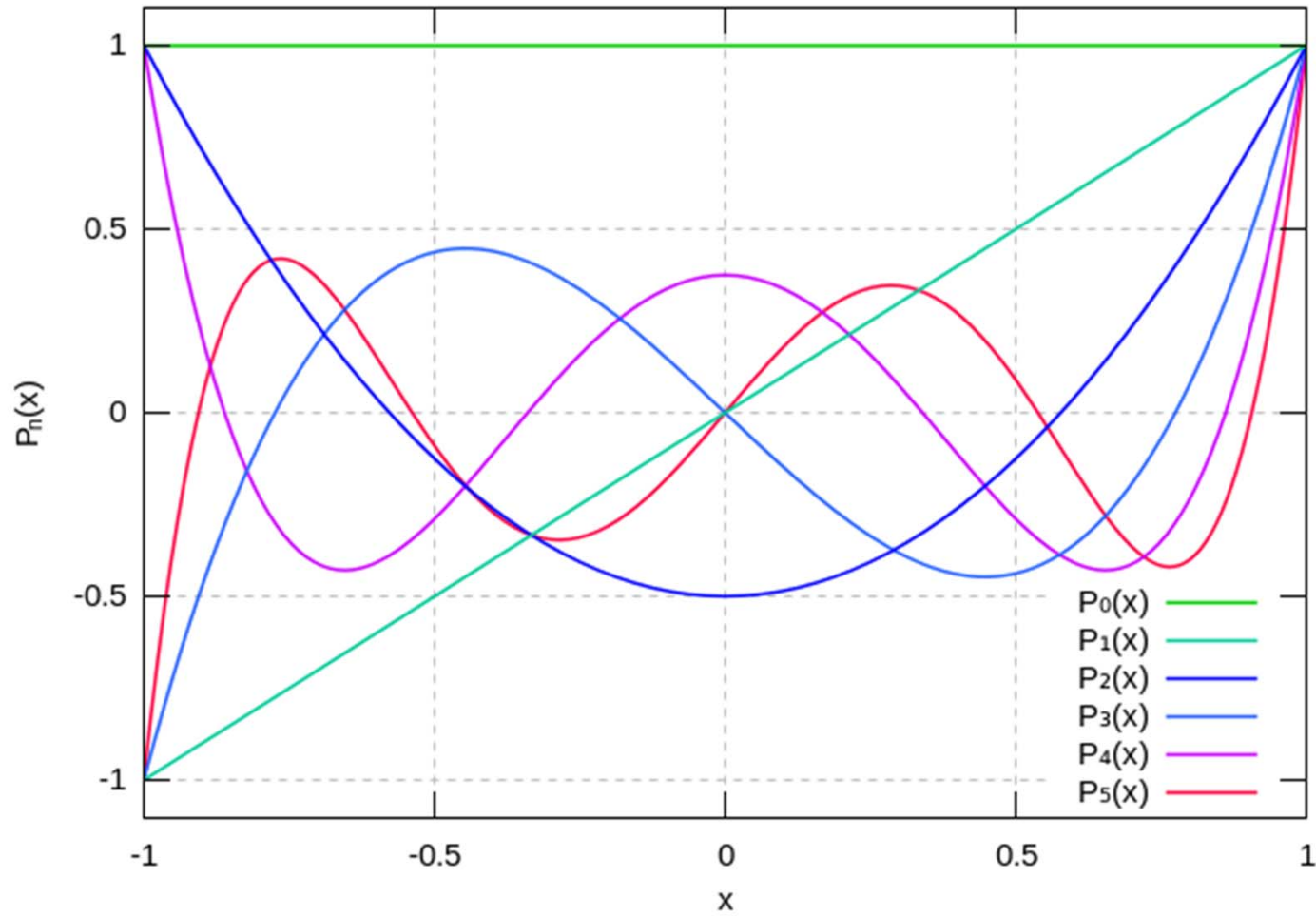


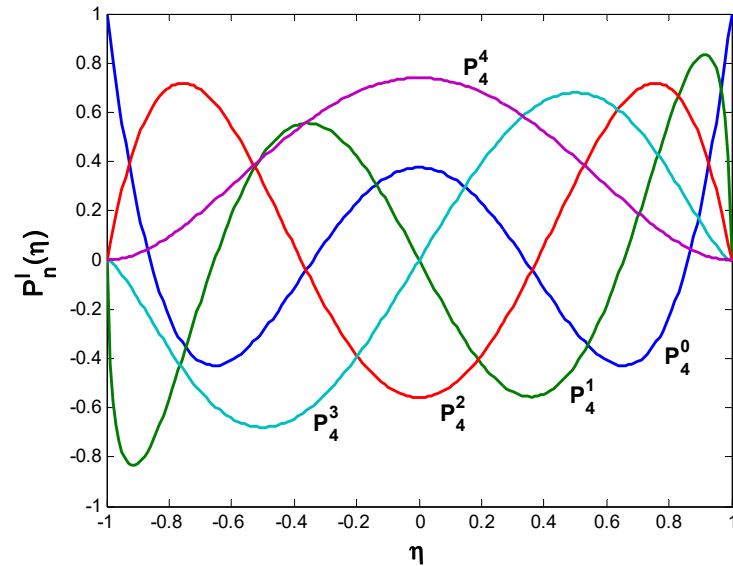
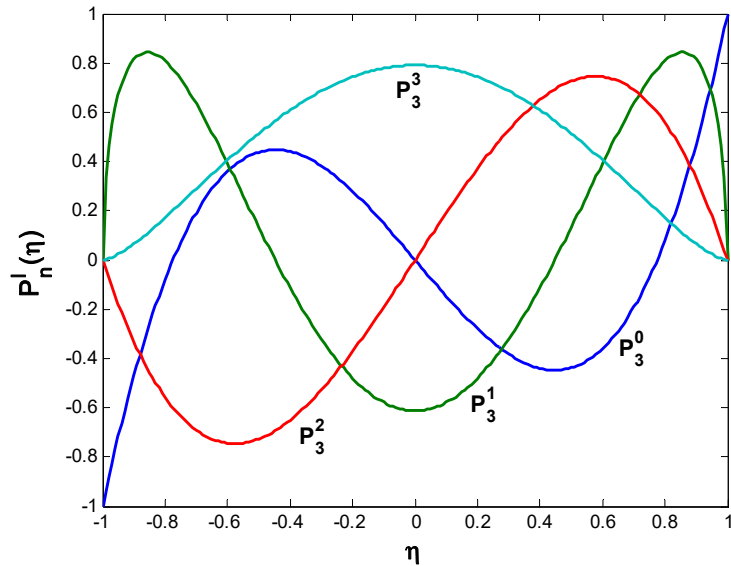
