

# **Povrchové plazmony v integrované fotonice**

úfe

## **Povrchové plazmony v integrované fotonice**

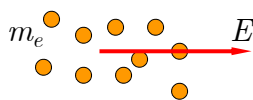
Typické aplikace:

1. vlnovodné polarizátory
2. SPR senzory
3. povrchové plazmony pro přenos informace („plazmonika“)

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## Permitivita kovu (Drudeho model)

„volný“ elektronový plyn v elektromagnetickém poli



Pohybová rovnice:  $-m_e \ddot{x} - m_e \gamma \dot{x} - eE = 0$

Pro harmonické pole  $E = E_0 \exp(-i\omega t)$

získáme ustálené řešení:  $x_0 = \frac{-eE_0}{m_e \omega^2 + im_e \gamma \omega}$

Polarizace:  $P_0 = -n_e e x_0 = \frac{-e^2 n_e}{m_e \omega^2 + im_e \gamma \omega} E_0 = \epsilon_0 \chi E_0$

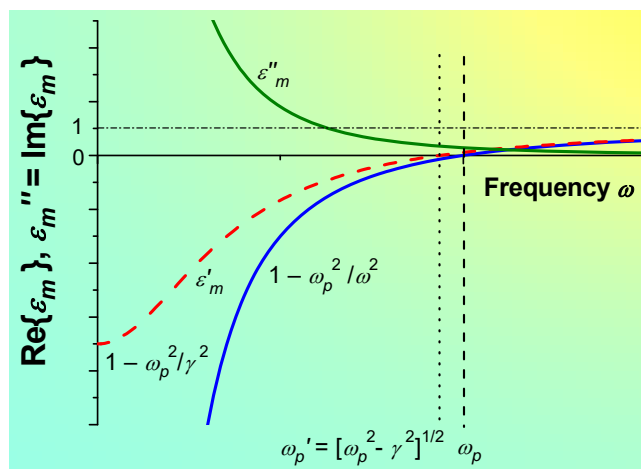
Permitivita:  $\epsilon_m = 1 + \chi = 1 - \frac{e^2 n_e / (m_e \epsilon_0)}{\omega^2 + i\gamma \omega} = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma \omega}$

Plazmová frekvence  $\omega_p = e \sqrt{\frac{n_e}{m_e \epsilon_0}}$

life

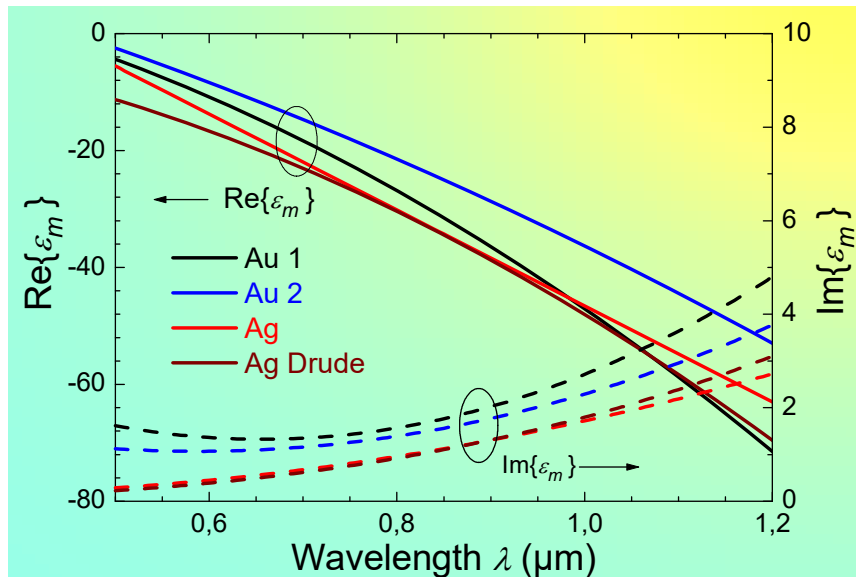
## Disperze kovu (Drudeho model)

$$\epsilon_m = \epsilon'_m + i\epsilon''_m = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2} + i \frac{\omega_p^2 \gamma}{\omega(\omega^2 + \gamma^2)}$$



life

## Disperze kovu (experimentální data)



## Povrchová plazmová vlna

(povrchový plazmon-polariton, povrchový plazmon)

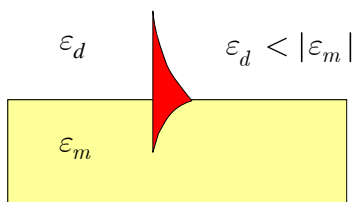
Vzájemně vázaná elektromagnetická a nábojová povrchová vlna  
localizovaná na rozhraní mezi dielektrikem a kovem

Pól  $R(N^2) \Rightarrow N^2$  povrchové vlny

TE:  $\sqrt{\epsilon_d - N^2} + \sqrt{\epsilon_m - N^2} = 0$  neexistuje řešení

TM:  $\epsilon_m \sqrt{\epsilon_d - N^2} + \epsilon_d \sqrt{\epsilon_m - N^2} = 0$  **povrchový plazmon**

$$N_{SP} = \sqrt{\frac{\epsilon_d \epsilon_m}{\epsilon_d + \epsilon_m}}$$



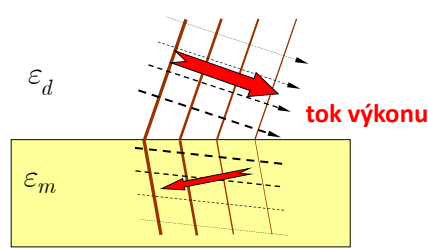
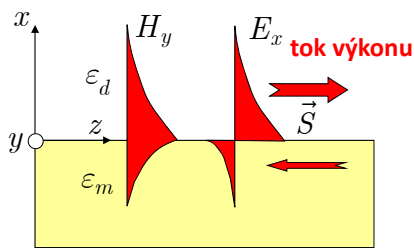
## Rozložení pole povrchového plazmonu

$$H_y(x, z) = H_0 e^{ik_0 N z} \begin{cases} e^{-k_0 \sqrt{N^2 - \epsilon_d} x}, & x > 0 & 1/k_0 \sqrt{N^2 - \epsilon_d} = 265 \text{ nm} \\ e^{k_0 \sqrt{N^2 - \epsilon_m} x}, & x < 0 & 1/k_0 \sqrt{N^2 - \epsilon_m} = 26 \text{ nm} \end{cases}$$

$$E_x(x, z) = Z_0 N H_0 e^{ik_0 N z} \begin{cases} \frac{1}{\epsilon_d} e^{-k_0 \sqrt{N^2 - \epsilon_d} x}, & x > 0 \\ \frac{1}{\epsilon_m} e^{k_0 \sqrt{N^2 - \epsilon_m} x}, & x < 0 \end{cases}$$

Pro  $\gamma = 0$ ,  $\text{Im}\{N\} = 0$

Pro  $\gamma > 0$ ,  $\text{Im}\{N\} > 0$



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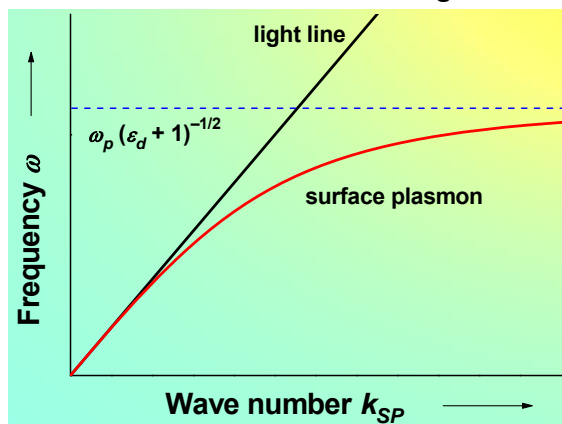
## Disperzní vlastnosti povrchového plazmonu

Pro  $\gamma = 0$ ,  $\omega < \omega_p / \sqrt{\epsilon_d + 1}$

$$k_{SP} = \frac{\omega}{c} N_{SP} = \frac{\omega n_d}{c} \sqrt{\frac{\omega_p^2 - \omega^2}{\omega_p^2 - \omega^2 (\epsilon_d + 1)}}$$

"light line"

faktor  $< 1$



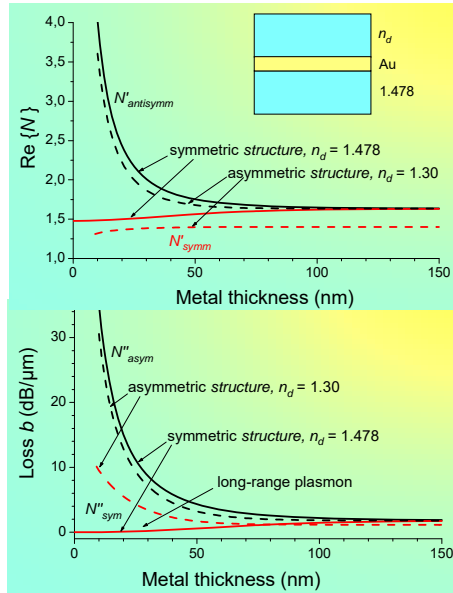
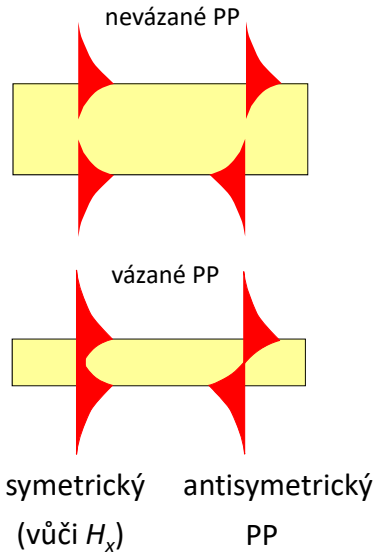
$\text{Re}\{N_{SP}\} > n_d \Rightarrow$

PP je **pomalá vlna**

nemůže být excitována  
zářením z dielektrika

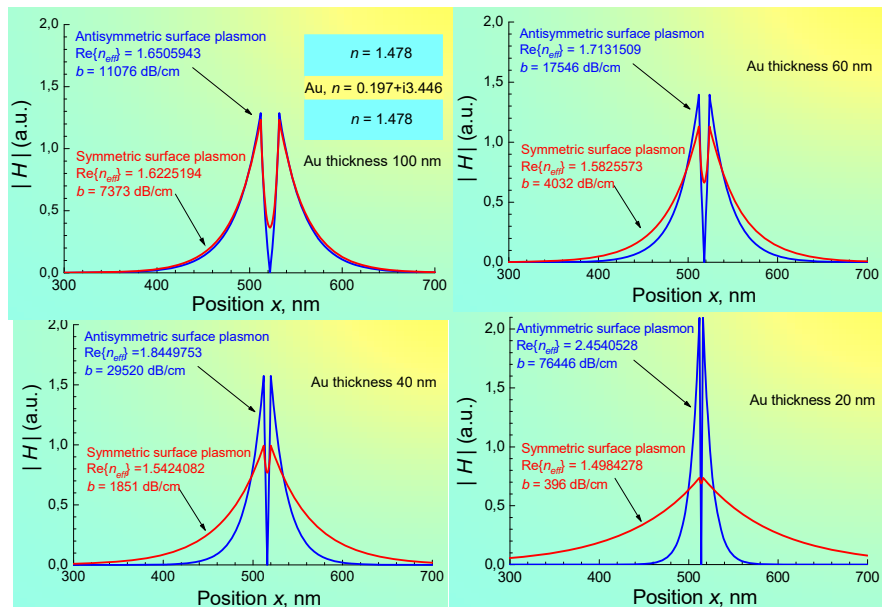
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## Povrchové plazmony na kovové vrstvě



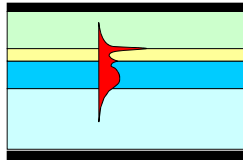
## Rozložení polí PP na kovových vrstvách

Závislost na tloušťce kovové vrstvy

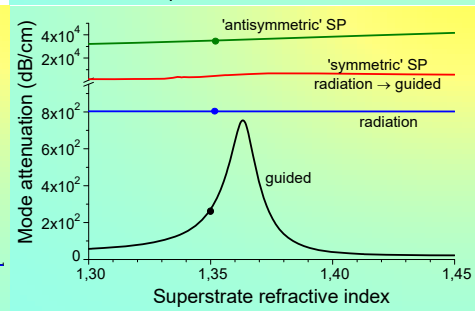
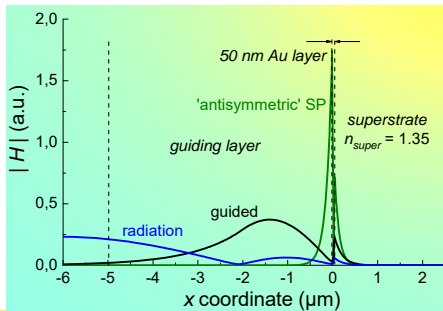
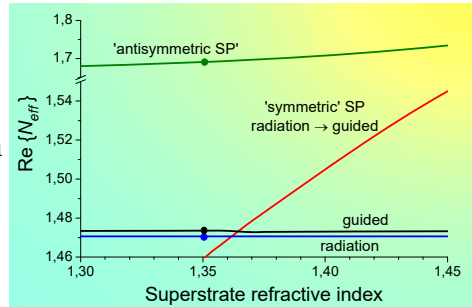


## Vidy planárních vlnodů s kovovou vrstvou

1.

