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Singular equilibrated stress fields for no-tension panels

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Abstract In this work we study the equilibrium problem for rectangular panels made of a no-tension material, clamped at the bottom, subjected to distributed vertical loads on the top, and to different types of lateral loads. Admissible and equilibrated stress fields are interpreted as tensor–valued measures with zero divergence. Such stress fields are explicitly determined under the assumption that the measure is absolutely continuous outside a smooth curve which supports a δ type singularity of the stress.

1 Introduction

In this paper, which follows [7–8] and [9], we study the equilibrium problem of a panel made of a no tension material [4]. The panel is free from body forces, clamped at its bottom and subjected to loads prescribed on the boundary. The stress field is plane and negative-semidefinite. If horizontal and vertical loads are distributed on the panel's top and the stress determinant is null [5], the equilibrium equations constitute a system of conservation laws, formally identical to the nonlinear system ruling the dynamics of the one-dimensional isentropic flow of a pressureless compressible gas [2].

For sufficiently regular distributions of loads with sufficiently small ratio between the tangential and normal component, the stress field can be explicitly determined by means of a representation formula [7]. As this ratio increases, it can happen that the representation formula loses its validity despite the fact that no collapse of the panel occurs. Under these circumstances the solution of the equilibrium equations cannot be unique and it displays some singularities. In [8] and [9], some cases are examined in which the solution is regular except on a finite number of curves where the stress field is unbounded. The solution is based on the fact that under appropriate hypotheses, the system of equilibrium equations is equivalent to a single scalar conservation law [2, 1]. Then the singularity curves can be determined by means of the Rankine-Hugoniot jump condition corresponding to this scalar equation. This method is not directly applicable if distributed loads are present on the lateral sides of the panel.

In this paper we suppose that the stress field is a tensor measure with divergence measure in the interior Ω of the panel [3, 6] to account for the singularities in the stress field. We limit ourselves to the cases in which only a single singularity curve γ is present. Thus the stress field $\boldsymbol{T}$ is the sum of a measure absolutely continuous with respect to Lebesgue's measure with a smooth density $\boldsymbol{T}_r$ on $\Omega \sim \gamma$, and a measure concentrated on γ, whose density is a smooth superficial tensor field $\boldsymbol{T}_s$. The equilibrium requires that $\boldsymbol{T}_r$ has null divergence outside γ, and that the surface divergence of $\boldsymbol{T}_s$ be balanced by the jump of the normal component of $\boldsymbol{T}_r$ across γ. In the examples presented in this paper, the form of the curve γ and superficial stress field $\boldsymbol{T}_s$ are obtained by means of this relation, once $\boldsymbol{T}_r$ has been determined.

2 Preliminaries

We denote by $\mathscr{E}$ the two-dimensional Euclidean point space, by $\mathscr{V}$ the associated real vector space with standard basis $\boldsymbol{e}_1, \boldsymbol{e}_2$, by Lin the space of all linear transformations of $\mathscr{V}$ and by $\Omega \subset \mathscr{E}$ an open set. Let $\gamma : [0, \bar{\tau}] \to \Omega$ be a smooth curve in Ω, with unit normal vector $\boldsymbol{n}$, and $\boldsymbol{u} : \gamma \to \mathscr{V}$ a smooth vector field on γ. The surface gradient of $\boldsymbol{u}$ is the map $\nabla_\gamma \boldsymbol{u} : \gamma \to \mathrm{Lin}$, such that

$$\nabla_\gamma \boldsymbol{u}(\boldsymbol{p})\boldsymbol{n}(\boldsymbol{p}) = \boldsymbol{0}, \quad \frac{d}{d\tau}\boldsymbol{u}(\gamma(\tau)) = \nabla_\gamma \boldsymbol{u}\big(\gamma(\tau)\big)\frac{d}{d\tau}\gamma(\tau), \tag{1}$$

for all $\boldsymbol{p} \in \gamma$ and for all $\tau \in [0, \bar{\tau}]$, respectively. Moreover, the surface divergence of $\boldsymbol{u}$ is the trace of its surface gradient,

$$\mathrm{div}_\gamma \boldsymbol{u} = \mathrm{tr}\, \nabla_\gamma \boldsymbol{u} \tag{2}$$