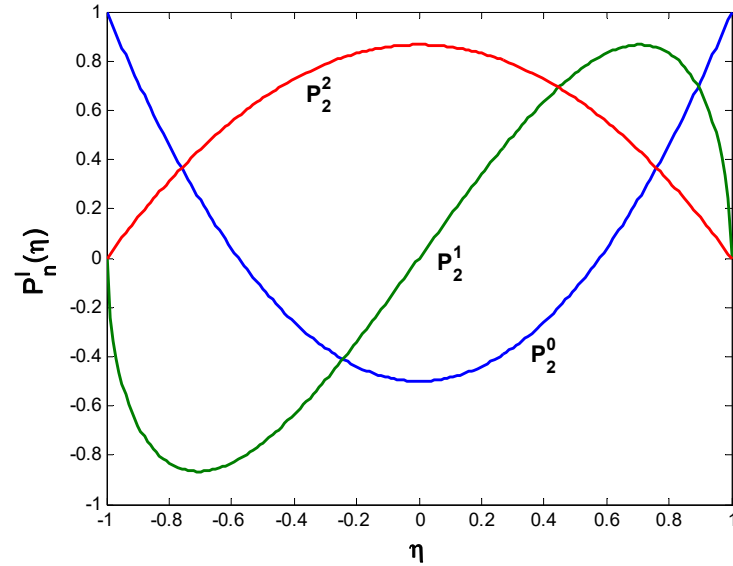
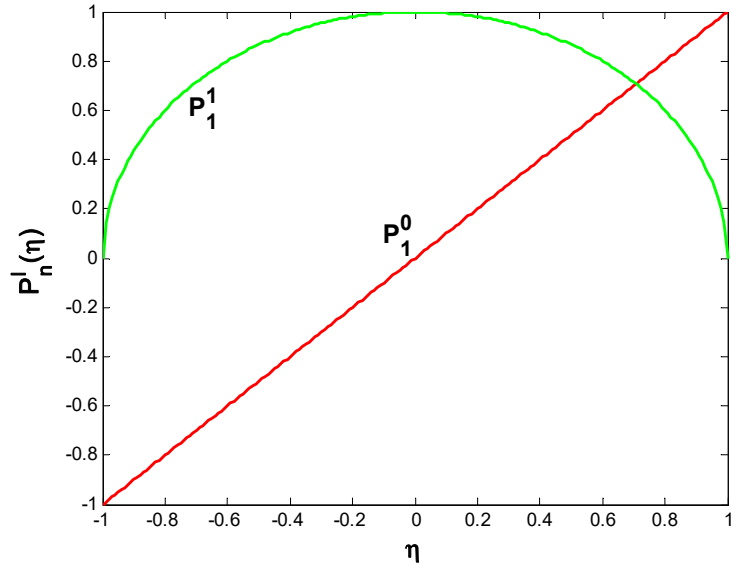
Legendre polynomials

