\tfrac{1}{2}Q_1 + Q_2\right)\sin 2\vartheta\sin\varphi\right] +$$
$$+ \vec{y}^0\left[\left(Q_1 + \tfrac{1}{2}Q_2\right)\sin 2\vartheta\cos\varphi\right] +$$
$$+ \vec{z}^0\left[\tfrac{1}{2}\left(Q_2 - Q_1\right)\sin^2\vartheta\sin 2\varphi\right].$$

$$F(\vartheta,\varphi) = \left|\vec{r}^0 \times \vec{Q}\right|^2$$
$$= (Q_1/2 + Q_2)^2 \sin^2(2\vartheta)\sin^2\varphi$$
$$+ (Q_1 + Q_2/2)^2 \sin^2(2\vartheta)\cos^2\varphi$$
$$+ \left((Q_2 - Q_1)/2\right)^2 \sin^4\vartheta\sin^2(2\varphi)$$

Coordinate axes :

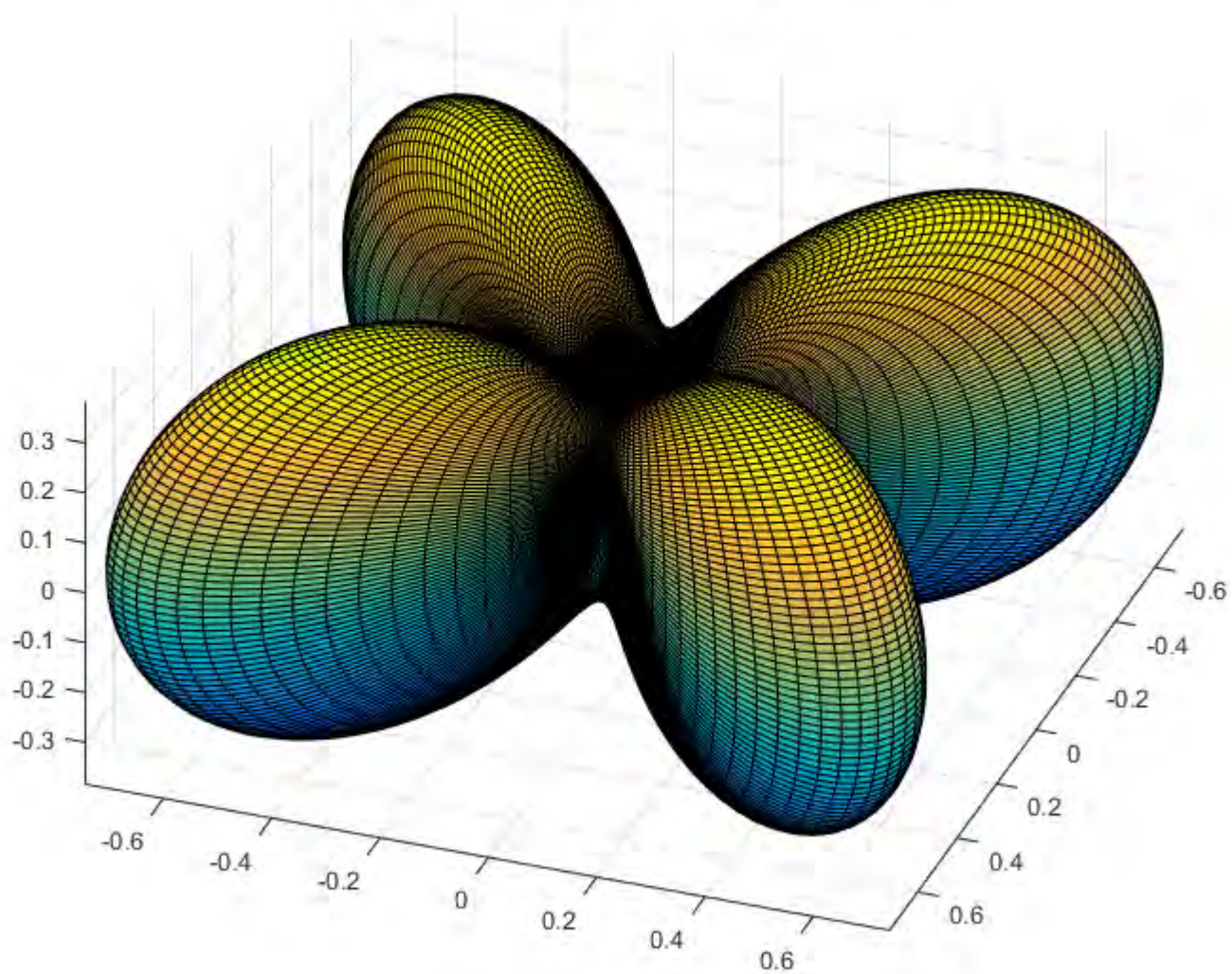$x : \vartheta = \pi/2, \quad \varphi = 0 \Rightarrow$ no radiation!
$y : \vartheta = \pi/2, \quad \varphi = \pi/2 \Rightarrow$ no radiation!
$z : \vartheta = 0 \Rightarrow$ no radiation!