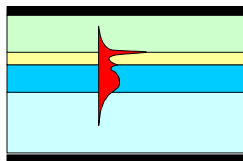


$$d_{Au} = 50 \text{ nm}$$

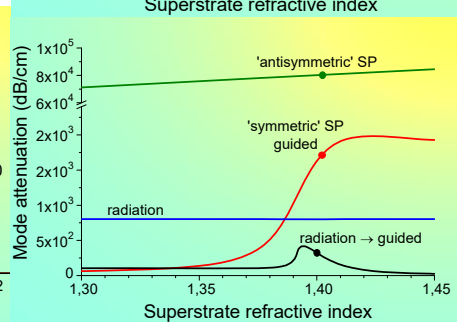
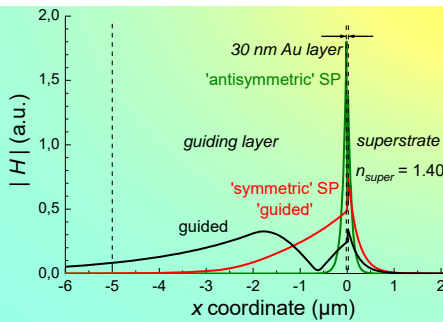
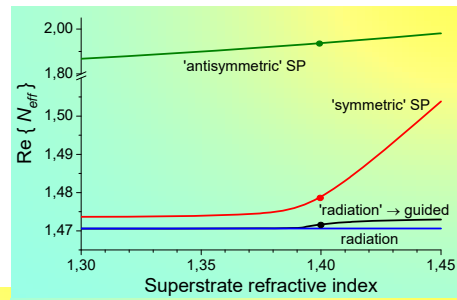


## Vidy planárních vlnodů s kovovou vrstvou

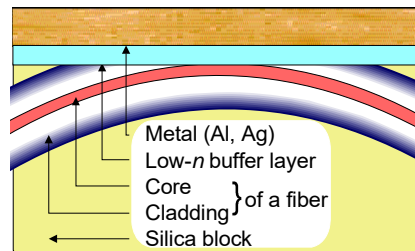
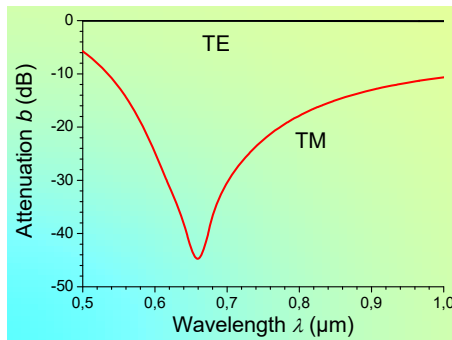
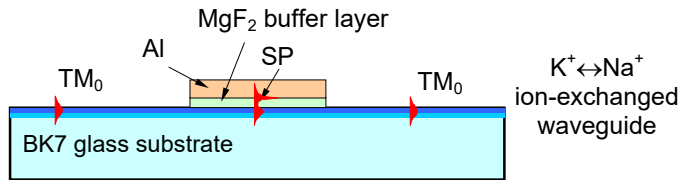
2.



$$d_{Au} = 30 \text{ nm}$$



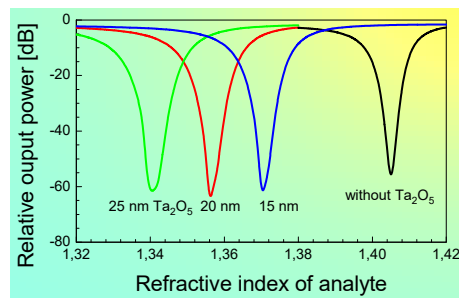
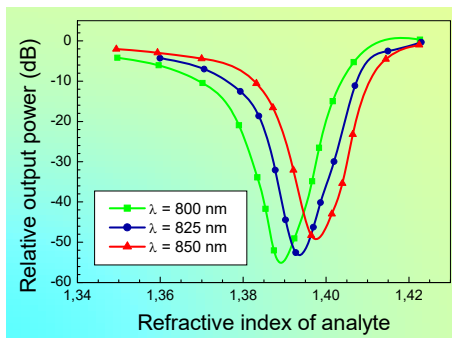
## Vlnodný polarizátor založený na rezonanční excitaci PP



## Průchod optického záření senzorem s PP

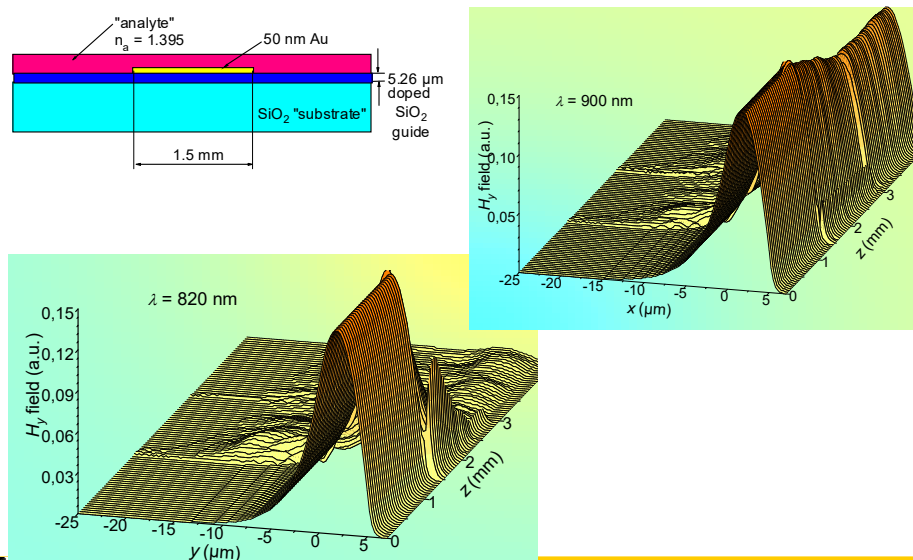
- závislost na indexu lomu analytu (zkoumaného prostředí)

2D (planární) model

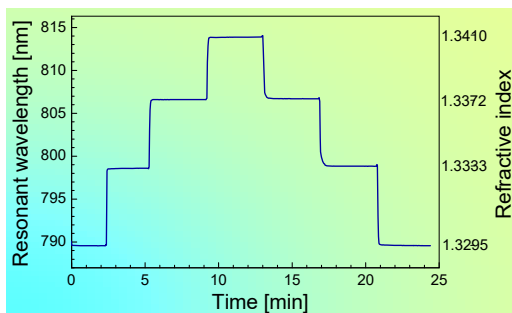
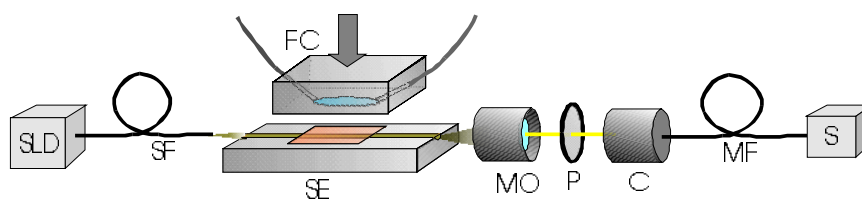


měření laditelným  
Ti:safírovým laserem

## Rozložení optického záření ve vlnovodu s úsekem, na němž se může šířit PP



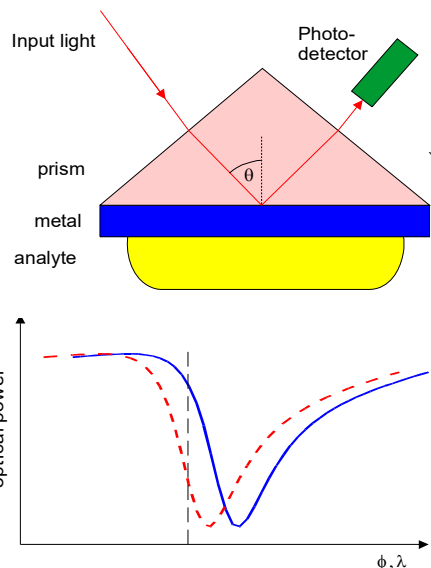
## Experimentální uspořádání integrovaně-optického senzoru s PP



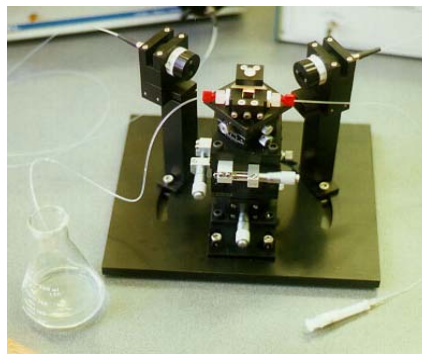
Rozlišení změn indexu lomu menších než  $1.2 \times 10^{-6}$



## Objemové senzory s PP

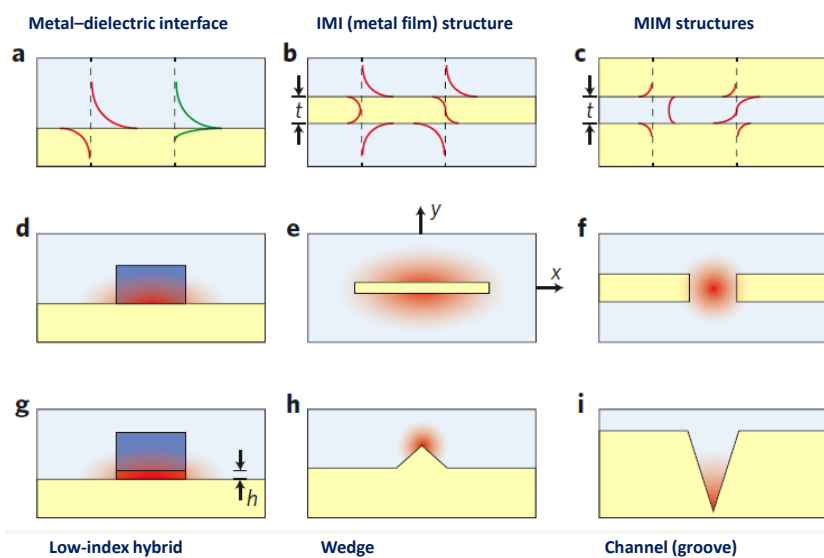


Rozlišení změn indexu lomu  
menších než  $5 \times 10^{-7}$



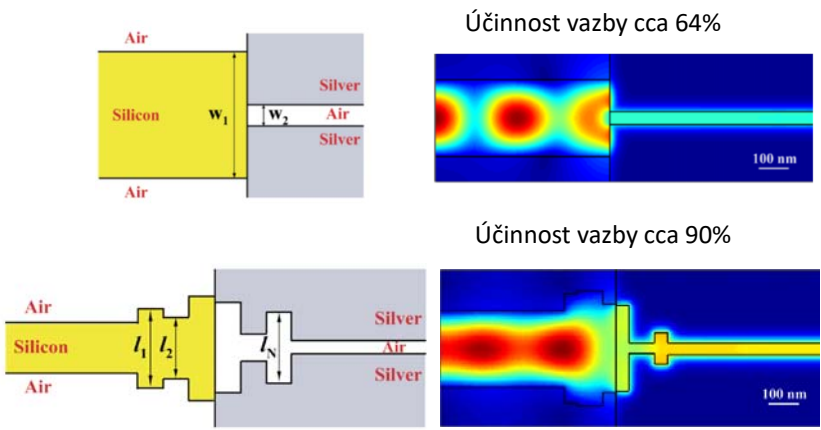
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## PLAZMONICKÉ VLNOVODY



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## Přechod mezi vlnovodem SOI a plazmonovým vlnovodem



G. Veronis, S. Fan, OWTNM 2006, p. 12  
(Stanford university)

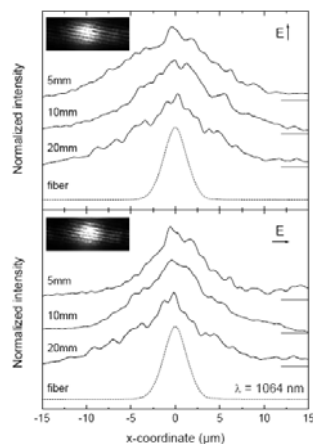
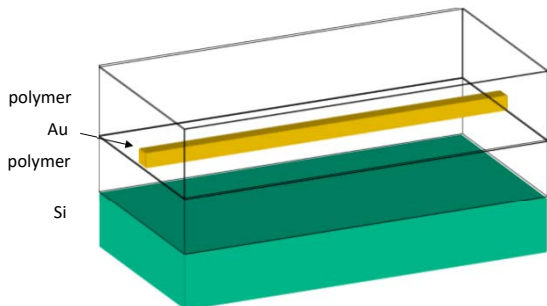
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## „Zlatý nanodrát“ jako vlnovod pro povrchové plazmony

(T. Rosenzweig, ECIO 2007)

Průřez „nanodrátu“  
100×100 nm,

útlum 4.3 dB/cm



Rozložení blízkého pole  
„plazmonů dalekého dosahu“

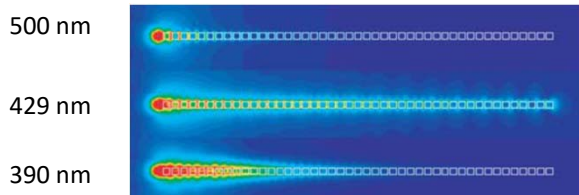
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## „Vlnovod“ tvořený řadou kovových nanočástic – vázané lokalizované plazmony

(S.A.Maier, ECIO 2007)

„Řetízek“ Au krychliček o straně 45 nm vzdálených od sebe 20 nm

Excitační vlnová délka



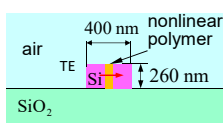
Na vlnové délce 429 je „překlenutelná vzdálenost“  
pro pokles výkonu na  $1/e^2$  celkem 2,2 mm  
(útlum cca 40 dB/cm)

life

2

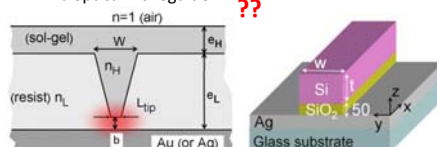
## Nové typy plazmonických vlnovodů

SOI “slot waveguide”



C. Koos & al., *Nat. Photonics* **3**(4), 16–219 (2009)

PIROW – plasmonic inverted  
rib optical waveguide

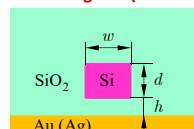


H. Benisty and M. Besbes,  
*J. Appl. Phys.* **108**(6), 063108 (2010).

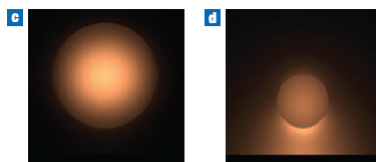
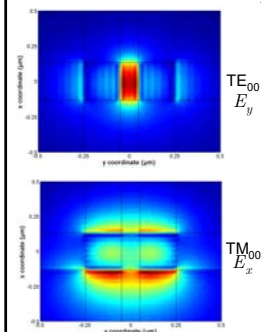
??