$$P_n(\eta) = \frac{(-1)^n}{2^n n!} \frac{d^n}{d\eta^n} (1 - \eta^2)^n, \quad n = 0, 1, 2, \dots$$
$$\int_{-1}^1 P_n(\eta) P_m(\eta) d\eta = \int_0^\pi P_n(\cos \theta) P_m(\cos \theta) \sin \theta d\theta = \frac{2}{2n+1} \delta_{mn}$$



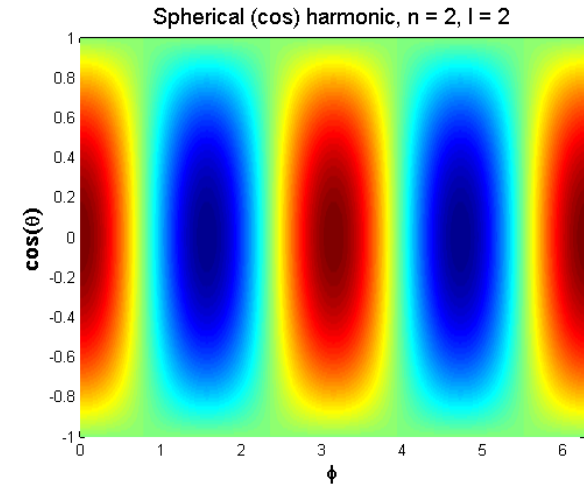
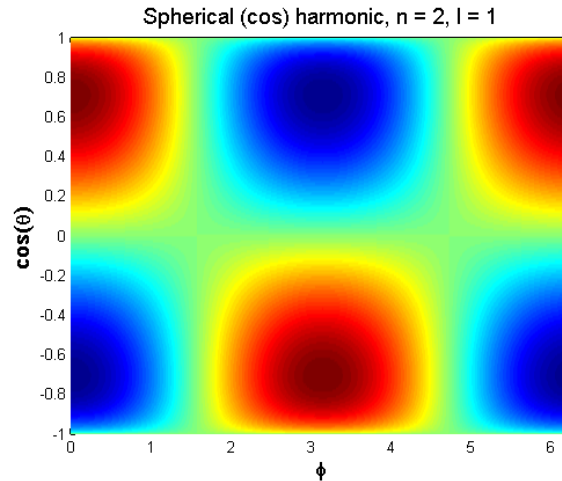
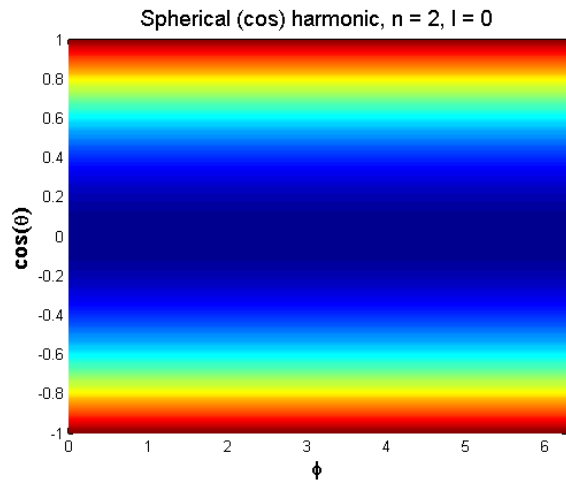
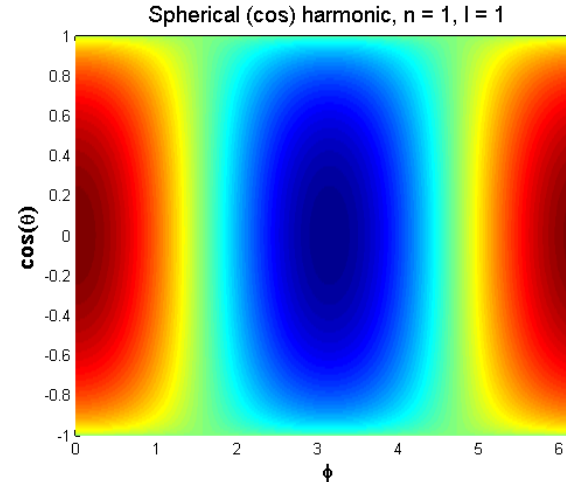
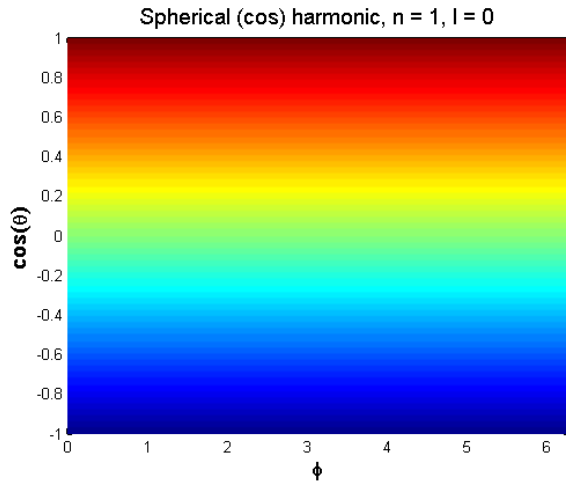
Associated Legendre functions

