Coupling within and across multiple scales of climate dynamics

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INTRODUCTION

- Interacting dynamical systems
- Statistical physics
- Graph theory
- COMPLEX NETWORKS
- **Multivariate time series $\longrightarrow$ networks**
  - Nodes: measuring sites
  - Edges: dependence, **“connectivity” measures**
    - weighted graph
    - threshold $\rightarrow$ binary graph

INTRODUCTION

- **Multivariate time series $\longrightarrow$ networks**
  - Edges: dependence, “connectivity” measure
  - linear cross-correlation – the measure of first choice
- Dependence measures
  - Pearson correlation – linearity – Gaussianity
  - Nonlinearity? hidden connectivity patterns?
- Dependence measure – connectivity $\longrightarrow$ topology
  - **dynamics (serial correlations)**
  - temporal and spatial sampling (time lags)
- Connectivity in various time scales
  - scale-specific networks
  - cross-scale interactions

Nonlinearity in air temperature

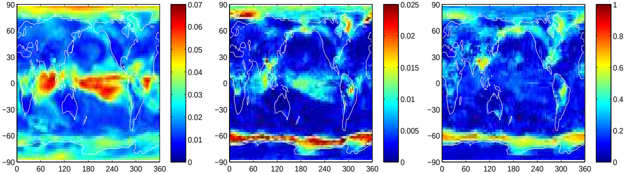


Figure 5: Average contribution of nonlinear dependences. Left: average mutual information for each location. Middle: average nonlinear contribution to mutual information I_E. Right: average nonlinear contribution relative to total mutual information (I_E/I).

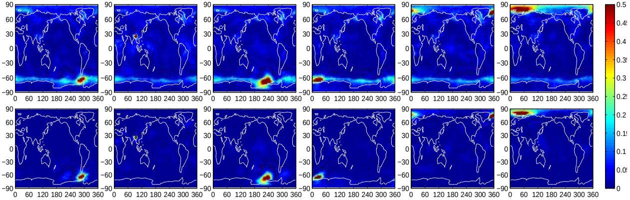


Figure 6: Dependence patterns of six locations with high relative nonlinear dependence contribution. Top: total mutual information. Bottom: linear mutual information

Nonlinearity in air temperature

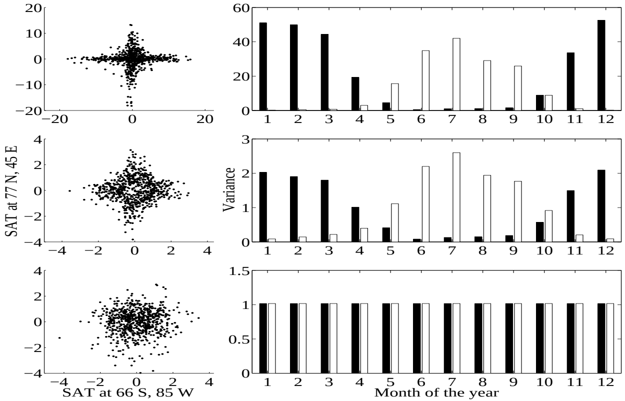


Figure 7: Example of apparent nonlinear coupling between remote areas. The apparent nonlinearity is attributable to yearly cyclicity in variance, see text for details. Top: original data anomalies, middle: univariately normalized anomalies, bottom: monthly variance normalized anomalies. Left column: scatterplots of time series values, right column: variances of data for each month (black: 77 N, 45 E; white: 66 S, 85 W).