H.-S. Chu & al., *J. Opt. Soc. Am. B* **28**(12), 2895 (2011) (others, too)

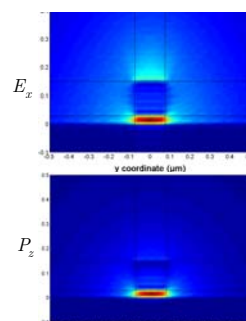
Hybrid dielectric-plasmonic  
slot waveguide (HDPSW)



R. F. Oulton & al., *New J. Phys.* **10**, 105018 (2008)



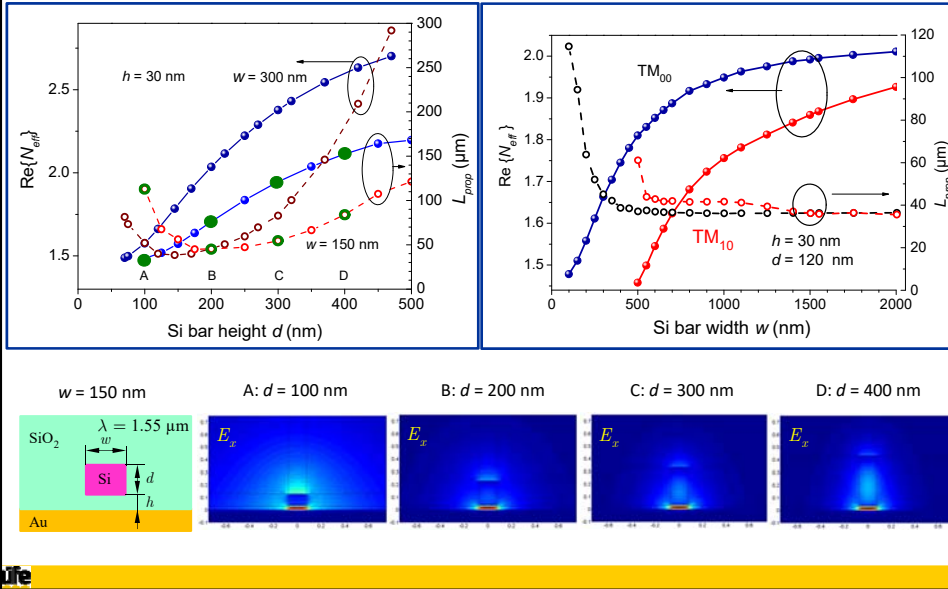
R. F. Oulton & al., *Nat. Photonics* **2**, 496 (2008);



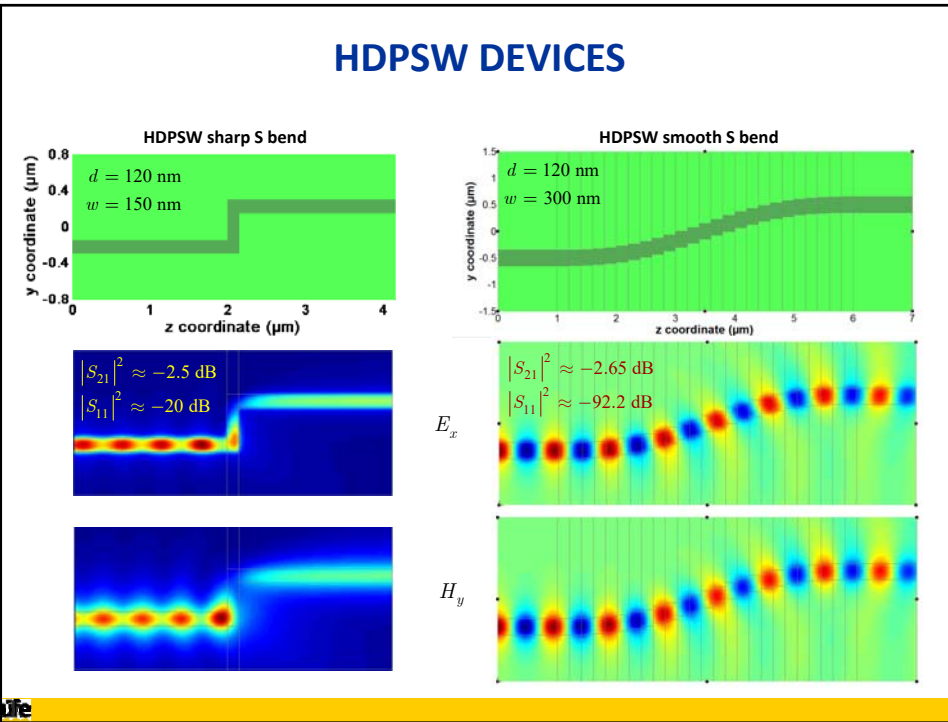
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# Hybrid dielectric-plasmonic slot waveguide

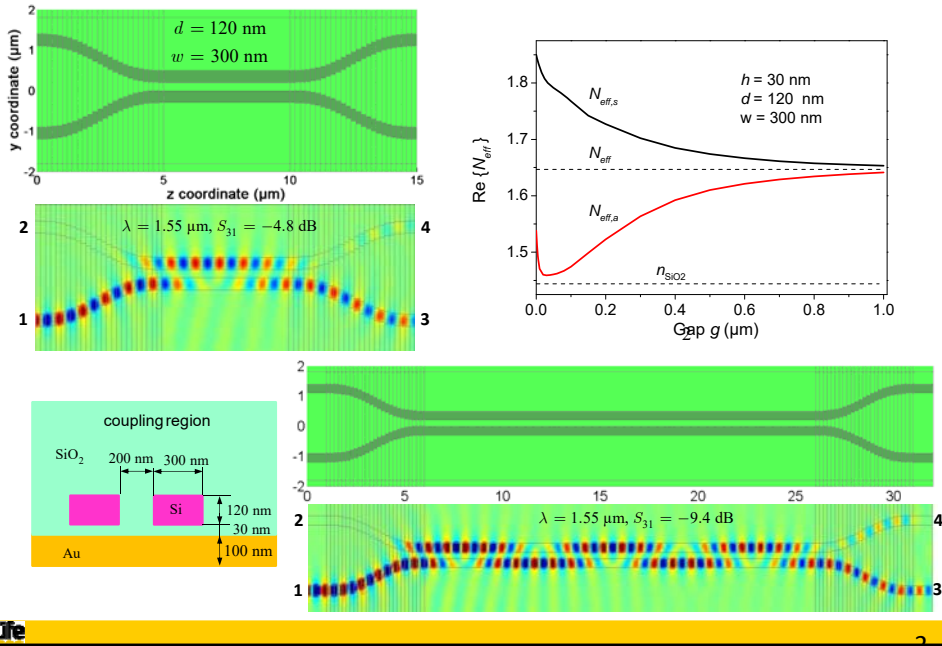
Influence of basic geometric parameters



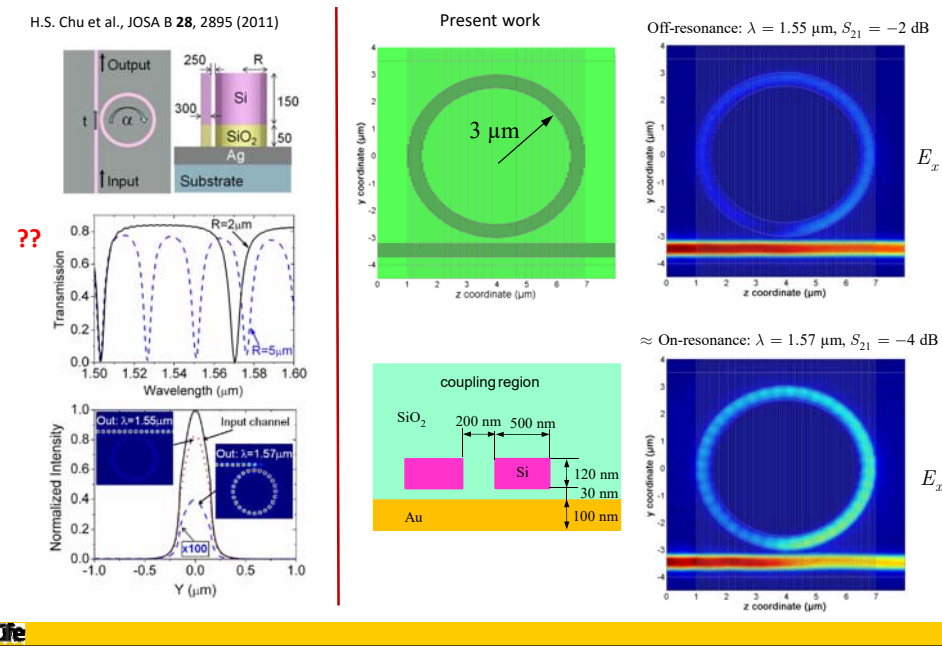
# HDPSW DEVICES



## DIRECTIONAL COUPLER

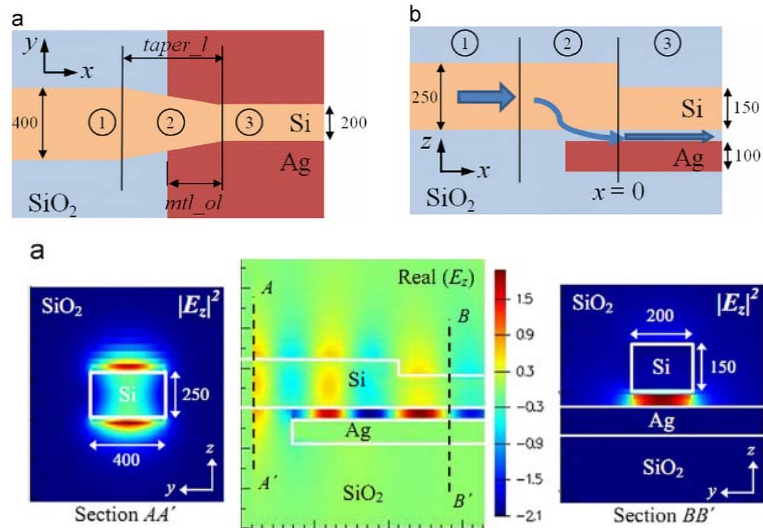


## RING MICRORESONATOR



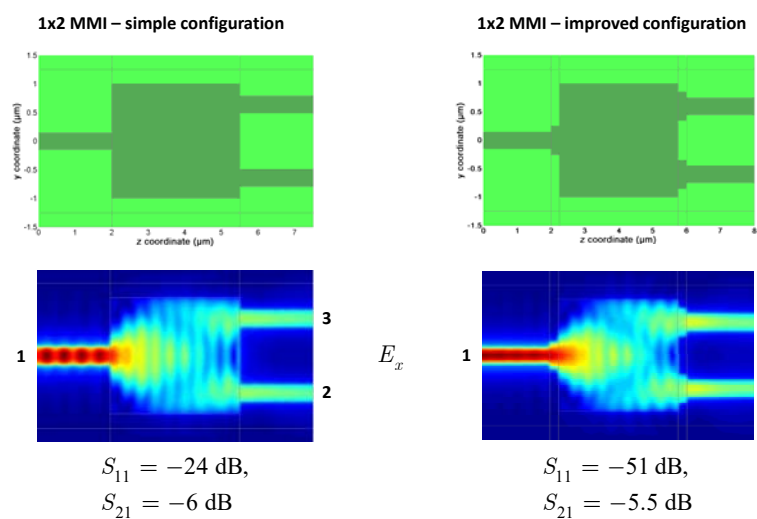
## VAZBA MEZI SOI NANODRÁTEM A HDPSW

R. Mote et al., *Optics Communications* 285 (2012) 3709–3713



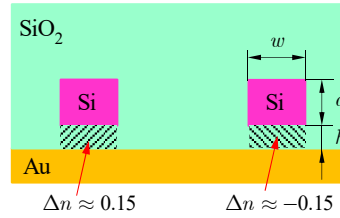
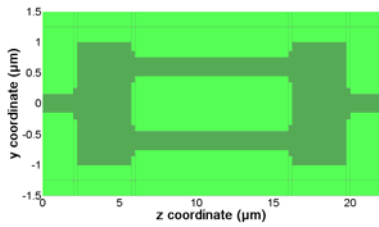
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## MULTIMODE INTERFERENCE COUPLER

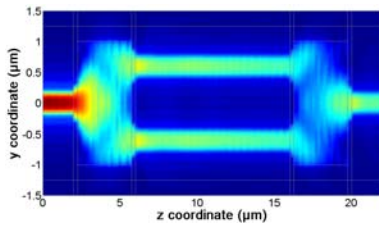


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## MACH-ZEHNDER INTERFEROMETER

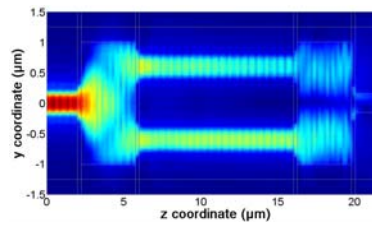


“On” state



$S_{11} = -37$  dB  
 $S_{21} = -6$  dB

“Off” state

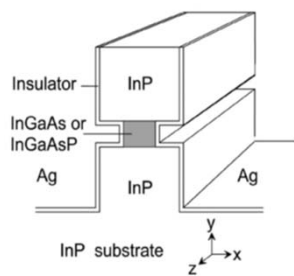


$S_{11} = -25$  dB  
 $S_{21} = -21$  dB

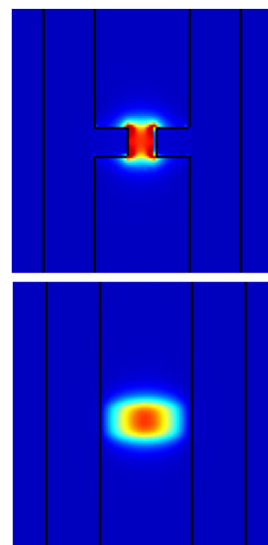
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## Koncept „plazmonového polovodičového laseru“

(M. Hill, ECIO 2007)



Rozměry aktivní oblasti laseru  
 $26 \times 26 \times 82$  nm



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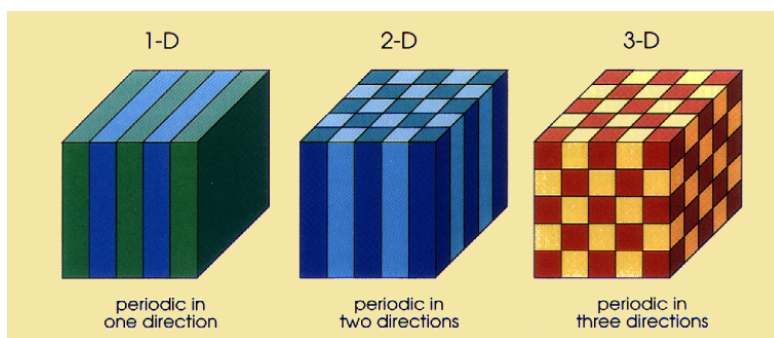
## Fotonické krystaly

### a integrovaná fotonika

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## Fotonické krystaly

1D, 2D nebo 3D periodické struktury s velkým kontrastem permittivity



E. Yablonovitch: „Inhibited spontaneous emission in solid-state physics and electronics“, *Phys. Rev. Lett.*, vol. 58, pp. 2059–2062, 1987

J. D. Joannopoulos *et al.*: *Photonic Crystals: molding the flow of light*, Princeton University Press 1995

S. G. Johnson, J. D. Joannopoulos: *Photonic Crystals, The road from theory to practice*, Kluwer Academic Publishers 2003

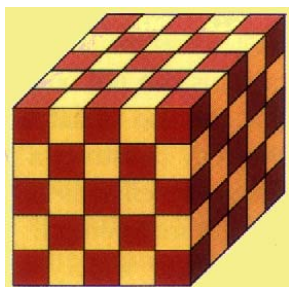
J.-M. Lourtioz *et al.*: *Photonic Crystals : Towards Nanoscale Photonic Devices*, Springer 2005

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**Fotony** se v periodickém dielektrickém prostředí pohybují „podobně“ jako **elektrony** v periodickém potenciálovém poli

Za jistých podmínek existuje **zakázaný pás energií fotonů**. Fotony s energií uvnitř zakázaného pásu se v periodickém prostředí nemohou šířit, záření se tudíž **totálně odráží zpět**



Z pohledu vlnové optiky jde o **braggovský odraz vlny od periodického prostředí**. Totální odraz je možno využít k vytvoření **optických vlnodů ve fotonických krystalech**

Vytvořit trojrozměrné periodické prostředí je však technologicky obtížné.