$$P_n^l(\eta) = \frac{(-1)^{n+l}}{2^n n!} (1 - \eta^2)^{l/2} \frac{d^{n+l}}{d\eta^{n+l}} (1 - \eta^2)^n, \quad l = 0, 1, \dots, n, \quad \int_{-1}^1 P_n^l(\eta) P_{n'}^l(\eta) d\eta = \frac{2(n+l)!}{(2n+1)(n-l)!} \delta_{nn'}, \quad \int_{-1}^1 P_n^l(\eta) P_n^{l'}(\eta) \frac{d\eta}{1-\eta^2} = \frac{(n+l)!}{l(n-l)!} \delta_{ll'}$$

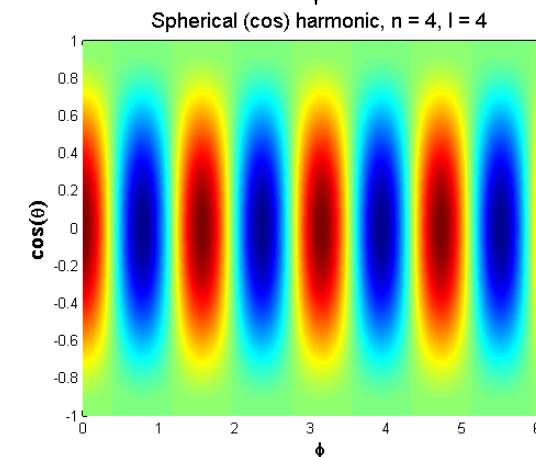
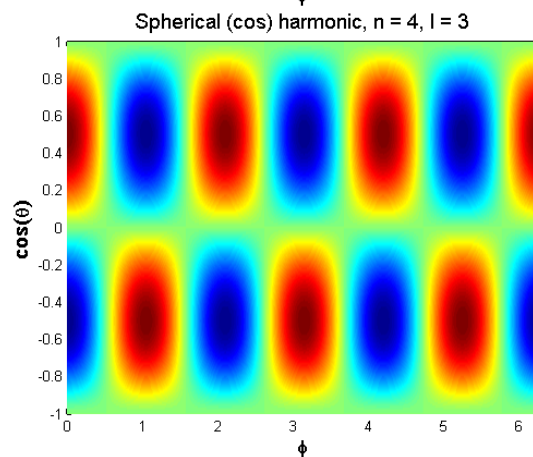
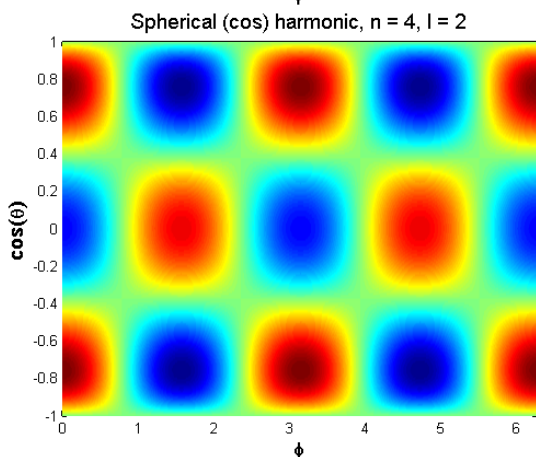
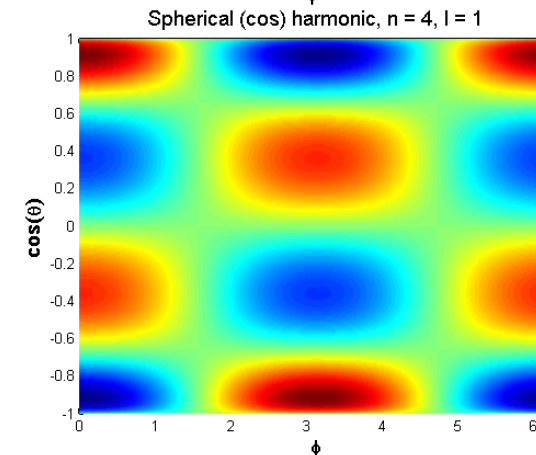
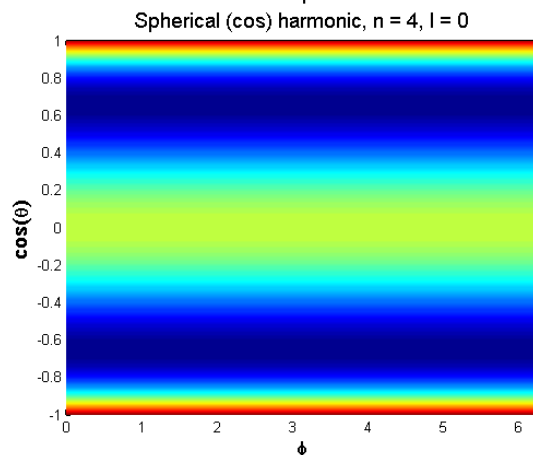
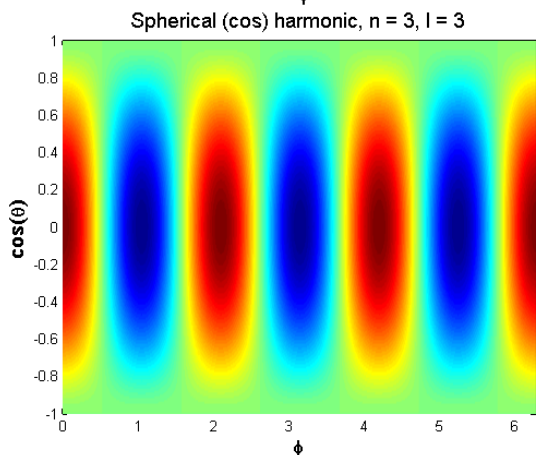
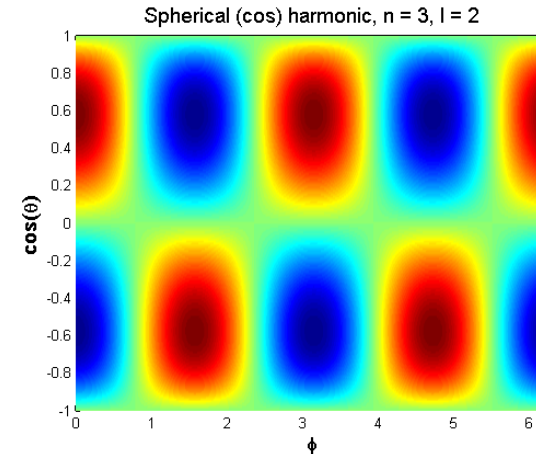
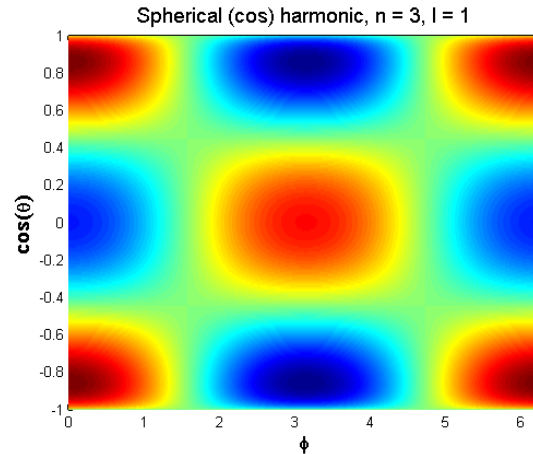
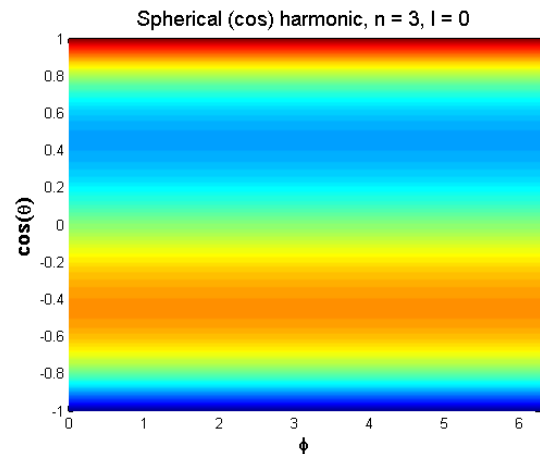