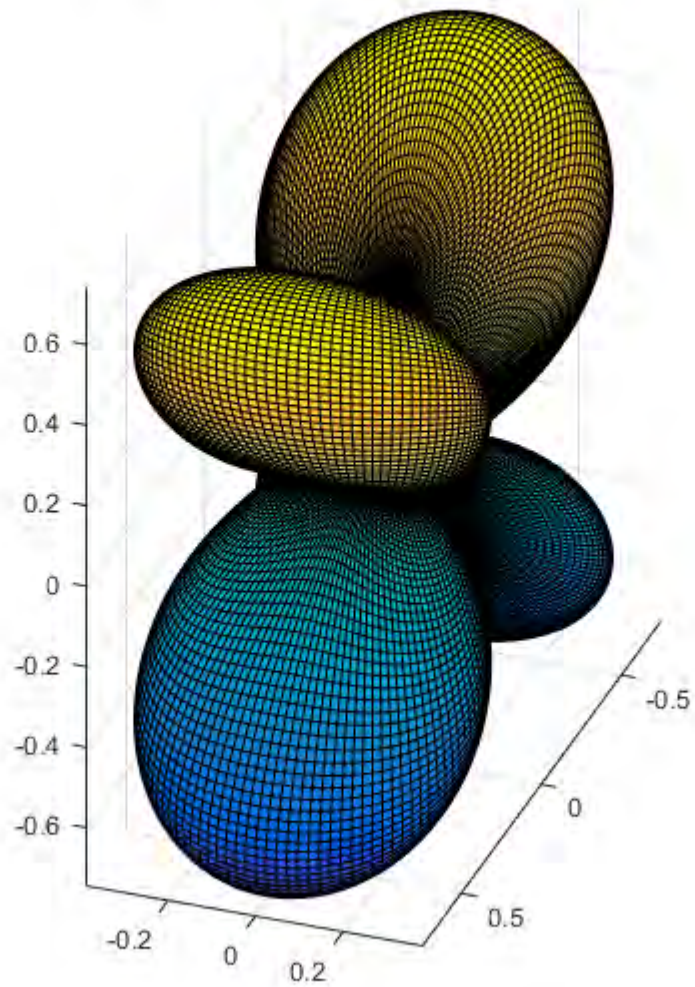**Electric quadrupole does not radiate in the direction of its main axes!**

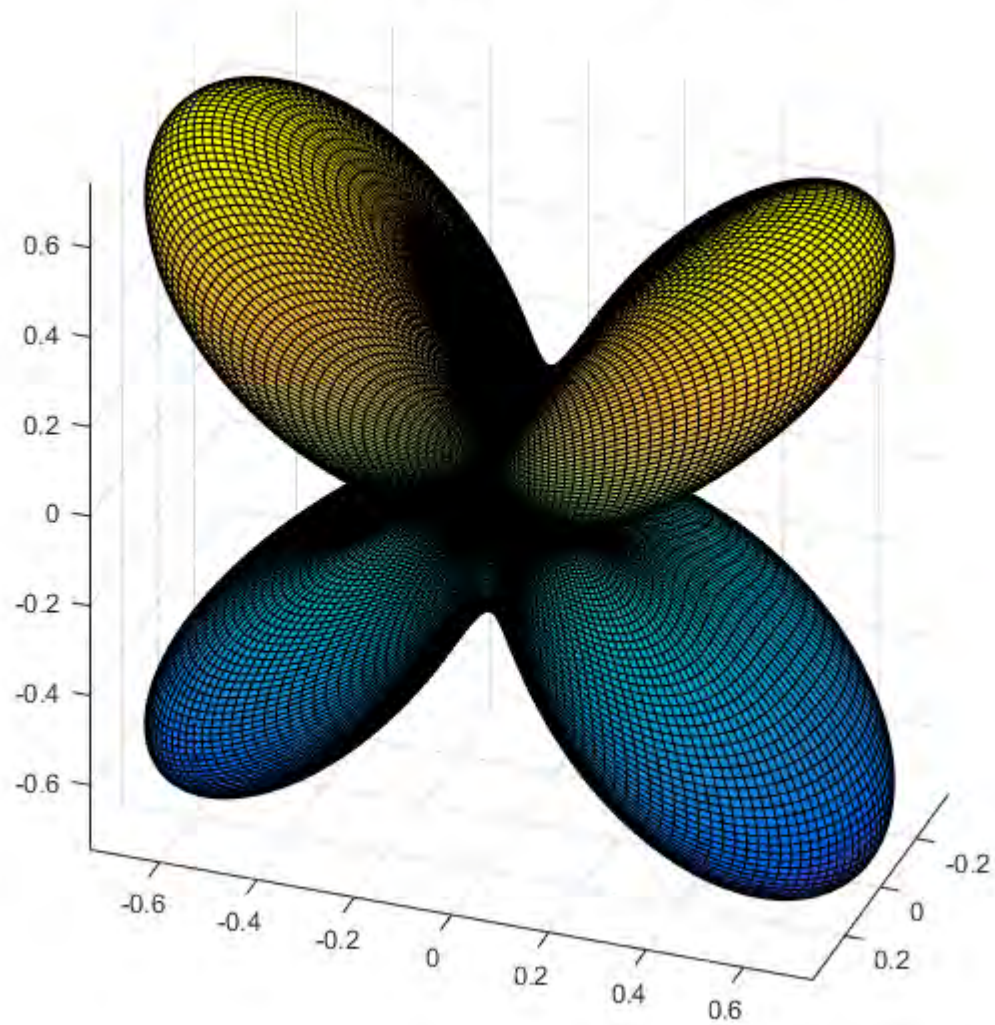**Quadrupole radiation diagram**
$Q_1 = 1.0, Q_2 = -1.0, Q_3 = -0.0$