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### “Pohybové rovnice” pro elektrony a fotony v krystalech

Schrödingerova rovnice pro elektron v periodickém potenciálu:

$$\left[ -\frac{\hbar^2}{2m_e} \Delta + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E \psi(\mathbf{r}) \quad V(\mathbf{r} + \mathbf{a}) = V(\mathbf{r}) \quad K = \frac{2\pi}{|\mathbf{a}|}$$

$$\psi(\mathbf{r}) = \sum_m u_m(\mathbf{r}) e^{im\mathbf{K}\cdot\mathbf{r}}$$

periodický potenciál    vlnová funkce    energie fotonu    (Floquetova)-Blochova vlna,

Aproximativní (jednočásticové) přiblížení

“Vlnová rovnice” pro fotony v periodické permitivitě

$$\nabla \times \mathbf{E} = i\omega\mu_0 \mathbf{H}, \quad \nabla \times \mathbf{H} = -i\omega\varepsilon_0 \varepsilon(\mathbf{r}) \mathbf{E},$$

$$\nabla \times \left( \frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \left( \frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r})$$

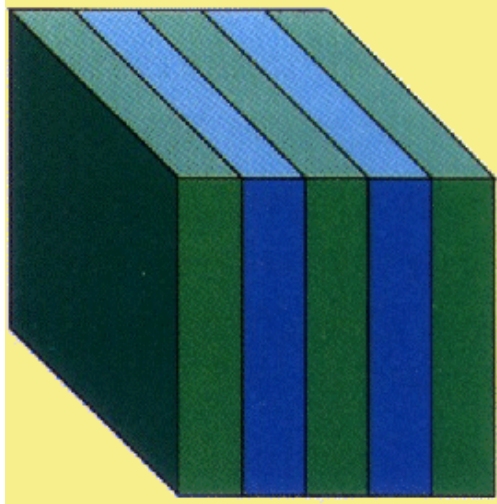
Přesná (“mnohočásticová”) teorie

Rovnice pro vlastní hodnoty energie fotonů a F-B funkce

Tento přístup je jednoduchý a průzračný, ale standardně nebere v úvahu *disperzi permitivity*  $\varepsilon(\mathbf{r}, \omega)$

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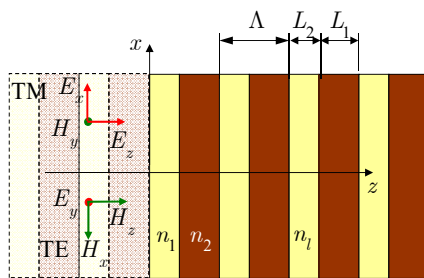
## Periodická vrstevnatá struktura jako jednorozměrný fotonický krystal



Uře

### Jednorozměrný fotonický krystal

Existence zakázaného pásu odvozená metodou přenosové matice  
(fotonická analogie Kronigova - Penneyova modelu krystalu)



Normalizace vln. vektorů:  $k_0 = 2\pi / \lambda$

$$\mathbf{k}_l = k_0 (\gamma \mathbf{x}^0 + N_l \mathbf{z}^0), \quad l = 1, 2$$

$$\gamma^2 + N_l^2 = \epsilon_l = \begin{cases} n_1^2 \\ n_2^2 \end{cases} \quad \gamma - \text{příčná konst. šíření **stejná**}$$

norm. konst. šíření

$$\begin{pmatrix} E_y(x_l + L_l) \\ H_x(x_l + L_l) \end{pmatrix} = \begin{pmatrix} \cos \varphi_l & -i \frac{Z_0}{\gamma_l} \sin \varphi_l \\ -i Y_0 \gamma_l \sin \varphi_l & \cos \varphi_l \end{pmatrix} \cdot \begin{pmatrix} E_y(x_l) \\ H_x(x_l) \end{pmatrix} = \mathbf{A}_l^{TE} \cdot \begin{pmatrix} E_y(x_l) \\ H_x(x_l) \end{pmatrix}$$

$$\varphi_l = k_0 N_l L_l$$

$$\begin{pmatrix} H_y(x_l + L_l) \\ E_x(x_l + L_l) \end{pmatrix} = \begin{pmatrix} \cos \varphi_l & i Y_0 \frac{n^2}{\gamma_l} \sin \varphi_l \\ i Z_0 \frac{\gamma_l}{n^2} \sin \varphi_l & \cos \varphi_l \end{pmatrix} \cdot \begin{pmatrix} H_y(x_l) \\ E_x(x_l) \end{pmatrix} = \mathbf{A}_l^{TM} \cdot \begin{pmatrix} H_y(x_l) \\ E_x(x_l) \end{pmatrix}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$Y_0 = \sqrt{\frac{\epsilon_0}{\mu_0}}$$

Uře

## Elektromagnetické Floquetovy – Blochovy vidy

Průchod  $l$ -tou vrstvou je popsán přenosovou maticí  $\mathbf{A}_l$ ,  
přenosová matice jedné celé periody je  ${}^{\Lambda}\mathbf{A} = \mathbf{A}_2 \cdot \mathbf{A}_1$ . Je třeba rozlišovat TE a TM vidy.

Floquetův-Blochův „vid“ (vlna) je definován pomocí vlastní funkce matice  ${}^{\Lambda}\mathbf{A}$ ,

$${}^{\Lambda}\mathbf{A} \cdot \begin{pmatrix} E^F_{y1} \\ H^F_{x1} \end{pmatrix} = s \begin{pmatrix} E^F_{y1} \\ H^F_{x1} \end{pmatrix}, \quad s = \exp(i\varphi^F), \quad \varphi^F = k^F \Lambda, \quad \text{resp.}$$

$k^F$  je konstanta šíření F-B vidu.

$${}^{\Lambda}\mathbf{A} \cdot \begin{pmatrix} H^F_{y1} \\ E^F_{x1} \end{pmatrix} = s \begin{pmatrix} H^F_{y1} \\ E^F_{x1} \end{pmatrix},$$

$$k^F \text{ je určen až na aditivní konstantu} \quad K = 2\pi / \Lambda : \quad \exp(ik^F \Lambda) = \exp[i(k^F + K)\Lambda]$$

Proto stačí určit  $k^F$  v intervalu  $-K/2 < k^F \leq K/2 \Rightarrow$  první Brillouinova zóna.

úře

## Vlastní hodnoty a fotonický zakázaný pás

Označme  $\Lambda = L_1 + L_2$ ,  $\varphi_1 = k_0 N_1 L_1$ ,  $\varphi_2 = k_0 N_2 L_2$ ,

matice  ${}^{\Lambda}\mathbf{A}$  má pak vlastní čísla

$$s = \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \pm \sqrt{\left[ \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \right]^2 - 1}.$$

FB vid se „šíří“, jen pokud  $|s| = 1$ , t.j., pokud

$$\left| \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \right| \leq 1.$$

Normovaná konstanta šíření je pak

$$k^{F'} = \frac{k^F}{K/2} = \frac{1}{\pi} \arccos \left[ \cos \left( \frac{\omega}{c} N_1 L_1 \right) \cos \left( \frac{\omega}{c} N_2 L_2 \right) - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \left( \frac{\omega}{c} N_1 L_1 \right) \sin \left( \frac{\omega}{c} N_2 L_2 \right) \right].$$

$$\text{Pokud} \quad \left| \cos \varphi_1 \cos \varphi_2 - \frac{1}{2} \left( \rho^2 + \frac{1}{\rho^2} \right) \sin \varphi_1 \sin \varphi_2 \right| > 1,$$

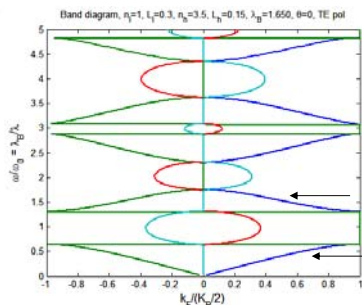
$k^F$  je komplexní, a vlna se nemůže šířit podél nekonečně dlouhého krystalu.

Tak vzniká **fotonický zakázaný pás**.

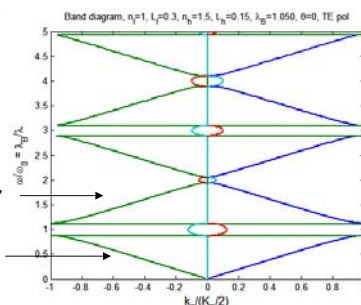
úře

## Pásová struktura jednorozměrného krystalu

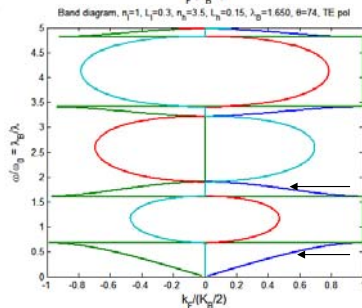
$n_1 = 1$   
 $n_2 = 3.5$   
 $\theta = 0^\circ$



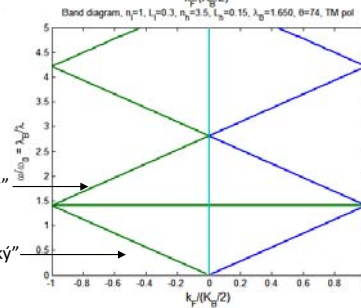
$n_1 = 1$   
 $n_2 = 1.5$   
 $\theta = 0^\circ$



TE  
 $\theta = 74^\circ$



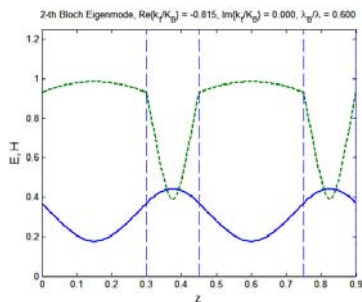
TM  
 $\theta = 74^\circ$



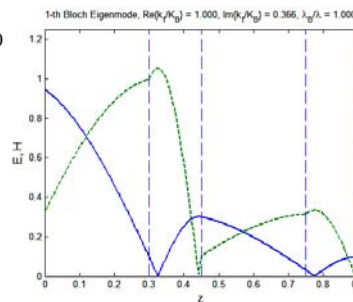
Uře

## Elektromagnetické Floquetovy – Blochovy vlny

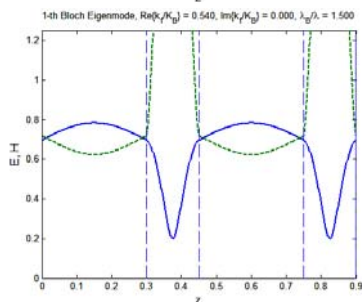
$n_1 = 1$   
 $n_2 = 3.5$   
 $\lambda_B / \lambda = 0.6$



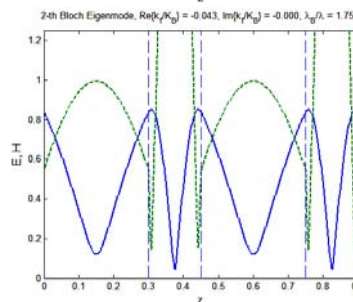
$\lambda_B / \lambda = 1.0$



$\lambda_B / \lambda = 1.5$

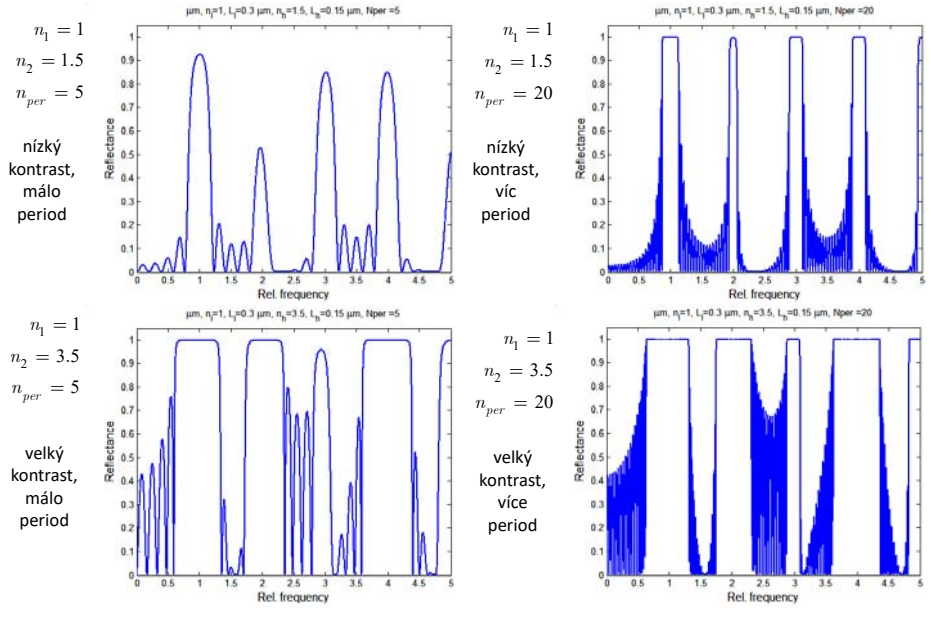


$\lambda_B / \lambda = 1.75$   
 blízko okraje  
 Brillouinovy  
 zóny



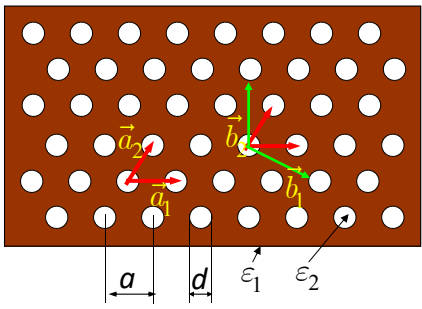
Uře

## Spektrální reflektance



Fotonické krystaly odpovídají často spíše „nanokrystalům“

## Dvozměrné „fotonické krystaly“



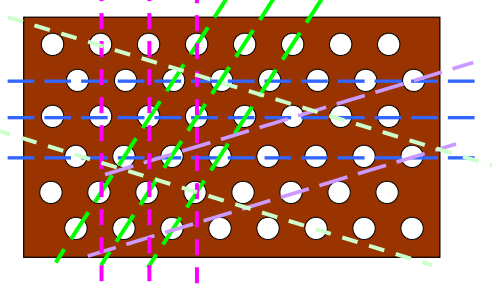
Elementární vektory prostorové mřížky  
 $\vec{a}_1 = (a, 0); \quad \vec{a}_2 = (a/2, \sqrt{3}a/2)$   
 Elementární vektory reciproké mřížky  
 $\vec{b}_1 = \left(\frac{1}{a}, \frac{1}{a\sqrt{3}}\right), \quad \vec{b}_2 = \left(0, \frac{2}{a\sqrt{3}}\right)$

Periodické uspořádání otvorů;  
 Blochův – Floquetův teorém

$$\begin{cases} E_z \\ H_z \end{cases} = u_{\vec{k}}(\vec{r}_{\parallel}) e^{i\vec{k} \cdot \vec{r}_{\parallel}} e^{i\vec{G} \cdot \vec{r}_{\parallel}}$$

