DESIGN AND TESTS OF THE CRYOSTAT WITH AN EXPERIMENTAL CELL FOR TURBULENT THERMAL CONVECTION

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Abstract
We have designed an experimental cell of He cryostat for the study of the turbulent natural convection at very high Rayleigh numbers (1e6 < *Ra* < 2e15) with cryogenic 4He gas as a working fluid. We plan to use the cell to investigate the functional dependence of the Nusselt number (*Nu*) on Rayleigh (*Ra*) and Prandtl (*Pr*) numbers under Boussinesq conditions. The main goal of our study is to resolve the question about the transition to an ultimate Kraichnan regime within a range of available *Ra*, as similar experiments with cryogenic 4He gas performed in Chicago, Grenoble, Oregon and Trieste gave controversial results. The Grenoble group claimed to observe the Kraichnan regime above *Ra* ~ 1e11 in their experiments. This phenomenon has not been observed in other laboratories. Using our newly developed cylindrical convection cell of variable aspect ratio *G* with very thin stainless steel wall is well suited to resolve this issue as well as remaining controversy about the *Nu*(*Ra*,*Pr*,*G*) dependence.

Introduction
Turbulent thermal convection plays a prominent role in the heat transport within atmospheric and oceanic circulation and engineering processes.

Rayleigh-Bénard convection (RBC) is a model configuration for convection studies [1]. It is defined as a thermally driven flow of fluid between perfectly conducting, isothermal and infinite top and bottom rigid plates of a characteristic vertical distance *L*. In the process the bottom plate is heated and the top one is cooled. The hotter liquid at the bottom of the cell expands and produces an unstable density gradient in the fluid layer (in gravitational field). If the density gradient is sufficiently strong, the hot fluid will rise up, which results in a convective flow and enhanced transport of heat between plates.

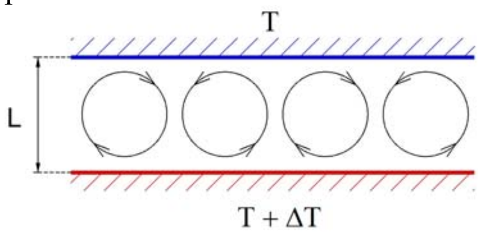

Figure 1: Schematic representation of Rayleigh-Bénard convection

Intensity of the convection is described by dimensionless Rayleigh number $Ra = g\alpha\Delta TL^3/\nu\kappa$, where α is the isobaric thermal expansion coefficient, g is the acceleration due to gravity, ΔT is the temperature difference between the bottom and top plates, L is the vertical dimension of the convection cell, ν is the kinematic viscosity, κ is the thermal diffusivity, their ratio is the Prandtl number $Pr = \nu/\kappa$.

From a dimensional analysis of Boussinesq equations, equations of motion of almost incompressible fluid, it follows that the description of the thermal convection depends upon two dimensionless parameters, the *Ra* number and *Pr* number. The convection heat transfer can be expressed by Nusselt number $Nu = H/H_0 = f\ (Ra,\ Pr)$, dimensionless ratio of the measured heat flux H to the conductive heat flux $H_0 = \lambda\Delta T/L$, where λ is the heat conductivity of the fluid. *Nu* generally also depends on the geometry of the convection cell. Laboratory model of RBC cell may be realized as a cylindrical vessel, closed on the top and bottom by high conductivity plates. Geometry of this cell is then characterized by aspect ratio $G = D/L$, where dimension L is the distance between plates and D is the inner diameter of the cell.

Cryogenic helium is a useful working fluid for experimental studies of thermal convection [2]. Helium properties have strong dependence on pressure and temperature in comparison with commonly used fluids (air, water, Hg) (Table 1).

