A NOTE ON THE a-BROWDER'S AND a-WEYL'S THEOREMS

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Abstract. Let T be a Banach space operator. In this paper we characterize a-Browder's theorem for T by the localized single valued extension property. Also, we characterize a-Weyl's theorem under the condition $E^a(T)=\pi^a(T)$, where $E^a(T)$ is the set of all eigenvalues of T which are isolated in the approximate point spectrum and $\pi^a(T)$ is the set of all left poles of T. Some applications are also given.

Keywords: B-Fredholm operator, Weyl's theorem, Browder's thoerem, operator of Kato type, single-valued extension property

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1. Introduction and definitions

Throughout this paper, $\mathcal{L}(X)$ denotes the algebra of all bounded linear operators acting on a Banach space X. For $T\in\mathcal{L}(X)$, let T^*, $N(T)$, $R(T)$, $\sigma(T)$, $\sigma_{\rm p}(T)$ and $\sigma_{\rm ap}(T)$ denote respectively the adjoint, the null space, the range, the spectrum, the point spectrum and the approximate point spectrum of T. Let $\alpha(T)$ and $\beta(T)$ be the nullity and the deficiency of T defined by

$$\alpha(T)=\dim N(T) \text{ and } \beta(T)=\operatorname{codim} R(T).$$

If the range $R(T)$ of T is closed and $\alpha(T)<\infty$ or $\beta(T)<\infty$, then T is called an *upper semi-Fredholm* or a *lower semi-Fredholm operator*, respectively.

In the sequel $\mathrm{SF}_+(X)$ (resp. $\mathrm{SF}_-(X)$) will denote the set of all upper (resp. lower) semi-Fredholm operator.

If $T\in\mathcal{L}(X)$ is either upper or lower semi-Fredholm, then T is called a *semi-Fredholm operator*, and the *index* of T is defined by $\operatorname{ind}(T)=\alpha(T)-\beta(T)$. If both $\alpha(T)$ and $\beta(T)$ are finite, then T is a *Fredholm operator*.

An operator T is called *Weyl* if it is Fredholm of index zero. For $T \in \mathcal{L}(X)$ and $n \in \mathbb{N}$ define $c_n(T)$ and $c'_n(T)$ by $c_n(T) = \dim R(T^n)/R(T^{n+1})$ and $c'_n(T) = \dim N(T^{n+1})/N(T^n)$. The *descent* $q(T)$ and the *ascent* $p(T)$ are given by

$$q(T) = \inf\{n\colon\ c_n(T) = 0\} = \inf\{n\colon\ R(T^n) = R(T^{n+1})\},$$
$$p(T) = \inf\{n\colon\ c'_n(T) = 0\} = \inf\{n\colon\ N(T^n) = N(T^{n+1})\}.$$

A bounded linear operator T is called *Browder* if it is Fredholm of finite ascent and descent. The essential spectrum $\sigma_{\mathrm{e}}(T)$, Weyl spectrum $\sigma_{\mathrm{w}}(T)$, and Browder spectrum $\sigma_{\mathrm{b}}(T)$ of $T \in \mathcal{L}(X)$ are defined by

$$\sigma_{\mathrm{e}}(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \text{ is not Fredholm}\},$$
$$\sigma_{\mathrm{w}}(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \text{ is not Weyl}\},$$
$$\sigma_{\mathrm{b}}(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \text{ is not Browder}\}.$$

Evidently

$$\sigma_{\mathrm{e}}(T) \subseteq \sigma_{\mathrm{w}}(T) \subseteq \sigma_{\mathrm{b}}(T).$$

For a subset $K \subseteq \mathbb{C}$, we write $\operatorname{acc} K$ or $\operatorname{iso} K$ for the accumulation or isolated points of K, respectively.