Similarly, if $\boldsymbol{T} : \gamma \to \mathrm{Lin}$ is a smooth tensor field on γ, its surface divergence is defined by

$$\boldsymbol{a} \cdot \mathrm{div}_\gamma \boldsymbol{T} = \mathrm{div}_\gamma(\boldsymbol{T}^{\mathrm{T}} \boldsymbol{a}) \tag{3}$$

for every $\boldsymbol{a} \in \mathscr{V}$. We say that vector field $\boldsymbol{u}$ is tangential if $\boldsymbol{u} \cdot \boldsymbol{n} = 0$, in γ, and that tensor field $\boldsymbol{T}$ is superficial if $\boldsymbol{T}\boldsymbol{n} = \boldsymbol{0}$, in γ.

Now, let s be the natural (arc) parameter of γ, $\boldsymbol{t}(s)$ its unit tangent vector and $\boldsymbol{u}(s) = u(s)\boldsymbol{t}(s)$ a tangential vector field on γ. In view of $(1)_2$ we have

$$(\nabla_\gamma \boldsymbol{u})\boldsymbol{t} = \frac{d}{ds}(u\boldsymbol{t}). \tag{4}$$

Then, with the help of $(1)_1$, from (2) we get

$$\mathrm{div}_\gamma \boldsymbol{u} = \mathrm{tr}\, \nabla_\gamma \boldsymbol{u}) = \boldsymbol{t} \cdot (\nabla_\gamma \boldsymbol{u})\boldsymbol{t} = \boldsymbol{t} \cdot \frac{d}{ds}(u\boldsymbol{t}) = \frac{du}{ds}, \tag{5}$$

because $\boldsymbol{t} \cdot \boldsymbol{t} = 1$ and $\boldsymbol{t} \cdot d\boldsymbol{t}/ds = 0$.

If $\boldsymbol{T} : \gamma \to \mathrm{Lin}$ is a symmetric tensor field on γ it is easy to verify that $\boldsymbol{T}$ is superficial if and only if we have

$$\boldsymbol{T}(s) = \sigma(s)\, \boldsymbol{t}(s) \otimes \boldsymbol{t}(s), \tag{6}$$

with σ a scalar field on γ. Then, in view of (3) and (5), for every $\boldsymbol{a} \in \mathscr{V}$, we deduce

$$\boldsymbol{a} \cdot \mathrm{div}_\gamma \boldsymbol{T} = \boldsymbol{a} \cdot \mathrm{div}_\gamma \big(\sigma \boldsymbol{t} \otimes \boldsymbol{t}\big) = \mathrm{div}_\gamma \big(\sigma (\boldsymbol{a} \cdot \boldsymbol{t})\boldsymbol{t}\big) = \frac{d}{ds}(\sigma (\boldsymbol{a} \cdot \boldsymbol{t})) = \boldsymbol{a} \cdot \frac{d}{ds}(\sigma \boldsymbol{t}) \tag{7}$$

and thus

$$\mathrm{div}_\gamma \boldsymbol{T} = \frac{d}{ds}(\sigma \boldsymbol{t}). \tag{8}$$

Let $\mathscr{B}(\Omega)$ be the σ algebra of all Borel subsets of Ω and X be a finite dimension real vector space with a dot product. We say that $M\colon \mathscr{B}(\Omega)\to X$, is a X valued (Borel) measure on Ω if M is a σ additive function on $\mathscr{B}(\Omega)$, i.e. if for all sequence B_i of disjoint elements of $\mathscr{B}(\Omega)$, we have

$$M\Big(\bigcup_{i=1}^{\infty} B_i\Big) = \sum_{i=1}^{\infty} M(B_i). \tag{9}$$

In particular, we call M, respectively, a (finite) signed measure, a vector measure or a tensor measure on Ω, if $X = \mathbb{R}$, $X = \mathscr{V}$ or $X = \mathrm{Lin}$. Thus, if $\boldsymbol{v} = v_1\boldsymbol{e}_1 + v_2\boldsymbol{e}_2$ belongs to $C_0(\Omega, \mathscr{V})$, the space of all continuous vector fields with compact support in Ω, and $\boldsymbol{\mu}$ is a vector measure on Ω, there are two unique signed measures, μ_1 and μ_2, on Ω such that

$$\int_\Omega \boldsymbol{v}\cdot d\boldsymbol{\mu} = \int_\Omega v_1\, d\mu_1 + \int_\Omega v_2\, d\mu_2. \tag{10}$$

