Gas kinetic scheme for Baer-Nunziato compressible two phase flow

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Baer-Nunziato model

$$\frac{\partial \phi_s}{\partial t} + u_s \frac{\partial \phi_s}{\partial x} = 0$$

$$\begin{pmatrix} \phi_s \rho_s \\ \phi_s \rho_s u_s \\ \phi_s \rho_s E_s \end{pmatrix}_t + \begin{pmatrix} \phi_s \rho_s u_s \\ \phi_s(\rho_s u_s^2 + p_s) \\ \phi_s u_s(\rho_s E_s + p_s) \end{pmatrix}_x = p_g \frac{\partial \phi_s}{\partial x} \begin{pmatrix} 0 \\ 1 \\ u_s \end{pmatrix}$$

$$\begin{pmatrix} \phi_g \rho_g \\ \phi_g \rho_g u_g \\ \phi_g \rho_g E_g \end{pmatrix}_t + \begin{pmatrix} \phi_g \rho_g u_g \\ \phi_g(\rho_s u_g^2 + p_g) \\ \phi_g u_g(\rho_g E_g + p_g) \end{pmatrix}_x = -p_g \frac{\partial \phi_s}{\partial x} \begin{pmatrix} 0 \\ 1 \\ u_s \end{pmatrix}$$

Let $\quad \rho^{(1)} = \rho_s \phi_s, \rho^{(2)} = \rho_g \phi_g; \quad p^{(1)} = p_s \phi_s, p^{(2)} = p_g \phi_g;$
and $u^{(1)} = u_s, u^{(2)} = u_g; \quad E^{(1)} = E_s, E^{(2)} = E_g, z = 1/\phi_s.$

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and $u^{(1)} = u_s, u^{(2)} = u_g; \quad E^{(1)} = E_s, E^{(2)} = E_g, z = 1/\phi_s.$

$$\left(\rho^{(1)}z\right)_t + \left(\rho^{(1)}zu^{(1)}\right)_x = 0$$

$$\begin{pmatrix} \rho^{(1)} \\ \rho^{(1)}u^{(1)} \\ \rho^{(1)}E^{(1)} \end{pmatrix}_t + \begin{pmatrix} \rho^{(1)}u^{(1)} \\ \rho^{(1)}u^{(1)}u^{(1)} + p^{(1)} \\ u^{(1)}(\rho^{(1)}E^{(1)} + p^{(1)}) \end{pmatrix}_x = p_g\frac{\partial\phi_s}{\partial x}\begin{pmatrix} 0 \\ 1 \\ u^{(1)} \end{pmatrix}$$

$$\begin{pmatrix} \rho^{(2)} \\ \rho^{(2)}u^{(2)} \\ \rho^{(2)}E^{(2)} \end{pmatrix}_t + \begin{pmatrix} \rho^{(2)}u^{(2)} \\ \rho^{(2)}u^{(2)}u^{(2)} + p^{(2)} \\ u^{(2)}(\rho^{(2)}E^{(2)} + p^{(2)}) \end{pmatrix}_x = -p_g\frac{\partial\phi_s}{\partial x}\begin{pmatrix} 0 \\ 1 \\ u^{(1)} \end{pmatrix}$$

$$w_t^{(i)} + F_x^{(i)} = s^{(i)}, i = 0, 1, 2. \tag{1}$$

Micro-modelling from the Boltzmann equation

$$f_t^{(1)} + u f_x^{(1)} + a f_u^{(1)} = Q^{(1)}$$
$$f_t^{(2)} + u f_x^{(2)} - a f_u^{(1)} = Q^{(2)}$$

Micro-modelling from the Boltzmann equation

$$f_t^{(1)} + uf_x^{(1)} + af_u^{(1)} = Q^{(1)}$$
$$f_t^{(2)} + uf_x^{(2)} - af_u^{(1)} = Q^{(2)}$$

Relation to the macro model

$$w^{(i)} = \int \psi^{(i)} g^{(i)} dvd\xi, \quad F^{(i)} = \int v\psi^{(i)} g^{(i)} dvd\xi,$$
$$\psi^{(i)} = \left(1, v, \frac{1}{2}(v^2 + \xi^2)\right)^T, i = 1, 2; \quad \psi^{(0)} = z.$$

$$g^{(i)} = \rho^{(i)} \left(\frac{\lambda}{\pi}\right)^{\frac{N+3}{2}} exp\left(-\lambda^{(i)}\left[(v - u^{(i)})^2 + \xi^2\right]\right)$$