Quadrupole radiation diagram
$Q_1 = 1.0, Q_2 = 0.0, Q_3 = -1.0$
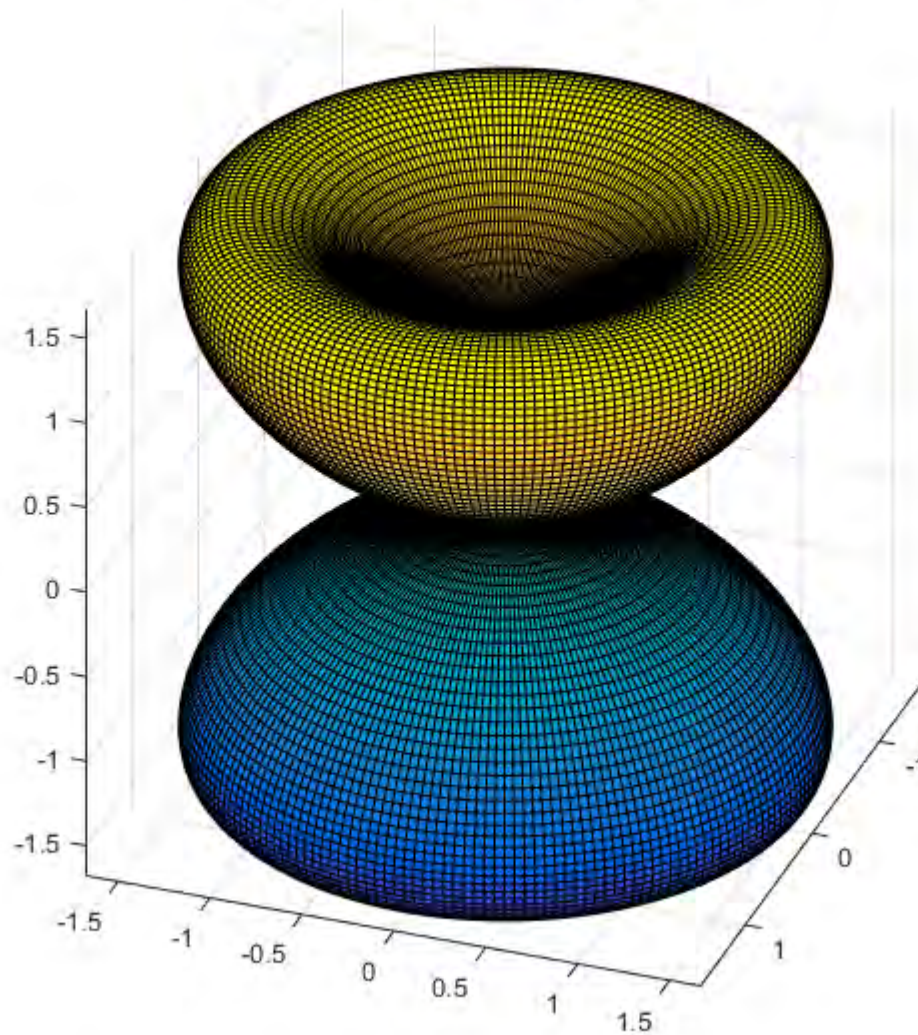
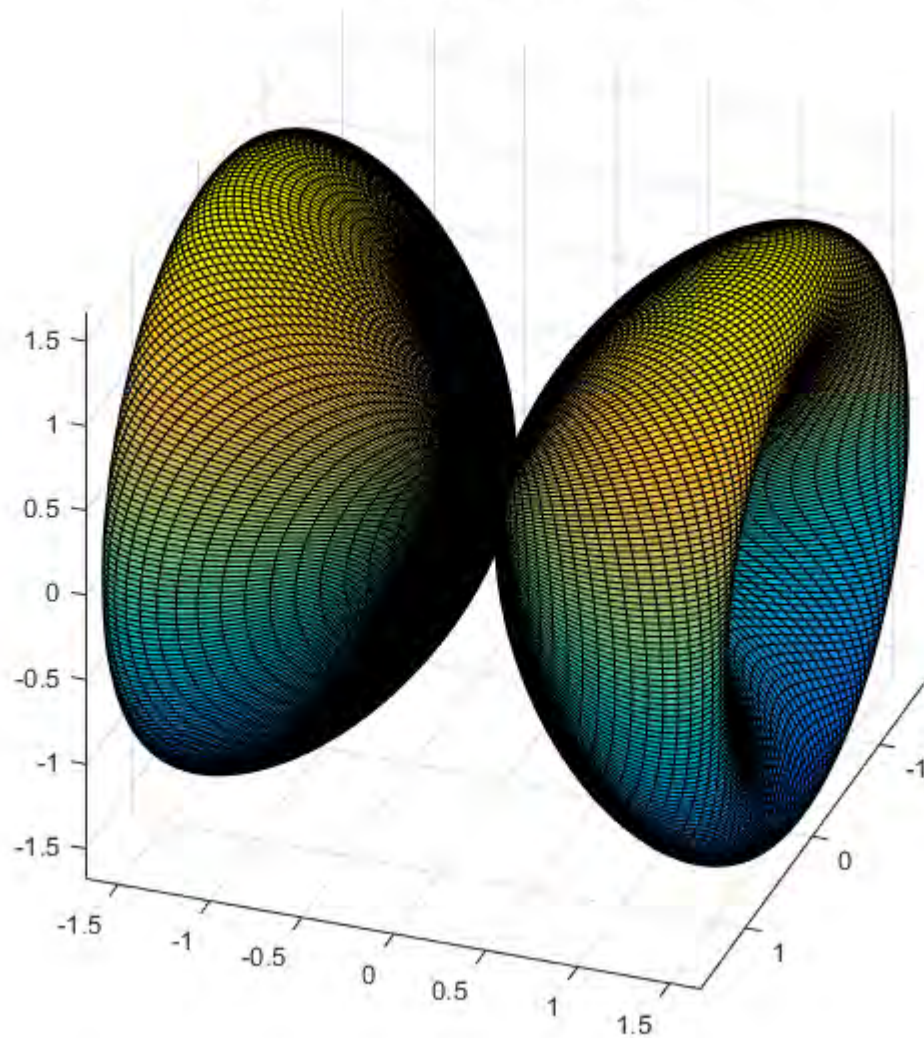Quadrupole radiation diagram
$Q_1 = 0.0, Q_2 = 1.0, Q_3 = -1.0$