CLIMATE NETWORKS

- **Multivariate time series**: gridded “reanalysis data” of atmospheric variables: air temperature, pressure, humidity, precipitation...
- Here: near-surface air temperature **anomalies**
  subtraction of seasonal means (mean Jan, mean Feb ...)
  removal of the annual cycle
  = **fluctuations** around seasonal means
- grid 2.5° x 2.5° $\longrightarrow$ 10^4 nodes
- Pearson correlation $\longrightarrow$ weighted network
- thresholding $\longrightarrow$ binary network
- $\longrightarrow$ graph-theoretical analysis

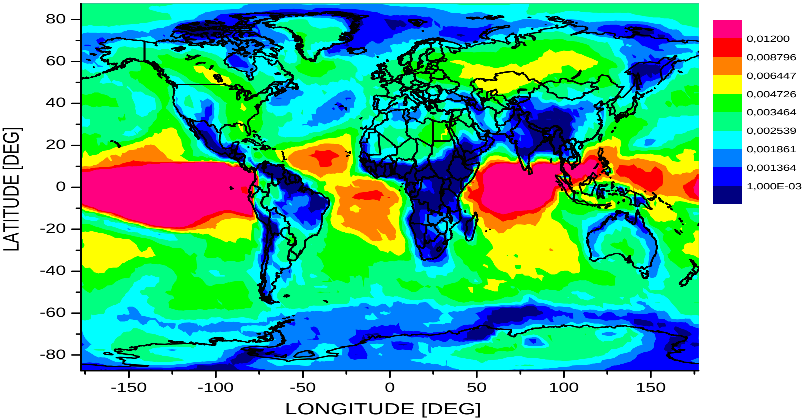
CLIMATE NETWORKS

- absolute correlations $C_{i,j} = |c_{i,j}|$
- adjacency matrix $A_{i,j} = 1$ iff $C_{i,j} > c_T$
- threshold c_T chosen to get network density $\varrho = 0.005$
- degree centrality $k_i = \sum_{j=1}^{N_N} A_{i,j}$
- area weighted connectivity $AWC_i = \frac{\sum_{j=1}^{N_N} A_{i,j} \cos(\lambda_j)}{\sum_{j=1}^{N_N} \cos(\lambda_j)}$

  λ_j – latitude of node j

Area Weighted Connectivity $\varrho = 0.005$ for

NCEP/NCAR SAT anomalies – absolute correlations



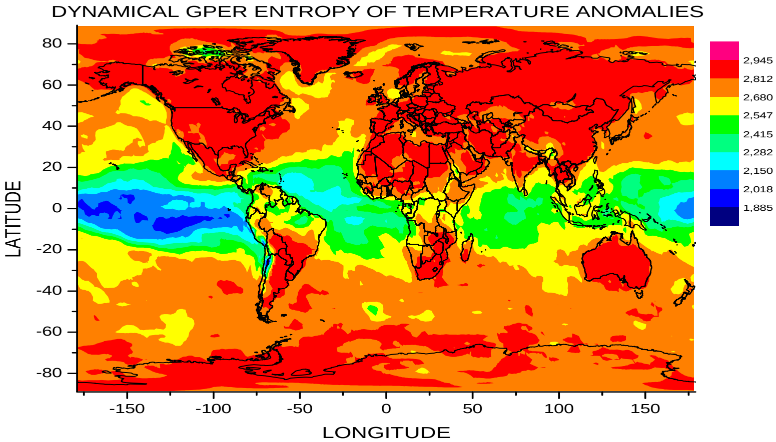
Connectivity vs. dynamics

- stochastic process $\{X_i\}$:
  indexed sequence of random variables, characterized by $p(x_1, \ldots, x_n)$
- **entropy rate** of $\{X_i\}$ is defined as

$$h = \lim_{n \to \infty} \frac{1}{n} H(X_1, \ldots, X_n)$$

- dynamical systems: *Kolmogorov-Sinai entropy*
- for a Gaussian process with spectral density function $f(\omega)$

$$h_G = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log f(\omega) d\omega$$



Entropy rates of temperature anomaly time series for each node.

SURROGATE DATA

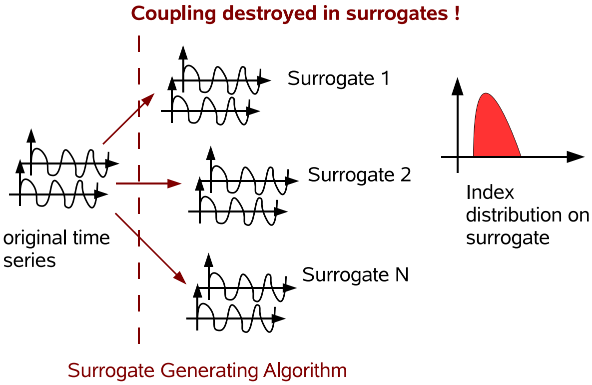
- generated by a model
- obtained by constrained randomization of the original data

- IID (scrambled) surrogate data
- **FT (AAFT, IAAFT ...) surrogate data**
- wavelet
- recurrence
- constrained randomization ...