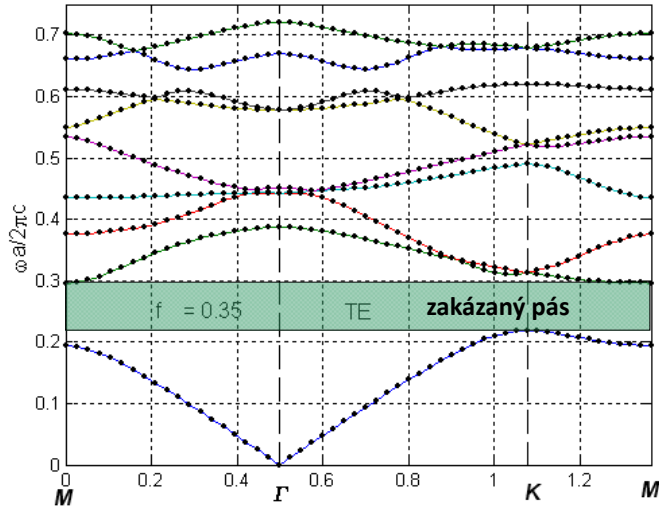
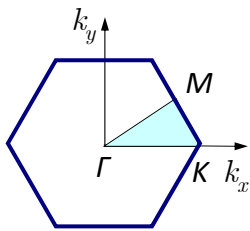
$$u_{\vec{k}}(\vec{r}_{\parallel}) = u_{\vec{k}}(\vec{r}_{\parallel} + \vec{a}_1) = u_{\vec{k}}(\vec{r}_{\parallel} + \vec{a}_2)$$

$$\vec{G} = m\vec{b}_1 + n\vec{b}_2; \quad m, n \text{ celé}$$



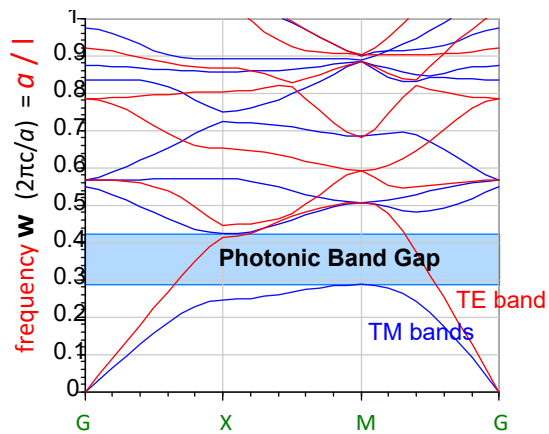
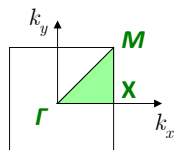
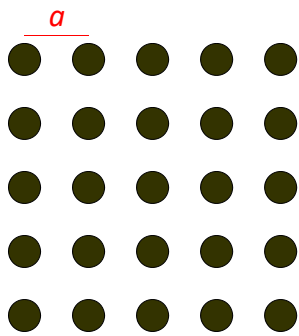
## Pásový diagram energií fotonů 2D krystalu s trojúhelníkovou mřížkou

první  
Brillouinova  
zóna  
prostoru  
vlnových vektorů



UFe

## Pásový diagram energií fotonů 2D krystalu se čtvercovou mřížkou

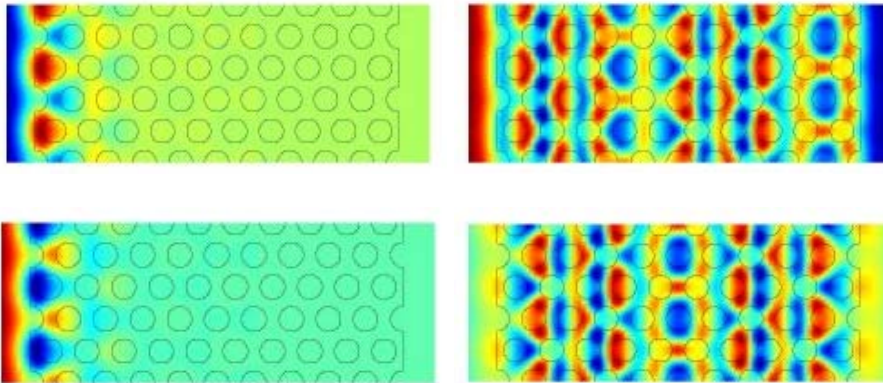


UFe

## Odraz rovinné vlny od 2D fotonického krystalu s trojúhelníkovou mřížkou otvorů v InP

Uvnitř zakázaného pásu

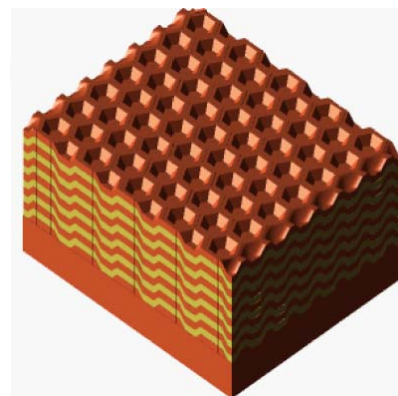
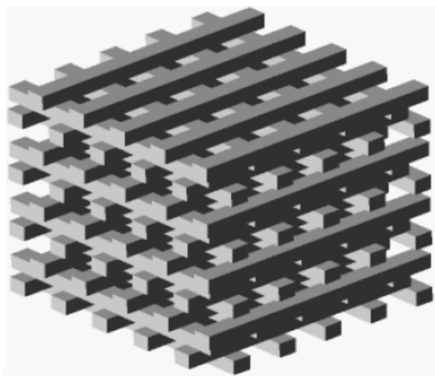
Vně zakázaného pásu



(Ing. Jiří Petráček, Dr., VUT Brno)

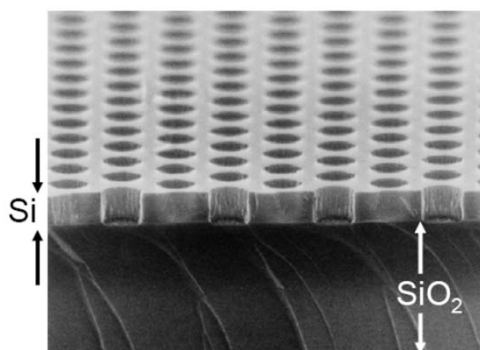
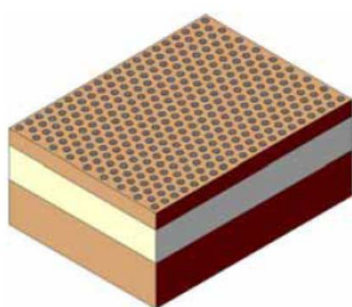
úfe

## Trojrozměrné fotonické krystaly



úfe

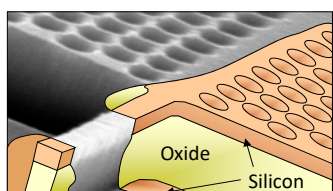
## „2.5-dimenzionální“ fotonické krystaly



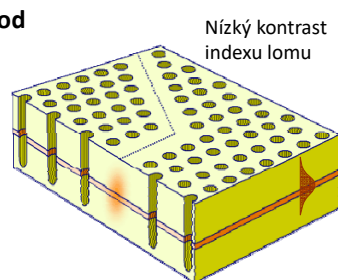
úfe

## Fotonické krystaly a vlnovody

### 1. 2D fotonický krystal + vertikální vlnovod

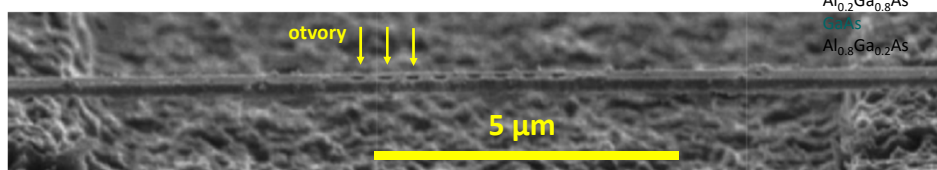


Vysoký  
kontrast  
indexu  
lomu



Nízký kontrast  
indexu lomu

### 2. Čárový 2D dielektrický vlnovod s 1D „fotonickým krystalem“

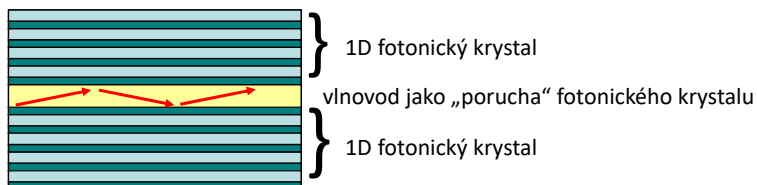


$\text{Al}_{0.2}\text{Ga}_{0.8}\text{As}$   
 $\text{Al}_{0.8}\text{Ga}_{0.2}\text{As}$

úfe



## Vlnovody v 1D fotonickém krystalu



Princip znám od 80. let jako „braggovský vlnovod“  
(antiresonant reflecting optical waveguide, **ARROW**)

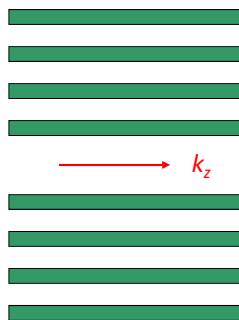
Rozdíly **ARROW** vlnovodu vůči konvenčnímu vlnovodu:

1. pro příslušný úhel dopadu vlny **musí** existovat **zakázaný pás**
2. počet period musí být dostatečný, jinak vzniká **útlum vytékáním** („tunelováním“); v krystalu konečných rozměrů existují **pouze vytékající vlny** s komplexní konstantou šíření

úfe

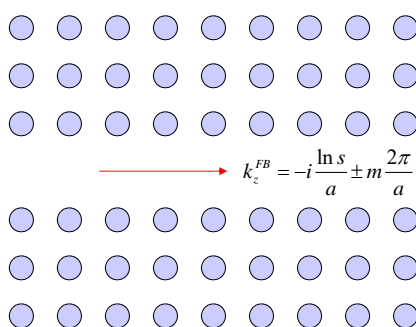
## Vlnovod ve fotonickém krystalu

Braggovský vlnovod  
(ARROW waveguide)



Anti-Reflecting Resonant  
Optical Waveguide

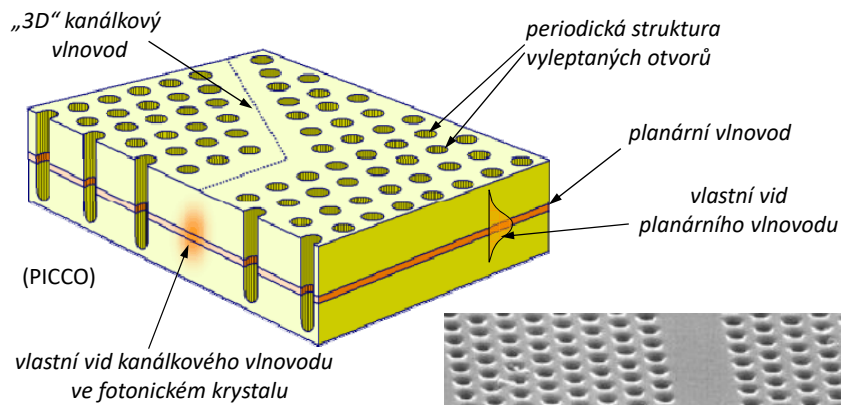
„Čárový defekt“ jako vlnovod  
1D periodicitu



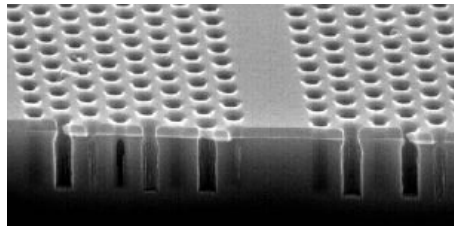
$k_z^{FB} a = \varphi \pm 2m\pi$  ... fázový posun při šíření  
o jednu periodu

úfe

## Realizace 2D fotonických krystalů: 2D krystal v planárním vlnovodu

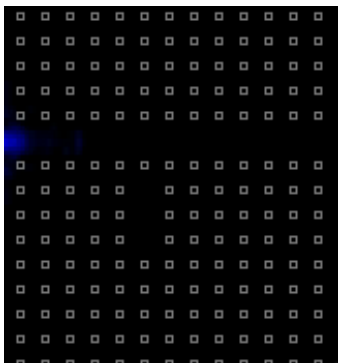


**Zásadní problém:**  
ztráty vyzářováním  
z roviny vlnovodu

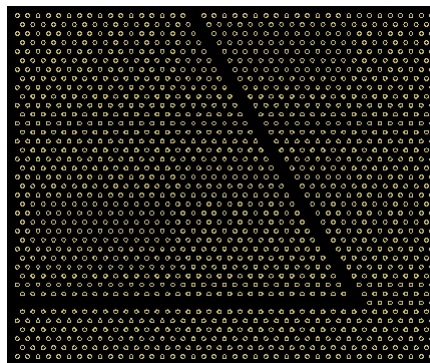


UFe

## Numerické modelování šíření vln ve fotonických krystalech



Buzení mikrodutiny  
ve fotonickém krystalu  
femtosekundovým impulsem  
(FDTD, Uni Twente, NL)



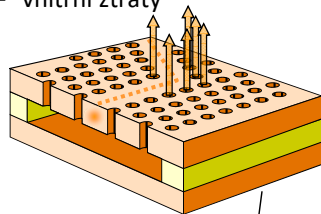
Šíření femtosekundového impulzu  
vlnovodným ohybem ve fotonickém  
krystalu (**F. Lederer et al.**,  
Friedrich-Schiller-Universität Jena, D)

UFe

## Ztráty vyzářováním z roviny krystalu

- Vysoký „vertikální“ kontrast indexu lomu

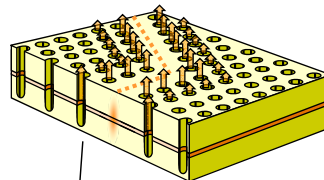
– vnitřní ztráty



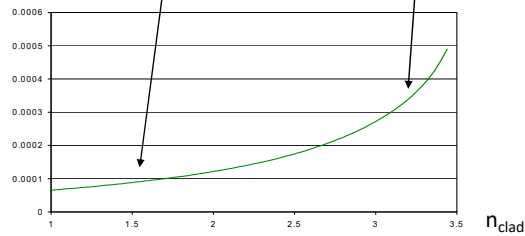
– ztráty vlivem drsnosti

- Malý „vertikální“ kontrast indexu lomu

– vnitřní ztráty



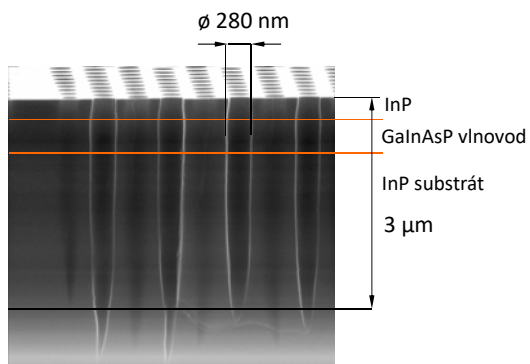
– ztráty vlivem drsnosti



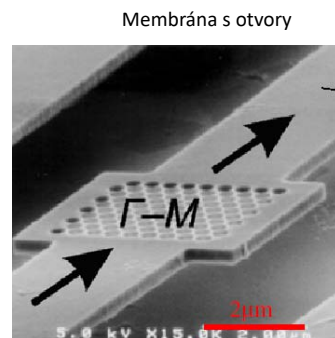
úfe

## Potlačení ztrát vyzářováním do substrátu

1. Leptání hlubokých otvorů: záření „nevnímá“ substrát (kromě vlnovodu ??)



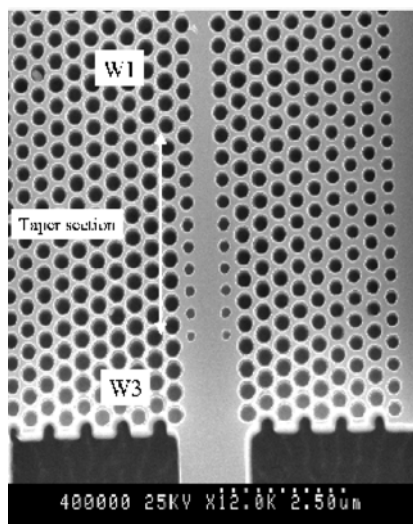
2. Úplné odstranění substrátu (technologicky náročné)