Similarly, we interpret $\int_\Omega \boldsymbol{S}\cdot d\boldsymbol{T}$, with $\boldsymbol{S}\in C_0(\Omega,\mathscr{V})$ and $\boldsymbol{T}$ a tensor measure on Ω. If $\varphi\in C_0(\Omega,\mathbb{R})$ is a real valued function with compact support in Ω and $\boldsymbol{M}$ is a vector (resp. tensor) measure, then

$$\int_\Omega \varphi\, d\boldsymbol{M} \tag{11}$$

is the element of $\mathscr{V}$ (resp. Lin) defined by

$$\boldsymbol{a}\cdot\int_\Omega \varphi\, d\boldsymbol{M} = \int_\Omega (\varphi\boldsymbol{a})\cdot d\boldsymbol{M}, \tag{12}$$

for every $\boldsymbol{a}\in\mathscr{V}$ (resp. Lin). Moreover, if $\boldsymbol{T}$ is a tensor measure and $\boldsymbol{v}\in C_0(\Omega,\mathscr{V})$, then

$$\int_\Omega \boldsymbol{v}\, d\boldsymbol{T} \tag{13}$$

is the element of $\mathscr{V}$ such that

$$\boldsymbol{a}\cdot\int_\Omega \boldsymbol{v}\, d\boldsymbol{T} = \int_\Omega \boldsymbol{a}\otimes\boldsymbol{v}\cdot d\boldsymbol{T} \tag{14}$$

for every $\boldsymbol{a}\in\mathscr{V}$.

Let $\boldsymbol{T}:\mathscr{B}(\Omega)\to\mathrm{Lin}$ be a tensor measure on Ω. We say that $\boldsymbol{T}$ is a tensor measure with divergence measure if there exists a vector measure $\boldsymbol{\mu}$ on Ω such that, for every $\varphi\in C_0^\infty(\Omega,\mathbb{R})$,

$$\int_\Omega \nabla\varphi\, d\boldsymbol{T} = -\int_\Omega \varphi\, d\boldsymbol{\mu}, \tag{15}$$

or, in view of (14) and (12), if

$$\int_\Omega \boldsymbol{a}\otimes\nabla\varphi\cdot d\boldsymbol{T} = -\int_\Omega (\boldsymbol{a}\varphi)\cdot d\boldsymbol{\mu} \tag{16}$$

for every $\boldsymbol{a}\in\mathscr{V}$. If this is the case we write $\operatorname{div}\boldsymbol{T} = \boldsymbol{\mu}$, and we say that $\boldsymbol{T}$ is a balanced tensor measure if $\operatorname{div}\boldsymbol{T} = \boldsymbol{0}$.

Let $\Omega\subset\mathscr{E}$ be an open set and $\boldsymbol{\gamma}$ be a smooth curve in Ω, with the unit normal $\boldsymbol{n}$. If $\boldsymbol{T}:\Omega\sim\boldsymbol{\gamma}\to\mathrm{Lin}$ is a tensor field we say that $\boldsymbol{T}$ is piecewise differentiable if $\boldsymbol{T}$ is continuously differentiable on $\Omega\sim\boldsymbol{\gamma}$ and for every point $\boldsymbol{p}\in\boldsymbol{\gamma}$ the limits

$$T^{\pm}(p) = \lim_{t \to 0,\, t > 0} T(p \pm t n(x)) \tag{17}$$

exist and the functions $T^{\pm}$ are continuously differentiable on γ.