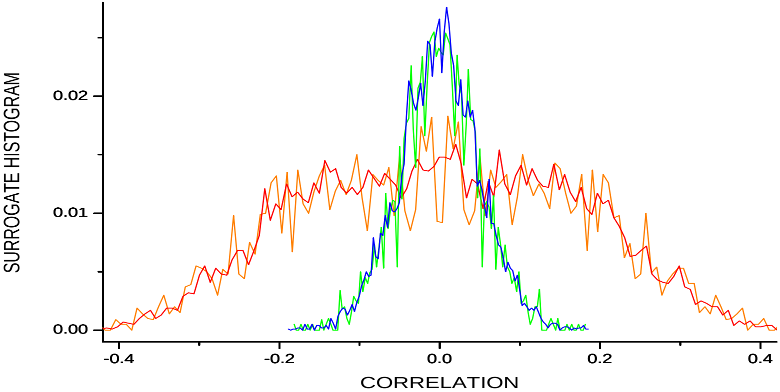
FT surrogates: preserve magnitudes of Fourier coefficients (spectra), randomize Fourier phases

Significance testing using surrogate data

- Use of bootstrap-like strategy (surrogate time series)
- Ideally preserve all properties except tested (coupling)



Connectivity vs. dynamics

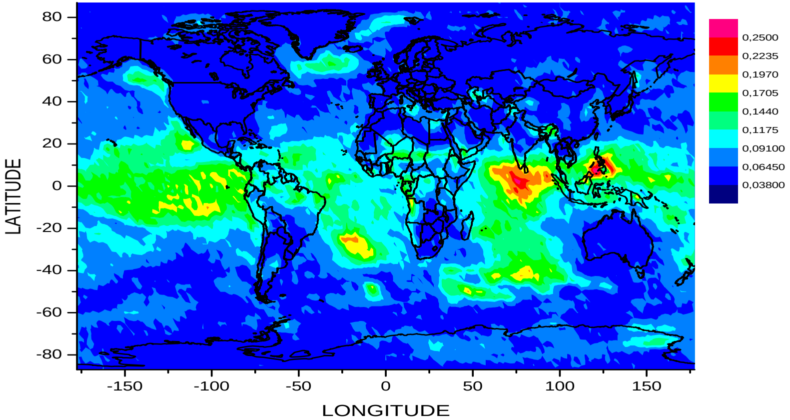


Surrogate cross-correlation for high-ER (green, blue) and low-ER (orange, red) NCEP/NCAR grid-points. FT (green, orange), AAFT (blue, red).

Connectivity vs. dynamics

Mean absolute correlation of NCEP/NCAR SAT anomalies

with FT surrogate data



Mutual information

- two variables X and Y:
- p(x), H(X), p(y), H(Y), joint PDF p(x,y), joint entropy H(X,Y)
- mutual information

$$I(X;Y) = H(X) + X(Y) - H(X,Y)$$

- static p(x) – entropy H(X)
- characterization of dynamics – entropy rate

- static joint p(x,y) – mutual information I(X;Y) (correlation)
- similarity of dynamics – mutual information rate

Mutual information rate

- stochastic processes $\{X_i\}$, $\{Y_i\}$, characterized by $p(x_1, \ldots, x_n)$ and $p(y_1, \ldots, y_n)$
- **mutual information rate**

$$i(X_i; Y_i) = \lim_{n \to \infty} \frac{1}{n} I(X_1, \ldots, X_n; Y_1, \ldots, Y_n)$$