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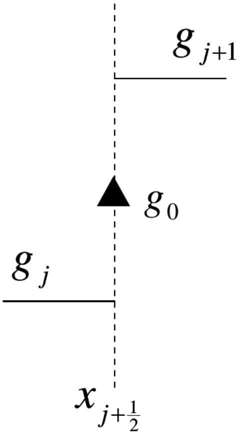
By choosing $a = \frac{p_g}{\rho^{(1)}} \frac{\partial \phi_s}{\partial x}$ we obtain the BN model

$$w_t^{(i)} + F_x^{(i)} = s^{(i)}, i = 0, 1, 2.$$



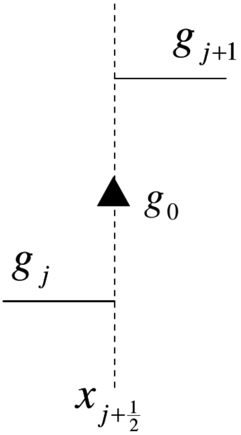
$$w_j^{n+1,(i)} = w_j^{n,(i)} + \frac{1}{\Delta x}\int_0^{\Delta t} (F_{j-1/2}^{(i)}(t) - F_{j+1/2}^{(i)}(t))dt + \Delta s_j^{(i)}$$

$$F_{j+1/2}^{(i)}(t) = \int v\psi^{(i)} f_{j+1/2}^{(i)}(t) dv d\xi$$

$$f_{j+1/2}^{(1)}(x_{j+1/2}, t, v) = \left(1 - 2\lambda a t (v - u^{(1)})\right) f_0^{(1)}(x_{j+1/2} - vt)$$

$$f_0^{(1)}(x_{j+1/2} - vt) = \begin{cases} g_j & v > 0 \\ g_{j+1}, & v < 0 \end{cases}$$

Gas kinetic solver



$$w_j^{n+1,(i)} = w_j^{n,(i)} + \frac{1}{\Delta x}\int_0^{\Delta t}(F_{j-1/2}^{(i)}(t) - F_{j+1/2}^{(i)}(t))dt + \Delta s_j^{(i)}$$

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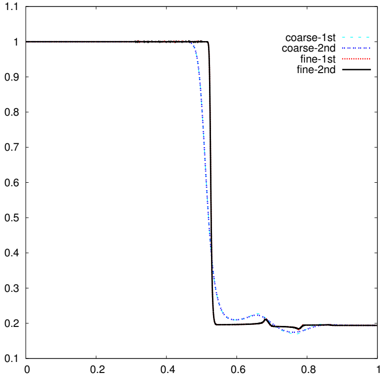
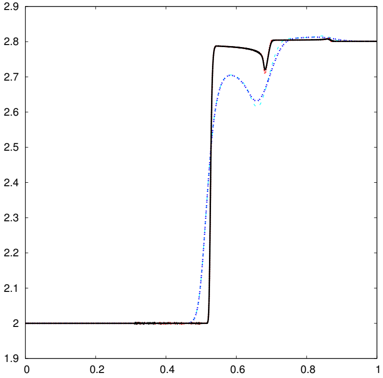
$$f_0^{(1)}(x_{j+1/2} - vt) = \begin{cases} g_j & v > 0 \\ g_{j+1}, & v < 0 \end{cases}$$

$$f_{j+1/2}^{(2)}(x_{j+1/2}, t, v) = f_0^{(2)}(x_{j+1/2} - vt) + a(f_u)_{j+1/2}^{(1)} t$$

