Numerical instability in PIC simulations of weakly collisional magnetized plasmas

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Stability of weakly collisional magnetized plasma in the **E**×**B** fields is studied with Particle-In-Cell simulations. In addition to the physical instability that was expected from theoretical considerations, numerical instability has been identified. The simulation stability is usually related to the resolution of the Debye length. However, the observed numerical instability is likely due to the low resolution of the electron Larmor radius (i.e. for grid spacing being larger than the Larmor radius but smaller than Debye length). The effect of the instability is the plasma filamentation, and large growth rate of potential fluctuations even for the physically stable cases.

1. Introduction

As instability we can designate a phenomenon during which the amplitude of small initial perturbation grows instead of being damped. In an unstable system the source of free energy must exist, as well as there must be a positive feedback of the system. Plasma instabilities can be divided into two main categories: (i) Macroinstabilities which are dominant on large scales, have long wavelengths and are caused by space configuration. (ii) Micro-instabilities, which are dominant on small scales, have large wavenumbers and are driven by distortions in the velocity phase-space.

Theoretical description of plasma instabilities can be done with magnetohydrodynamics (MHD) theory, which is suitable for macroinstabilities, or the kinetic theory, which can be used for both micro- and macroinstabilities. Since instabilities are highly nonlinear phenomena, the fully analytical solution might be very complicated and difficult in many cases and the problem must be linearized or solved by using some quasilinear theory.

Due to difficulties with analytical solutions, numerical simulations, which allow for detailed study of nonlinear phenomena such as instabilities or turbulences, became more important in last decades. However, when using numerical simulation one should be aware of particular issues that are important for numerical stability of the simulations.

The most important parameters which affect numerical stability are spatial grid resolution Δx and size of the time step Δt. These arise due to approximations of derivatives by finite differences. The role of spatial and temporal spacing for numerical stability has been well studied already by 1980's [1]. For instance numerical instabilities due to spatial spacing will occur if $k\,\Delta x < 1$, or $\lambda_D/\Delta x < 1$, where k is the characteristic wavenumber in the system, and λ_D is the Debye length.

In our recent study, we focused on the influence of collision type on the stability of weakly collisional plasma in **E**×**B** fields [2]. In the numerical study we encountered numerical instabilities even for the cases, which should otherwise be numerically stable according to the well-established stability conditions [1]. In this work we identify these instabilities and suggest their origin.

2. Numerical simulations

We use the self-consistent electrostatic 3D Particle-in-Cell (PIC) numerical simulations. Our code allows us to set external static magnetic and electric field in arbitrary direction and set collisions with neutrals using the Monte Carlo null collision method [3].

For the simulation we set the external magnetic field **B** magnitude to $B = 0.005$ T in the x–direction, and the external electric field **E** magnitude to $E = 550\ \mathrm{V{\cdot}m^{-1}}$ in the y–direction. The plasma particle trajectories are calculated using the leap-frog method combined with the Boris algorithm [4].

The neutrals are assumed to be cold, of the same mass as ions, and collision frequency between neutrals and plasma particles is set to be constant. For electron-neutral collisions we assume only elastic collisions, while for ion-neutral collision the charge exchange (C.E.) and elastic collisions (E.S.) are considered.

Tab. 1 – Spacing, ratio of gyroradius and spacing, and collision type for all simulated systems.

	Case 1 C.E	Case 1 E.S.	Case 2 C.E.	Case 2 E.S.
Δx	3.9 mm	3.9 mm	7.8 mm	7.8 mm
$R_L/\Delta x$	1.05	1.05	0.52	0.52
Coll.	Ch. Ex.	Elast.	Ch. Ex.	Elast.

We use the box size of $L = 0.5$ m in each direction. With the time-step of $\Delta t \approx 0.04\ \tau_{Le}$, where τ_{Le} is the electron gyroperiod, we resolve well the electron gyromotion, which is the fastest motion in the system. The simulated plasma density is $n = 4.3\times10^{13}\ \mathrm{m}^{-3}$, electron temperature $T_e = 74.1$ eV, the electron to ion temperature ratio $T_e/T_i = 4$, and electron to ion mass ration $m_e/m_i = 1/500$. Thus, the crossfield drift under these conditions is subsonic. To simulate physical processes we use grid spacing chosen to resolve electron gyroradius, which is smaller than the Debye length in our system thus it the smallest scale. However, in cases where the electron gyroradius is not resolved we do observe numerical instabilities.

We simulate 4×10^7 particles per plasma specie, and parallelization is done using the Message-Passing-Interface (MPI). We have verified that such number of simulated particles is sufficient and the numerical noise is acceptable. Due to the finite number of the simulated particles, we cannot reach the Vlasov limit in the collisionless case, which is a general shortcoming of the PIC method.

3. Results

In this section we present results of our observations of numerical instabilities. For the simulated plasma system described above, we have monitored the RMS values of electrostatic potential fluctuations. While in simulations with the gyroradius to grid spacing ratio $R_L/\Delta x = 1.05$ we observed purely physical behaviour as it was expected from theoretical considerations, in simulation with the ratio $R_L/\Delta x = 0.52$ there was a growth of artificial instability. Parameters of the simulated cases are summarized in Tab. 1 and results are shown in Figs. 1 a 2. Fig. 1 shows potential fluctuations in the simulation with charge exchange collisions where the physical growth of fluctuations [5] after one ion gyroperiod for the Case 1 C.E., and unphysical growth for the Case 2 C.E.