Mutual information rate

- for Gaussian stochastic processes $\{X_i\}$, $\{Y_i\}$, characterized by power spectral densities (PSD) $\Phi_X(\omega)$, $\Phi_Y(\omega)$ and cross PSD $\Phi_{X,Y}(\omega)$
- **mutual information rate**

$$i(X_i; Y_i) = -\frac{1}{4\pi} \int_0^{2\pi} \log(1 - |\gamma_{X,Y}(\omega)|^2) d\omega$$

- magnitude-squared coherence

$$|\gamma_{X,Y}(\omega)|^2 = \frac{|\Phi_{X,Y}(\omega)|^2}{\Phi_X(\omega)\Phi_Y(\omega)}$$

ELSEVIER

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PHYSICS LETTERS A

On entropy rates of dynamical systems and Gaussian processes

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Abstract

The possibility of a relation between the Kolmogorov–Sinai entropy of a dynamical system and the entropy rate of a Gaussian process isospectral to time series generated by the dynamical system is numerically investigated using discrete and continuous chaotic dynamical systems. The results suggest that such a relation as a nonlinear one-to-one function may exist when the Kolmogorov–Sinai entropy varies smoothly with variations of the system parameters, but is broken in critical states near bifurcation points.

1. Entropy rates

Entropy rates will be considered as a tool for quantitative characterization of dynamic processes evolving in time. Let $\{x_i\}$ be a time series, i.e., a series of mea-

$$H(X_1, \ldots, X_n) = -\sum_{x_1} \ldots \sum_{x_n} p(x_1, \ldots, x_n) \log p(x_1, \ldots, x_n). \quad (2)$$

Gaussian ER and nonlinear DS

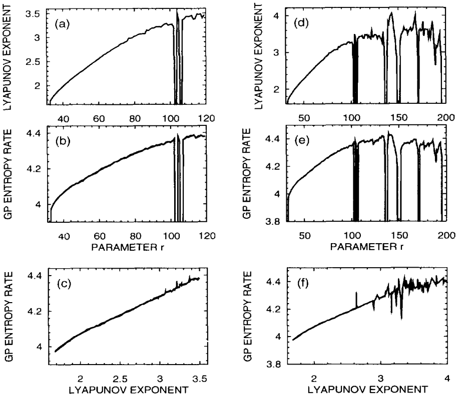


Fig. 3. Further results for the Lorenz system: (a) The positive Lyapunov exponents computed from the Lorenz equations for the parameter r varying from 33 to 120 in steps of 1. (b) The GP entropy rates estimated from 15 realizations of 16k time series (mean: thick line; mean ± SD: thin lines, coinciding with the mean) for different values of the parameter r varying as in (a). (c) Plot of GPER (the same line codes as before) versus LE. (d), (e), (f) The same as (a), (b), (c), respectively, except for the parameter r varying from 33 to 200 in steps of 1.

$r > 65$ enters the bifurcation region (Figs. 3a, 3b and

plot was obtained by increasing the parameter a from

Route to synchronization

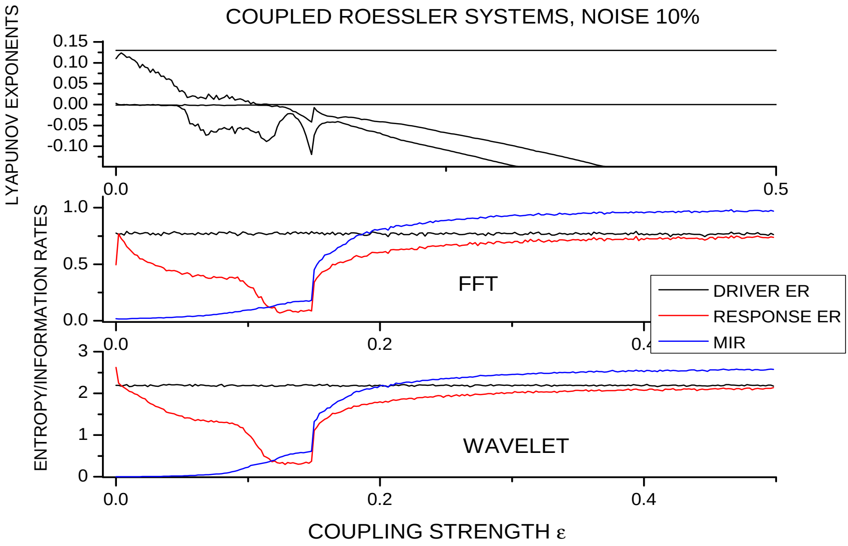
- unidirectionally coupled Rössler systems