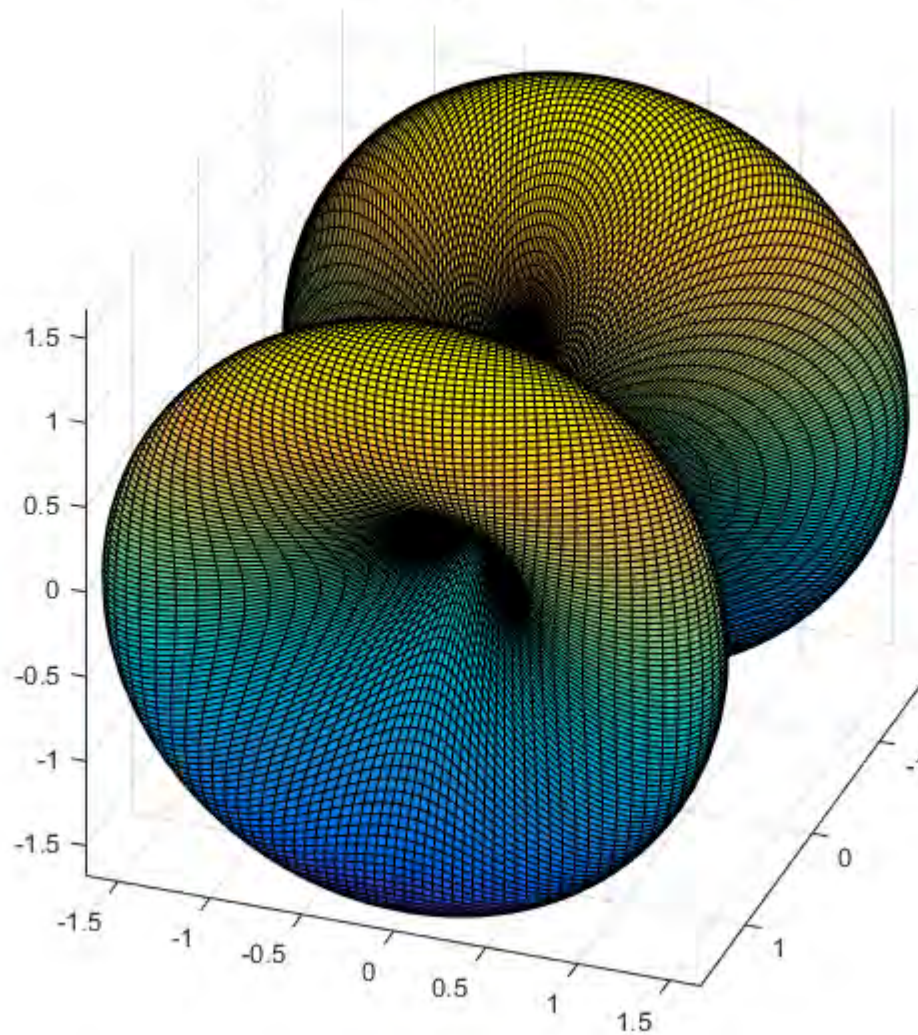Quadrupole radiation diagram
$Q_1 = 1.0, Q_2 = 1.0, Q_3 = -2.0$