úfe

## Vazba s vlnovodem ve fotonickém krystalu

CNRS – LPN, Anne Talneau, Ph. Lalanne

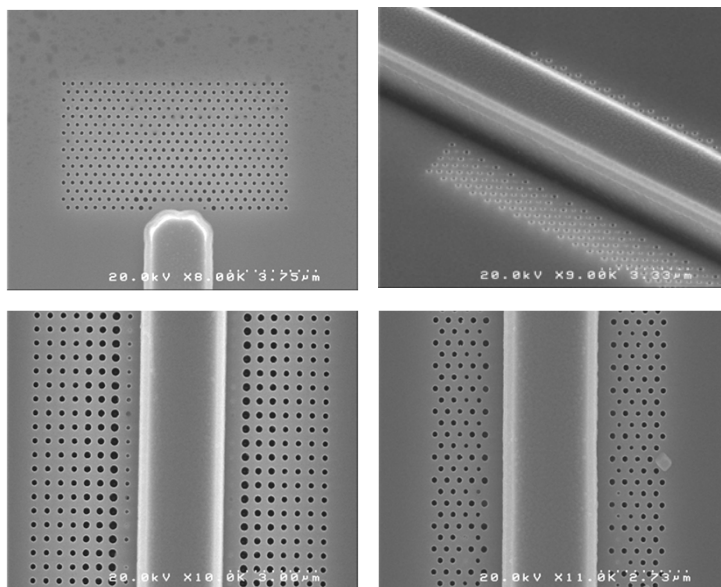


CNRS French patent  
(2001)

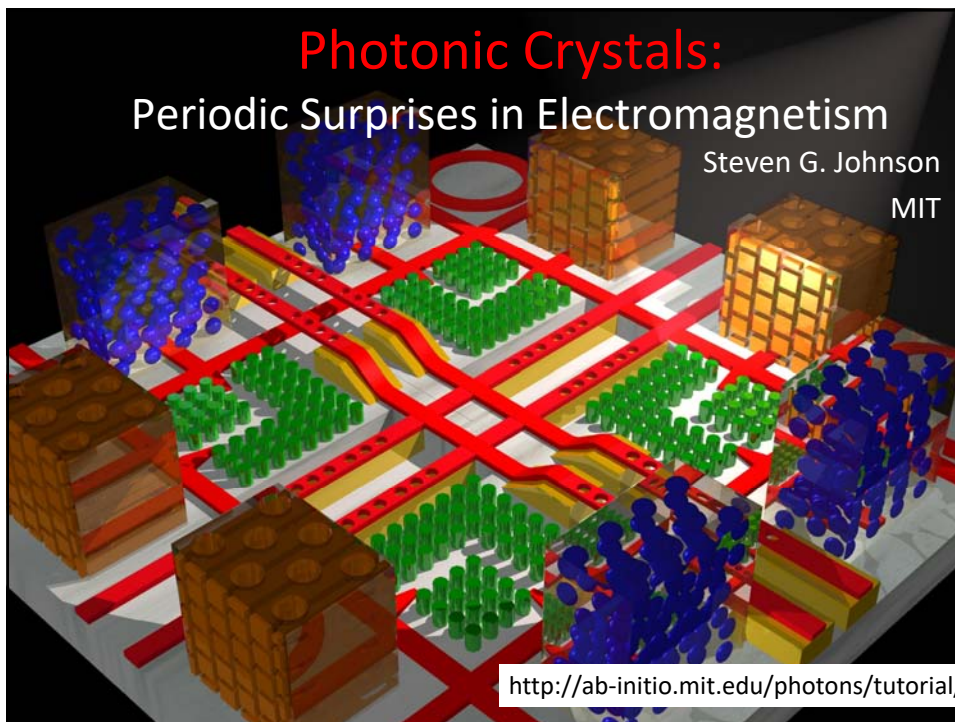
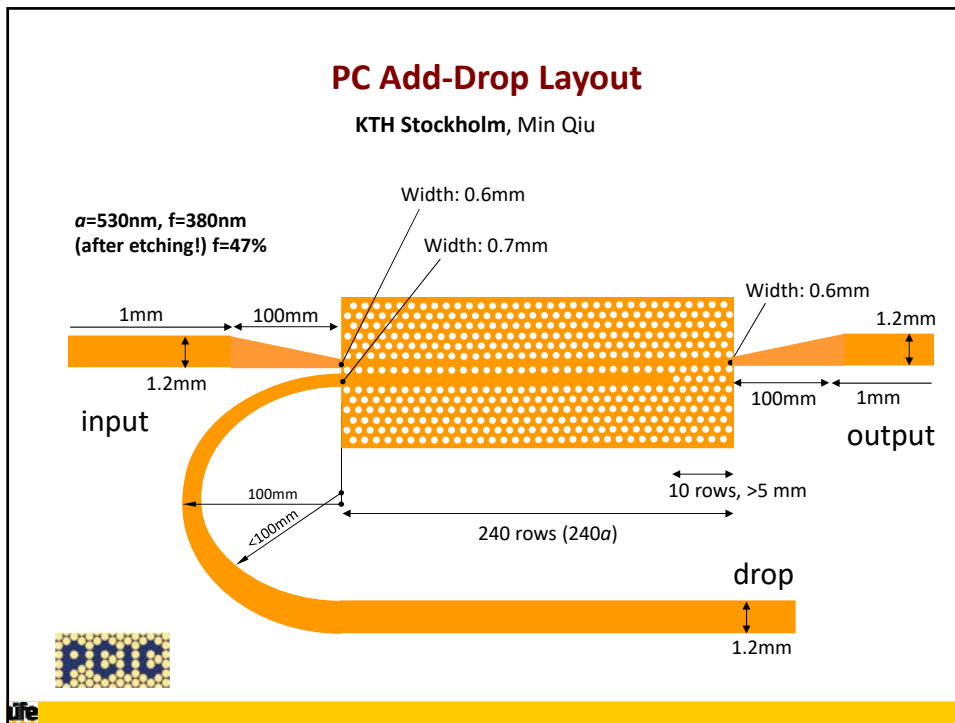
Ufe

## 2D fotonické krystaly jako zrcadla polovodičových laserů

(Alcatel, 2002-3)

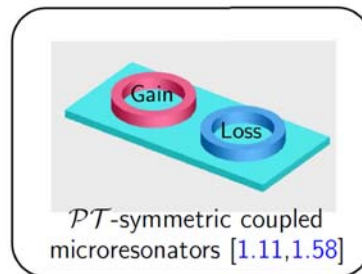
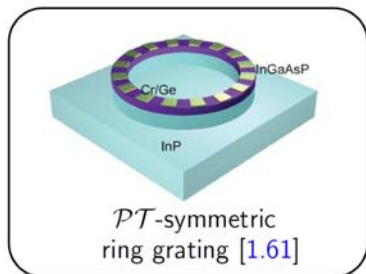
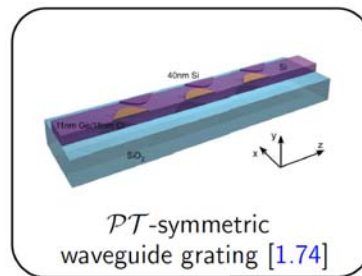
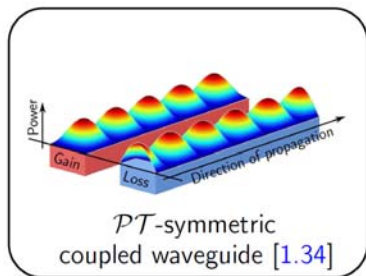


Ufe



## Vlnodné struktury se ztrátami a ziskem jako analogie kvantově-mechanických struktur s (porušenou) symetrií parita-čas

Life



1.34: C. E. Rüter et al., *Nat. Phys.* 6, 192-195 (2010).  
1.61: L. Feng et al., *Science* 346, 972-975 (2014).

1.74: L. Feng et al., *Science* 333, 729-733 (2011).  
1.11: L. Chang et al., *Nat. Photonics* 8, 524-529 (2014).  
1.58: H. Hodaei et al., *Science* 346, 975-978 (2014).

Life

## FORMAL ANALOGY BETWEEN A PHOTONIC WAVEGUIDE AND A POTENTIAL WELL IN QUANTUM MECHANICS

### Eigenmode equation for TE modes of a planar waveguide

$$\frac{1}{k_0^2} \frac{d^2 E(x)}{dx^2} + \varepsilon(x) E(x) = N^2 E(x)$$

mode field distribution

$E(x)$

$\Leftrightarrow$

$\psi(x)$

wave function

wave number

$k_0$

$\Leftrightarrow$

$\frac{\sqrt{2m}}{\hbar}$

mass; Planck constant

relative permittivity profile

$\varepsilon(x)$

$\Leftrightarrow$

$-V(x)$

potential

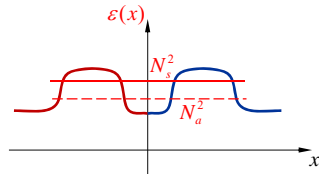
effective refractive index

$N^2$

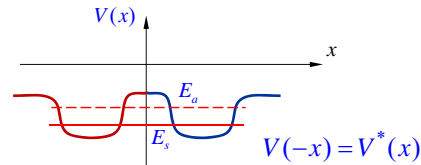
$\Leftrightarrow$

$-E$

particle energy



Loss/gain structure:  $\varepsilon(-x) = \varepsilon^*(x)$

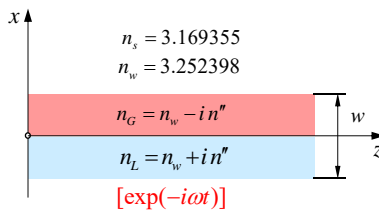


" $\mathcal{PT}$  symmetry": **complex potential(!)**,

life

## WAVEGUIDE STRUCTURE WITH LOSS/GAIN: HISTORICAL REMARKS

≈ 1995: **COST 240 Action**: Loss/gain waveguide modelling task by **H. P. Nolting (HHI)** (aimed at benchmarking of BPM methods)

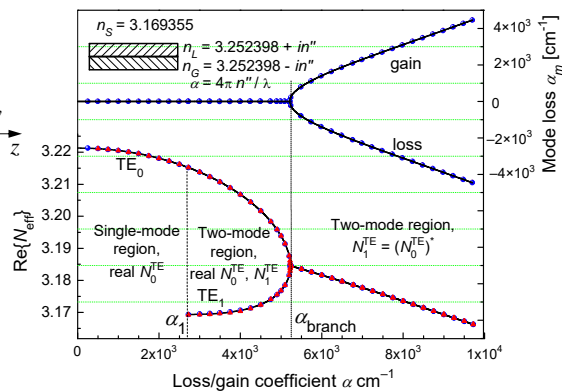


$w = 1 \mu\text{m}$ ,

$$n'' = \frac{\alpha}{2k_0} = \frac{\lambda}{4\pi} \alpha \times 10^{-4} [-; \mu\text{m}, \text{cm}^{-1}],$$

$\lambda = 1.55 \mu\text{m}$ ,

$\alpha \dots$  "loss/gain coefficient"  $[\text{cm}^{-1}]$



1. H.-P. Nolting, G. Sztэфka, J. tyroky, "Wave Propagation in a Waveguide with a Balance of Gain and Loss," Integrated Photonics Research '96, Boston, USA, 1996, pp. 76-79.  
2. G. Guekos, Ed., Photonic Devices for telecommunications: how to model and measure. Berlin: Springer, 1998, pp. 76-78.

life

## DISPERSION EQUATION (TE modes)

$$\Phi(N, \alpha) = \gamma_G [\gamma_S \cos(k_0 \gamma_G w) - \gamma_G \sin(k_0 \gamma_G w)] [\gamma_S \sin(k_0 \gamma_L w) + \gamma_L \cos(k_0 \gamma_L w)] + \gamma_L [\gamma_S \cos(k_0 \gamma_L w) - \gamma_L \sin(k_0 \gamma_L w)] [\gamma_S \sin(k_0 \gamma_G w) + \gamma_G \cos(k_0 \gamma_G w)] = 0$$

$$\gamma_S = \sqrt{N^2 - n_s^2}, \quad \gamma_L = \sqrt{n_L^2 - N^2}, \quad \gamma_G = \sqrt{n_G^2 - N^2}, \quad k_0 = \frac{2\pi}{\lambda}$$

"Exceptional" point:  $\frac{dN}{d\alpha} \rightarrow \infty$ .

Since  $\Phi(N, \alpha) \equiv 0$ ,  $\frac{\partial \Phi}{\partial N} \frac{\partial N}{\partial \alpha} + \frac{\partial \Phi}{\partial \alpha} = 0 \Rightarrow \frac{\partial N}{\partial \alpha} = -\frac{\partial \Phi / \partial \alpha}{\partial \Phi / \partial N} \rightarrow \infty \Rightarrow \frac{\partial \Phi}{\partial N} = 0$ .