For the aim of this paper, we need the following result [6]. Let $\Omega \subset \mathscr{E}$ be an open set, $\partial\Omega$ its boundary and $\gamma : [0, \bar{\tau}] \to \mathscr{E}$ be a smooth curve such that $\gamma(]0, \bar{\tau}[) \subset \Omega$, $\gamma(0) \in \partial\Omega$ and $\gamma(\bar{\tau}) \in \partial\Omega$. Let T be a tensor measure on Ω defined by

$$\int_\Omega S \cdot dT = \int_\Omega S \cdot T_r \, dA + \int_\gamma S \cdot T_s \, ds, \tag{18}$$

for each $S \in C_0(\Omega, \mathrm{Lin})$, where T_r is a piecewise differentiable tensor field on $\Omega \sim \gamma$ and T_s is a continuously differentiable tensor field on γ. Then T is a tensor measure with divergence measure on Ω if and only if T_s is superficial; moreover if this is the case, T is balanced if and only if

$$\operatorname{div} T_r = 0 \quad \text{on} \quad \Omega \sim \gamma, \tag{19}$$

$$[T_r] n + \operatorname{div}_\gamma T_s = 0 \quad \text{on} \quad \gamma, \tag{20}$$

with $[T_r] = T_r^+ - T_r^-$ the jump of T_r across γ.

Now, let us suppose that curve γ is the graph of a function $y = \omega(x)$, with $x \in [x_0, x_1]$, i.e. $\gamma = \{(x, \omega(x)) \in \mathscr{E} : x \in [x_0, x_1]\}$. Then, for e_1 and e_2 the unit normal vectors of the standard basis in $\mathscr{V}$, the unit tangent t and the unit normal n of γ are

$$t = J^{-1}(e_1 + \omega' e_2), \qquad n = J^{-1}(-\omega' e_1 + e_2), \tag{21}$$

where a prime denotes differentiation with respect to x and

$$J = \frac{ds}{dx} = \sqrt{1 + (\omega')^2}. \tag{22}$$

Moreover, from (6) we get

$$T_s = \sigma J^{-2}\{(\omega')^2 e_1 \otimes e_1 - \omega'(e_1 \otimes e_2 + e_2 \otimes e_1) + e_2 \otimes e_2\}. \tag{23}$$

Assuming

$$[T_r] = \delta_{11} e_1 \otimes e_1 + \delta_{12}(e_1 \otimes e_2 + e_2 \otimes e_1) + \delta_{22} e_2 \otimes e_2, \tag{24}$$

from $(21)_2$ we obtain

$$[T_r] n = J^{-1}\{(\delta_{12} - \omega' \delta_{11}) e_1 + (\delta_{22} - \omega' \delta_{12}) e_2\} \tag{23}$$

and from (8) and (22)

$$\operatorname{div}_\gamma T_s = \frac{d}{ds}\left(\frac{\sigma}{J} e_1 + \frac{\sigma}{J}\omega' e_2\right) = J^{-1}\frac{d}{dx}\left(\frac{\sigma}{J} e_1 + \frac{\sigma}{J}\omega' e_2\right) \tag{24}$$

Then, for

$$\frac{\sigma}{J} = \beta, \tag{25}$$

from (20), (23) and (24) we deduce the system of ordinary differential equations

$$\beta' - \omega' \delta_{11} + \delta_{12} = 0, \tag{26}$$

$$(\beta \omega')' - \omega' \delta_{12} + \delta_{22} = 0, \tag{27}$$

of which we will see some applications in the next paragraph.

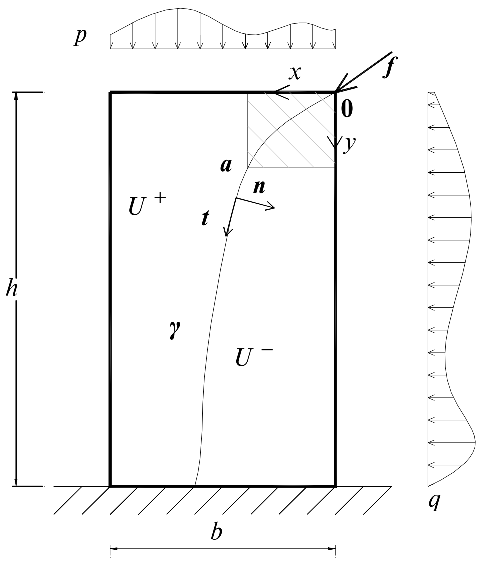

Fig. 1. The panel under general load conditions.