Spherical harmonics

$$Y_n^l(\theta, \varphi) = \sqrt{\frac{(2n+1)(n-l)!}{4\pi(n+l)!}} P_n^l(\cos\theta) e^{il\varphi}, \quad \operatorname{Re}\{Y_n^l(\theta, \varphi)\} = \sqrt{\frac{(2n+1)(n-l)!}{4\pi(n+l)!}} P_n^l(\cos\theta) \cos l\varphi, \quad \int_0^{2\pi} \int_0^\pi Y_n^l(\theta, \varphi) Y_{n'}^{l'*}(\theta, \varphi) \sin\theta d\theta d\varphi = \delta_{nn'} \delta_{ll'}$$



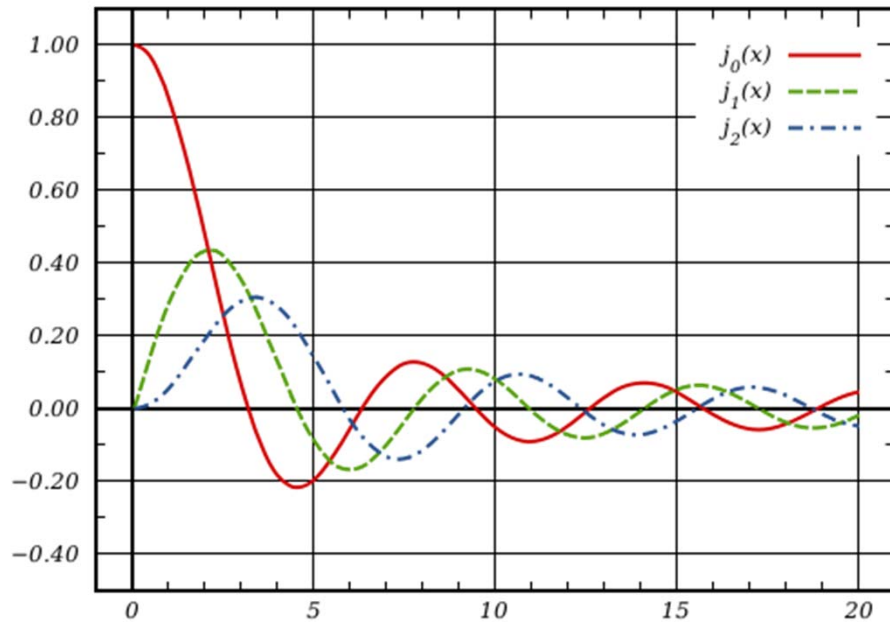
Spherical harmonics



Spherical Bessel functions

$$j_n(\rho) = \sqrt{\frac{\pi}{2\rho}} J_{n+\frac{1}{2}}(\rho), \quad y_n(\rho) = \sqrt{\frac{\pi}{2\rho}} Y_{n+\frac{1}{2}}(\rho), \quad h_n^{(1,2)} = j_n(\rho) \pm y_n(\rho)$$