$$\begin{aligned}\dot{x}_1 &= -\omega_1 x_2 - x_3 \\ \dot{x}_2 &= \omega_1 x_1 + a_1\, x_2 \\ \dot{x}_3 &= b_1 + x_3(x_1 - c_1)\end{aligned}$$

$$\begin{aligned}\dot{y}_1 &= -\omega_2 y_2 - y_3 + \epsilon(x_1 - y_1) \\ \dot{y}_2 &= \omega_2 y_1 + a_2\, y_2 \\ \dot{y}_3 &= b_2 + y_3(y_1 - c_2)\end{aligned}$$

$a_1 = a_2 = 0.15$, $b_1 = b_2 = 0.2$, $c_1 = c_2 = 10.0$
frequencies $\omega_1 = 1.015$, $\omega_2 = 0.985$.

Route to synchronization and MIR, ER



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Synchronization as adjustment of information rates: Detection from bivariate time series

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An information-theoretic approach for studying synchronization phenomena in experimental bivariate time series is presented. ''Coarse-grained'' information rates are introduced and their ability to indicate generalized synchronization as well as to establish a ''direction of information flow'' between coupled systems, i.e., to discern the driving from the driven (response) system, is demonstrated using numerically generated time series from unidirectionally coupled chaotic systems. The method introduced is then applied in a case study of electroencephalogram recordings of an epileptic patient. Synchronization events leading to seizures have been found on two levels of organization of brain tissues and ''directions of information flow'' among brain areas have been identified. This allows localization of the primary epileptogenic areas, also confirmed by magnetic resonance imaging and pasitron emission tomography scans.

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I. INTRODUCTION

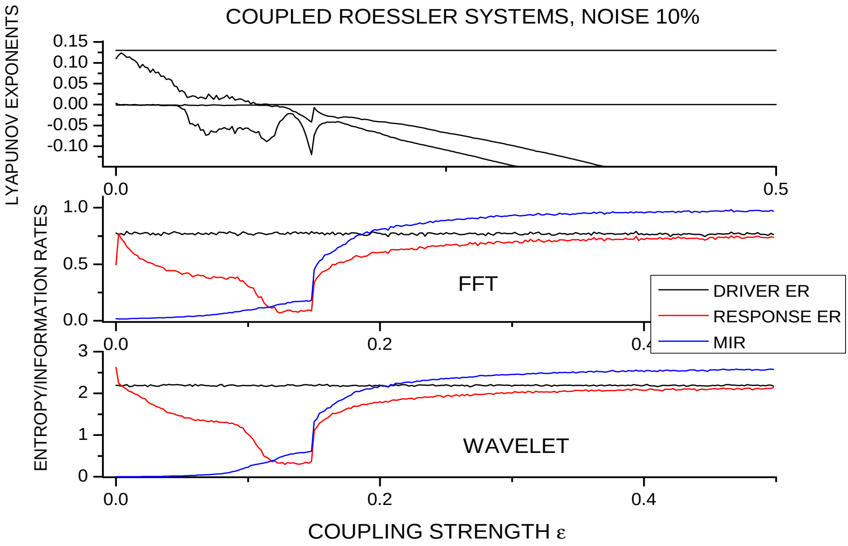
During the last decade there has been considerable interest in the study of the cooperative behavior of coupled chaotic systems [1]. Synchronization phenomena have been observed in many physical and biological systems, even in

electroencephalogram (EEG) recordings of an epileptic patient. A conclusion is given in Sec. V.