We say that *Weyl's theorem* holds for $T \in \mathcal{L}(X)$ if

$$\sigma(T) \setminus \sigma_{\mathrm{w}}(T) = E_0(T),$$

where $E_0(T)$ is the set of isolated points of $\sigma(T)$ which are eigenvalues of finite multiplicity, and that *Browder's theorem* holds for $T \in \mathcal{L}(X)$ if

$$\sigma_{\mathrm{w}}(T) = \sigma_{\mathrm{b}}(T).$$

For $T \in \mathcal{L}(X)$, let $\mathrm{SF}_+^-(X)$ be the class of all $T \in \mathrm{SF}_+(X)$ with $\operatorname{ind} T \leqslant 0$. The *essential approximate point spectrum* $\sigma_{\mathrm{SF}_+^-}(T)$ and the *Browder essential approximate point spectrum* $\sigma_{\mathrm{ab}}(T)$ (see [24], [25]) are defined by

$$\sigma_{\mathrm{SF}_+^-}(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \text{ is not in } \mathrm{SF}_+^-(X)\},$$
$$\sigma_{\mathrm{ab}}(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \notin \sigma_{\mathrm{SF}_+^-}(T) \text{ or } p(T - \lambda) = \infty\}.$$

We say that *a-Weyl's theorem* holds for $T \in \mathcal{L}(X)$ if

$$\sigma_{\mathrm{ap}}(T) \setminus \sigma_{\mathrm{SF}_+^-}(T) = E_0^a(T),$$

where $E_0^a(T)$ is the set of isolated points of $\sigma_{\rm ap}(T)$ which are eigenvalues of finite multiplicity, and that *a-Browder's theorem* holds for $T \in \mathcal{L}(X)$ if

$$\sigma_{\mathrm{SF}_+^-}(T) = \sigma_{\rm ab}(T).$$

In [10], [26], it is shown that for any $T \in \mathcal{L}(X)$ we have the implications

$$\begin{aligned}&a\text{-Weyl's theorem} \Rightarrow \text{Weyl's theorem} \Rightarrow \text{Browder's theorem},\\ &a\text{-Weyl's theorem} \Rightarrow a\text{-Browder's theorem} \Rightarrow \text{Browder's theorem}.\end{aligned}$$

For a bounded linear operator T and a nonnegative integer n define $T_{[n]}$ to be the restriction of T to $R(T^n)$ viewed as a map from $R(T^n)$ into $R(T^n)$ (in particular, $T_{[0]} = T$). If for some integer n the range space $R(T^n)$ is closed and $T_{[n]}$ is an upper or a lower semi-Fredholm operator, then T is called an *upper* or a *lower semi-B-Fredholm* operator, respectively. In this case the *index* of T is defined as the index of the semi-Fredholm operator $T_{[n]}$, see [8], [9]. Moreover, if $T_{[n]}$ is a Fredholm operator, then T is called a *B-Fredholm* operator. A *semi-B-Fredholm* operator is an upper or a lower semi-B-Fredholm operator. An operator $T \in \mathcal{L}(X)$ is said to be a *B-Weyl operator* if it is a B-Fredholm operator of index zero. The *semi-B-Fredholm spectrum* $\sigma_{\rm SBF}(T)$ and the *B-Weyl spectrum* $\sigma_{\rm BW}(T)$ of T are defined by

$$\begin{aligned}\sigma_{\rm SBF}(T) &= \{\lambda \in \mathbb{C}\colon\ T - \lambda I \text{ is not a semi-B-Fredholm operator}\},\\ \sigma_{\rm BW}(T) &= \{\lambda \in \mathbb{C}\colon\ T - \lambda I \text{ is not a B-Weyl operator}\}.\end{aligned}$$