Table 1: Fluid parameters for various fluids [2]

Fluid	**Temperature**	$\alpha/\nu\kappa$
Air	20°C	0.122
Water	20°C	14.4
SF6	50 °C (50 bar)	7.5 × 10e5
Mercury	20 °C (SVP)	3.43
Helium gas	5.5 K (2.8 bar)	1.41 × 10e8

Ra number of He can be varied over a wide range in the vicinity of He critical point (5.20 K, 2.26 bar) and simultaneously it is possible to reach very high values of *Ra* in a relatively small cell. Niemela et al. (2000) reached *Ra* about 1e17, which is the value of the order characteristic for the turbulence in atmosphere (table 2) [3]. Similar results can be achieved with heavy gases as SF6, but a disadvantage of this gas is a necessity of operation at high pressures (up to 50 bar at 300 K) for the highest *Ra* values [4].

Table 2: Exaples of high Rayleigh numbers [2]

Example	**Ra**
Atmosphere	≈ 10e17
Ocean (L ≈ 1 km, ΔT ≈ 1 °C)	≈ 10e20
Sun	≈ 10e21

Kraichnan regime and *Nu*(*Ra*) dependence

The flow in the cell becomes turbulent typically at Rayleigh numbers *Ra* exceeding 1e6, except for the boundary layer. The boundary layer should undergo a laminar-turbulent transition for high *Ra* numbers. Kraichnan predicted that in this region of "ultimate state of thermal convection" the Nusselt number (*Nu*) should scale according to asymptotic relation $Nu \sim Ra$^b Pr^c with b = 1/2 and c = -1/4 with a (log *Ra*)^(-3/2) correction [10]. In the region at lower *Ra*, where Kraichnan regime (*Ra* scaling 1/2) is not expected, various theories predicted the $Nu \sim Ra$^*b* with two different values of power exponent

($b = 1/3$ or $b = 2/7$) or more complicated relation for *Nu* was derived. Nowadays theoretically acquired dependences *Nu* ~ *Ra^2/7* and *Nu* ~ *Ra^1/3* and the theoretical results by Grosmann and Lohse who suggested that *Nu*(*Ra*) dependence does not follow a strict power-law are discussed on the basis of experimental data.

Some contradictory experimental results

Chavanne et al. measured *Nu*(*Ra*) dependence in a cell with *G* = 1/2 and at *Ra* from 1e7 up to 6e12 under Boussinesq condition with constant *Pr* ~ 0.7 [5]. Under those conditions they obtained exponent *b* near to 2/7. They achieved the highest *Ra* up to 1e14 under non-Boussinesq conditions with *Pr* from 0.65 up to 35. Above about *Ra* = 3e11 they observed a new regime characterized by *Nu* ~ *Ra*^0.38 and different character of temperature fluctuation (figure 2). This finding they interpreted as the transition to Kraichnan regime [8, 9]. Grenoble group claimed to observe the Kraichnan regime above *Ra* ~ 1e11 in experiments with three different convection cells (with brass bottom plate, smooth and rough copper plates).

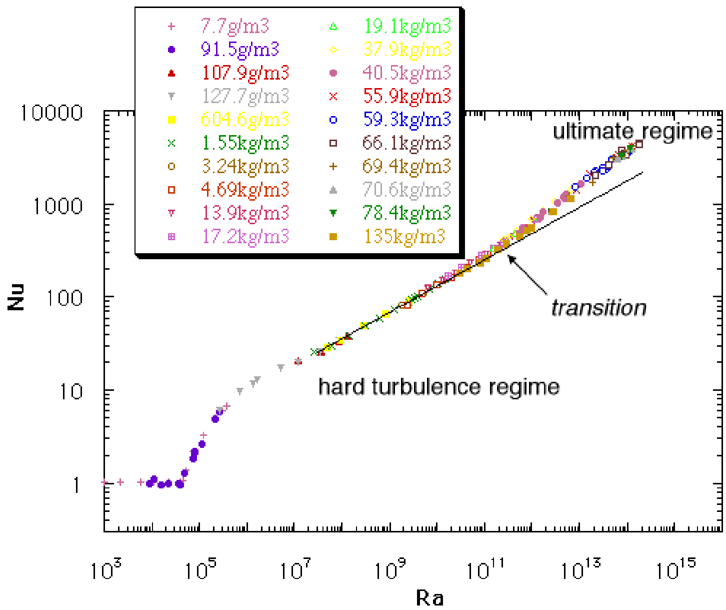

Figure 2: Log-log plot of the Nusselt number (*Nu*) versus Rayleigh number (*Ra*) – Grenoble group 1996 [5]