3 Rectangular panels

Let us now consider a rectangular panel of width b and height h, clamped at its base $y = h$ and subjected to a vertical load p distributed on the panel's top $y = 0$, a horizontal load q distributed on its right end side $x = 0$ and a force $\boldsymbol{f} = f_1\boldsymbol{e}_1 + f_2\boldsymbol{e}_2$ concentrated at the point o with coordinates $x = 0, y = 0$, (fig. 1). Denoting by Ω the inner part of the panel,

$$\Omega = \{(x, y) \in \mathscr{E} : 0 < x < b,\ 0 < y < h\}, \tag{28}$$

we propose ourselves to determine a curve $\boldsymbol{\gamma}$

$$y = \omega(x) \quad \text{with} \quad \omega(0) = 0, \tag{29}$$

and a continuously differentiable, negative-semidefinite, superficial stress field $\boldsymbol{T}_s$ on $\boldsymbol{\gamma}$, such that, denoted by $\Omega^+ = \{(x, y) \in \Omega :\ 0 < x < \omega^{-1}(y)\}$ and $\Omega^- = \{(x, y) \in \Omega :\ \omega^{-1}(y) < x < b\}$ the two regions in which $\boldsymbol{\gamma}$ divides Ω, and assumed for $\boldsymbol{T}_r$ the expression

$$\boldsymbol{T}_r = \begin{cases} -p(x)\boldsymbol{e}_2 \otimes \boldsymbol{e}_2 & \text{in} \quad \Omega^-, \\ -q(y)\boldsymbol{e}_1 \otimes \boldsymbol{e}_1 & \text{in} \quad \Omega^+, \end{cases} \tag{30}$$

the stress tensor measure $\boldsymbol{T}$ defined by (18) are balanced and in equilibrium with the external loads. Since, according to (30), $\boldsymbol{T}_r$ is equilibrated with distributed loads p and q and satisfies (19), it is enough to determine $\boldsymbol{\gamma}$ and $\boldsymbol{T}_s$ to satisfy (20) and the equilibrium boundary condition

$$\boldsymbol{T}_s(\boldsymbol{0})\boldsymbol{t}(\boldsymbol{0}) = -\boldsymbol{f} \tag{31}$$

To this goal we observe that in this case, according to (24) and $(29)_1$, we get

$$\delta_{11} = -q(\omega(x)), \quad \delta_{22} = p(x), \quad \delta_{12} = 0 \tag{32}$$

and therefore from (26) and (27) we deduce

$$\beta' + q(\omega(x))\omega' = 0, \tag{33}$$

$$(\beta\omega')' + p(x) = 0, \tag{34}$$

from which, denoted by P and Q the primitives of p and q, respectively, such that $P(0) = 0$ and $Q(0) = 0$, we get

$$\beta(x) = \beta(0) - Q(\omega(x)), \tag{35}$$

$$\beta(x)\omega'(x) = \beta(0)\omega'(0) - P(x) \tag{36}$$

With the help of (35), (36) becomes

$$\big(Q(\omega(x)) - \beta(0)\big)\,\omega'(x) = P(x) - \beta(0)\omega'(0); \tag{37}$$

moreover, in view of (6), $(21)_1$ and (25), the equilibrium boundary condition (31) becomes

$$\beta(0)(\boldsymbol{e}_1 + \omega'(0)\boldsymbol{e}_2) = -\left(f_1\boldsymbol{e}_1 + f_2\boldsymbol{e}_2\right), \tag{38}$$

from which we deduce

$$\beta(0) = -f_1, \quad \beta(0)\omega'(0) = -f_2. \tag{39}$$

Then, (37) implies

$$\big(Q(\omega(x)) + f_1\big)\,\omega'(x) = P(x) + f_2 \tag{40}$$

that can be integrated with the help of initial condition $(29)_2$. We note that since $\boldsymbol{T}_s$ is negative-semidefinite, $\sigma(0) \leq 0$ in view of (6), and from (22) and (25) we obtain $\beta \leq 0$. Moreover since curve $\boldsymbol{\gamma}$, except its ends, is all contained in Ω, $\omega'(0) \geq 0$ has to hold.