Exceptional point is given by the simultaneous solution of the following two equations:

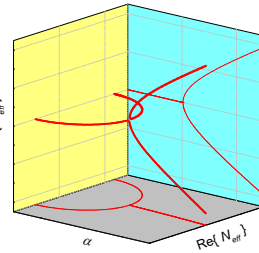
$$\Phi(N_B, \alpha_B) = 0, \quad \Phi'_N = \frac{d\Phi(N_B, \alpha_B)}{dN} = 0$$

Taylor expansion of  $\Phi(N, \alpha)$  in the vicinity of  $N_B, \alpha_B$  sounds

$$\Phi(N, \alpha) \approx \Phi'_\alpha (\alpha - \alpha_B) + \frac{1}{2} \Phi''_N (N - N_B)^2 = 0$$

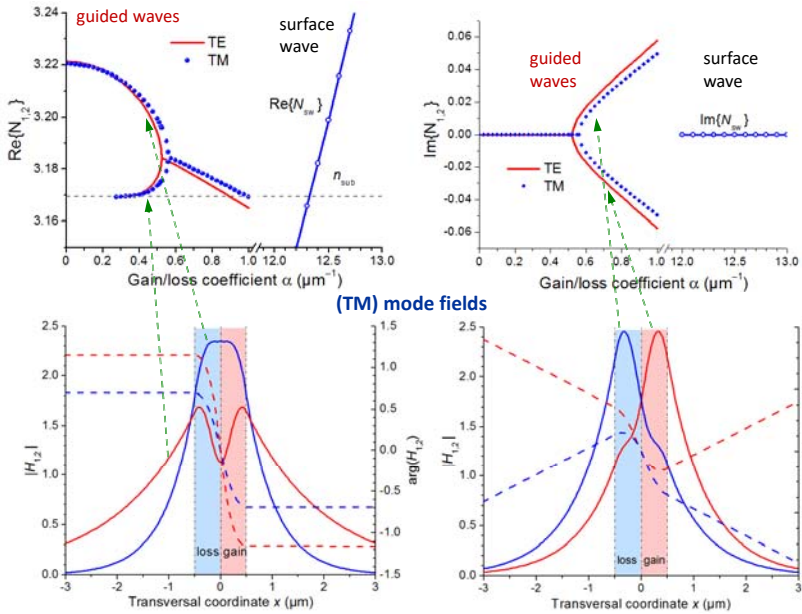
from which it follows

$$N \approx N_B \pm iC \sqrt{(\alpha - \alpha_B)}, \quad C = \sqrt{\frac{2\Phi'_\alpha}{\Phi''_N}}$$



life

## 2D ANALYSIS (PLANAR WAVEGUIDES)

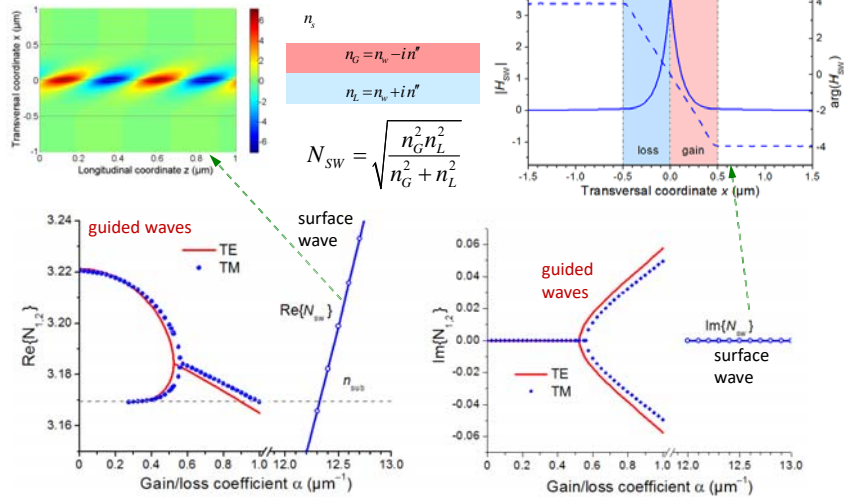


life



## WAVEGUIDE STRUCTURE WITH LOSS/GAIN: SURFACE WAVE

Non-attenuated TM surface wave



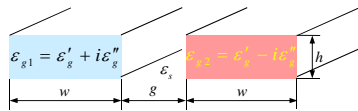
J. Čtyřoký et al., "Waveguide structures with antisymmetric gain/loss profile," *Optics Express*, vol. 18, pp. 21585-21593, 2010.



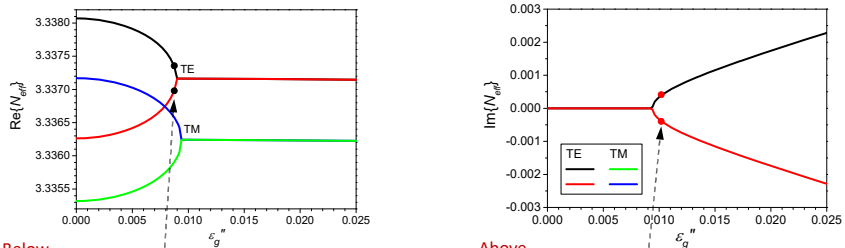
## COUPLED WAVEGUIDES WITH LOSS/GAIN

**Balanced loss/gain switching:**

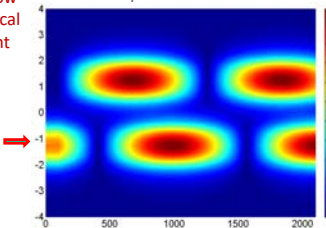
$$\mathcal{E}(-x, y) = \mathcal{E}^*(x, y)$$



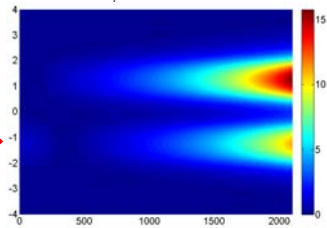
$\epsilon_s = 10.89, \epsilon'_s = 11.56$   
 $w = 1.5 \mu\text{m}, h = 0.75 \mu\text{m}$   
 $g = 1 \mu\text{m}, \lambda = 1.55 \mu\text{m}$



Below critical point



Above critical point



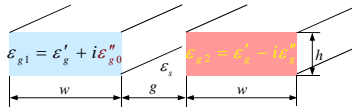
C. E. Rüter et al. "Observation of parity-time symmetry in optics," *Nature Physics*, vol. 6, pp. 192-195, 2010.



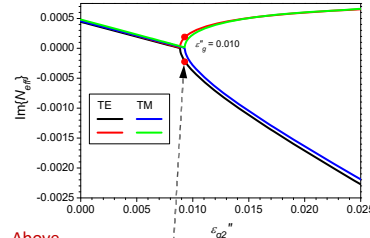
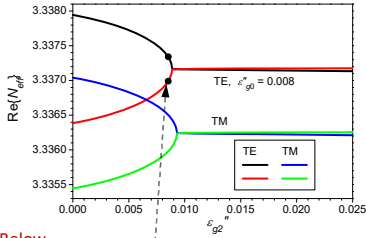
# COUPLED WAVEGUIDES WITH LOSS/GAIN

**Fixed loss/variable gain switching:**

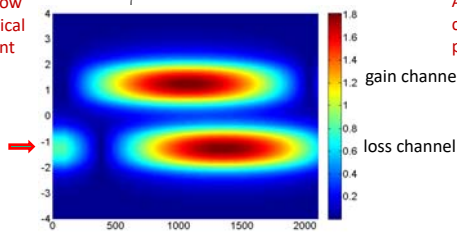
$$\mathcal{E}(-x, y) \neq \mathcal{E}^*(x, y)$$



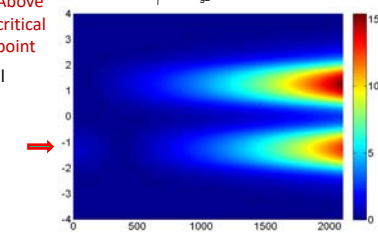
$$\begin{aligned} \epsilon_s &= 10.89, \quad \epsilon_g' = 11.56 \\ w &= 1.5 \mu\text{m}, \quad h = 0.75 \mu\text{m}, \\ g &= 1 \mu\text{m}, \quad \lambda = 1.55 \mu\text{m}. \end{aligned}$$



Below critical point



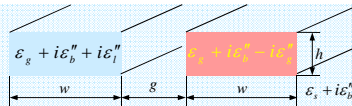
Above critical point



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# COUPLED WAVEGUIDES WITH LOSS/GAIN

**Un/balanced loss/gain:**



$$\begin{aligned} \epsilon_s &= 10.89, \quad \epsilon_g = 11.56 \\ w &= 1.5 \mu\text{m}, \quad h = 0.75 \mu\text{m}, \\ g &= 1 \mu\text{m}, \quad \lambda = 1.55 \mu\text{m}. \end{aligned}$$

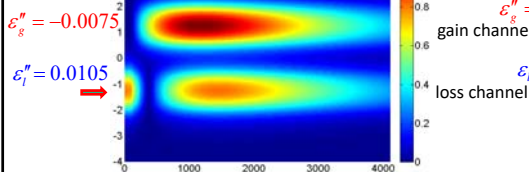
Structure with uniform background loss,  $\epsilon_b'' = 0.002$

Output **power increase** (from both waveguides!) by **increasing loss** of the lossy channel:

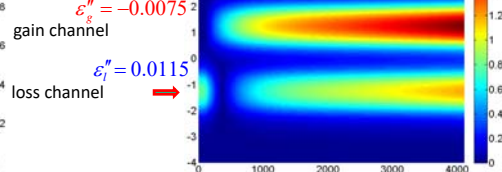
Lower loss, "subcritical" regime

Higher loss, "supercritical" regime

Below critical point



Above critical point



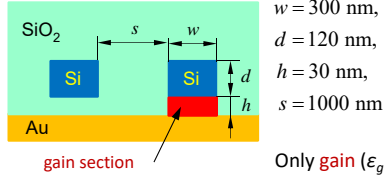
A. Guo et al., "Observation of PT-Symmetry Breaking in Complex Optical Potentials," *Physical Review Letters*, vol. 103, no. 9, pp. 093902-1-4, 2009.

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# PLASMONIC LOSS/GAIN STRUCTURES

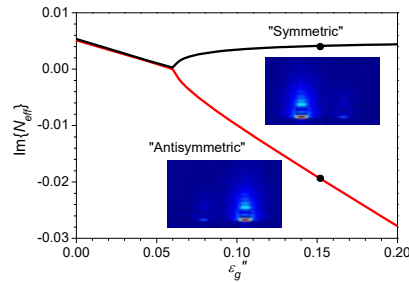
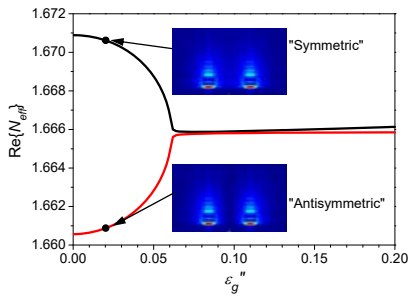
An **unbalanced** plasmonic loss/gain structure:

Hybrid dielectric-plasmonic slot waveguide directional coupler **with gain section**



$$\varepsilon(-x, y) \neq \varepsilon^*(x, y)$$

Only gain ( $\varepsilon_g''$ ) in the gain section is now tuned:  $\varepsilon_{\text{gain}} = \varepsilon_{\text{SiO}_2} - i\varepsilon_g''$



Life

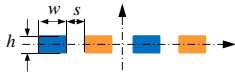
# MORE COMPLEX GAIN-LOSS STRUCTURES

Linear arrays of coupled waveguides with loss and gain

(quasi-TE polarization)

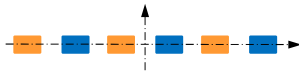
$$\varepsilon(-x, y) = \varepsilon^*(x, y)$$

4 coupled channel waveguides



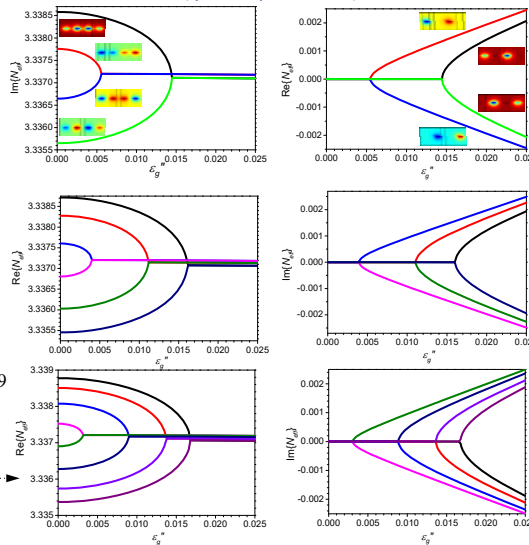
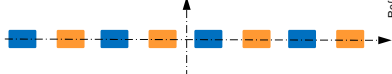
$w = 115 \text{ }\mu\text{m}$ ,  
 $h = 0.75 \text{ }\mu\text{m}$ ,  
 $s = 1 \text{ }\mu\text{m}$

6 coupled channel waveguides



$$\varepsilon_{g,1} = 11.56 + i\varepsilon_g'', \quad \varepsilon_{g,2} = 11.56 - i\varepsilon_g'', \quad \varepsilon_s = 10.89$$

8 coupled channel waveguides



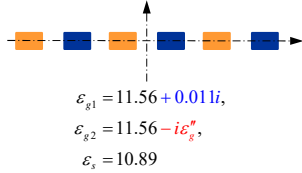
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## LINEAR ARRAY WITH UNBALANCED LOSS/GAIN

Coupled waveguides with *fixed loss* and *variable gain*

$$\varepsilon(-x, y) \neq \varepsilon^*(x, y)$$

6 coupled channel waveguides

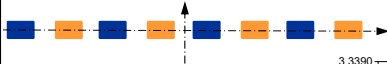


$$\varepsilon_{g1} = 11.56 + 0.011i,$$

$$\varepsilon_{g2} = 11.56 - i\varepsilon_g^*,$$

$$\varepsilon_s = 10.89$$

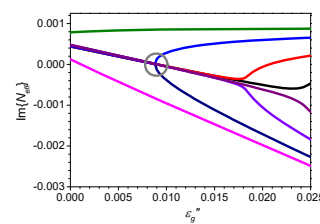
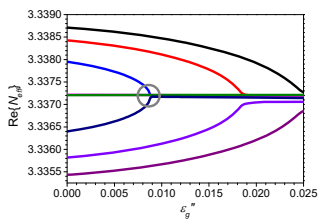
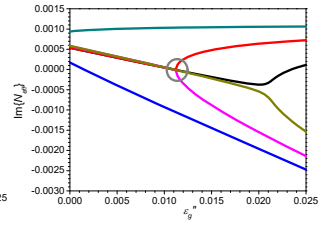
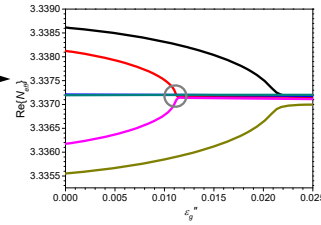
8 coupled channel waveguides



$$\varepsilon_{g1} = 11.56 + 0.009i,$$

$$\varepsilon_{g2} = 11.56 - i\varepsilon_g^*,$$

$$\varepsilon_s = 10.89$$



“Switching” by pure gain modulation is feasible also in loss/gain waveguide arrays

Life

## MORE COMPLEX GAIN-LOSS STRUCTURES

“Circular” arrays of coupled waveguides with loss and gain

4 waveguides

$$w = 1 \mu\text{m}$$

$$r = 1.5w$$

$$\varepsilon_{g1} = 11.56 + i\varepsilon_g^*,$$

$$\varepsilon_{g2} = 11.56 - i\varepsilon_g^*,$$

$$\varepsilon_s = 10.89$$

6 waveguides

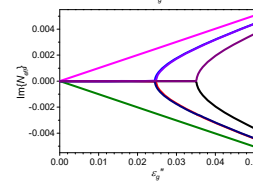
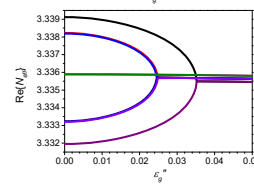
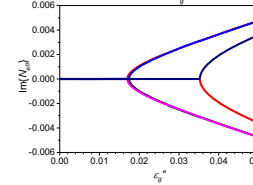
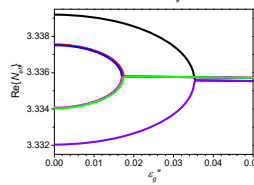
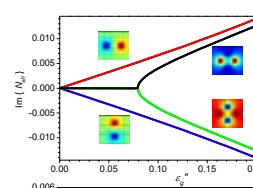
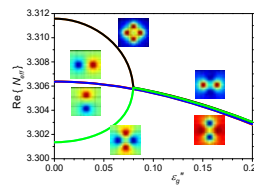
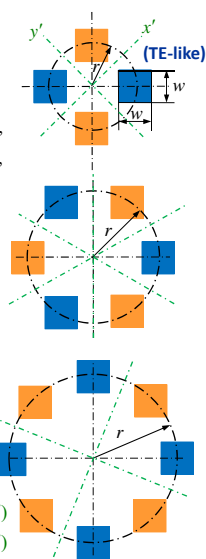
$$r = 2w$$