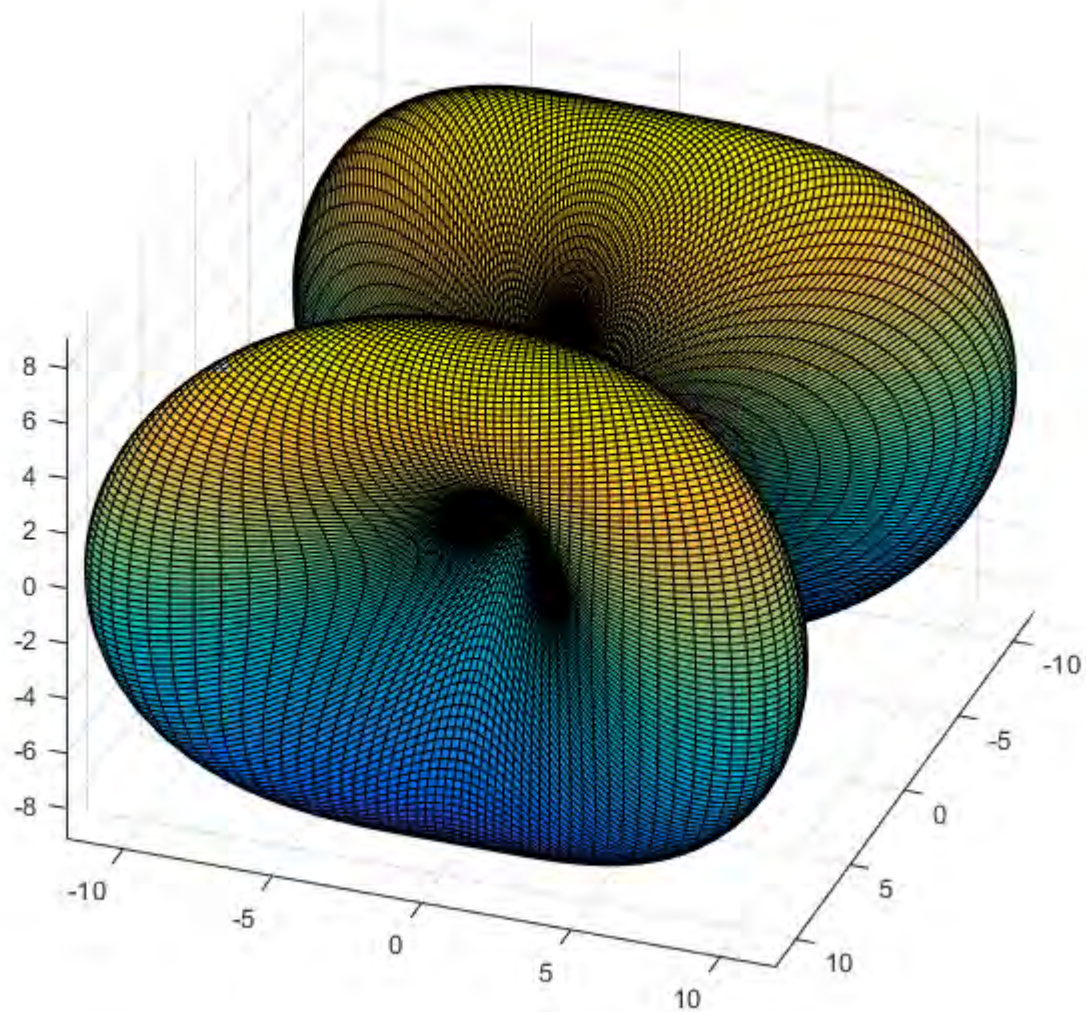**Quadrupole radiation diagram**
$Q_1 = 1.0, Q_2 = -2.0, Q_3 = 1.0$

Quadrupole radiation diagram
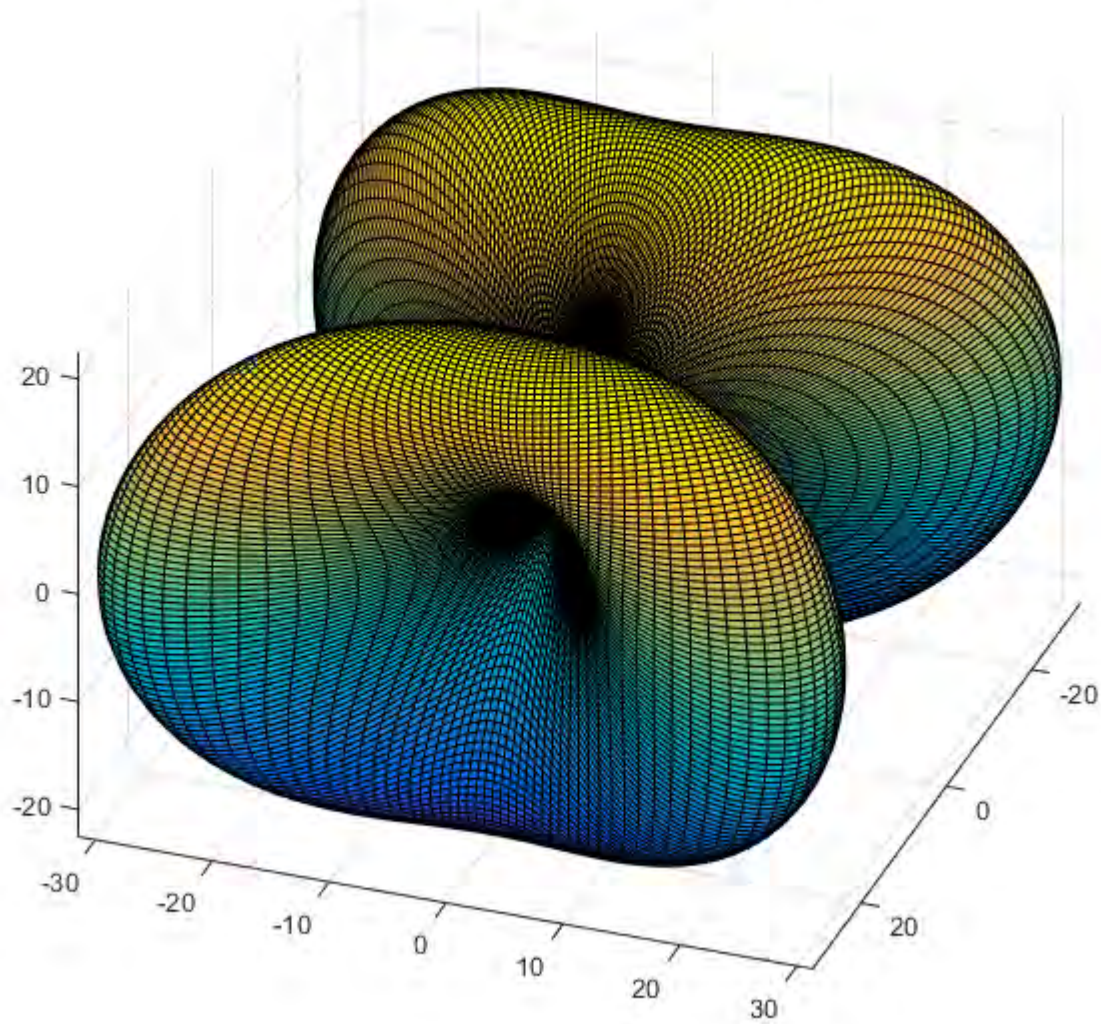$Q_1 = -2.0$, $Q_2 = 1.0$, $Q_3 = 1.0$