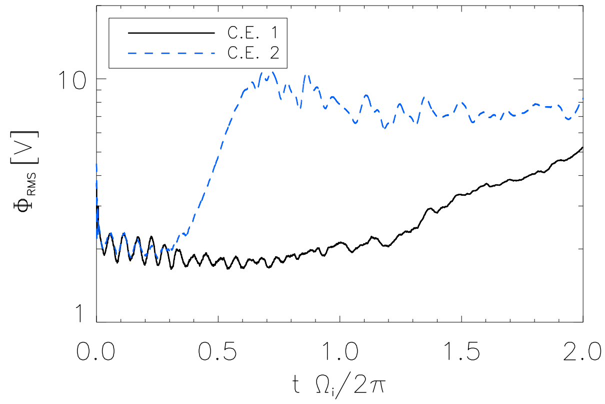

Fig. 1 – Temporal evolution of potential fluctuations for charge exchange collision case. Black solid line is for Case 1 C.E. and blue dashed line is for Case 2 C.E..

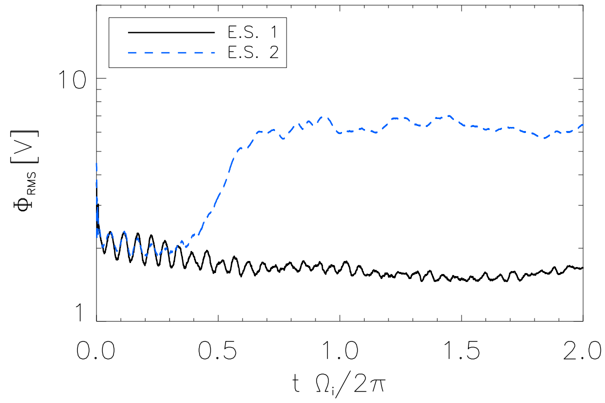

Fig. 2 – Temporal evolution of potential fluctuations for the case with elastic collisions. Black solid line is for Case 1 E.S. and blue dashed line is for Case 2 E.S.

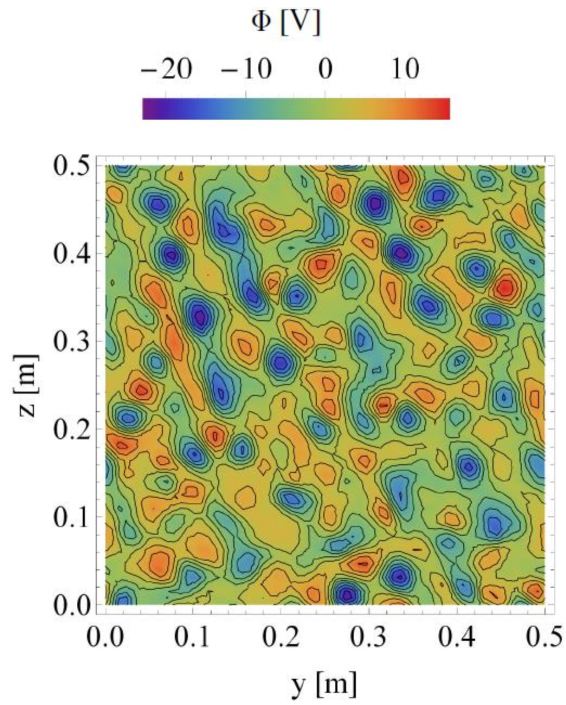

Fig. 3 – Filament structures in potential density in later stages of simulation with elastic collisions.

Similar situation is depicted in Fig. 2 for simulation with elastic collisions (E.S.). There is a visible damping in the system with better grid resolution and a large growth of fluctuations in the system with worse resolution. In Fig. 3 we show the cut through the potential in the direction perpendicular to the magnetic field for the case with elastic collisions with worse grid, where the filamentation of the plasma can easily be recognized. In all simulations the standard stability condition $\lambda_D/\Delta x > 1$ was fulfilled, and apart from the grid spacing, all other parameters remain unchanged.

4. Discussion

Since in our simulations we fulfilled the stability conditions given in [1], it is open question what is behind the observed numerical instabilities.

The only change in the two sets of simulations is in spatial resolution of electron Larmor radius. In our simulations the electron gyroradius for thermal velocity is $R_L \approx 4.1$ mm, so with worse grid resolution we have $R_L/\Delta x < 1$. For the better grid resolution we have $R_L/\Delta x > 1$. In previous studies it has been stated that the electron gyroradius is not crucial for the stability of the system [6]. In [1,7] authors focused on study of numerical stability for small timestep with respect to the electron gyroperiod and noted that not resolving R_L can have stabilizing effects. In our simulations, the temporal resolution of electron gyromotion is sufficient, and we do observe the numerical instability.

5. Conclusions

In our recent study covering numerical simulation of weakly collisional magnetized plasmas we observed numerical instability, which is likely to originate from insufficient resolution of the electron gyroradius. It is characterized by a rapid growth of the electrostatic potential fluctuations and forming filamentary structures in the electrostatic potential. This numerical instability should be addressed in more detail, and thus it is a focus of our current studies.

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