We say that the *generalized Weyl's theorem* holds for T if

$$\sigma(T) \setminus \sigma_{\rm BW}(T) = E(T),$$

where $E(T)$ is the set of all isolated eigenvalues of T, and the *generalized Browder's theorem* holds for T if

$$\sigma(T) \setminus \sigma_{\rm BW}(T) = \pi(T),$$

where $\pi(T)$ is the set of all poles of T (see [8, Definition 2.13]). The generalized Weyl's and generalized Browder's theorems have been studied in [3], [7], [8], [28]. Similarly, let $\mathrm{SBF}_+(X)$ be the class of all upper semi-B-Fredholm operators, and $\mathrm{SBF}_+^-(X)$ the class of all $T \in \mathrm{SBF}_+(X)$ such that $\mathrm{ind}(T) \leqslant 0$. Further, let

$$\sigma_{\mathrm{SBF}_+^-}(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \text{ is not in } \mathrm{SBF}_+^-(X)\},$$

which is called the *semi-essential approximate point spectrum*, see [8]. We say that T obeys the *generalized a-Weyl's theorem* if

$$\sigma_{\mathrm{SBF}_+^-}(T) = \sigma_{\rm ap}(T) \setminus E^a(T),$$

where $E^a(T)$ is the set of all eigenvalues of T which are isolated in $\sigma_{\mathrm{ap}}(T)$ ([8, Definition 2.13]). From [8], we know that

$$\text{generalized } a\text{-Weyl's theorem} \Rightarrow \text{generalized Weyl's theorem} \Rightarrow \text{Weyl's theorem},$$

$$\text{generalized } a\text{-Weyl's theorem} \Rightarrow a\text{-Weyl's theorem}.$$

Moreover, in [5] it is shown that, if $E(T) = \pi(T)$, then

$$\text{generalized Weyl's theorem} \Leftrightarrow \text{Weyl's theorem},$$

and if $E^a(T) = \pi^a(T)$, then

$$\text{genearlized } a\text{-Weyl's theorem} \Leftrightarrow a\text{-Weyl's theorem}.$$

For $T \in \mathcal{L}(X)$ we say that T is *Drazin invertible,* if there exist $B, U \in \mathcal{L}(X)$ such that U is nilpotent and $TB = BT$, $BTB = B$ and $TBT = T + U$. It is known that T is Drazin invertible if and only if it has finite ascent and descent, which is also equivalent to the fact that $T = T_0 \oplus T_1$, where T_0 is invertible and T_1 is nilpotent, see [16, Proposition A] and [19, Corollary 2.2]. The Drazin spectrum is defined by

$$\sigma_D(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \text{ is not Drazin invertible}\}.$$

As in [22], define a set $\mathrm{LD}(X)$ by

$$\mathrm{LD}(X) = \{T \in \mathcal{L}(X)\colon\ p(T) < \infty \text{ and } R(T^{p(T)+1}) \text{ is closed}\}.$$

An operator $T \in \mathcal{L}(X)$ is said to be *left Drazin invertible* if $T \in \mathrm{LD}(X)$. The left Drazin spectrum $\sigma_{\mathrm{LD}}(T)$ of T is defined by

$$\sigma_{\mathrm{LD}}(T) = \{\lambda \in \mathbb{C}\colon\ T - \lambda \text{ is not in } \mathrm{LD}(X)\}.$$

It is known, see [8, Lemma 2.12], that

$$\sigma_{\mathrm{SBF}_+^-}(T) \subseteq \sigma_{\mathrm{LD}}(T) \subseteq \sigma_{\mathrm{ap}}(T).$$

We say that $\lambda \in \sigma_{\mathrm{ap}}(T)$ is a *left pole* of T if $T - \lambda \in \mathrm{LD}(X)$, and that $\lambda \in \sigma_{\mathrm{ap}}(T)$ is a left pole of T of finite rank if λ is a left pole of T and $\alpha(T - \lambda) < \infty$. We denote by $\pi^a(T)$ the set of all left poles of T, and by $\pi_0^a(T)$ the set of all left poles of finite rank. We say that T obeys the *generalized a-Browder's theorem* if

$$\sigma_{\mathrm{SBF}_+^-}(T) = \sigma_{\mathrm{ap}}(T) \setminus \pi^a(T).$$

Recently, in [5] the authors proved that

$$\text{generalized Browder's theorem} \Leftrightarrow \text{Browder's theorem},$$
$$\text{generalized } a\text{-Browder's theorem} \Leftrightarrow a\text{-Browder's theorem}.$$

