WINTER SCHOOL SNA LECTURES

High-Performance Variants of Krylov Subspace Methods

Erin Carson

A series of two 90 min lectures will cover the following topics:
(1) Modeling the performance of Krylov subspace methods (global vs. point-to-point communication; communication costs of SpMVs and inner products). (2) Introduction to high-performance variants (s-step methods, pipelined methods; matrix-powers kernel, Tall-Skinny QR factorization, blocking vs. nonblocking communication). (3) Practical implementation challenges (matrix partitioning, preconditioning, numerical behavior). (4) High-performance variants in finite precision (bounds on attainable accuracy, effect on convergence rate). (5) Remedies for roundoff error (choice of polynomial basis, residual replacement strategies, variable s-step methods). (6) Guidance for choosing a method in practice

An introduction to Extended Finite Element Methods

Jaroslav Haslinger

One 90 min lecture
Extended Finite Element Methods (XFEM) were proposed by engineering community some twenty years ago originally for solving crack problems. To avoid remeshing of computational domains in problems with advancing cracks (necessary in standard FEMs), meshes which do not respect the geometry of the crack were used. To simulate discontinuity of the solution across the crack, finite element spaces are enriched by Heaviside type functions, and, additionally by appropriate singular functions in a vicinity of the crack tip owing to loss of smoothness of the solution there. The second part of this contribution deals with the EXFEM type method for solving 2nd order elliptic equations with the mixed Dirichlet - Neumann boundary conditions. In this case the partition of the computational domain Ω is given by "a print" of a classical partition of another domain Q containing Ω in its interior, i.e. the boundary of Ω and the mesh used in computations are independent. An appropriate stabilization approach ensures the optimal rate of convergence of this method.

On the Way from Matrix to Tensor Computations

Martin Plešinger

A series of two 90 min lectures
This course is a brief introduction to tensor computations. The whole story will be presented on the background of standard matrix computations, with emphasis on important analogies and differences. We start with explaining the concept of tensors, as we understand it, and motivation why to use such objects in practical computations. After introducing terminology, notation, and standard manipulation with tensors, we will discuss some basic decomposition of tensors. Then we focus on the so-called low-rank arithmetic of matrices and tensors, that brings us towards first real applications. In order to be able to manipulate with tensors of

really high orders, it will be necessary to come up with more complex structures - e.g. hierarchical decompositions, or even more general tensor networks. Finally, some useful Matlab tools for first time experiments, as well as some open questions will be mentioned.

Guaranteed eigenvalue bounds for elliptic partial differential operators

Tomáš Vejchodský

A series of three 90 min lectures
The talk will briefly review the Hilbert-Schmidt spectral theorem, its relation to eigenvalue problems for linear elliptic operators in Hilbert spaces, and the Courant-Fischer-Weyl min-max principle. The finite element method for linear elliptic partial differential eigenvalue problems will be recalled and the emphasis will be placed on lower bounds of eigenvalues. Namely, the Weinstein's bound, the Lehmann-Goerisch method, and recent lower bounds based on explicit estimates of the interpolation constant will be presented. The content of the talk will be illustrated by a number of examples.