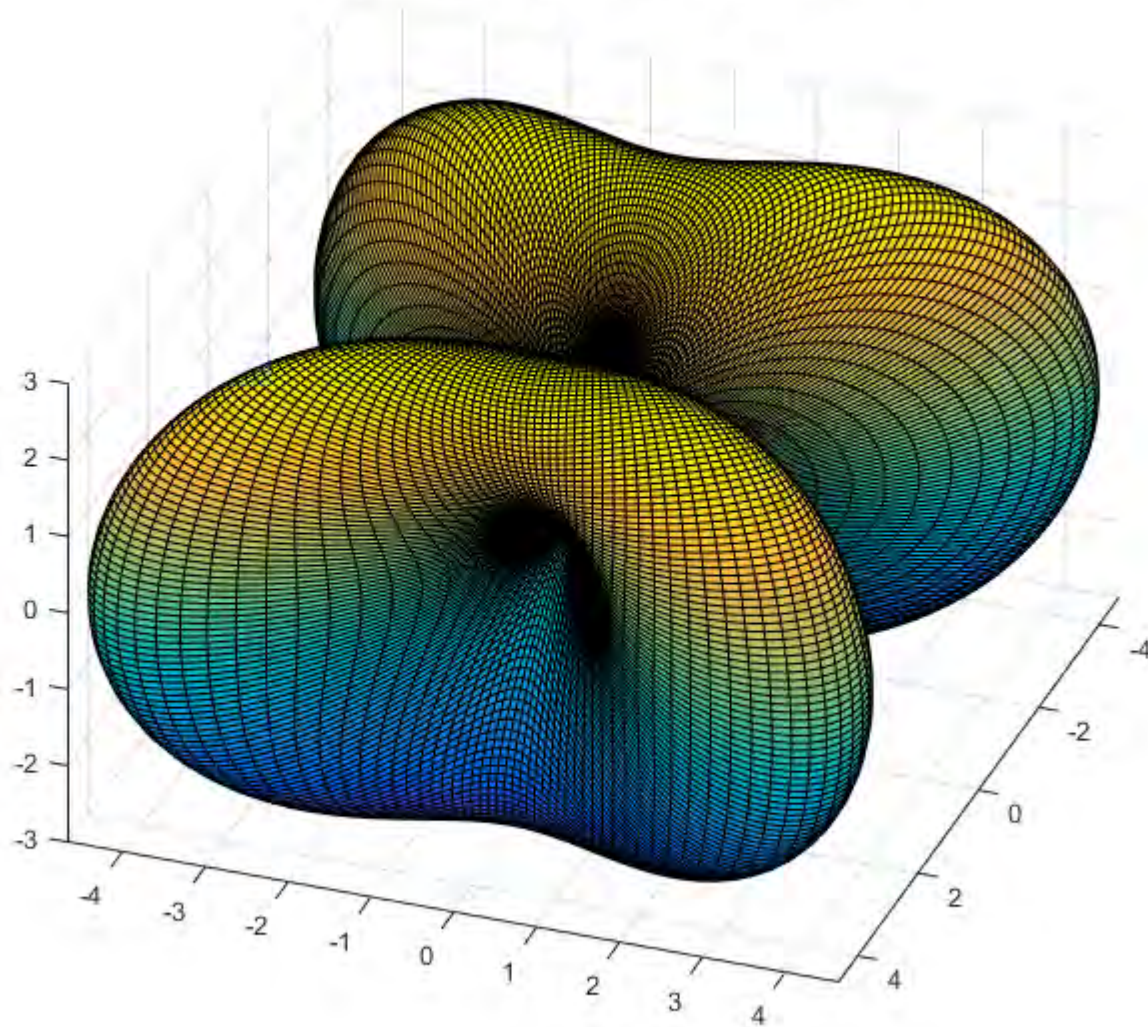Quadrupole radiation diagram
$Q_1 = 5.0, Q_2 = -3.0, Q_3 = -2.0$