The quasi-nilpotent part of T is the subspace

$$H_0(T) := \{x \in X \colon \lim_{n\to\infty} \|T^n x\|^{1/n} = 0\}.$$

The space $H_0(T)$ is hyperinvariant under T and satisfies $T^{-n}(0) \subseteq H_0(T)$ for all $n \in \mathbb{N}$. For its further properties, see [1], [20], [21].

An operator $T \in \mathcal{L}(X)$ is said to be *semi-regular* if $R(T)$ is closed and $N(T) \subseteq R(T^n)$ for every $n \in \mathbb{N}$. We say that T is of *Kato type* at a point $\lambda \in \mathbb{C}$ if there exists a pair of T-invariant closed subspaces (M, N) such that $X = M \oplus N$, the restriction $(T-\lambda)\big|_M$ is nilpotent and $(T-\lambda)\big|_N$ is semi-regular.

Let $\mathcal{O}(U, X)$ be the Fréchet space of all X-valued analytic functions on an open subset U of $\mathbb{C}$. We say that $T \in \mathcal{L}(X)$ has the *single-valued extension property* at $\lambda \in \mathbb{C}$ (the SVEP for short) if for every open disk $D(\lambda, r)$, the map

$$\begin{aligned} T_{D(\lambda,r)} \colon \mathcal{O}(D(\lambda, r), X) &\longrightarrow \mathcal{O}(D(\lambda, r), X) \\ f &\longmapsto (z - T)f \end{aligned}$$

is injective. Let $S(T)$ be the set of all λ on which T does not have the SVEP. We say that T has the SVEP if $S(T) = \emptyset$, see [12]. We note that $S(T) \subseteq \sigma_{\mathrm{p}}(T)$.

2. Preliminary results

Definition 2.1 [13]. Let $T \in \mathcal{L}(X)$ and $d \in \mathbb{N}$. Then T has a *uniform descent* for $n \geqslant d$ if

$$R(T) + N(T^n) = R(T) + N(T^d) \text{ for all } n \geqslant d.$$

If in addition, $R(T) + N(T^d)$ is closed, then T is said to have a *topological uniform descent* for $n \geqslant d$.

The following result which is proved in [6] is a generalization of the result of Finch [12].

Lemma 2.1. *Let $T \in \mathcal{L}(X)$. If T is an operator of topological uniform descent for $n \geqslant d$, then the following conditions are equivalent:*
(i) *T has the SVEP at 0.*
(ii) *0 is not an accumulation point of $\sigma(T)$.*

Theorem 2.1. *Let $T \in \mathcal{L}(X)$. Then T satisfies a-Browder's theorem if and only if T has the SVEP at $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$.*

P r o o f. Suppose that T satisfies a-Browder's theorem, that is

$$\sigma_{\mathrm{ap}}(T) \setminus \sigma_{\mathrm{SBF}_+^-}(T) = \pi^a(T).$$

Let us see that T has the SVEP at $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$. If $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$, then $\lambda \in \pi^a(T)$, and hence $\lambda \in \operatorname{iso} \sigma_{\mathrm{ap}}(T)$ (see [8, Remark 2.6]). This implies that T has the SVEP at $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$. For the opposite implication suppose that $T - \lambda$ has the SVEP for all $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$. Let us prove that $\sigma_{\mathrm{ap}}(T) \setminus \sigma_{\mathrm{SBF}_+^-}(T) = \pi^a(T)$. We know that $\sigma_{\mathrm{ap}}(T) \setminus \sigma_{\mathrm{SBF}_+^-}(T) \supseteq \pi^a(T)$. Hence it suffices to prove that $\sigma_{\mathrm{ap}}(T) \setminus \sigma_{\mathrm{SBF}_+^-}(T) \subseteq \pi^a(T)$. If $\lambda \in \sigma_{\mathrm{ap}}(T)$ and $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$, then $T - \lambda$ is of topological uniform descent. Since T has the SVEP at λ, hence according to Lemma 2.1 λ is isolated in $\sigma(T)$, and hence also in $\sigma_{\mathrm{ap}}(T)$. From [8, Theorem 2.8] we conclude that $\lambda \in \pi^a(T)$. Consequently,