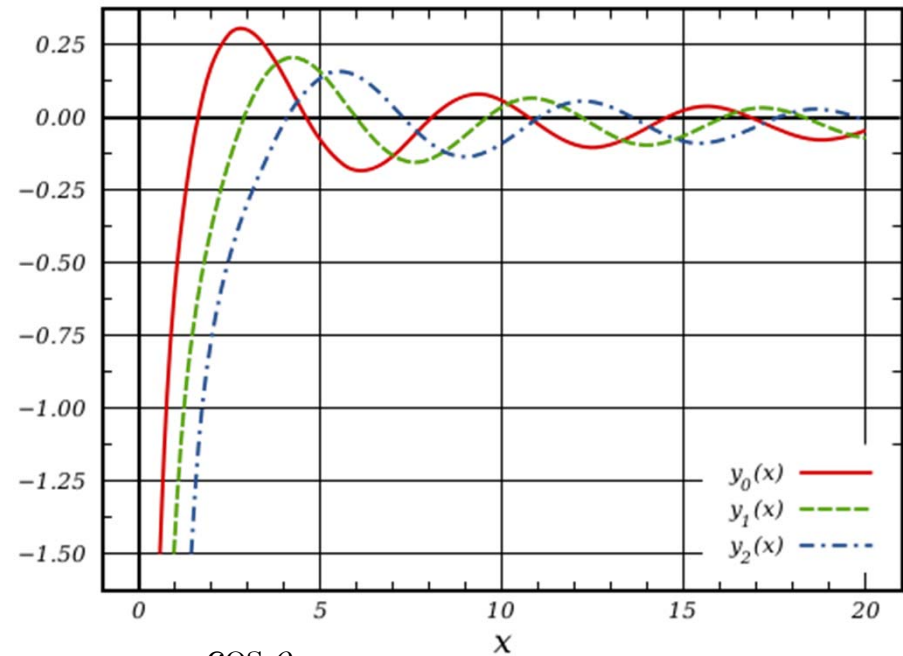
$$j_n(\rho) = (-\rho)^n \left(\frac{1}{\rho} \frac{d}{d\rho} \right)^n \frac{\sin \rho}{\rho}, \quad y_n(\rho) = -(-\rho)^n \left(\frac{1}{\rho} \frac{d}{d\rho} \right)^n \frac{\cos \rho}{\rho}$$



$$j_0(\rho) = \frac{\sin \rho}{\rho},$$

$$j_1(\rho) = \frac{\sin \rho}{\rho^2} - \frac{\cos \rho}{\rho},$$

$$j_2(\rho) = \left(\frac{3}{\rho^2} - 1 \right) \frac{\sin \rho}{\rho} - \frac{3 \cos \rho}{\rho},$$



$$y_0(\rho) = -\frac{\cos \rho}{\rho},$$

$$y_1(\rho) = -\frac{\cos \rho}{\rho^2} - \frac{\sin \rho}{\rho},$$

$$y_2(\rho) = \left(-\frac{3}{\rho^2} + 1 \right) \frac{\cos \rho}{\rho} - \frac{3 \sin \rho}{\rho},$$