Niemela et al. published another results for the same aspect ratio, but with the convection cell 5 times higher [3]. They obtained the dependence *Nu* ~ *Ra*^0.309 in the *Ra* number range from 1e6 up to 1e17. Their power exponent b was between 2/7 and 1/3. After correction for parasitic effects of a sidewall and copper plates the exponent b is closer to 1/3. Niemela and Sreenivasan [8, 9] have realized additional experiments to check the dependence *Nu*(*Ra*) with the aspect ratios *G* = 1 and 4. From this measurement they obtained the power exponent *b* close to 1/3 (figure 3).

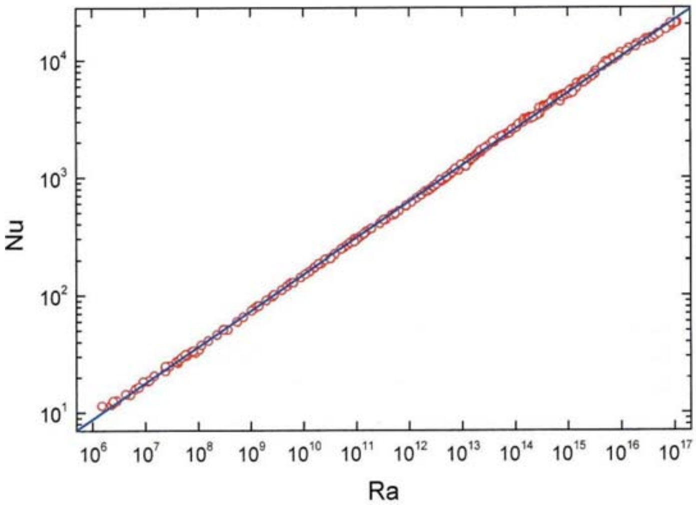

Figure 3: Log-log plot of the Nusselt number (*Nu*) versus Rayleigh number (*Ra*) – Oregon group 2000 [3]

Parasitic heat

Experimental data are influenced by construction details of the cell such as sidewall conduction, plates conductivity and capacity, roughness of the plates [8, 15, 16]. Measured data have to be analyzed very carefully.

With increasing precision of Nusselt number *Nu* the spurious effects previously neglected have to be taken into account. It was shown by a numerical model of Ahlers [15] that conduction of sidewalls of the cell may significantly contribute to heat transfer, increase measured *Nu* value and thus influence power law *Nu*(*Ra*). Roche et al. [16] derived a simple analytical formula for sidewall correction of *Nu* as a function of correction number *W*. This number is defined as a ratio of conduction of sidewall and conduction of the static gas column in the cell. They tested their formula by their results obtained for three different cells with aspect ratio 1/2: a large thick wall cell with $W = 3$, a large thin wall cell with $W = 0.6$ and a small cell, $W = 3$. After correction, data collapsed within 15% error interval into one dependence *Nu*(*Ra*) while the uncorrected data differed up to 40%. This model is equivalent to the broadening of the area of copper plate by the sidewall heating of the fluid.

Nevertheless, later discussion of sidewall effect by Niemela and Sreenivasan [8] based on numerical modelling of fluid structure and also on experiment, came to the conclusion that the corrections can not be precise and all that one can say is that the correction shifts the range *Nu* downward by few percent (it was done for their experiment) and that this shift diminishes at high *Ra* and thus the exponent in power law *Nu*(*Ra*) is slightly increased after correction. In contrary to Roche et al. they note that the correction does not depend on the thickness of the wall but depends on the ratio of the conductivities of the wall material and the gas.