8 waveguides

$$r = 2.55w$$

$$\varepsilon(-x', y') = \varepsilon^*(x', y')$$

$$\varepsilon(x', -y') = \varepsilon^*(x', y')$$



Life

## CIRCULAR ARRAYS WITH UNBALANCED LOSS/GAIN

Coupled waveguides with *fixed loss* and *variable gain*

6 waveguides

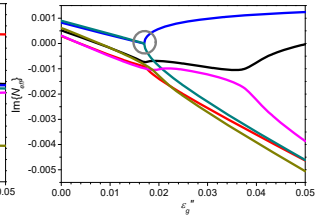
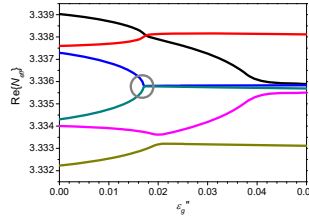
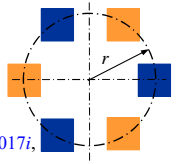
$$w = 1 \mu\text{m}$$

$$r = 2w$$

$$\epsilon_s = 10.89$$

$$\epsilon_{g1} = 11.56 + 0.017i$$

$$\epsilon_{g2} = 11.56 - i\epsilon_g''$$

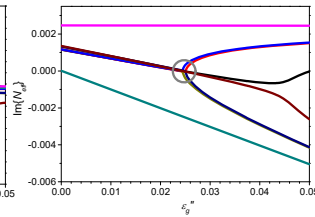
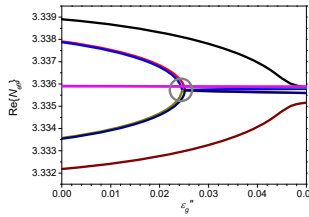
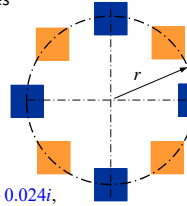


8 waveguides

$$r = 2.55w$$

$$\epsilon_{g1} = 11.56 + 0.024i$$

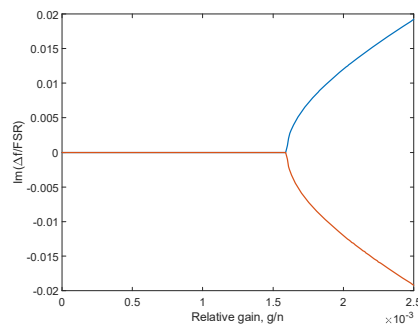
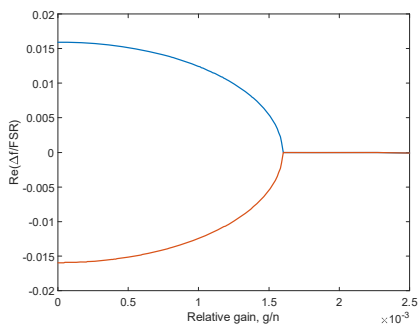
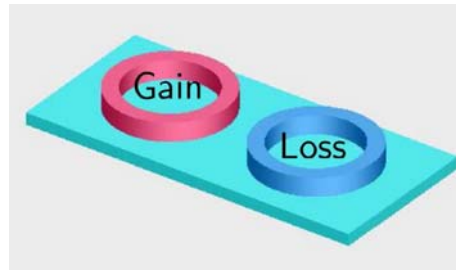
$$\epsilon_{g2} = 11.56 - i\epsilon_g''$$



“Switching” by pure gain modulation is feasible also in loss/gain waveguide arrays

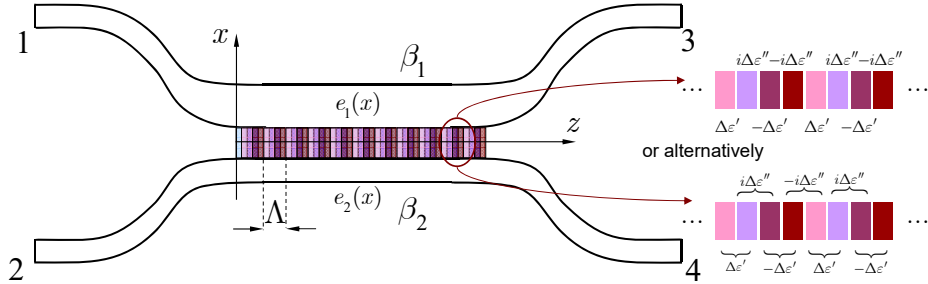
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## RESONANT FREQUENCIES OF A PAIR OF COUPLED $\mathcal{PT}$ -SYMMETRIC RING RESONATORS



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## ASYMMETRIC COMPLEX GRATING COUPLER



Grating-assisted directional coupler using **asymmetric complex grating**

$$E_y(x, z) \approx A_1(z)e_1(x)\exp(i\beta_1 z) + A_2(z)e_2(x)\exp(i\beta_2 z); \quad \text{complex, periodic in } z$$

$$\frac{dA_1(z)}{dz} \cong i\kappa_{11}(z)A_1(z) + i\kappa_{12}(z)e^{-i(\beta_1 - \beta_2)z}A_2(z), \quad \kappa_{mn}(z) = \frac{k_0}{2} \int_S \Delta\epsilon(x, z)e_m(x)e_n(x) dS$$

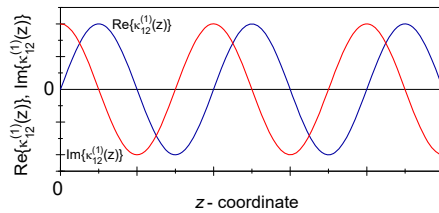
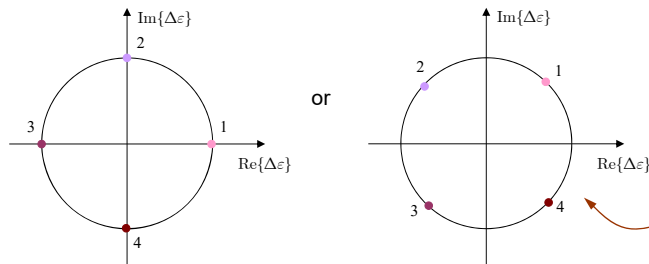
$$\frac{dA_2(z)}{dz} \cong i\kappa_{21}(z)e^{i(\beta_1 - \beta_2)z}A_1(z) + i\kappa_{22}(z)A_2(z), \quad = \kappa_{mn}^{(1)}e^{iKz} + \kappa_{mn}^{(2)}e^{2iKz} + \dots, \quad K = 2\pi/\Lambda$$

Fourier expansion contains only **positive exponentials** ("SSB modulation")

Life

## ASYMMETRIC COMPLEX GRATING

Complex permittivity perturbation in individual grating segments:



The second option seems technologically simpler since it requires only **two different values** of  $\Delta\epsilon'$  and  $\Delta\epsilon''$

Life

## SOLUTION OF COUPLED MODE EQUATIONS

Let us consider the following ideal case of the grating at synchronism,

$$\begin{aligned} \kappa_{11}(z) &= 0, \quad \kappa_{12}(z) = \kappa_{12}^{(1)} \exp(iKz), \\ \kappa_{21}(z) &= 0, \quad \kappa_{22}(z) = 0, \\ \Delta\beta &= K - (\beta_1 - \beta_2) = 0. \end{aligned}$$

Then, the coupled equations read

$$\begin{aligned} \frac{dA_1(z)}{dz} &\cong i\kappa_{12}^{(1)}A_2(z), \\ \frac{dA_2(z)}{dz} &\cong 0. \end{aligned}$$

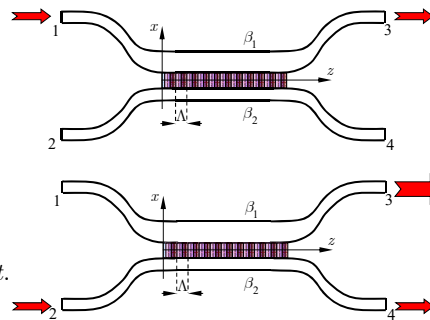
For  $A_1(0) \neq 0$ ,  $A_2(0) = 0$

we get the solution

$$\begin{aligned} A_1(z) &= A_1(0) = \text{const.}, \quad P_1(z) = P_1(0), \\ A_2(z) &= 0, \quad P_2 = 0. \end{aligned}$$

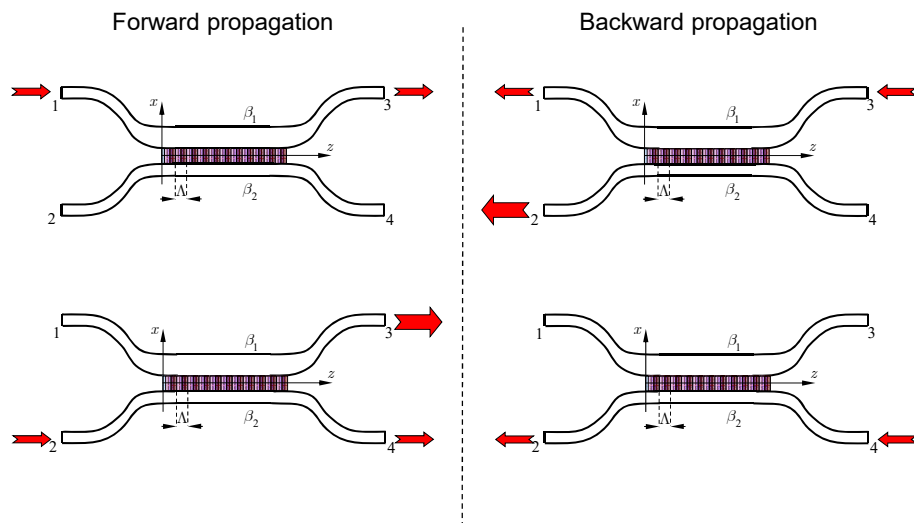
for  $A_1(0) = 0$ ,  $A_2(0) \neq 0$

$$\begin{aligned} A_1(z) &= i\kappa_{12}^{(1)}A_2(0)z, \quad P_1(z) = |\kappa_{12}^{(1)}|^2 P_2(0)z^2, \\ A_2(z) &= A_2(0) = \text{const.}, \quad P_2(z) = P_2(0) = \text{const.} \end{aligned}$$

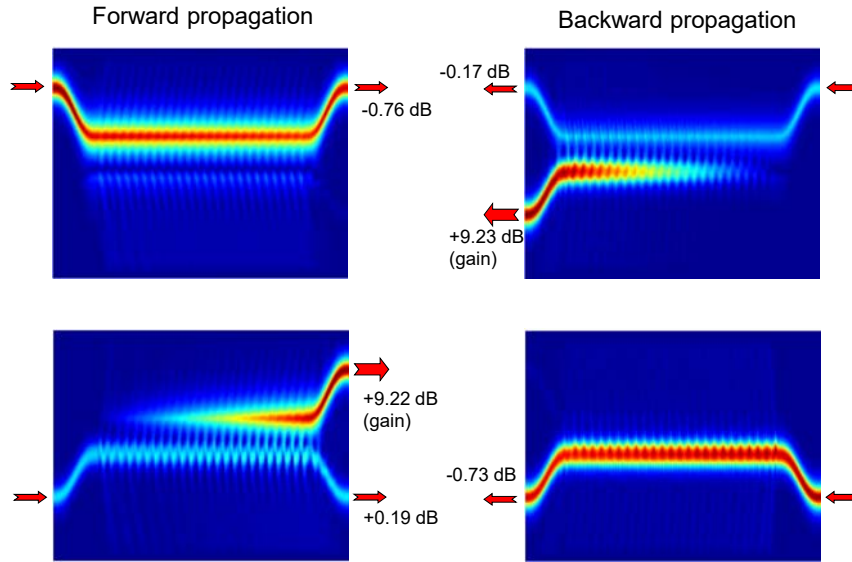


we get

## ACGC IS A RECIPROCAL DEVICE!



## “STANDARD” ACGC – RESULTS

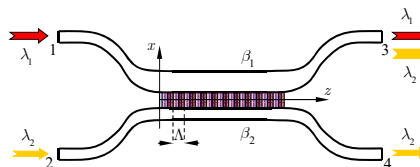


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## STRAIGHTFORWARD ACGC APPLICATIONS

### Wideband ADD multiplexor

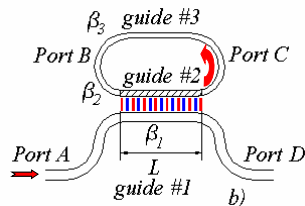
M. Greenberg and M. Orenstein,  
*PTL* 17, 1450-1452, 2005



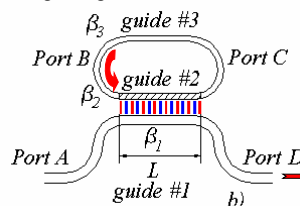
### Light trapping in a ring resonator (a “dynamic memory cell”)

M. Kulishov *et al.*, *OE* 13, 3567-3578, 2005

grating “switched on”:



grating “switched off”:



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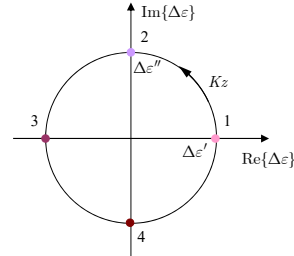


## “SSB” MODULATION

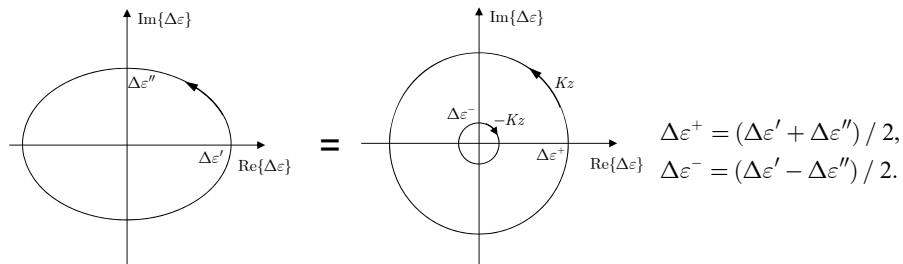
For “unidirectional” behaviour, the condition  $\Delta\varepsilon' = \Delta\varepsilon''$  is of key importance:

$$\dots \begin{matrix} i\Delta\varepsilon'' & -i\Delta\varepsilon'' & i\Delta\varepsilon'' & -i\Delta\varepsilon'' \\ \Delta\varepsilon' & -\Delta\varepsilon' & \Delta\varepsilon' & -\Delta\varepsilon' \end{matrix} \dots$$

$$\begin{aligned} \Delta\varepsilon(z) &= \Delta\varepsilon' \cos Kz + i\Delta\varepsilon'' \sin Kz \\ &= \frac{1}{2}(\Delta\varepsilon' + \Delta\varepsilon'')e^{iKz} + \frac{1}{2}(\Delta\varepsilon' - \Delta\varepsilon'')e^{-iKz} \end{aligned}$$



In the case of  $\Delta\varepsilon' \neq \Delta\varepsilon''$  we get



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## SOME RELEVANT REFERENCES

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