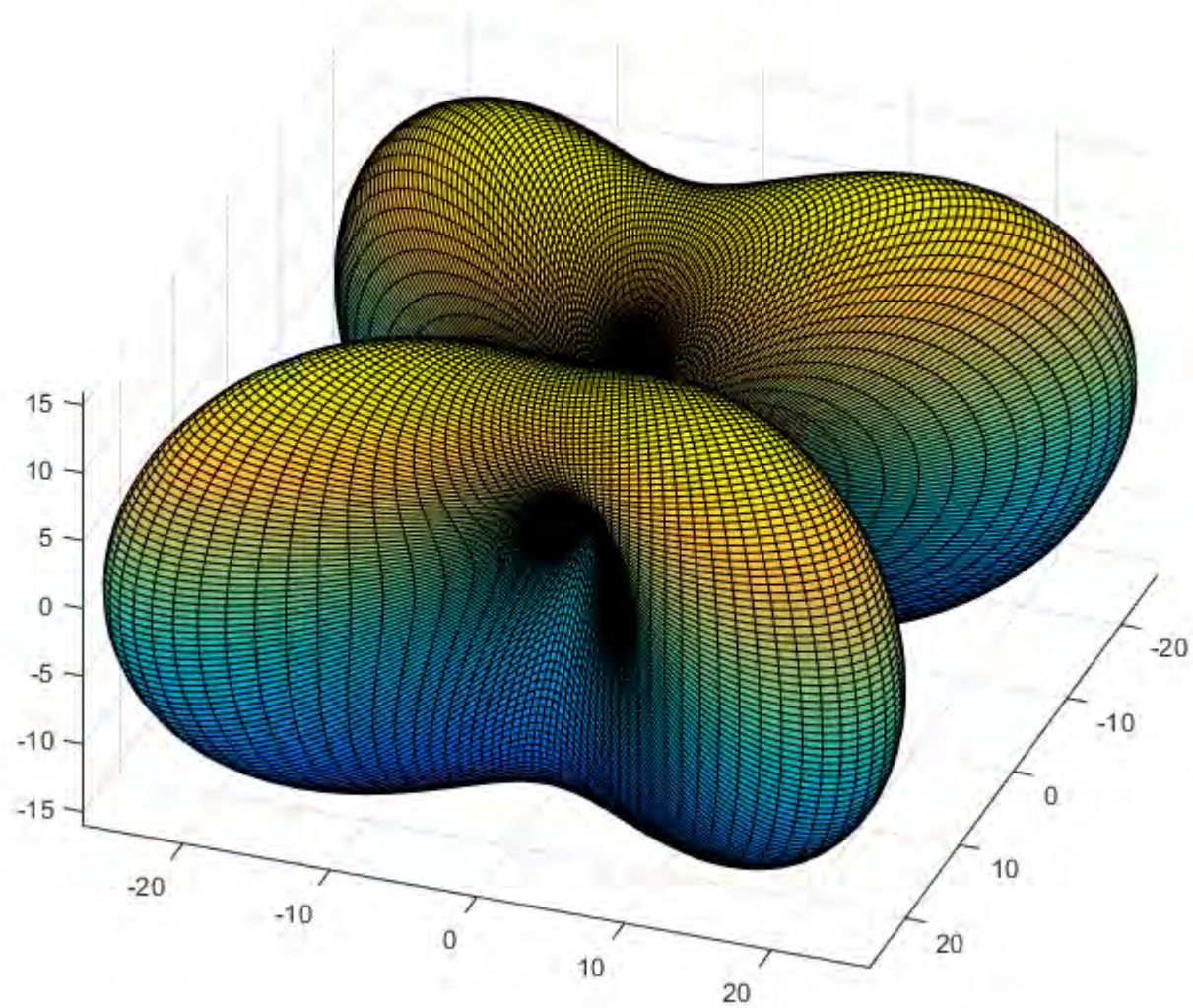**Quadrupole radiation diagram**
$Q_1 = 8.0, Q_2 = -5.0, Q_3 = -3.0$

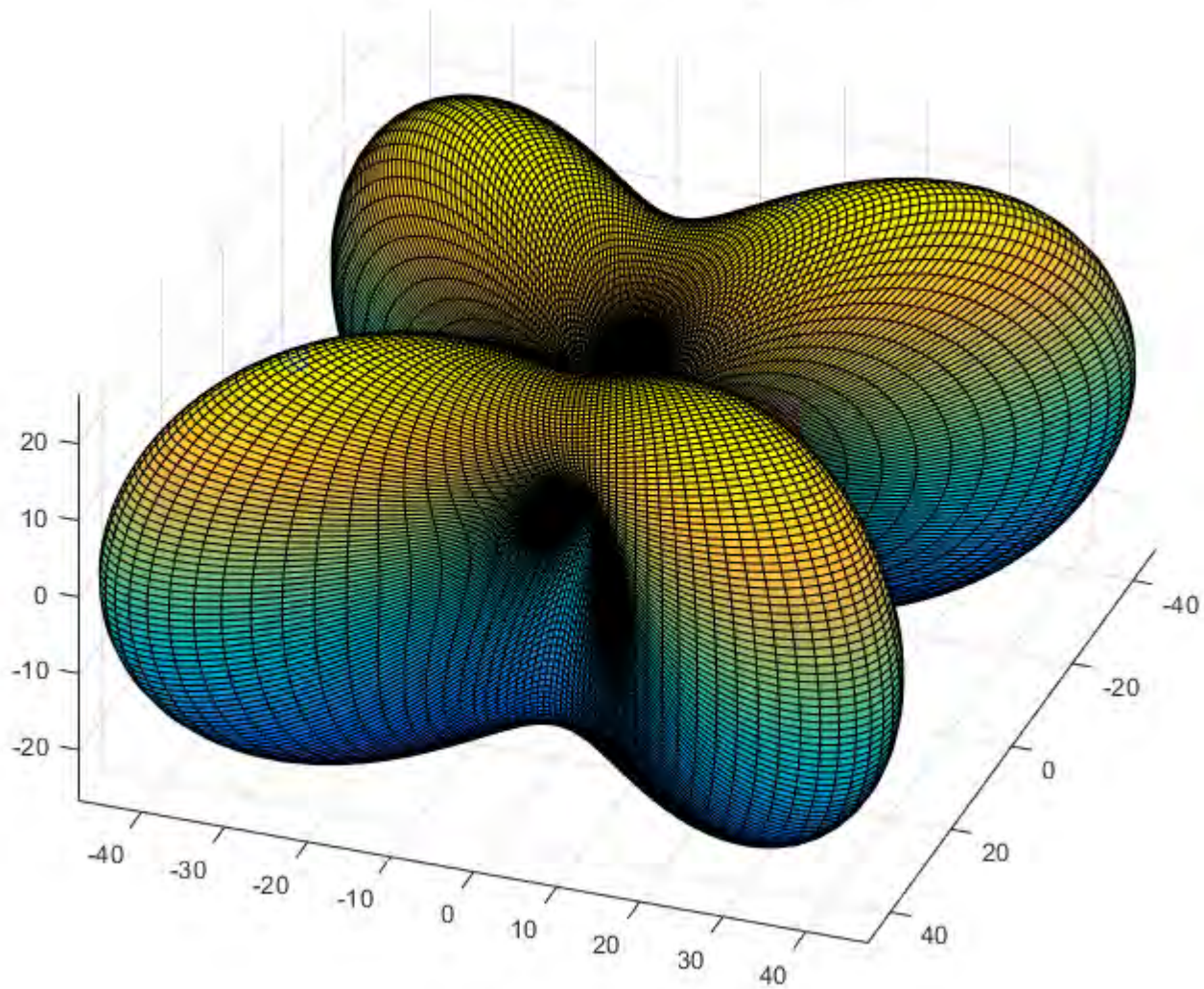Quadrupole radiation diagram
$Q_1 = 3.0, Q_2 = -2.0, Q_3 = -1.0$