$$\sigma_{\mathrm{ap}}(T) \setminus \sigma_{\mathrm{SBF}_+^-}(T) \subseteq \pi^a(T).$$

□

In [5], it is proved that a-Weyl's theorem and a-Browder's theorem are equivalent under the condition $E^a(T) = \pi^a(T)$.

Proposition 2.1 [5]**.** *Let $T \in \mathcal{L}(X)$ be such that $E^a(T) = \pi^a(T)$. Then the following properties are equivalent:*

i) *T satisfies a-Browder's theorem.*

ii) *T satisfies a-Weyl's theorem.*

The following result shows that a-Weyl's theorem and a-Browder's theorem are equivalent to the SVEP at $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$.

Theorem 2.2. *Let $T \in \mathcal{L}(X)$ be such that $E^a(T) = \pi^a(T)$. Then the following properties are equivalent:*

i) *T satisfies a-Weyl's theorem.*

ii) *T satisfies a-Browder's theorem.*

iii) *T has the SVEP at all $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$.*

P r o o f. Assume that $E^a(T) = \pi^a(T)$. Then i) and ii) are equivalent by Proposition 2.1 and from Theorem 2.1 we get that i) is equivalent to iii). □

In the case of Hilbert spaces we have the following lemma which will be used in the sequel.

Lemma 2.2 [8, Theorem 2.11]**.** *Let H be a Hilbert space, $T \in \mathcal{L}(H)$, and let λ be an isolated point in $\sigma_{\rm ap}(T)$. Then the following properties are equivalent:*

i) *λ is a left pole of T.*

ii) *There exist T-invariant subspaces M and N of H such that $T-\lambda = (T-\lambda)\big|_M \oplus (T-\lambda)\big|_N$ on $H = M \oplus N$ where $(T-\lambda)\big|_M$ is bounded below and $(T-\lambda)\big|_N$ is nilpotent.*

Theorem 2.3. *If $T \in \mathcal{L}(H)$, then $(T-\lambda)$ is Kato type for all $\lambda \in E^a(T)$ if and only if $E^a(T) = \pi^a(T)$.*

Proof. Suppose that $E^a(T) = \pi^a(T)$. If $\lambda \in E^a(T)$ then λ is isolated in $\sigma_{\rm ap}(T)$ and λ is a left pole of T. By Lemma 2.2, there exist T-invariant subspaces M and N of H such that $T-\lambda = (T-\lambda)\big|_M \oplus (T-\lambda)\big|_N$ on $H = M \oplus N$ where $(T-\lambda)\big|_M$ is bounded below and $(T-\lambda)\big|_N$ is nilpotent. Hence $(T-\lambda)$ is of Kato type for all $\lambda \in E^a(T)$. Conversely, let $\lambda \in E^a(T)$. Then, by assumption, there exist T-invariant subspaces M and N such that $X = M \oplus N$, where $(T-\lambda)\big|_M$ is nilpotent and $(T-\lambda)\big|_N$ is semi-regular. Since λ is isolated in $\sigma_{\rm ap}(T)$ and $S(T) \subseteq \sigma_{\rm ap}(T)$ then T has the SVEP at λ. In particular, $(T-\lambda)\big|_N$ has the SVEP at 0. Hence, $(T-\lambda)\big|_N$ is a semi-regular operator with the SVEP in 0. Thus it follows from [2, Theorem 2.11] that $(T-\lambda)\big|_N$ is injective. Now from Lemma 2.2 we have that $\lambda \in \pi^a(T)$. Hence $E^a(T) = \pi^a(T)$. □

Combining Theorem 2.1 with the preceding theorem we obtain the following result.