II. COARSE-GRAINED INFORMATION RATES

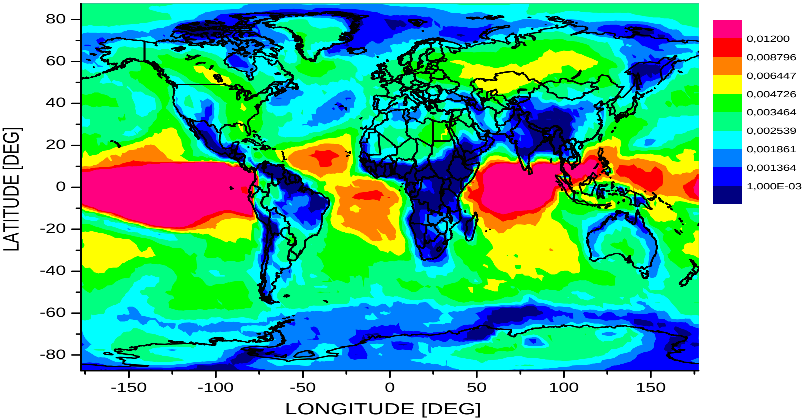
Consider discrete random variables X and Y with sets of

Route to synchronization and MIR, ER



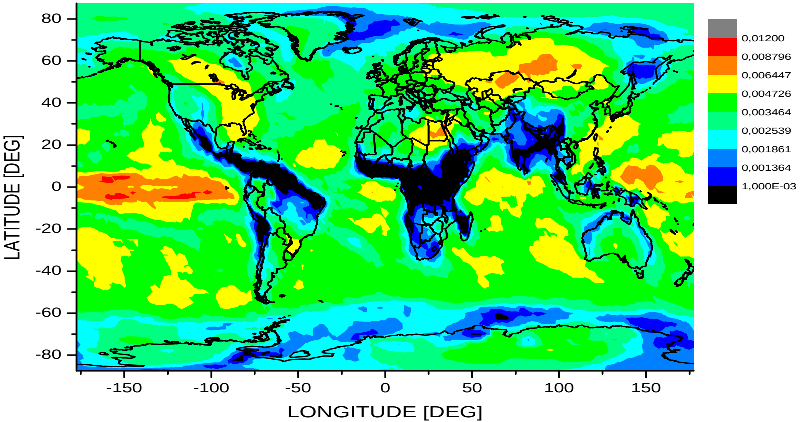
Area Weighted Connectivity $\varrho = 0.005$ for

NCEP/NCAR SAT anomalies – absolute correlations



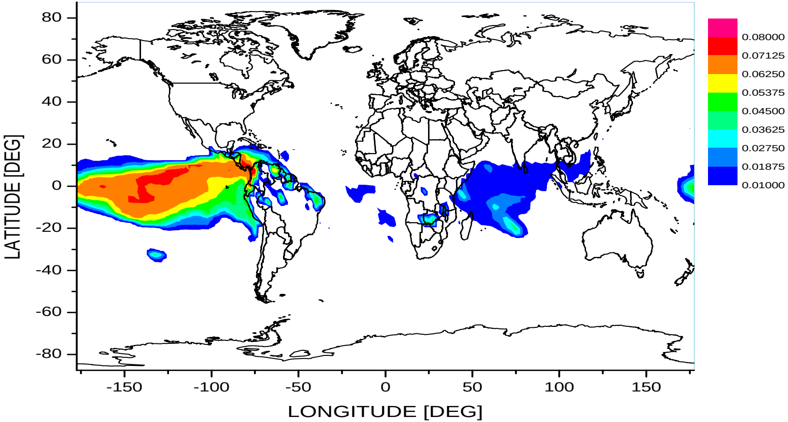
Area Weighted Connectivity $\varrho = 0.005$ for