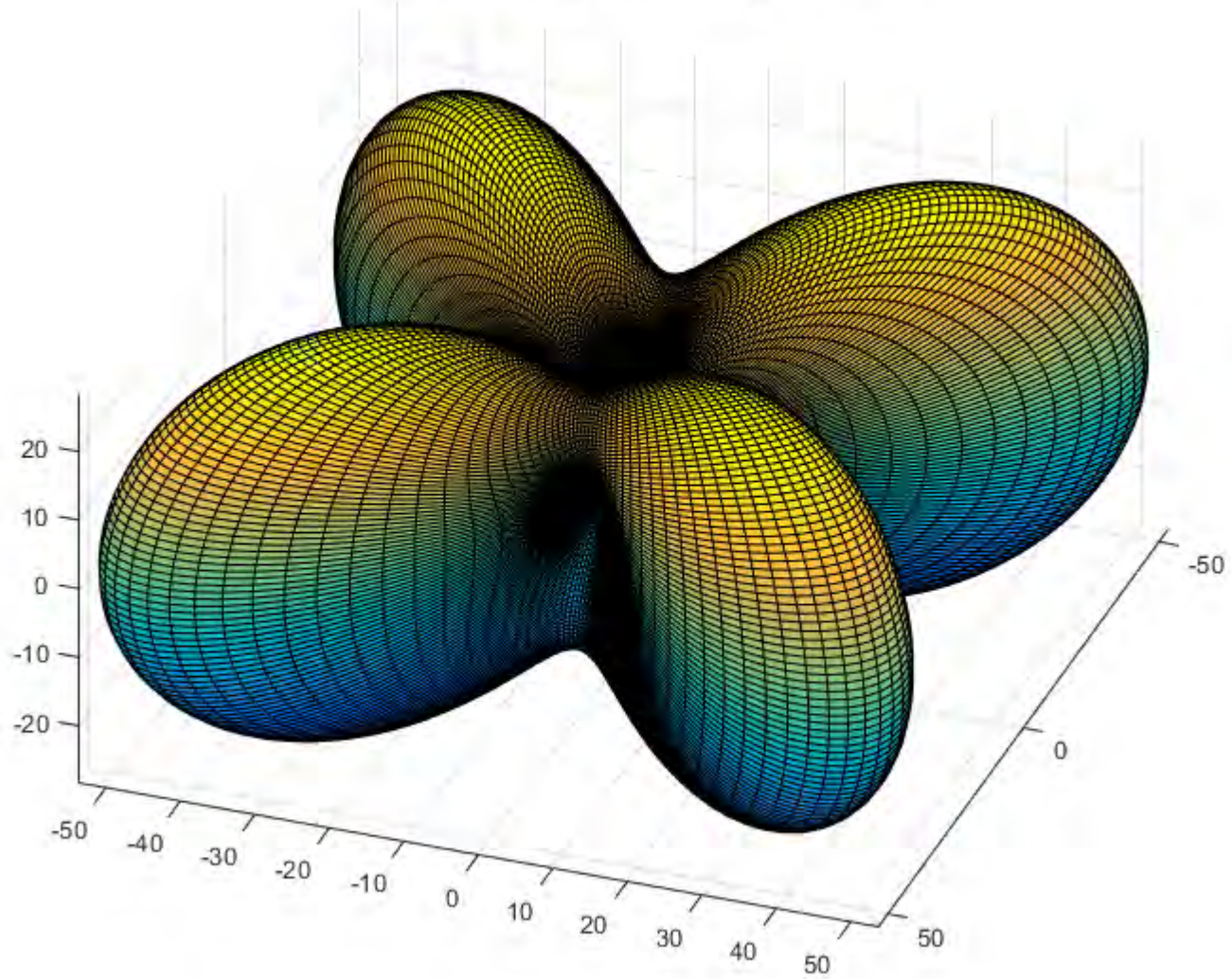**Quadrupole radiation diagram**
$Q_1 = 7.0, Q_2 = -5.0, Q_3 = -2.0$

Quadrupole radiation diagram
$Q_1 = 9.0, Q_2 = -7.0, Q_3 = -2.0$

Quadrupole radiation diagram
$Q_1 = 9.0, Q_2 = -8.0, Q_3 = -1.0$

Quadrupole radiation diagram
$Q_1 = 1.0, Q_2 = -1.0, Q_3 = -0.0$