Corollary 2.1. *Let $T \in \mathcal{L}(H)$. If $T-\lambda$ is of Kato type for all $\lambda \in E^a(T)$, then the following assertions are equivalent:*

i) *T satisfies a-Weyl's theorem.*

ii) *T satisfies a-Browder's theorem*

iii) *T has the SVEP at all $\lambda \notin \sigma_{\mathrm{SBF}_+^-}(T)$.*

3. Applications

Following [23], let $\mathcal{P}(X)$ be the class of all operators $T \in \mathcal{L}(X)$ such that for every complex number λ there exists an integer $d_\lambda \geqslant 1$ for which the following condition holds:

$$H_0(T-\lambda) = N(T-\lambda)^{d_\lambda}. \tag{3.1}$$

Theorem 3.1. *Let $T \in \mathcal{P}(X)$. Then T^* satisfies a-Weyl's theorem.*

P r o o f. Since T has finite ascent, then by [17, Proposition 1.8] T has the SVEP and so by Theorem 2.1 it satisfies a-Browder's theorem. Let $\lambda \in E^a(T^*)$; then λ is an isolated point of $\sigma_{\rm ap}(T^*)$ which is equal to $\sigma(T^*)$ since T has the SVEP ([18]). Since T^* satisfies the generalized a-Weyl's theorem [4], we have $\lambda \notin \sigma_{{\rm SBF}_+^-}(T^*)$. Hence it follows from [8, Theorem 2.8] that $\lambda \in \pi^a(T^*)$. Thus $E^a(T^*) \subseteq \pi^a(T^*)$. Since always $\pi^a(T^*) \subseteq E^a(T^*)$, we have $E^a(T^*) = \pi^a(T^*)$. Now the result follows from Theorem 2.2. □

An operator $T \in \mathcal{L}(X)$ is a *generalized scalar* operator if there exists a continuous algebra homomorphism $\varphi\colon \mathcal{C}^\infty(\mathbb{C}) \to \mathcal{L}(X)$ such that $\varphi(1) = I$ and $\varphi(Z) = T$. Since every generalized scalar operator belongs to $\mathcal{P}(X)$ ([23]), we have

Corollary 3.1. *Let $T \in \mathcal{L}(X)$ be a generalized scalar operator. Then T^* satisfies a-Weyl's theorem.*

Let $T \in \mathcal{L}(H)$. T is a *p-hyponormal* operator if $(TT^*)^p \leqslant (T^*T)^p$ for $0 < p \leqslant 1$. The class of p-hyponormal operators satisfies equality (3.1), hence the following corollary holds.

Corollary 3.2 [15]. *Let $T \in \mathcal{L}(H)$ be a p-hyponormal operator. Then T^* satisfies a-Weyl's theorem.*

We say that $T \in \mathcal{L}(H)$ is an *M-hyponormal* operator if there exists a positive number M such that $\|(T-\mu)^*x\| \leqslant M\|(T-\mu)x\|$ for all $x \in H$ and all $\mu \in \mathbb{C}$. The class of M-hyponormal operators satisfies equality (3.1), hence we have the following corollary.

Corollary 3.3 [15]. *Let $T \in \mathcal{L}(H)$ be an M-hyponormal operator. Then T^* satisfies a-Weyl's theorem.*

$T \in \mathcal{L}(H)$ is said to be a log-*hyponormal* operator if T is invertible and $\log(TT^*) \leqslant \log(T^*T)$. Since log-hyponormal operators satisfy equality (3.1), we have the following

Corollary 3.4 [15]. *Let $T \in \mathcal{L}(H)$ be a* log-*hyponormal operator. Then T^* satisfies a-Weyl's theorem.*

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