Therefore from (39) it follows that both f_1 and f_2 has to be not negative, that is to say that the force $\boldsymbol{f}$ must be directed towards the inside of the panel [5].

4 Examples

Example 1 In this example we suppose that the vertical distributed load is uniform, the horizontal one is linear and the concentrated force is null (fig. 2),

$$p(x) = p_0, \quad q(x) = q_0\left(1 - \frac{y}{h}\right), \quad \boldsymbol{f} = \boldsymbol{0}. \tag{41}$$

Then

$$Q(\omega) = q_0\omega\left(1 - \frac{\omega}{2h}\right), \quad P(x) = p_0 x \tag{42}$$

and from (40) and $(29)_2$ we obtain for $\boldsymbol{\gamma}$ the implicit equation

$$q_0\omega^2\left(1 - \frac{\omega}{3h}\right) = p_0 x^2 \tag{43}$$

We observe that $\boldsymbol{\gamma}$ intersects the panel base at $x = h\sqrt{\dfrac{2q_0}{3p_0}}$. In order that this solution to be valid the intersection point must be inner to the panel's base, that is to say $b \geq h\sqrt{\dfrac{2q_0}{3p_0}}$ and this, fixed p_0, imposes to q_0 not to exceed the value

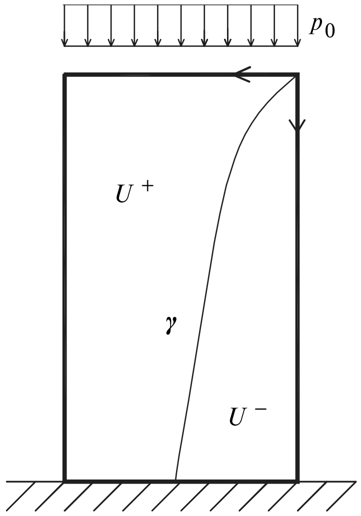

Fig. 1. Load-distribution laws on the boundary of the panel.

$$q_m = \frac{3}{2} p_0 \left(\frac{b}{h}\right)^2, \tag{44}$$

whose attainment gives raise to the overturn of the panel around to the corner of coordinates $x = b$, $y = h$. From (43) we deduce $x = \omega\sqrt{\frac{q_0}{p_0}(1 - \frac{\omega}{3h})}$ and then from (40), for $f_1 = f_2 = 0$, and (42) we obtain

$$\omega' = \sqrt{\frac{p_0}{q_0}} \frac{\sqrt{1 - \frac{\omega}{3h}}}{1 - \frac{\omega}{2h}} \tag{45}$$

In particular, at the panel bottom we have $\omega = h$ and therefore, taking into account (22) and (35), it turns out

$$\omega' = 2\sqrt{\frac{2p_0}{3q_0}}, \quad J = \sqrt{1 + \frac{8p_0}{3q_0}}, \quad \beta = -\frac{1}{2} q_0 \tag{46}$$

from which, with the help of (21) and (25), we obtain the reaction $\boldsymbol{T}_s \boldsymbol{t} = \sigma \boldsymbol{t}$ at the end of γ. Its intensity turns out to be $\frac{1}{2} q_0 h \sqrt{1 + \frac{8p_0}{3q_0}}$.

Example 2 In this second example, we still suppose that the vertical load is uniform while the horizontal one is null; moreover we assume the presence of a horizontal concentrated force (fig. 3),

$$p = p_0, \quad q = 0, \quad \boldsymbol{f} = f \boldsymbol{e}_1, \tag{48}$$

so that $P(x) = p_0 x$ and $Q(\omega) = 0$. Therefore, from (40), for $f_1 = f$, taking also into account $(29)_2$, we deduce

$$\omega(x) = \frac{p_0 x^2}{2f} \tag{49}$$

and then, in view of (22), (35), $(39)_1$ and (25) we get

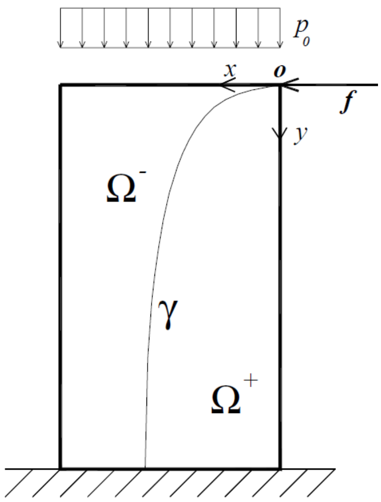

Fig. 2. Load-distribution laws on the boundary of the panel. Case 2.