NCEP/NCAR SAT anomalies – mutual information rate



NCEP/NCAR SAT anomalies – mutual information rate

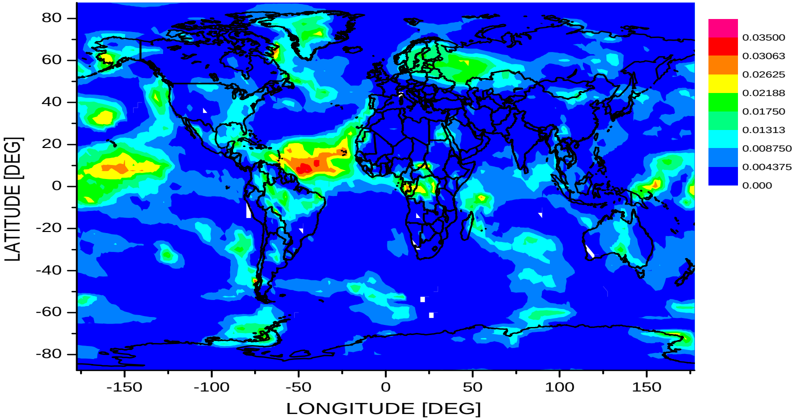
scale/period 4–6 years



Scale-specific climate network, AWC, $\varrho = 0.005$

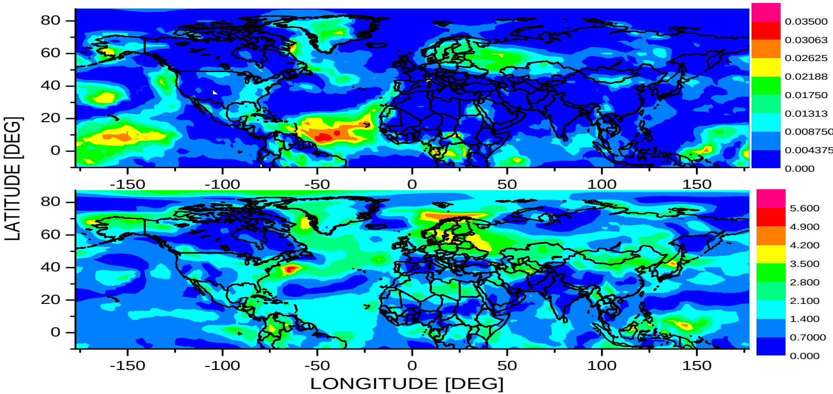
NCEP/NCAR SAT anomalies – mutual information rate