In comparison with apparatuses built in other laboratories our cell is characterised by very low correction number $W = 0.18$. Using the formula of Roche et al., we can estimate, that the correction of *Nu* for our apparatus which is about 2% at $Ra = 1e12$ and 10% at $Ra = 1e8$.

In addition, the wall of our cell is in very weak thermal contact with copper plates and thus the wall is heated by the fluid only.

Large scale circulation

For overall understanding of the turbulent convection, it is significant to know the behaviour of the fluid inside the cell. Organized features of motion inside the cell, such as large-scale circulation, plumes, jets are distinguished and studied [7]. The large scale circulation, so called “wind”, has a size of the order of the convection cell. Irregular reversal of the wind flow direction was observed [7]. A source of reversal of the wind direction is not clear. It is supposed, that the wind is interactively driven by the rising and falling large plumes or collection of plumes or small perturbations that are generated at the bottom and top plates [12]. The lifetimes of bi-directional states of the flow have quantitative statistical analogy with the lifetime of solar flare in the Sun’s outer layer [10].

Cryostat ConEV (Convection Experimental Vessel)

A cryostat with a cylindrical convection cell with very thin walls especially designed for this purpose [11] will be used for experiments to elucidate the phenomena described in the previous section.

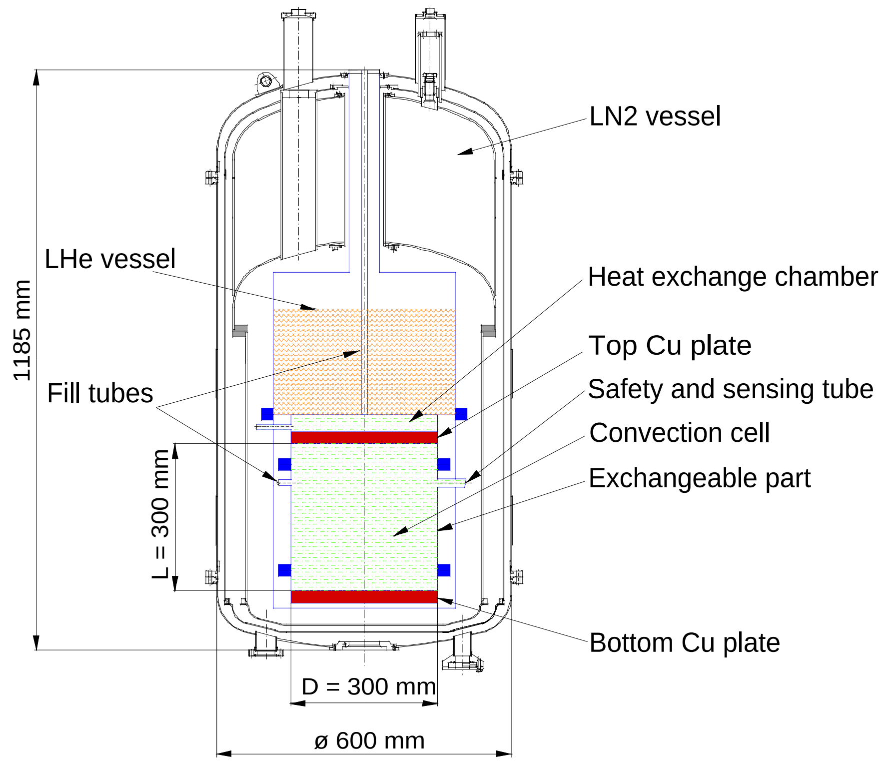

Figure 4: Schematic representation of the cryostat ConEV

The cryostat configuration is typical for NMR magnet cryostats. Liquid nitrogen (LN2) vessel (60 litres) is situated above liquid helium (LHe) vessel (28 litres) and convection cell (21 litres of cold helium gas). Convection cell is thermally connected with LHe vessel by an exchange chamber filled by gaseous He. Radiation heat flux from outer wall to LN2 vessel is reduced by the thermal shield that is cooled by cold gaseous nitrogen. The convection cell is shielded by an aluminium shield cooled by liquid He. Evaporation rates of LN2 and LHe are 5 l/day and 1.2 l/day, respectively.