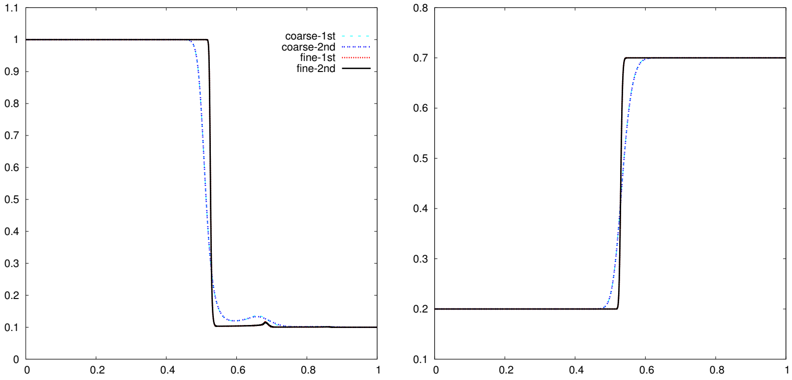
	ϕ_L	u_L	ρ_L	P_L	ϕ_R	u_R	ρ_R	P_R
solid	0.8	0.3	2	5	0.3	0.3	2	12.8567
gas	0.2	2	1	1	0.7	2.8011	0.1941	0.1



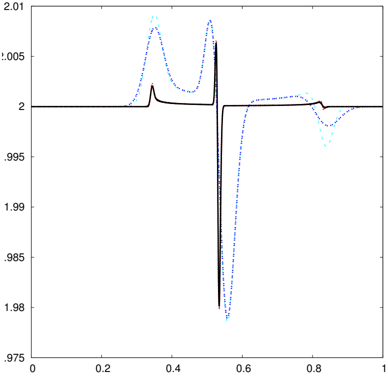
(a) ρ_g



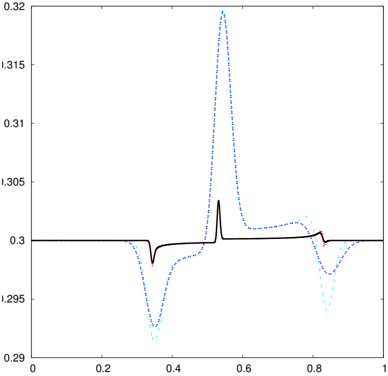
(b) u_g


1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
coarse-1st
coarse-2nd
fine-1st
fine-2nd
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

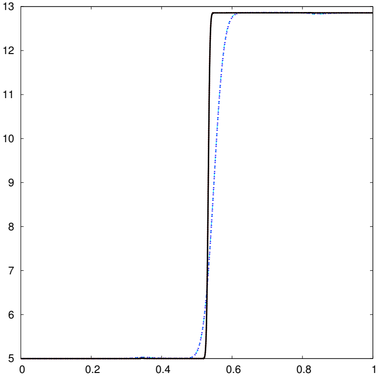
(a) p_g (b) ϕ_g


2.01
2.005
2
1.995
1.99
1.985
1.98
1.975
0
0.2
0.4
0.6
0.8
1

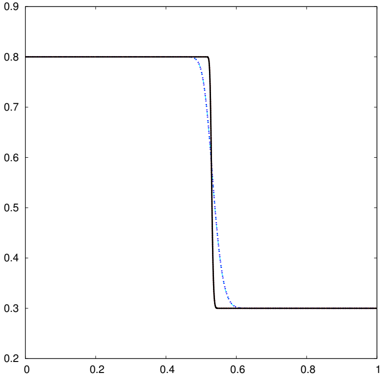
(a) ρ_s


0.32
0.315
0.31
0.305
0.3
0.295
0.29
0
0.2
0.4
0.6
0.8
1

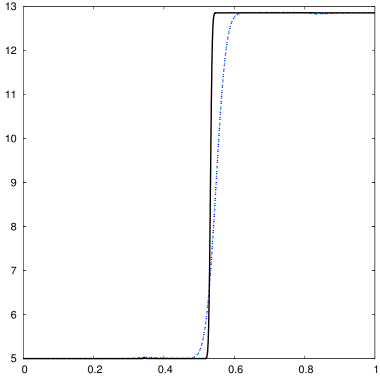
(b) u_s



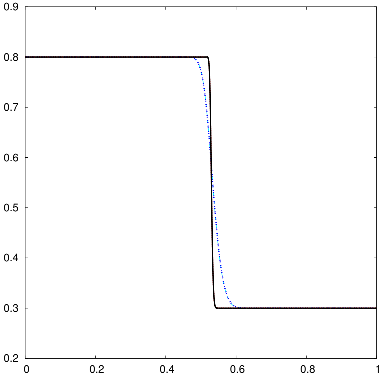
(a) p_s



(b) ϕ_s



(a) p_s



(b) ϕ_s

Thank you for your attention!