scale/period 7–8 years



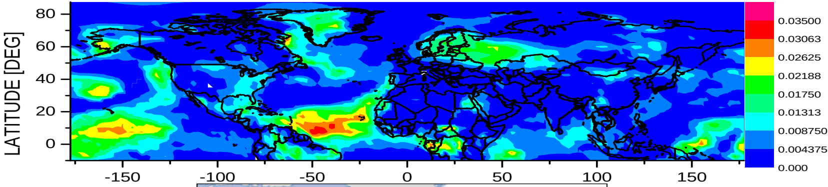
Top: AWC, $\varrho = 0.005$

Bottom NAO – SAT MIR coherence



NCEP/NCAR SAT anomalies – mutual information rate

scale/period 7–8 years and Gulf stream



FILTERING $\longrightarrow$ HILBERT TRANSFORM

COMPLEX CONTINUOUS WAVELET TRANSFORM

ANALYTIC SIGNAL

$$\psi(t) = s(t) + j\hat{s}(t) = A(t)e^{j\phi(t)} \qquad (1)$$

INSTANTANEOUS PHASE

$$\phi(t) = \arctan\frac{\hat{s}(t)}{s(t)} \qquad (2)$$

INSTANTANEOUS AMPLITUDE

$$A(t) = \sqrt{\hat{s}(t)^2 + s(t)^2} \qquad (3)$$

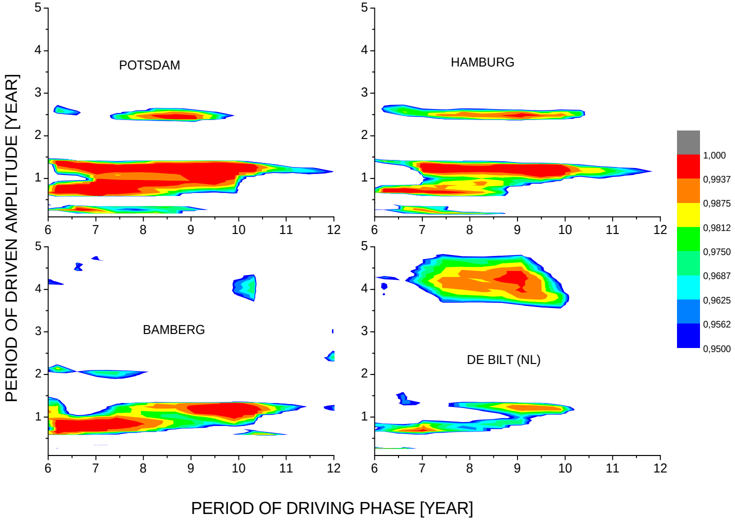
Cross-frequency interactions

- phase–phase
- amplitude–amplitude
- phase–amplitude
  - neurophysiology: phase of slow oscillations (δ, θ) modulates the amplitude of fast oscillations (γ)

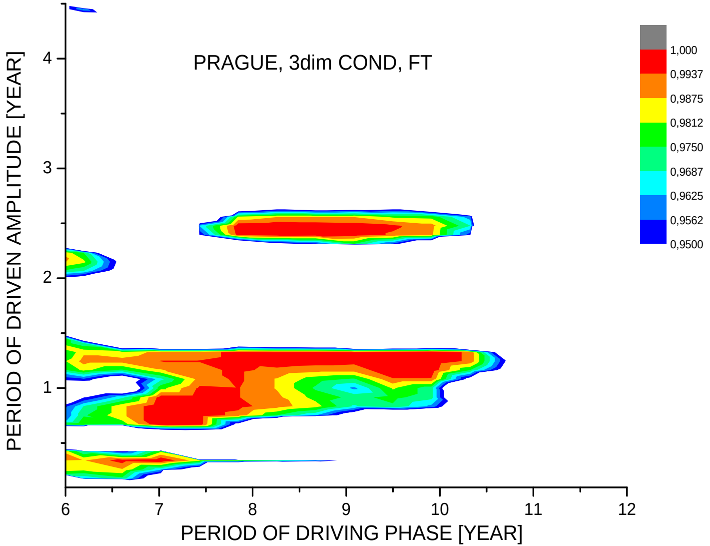
CAUSAL PHASE –> AMPLITUDE INTERACTIONS
in about a century long records of daily near-surface air temperature records from European stations

- phase ϕ_1 of slow oscillations (around 10 year period)
- amplitude A_2 of higher-frequency variability (periods 5 years and less)
- $I(\phi_1(t); A_2(t+\tau)|A_2(t), A_2(t-\eta), \ldots, A_2(t-m\eta))$
- testing using surrogate data approach
  - Fourier transform (FT) surrogate data (Theiler et al.)
  - multifractal (MF) surrogate data (Paluš)

CAUSAL PHASE –> AMPLITUDE INTERACTIONS



CAUSAL PHASE –> AMPLITUDE INTERACTIONS



CONCLUSION

- Interactions within different scales – scale specific networks
- Interactions across scales
  - cross-scale network in a site
  - cross-scale (tele)connections between sites/nodes
- Complex multigraphs with nodes connected by a number of links within and across scales

Relevant publications:

- **Paluš, M., Hartman, D., Hlinka, J., Vejmelka M.** *Discerning connectivity from dynamics in climate networks.* **Nonlin. Processes Geophys.**, 2011, Vol. 18, pp. 751-763.
- **Hlinka, J., Hartman, D., Paluš, M.** *Small-world topology of functional connectivity in randomly connected dynamical systems.* **Chaos**, 2012, Vol. 22, Issue 3, 033107.
- **Hlinka, J., Hartman, D., Vejmelka, M., Novotná, D., Paluš, M.** *Non-linear dependence and teleconnections in climate data: sources, relevance, nonstationarity.* **Climate Dynamics**, 2013.
- **Hlinka, J., Hartman, D., Vejmelka, M., Runge, J., Marwan, N., Kurths, J., Paluš, M.** *Reliability of inference of directed climate networks using conditional mutual information.* **Entropy**, 2013, Vol. 15(6), pp. 2023-2045.

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CONCLUSION

Thank you for your attention

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SW for interaction analysis
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