Convection cell (Figure 5)

Parameters:

- cylindrical experimental cell 300 mm in diameter *D* and up to 300 mm in height *L*
- *Ra* up to about 1e15 under Boussinesq condition
- cylindrical cell with the top and bottom made of thick 28 mm high conductivity copper plates (RRR = 290, i.e. conductivity of about 2kW/mK)
- the cell design allows to change the aspect ratio *G* = *D/L* from 1 to 2.5, via exchange of its middle part
- very thin cylindrical sidewall is not in direct contact with copper plates, this substantially reduces parasitic sidewall effect
- the cell was designed for measurements at pressures from 100 Pa to 250kPa at temperatures up to about 8 K

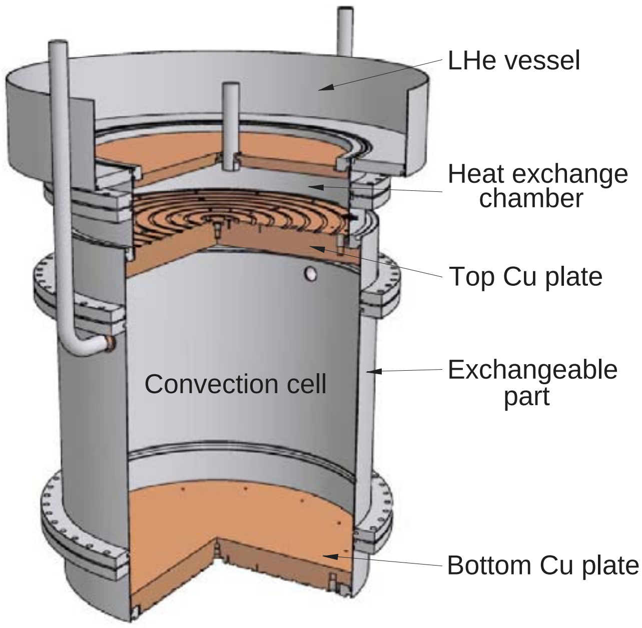

Figure 5: Convection cell

Methodological approach

The adjustment of *Ra* is achieved through changes in the gas density, heat flux applied to the bottom copper plate and temperature of the upper copper plate from 4.2 K up to 7 K. The temperature of the upper plate is stabilized with high precision and accuracy of temperature measurement in mK. The absolute value of the pressure of the gas within the cell is measured with an accuracy of about 0.2%. The state of He is evaluated from measured temperatures of copper plates and the pressure in the cell. The gas properties will be obtained from software NIST Reference Fluid Thermodynamic and Transport Properties Database (Version 8.0), for stable state of He in the cell. Rayleigh number is calculated from measured temperatures of copper plates and gas properties. The Nusselt number is calculated from the measured temperature difference and the applied heat flux to the bottom plate.

Characteristic of the wind, temperature and velocity fluctuations as a function of position, will be measured by very small Ge temperature sensors, cubes having a side of 250 micrometers, placed inside the cell. We plan to use about 12 sensors. The voltages of sensors will be detected by a lock-in amplifier. From measured data, probability density functions, correlation functions, power spectra and high order structure functions for various flow regimes in the cell will be calculated.

Formulation of our aims

1. To study of the dependence of *Nu(Ra)* in a large *Ra* number range from 1e6 up to 2e15 under Boussinesq condition.
2. To study, within a range of available numbers *Ra,* the transition to Kraichnan regime characterized by a power exponent $b = 1/2$.
3. To better understand the effect of variable *Pr* number on the dependence *Nu(Ra)*.
We will measure in the vicinity of the critical point of He for reaching high *Pr* numbers under Boussinesq condition. Expected *Pr* numbers are in a range of $0.7 < Pr < 20$.
4. To measure velocity of large-scale circulation and various aspects connected with this phenomenon.
5. All measurements will be done in the cell with variable aspect ratio *G*.

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