$$J = \sqrt{1 + \left(\frac{p_0 x}{f}\right)^2}, \quad \beta = -f, \quad \sigma = -f\sqrt{1 + \left(\frac{p_0 x}{f}\right)^2}, \tag{50}$$

from which, with the help of (23), we can determine $\boldsymbol{T}_s$. It has to be noted that the horizontal component of $\boldsymbol{T}_s \boldsymbol{t}$ is $-f$ and then it is constant along $\boldsymbol{\gamma}$. Moreover, from (49), for $x = b$ and $\omega = h$, we get the maximum intensity of force $\boldsymbol{f}$ compatible with the equilibrium $f_m = \frac{p_0 b^2}{2h}$ while from (49) and $(50)_2$ we get that the intensity of the concentrated reaction at the panel's base is $f\sqrt{1 + \frac{2p_0 h}{f}}$.

Example 3 As last example we consider the case in which the panel is subject only to the uniform distributed vertical load $p = p_0$. Using the results of the previous example we want to verify that, beyond to the regular stress state, $\boldsymbol{T} = -p_0 \boldsymbol{e}_2 \otimes \boldsymbol{e}_2$ defined in all Ω, it is possible to determine infinite equilibrated and compatible stress fields in the class of measures with divergence measure, each of which characterized by

(i) a superficial stress $\boldsymbol{T}_s$ defined on a curve γ ($y = \omega(x)$) that is symmetric with respect to y axis (fig. 4), that intersects this axis for $y = \lambda$, $(0 \leq \lambda < h)$ and the panel bottom for $|x| = \mu$ $(0 < \mu \leq \frac{1}{2}b)$,

$$\omega(0) = \lambda, \quad \omega(|\mu|) = h, \quad \omega'(0) = 0, \tag{48}$$

(ii) a stress field

$$\boldsymbol{T}_r = \begin{cases} -p_0 \boldsymbol{e}_2 \otimes \boldsymbol{e}_2, & \text{in } \Omega^-, \\ 0 & \text{in } \Omega^+, \end{cases} \tag{49}$$

where $\Omega^+ = \{(x, y) \in \Omega : |x| < \omega^{-1}(y),\ \lambda < y < h\}$ is the region below γ and Ω^- is the interior of its complementary in Ω.

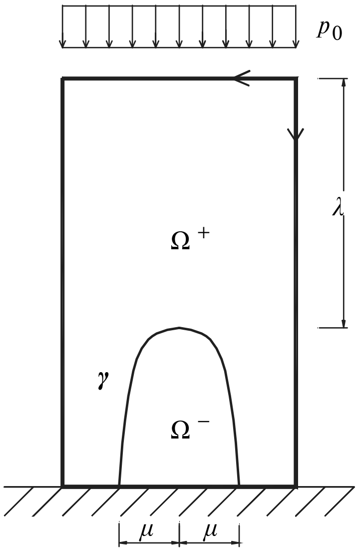

Fig. 3. Load-distribution laws on the boundary of the panel. Case 3.

In fact, from (33) and (34), with $p = p_0$ and $q = 0$ and with the help of (48) we obtain

$$\omega(x) = \lambda + \frac{h-\lambda}{\mu^2}x^2, \quad \beta = -\frac{p_0\mu^2}{2(h-\lambda)}, \tag{50}$$

from which, in view of (22), (25) and (23), it is easy to calculate J, σ and $\boldsymbol{T}_s$. We can observe that the interaction between the two parts of the panel, divided by the symmetry axis $x = 0$, is constituted only by a horizontal force concentrated at the γ apex, whose intensity β is an increasing function of λ, that becomes unbounded when λ tends to h.

This kind of solutions can be used for studying the equilibrium problem of panels with openings [9].

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