Colloquium FLUID DYNAMICS 2011
Institute of Thermomechanics AS CR, v.v.i., Prague, Czech Society for Mechanics, the ERCOFTAC Czech Pilot Centre

An Evaluation of Novel Integral Scheme for Calculations of Transitional Boundary Layers

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ABSTRACT: *Evaluation of new integral scheme for calculations of two-dimensional, incompressible transitional boundary layers has been performed. To precede these approximate calculations for three different cases of boundary layers characteristics, the mathematical model Abu-Darag [17] was utilized in order to enable the prediction of the main boundary layer integral parameters. The model was proposed to calculate the characteristics of the boundary layers under the effect of moderate free-stream turbulence levels by enhancing established integral techniques in conjunction with intermittency weighted model of the transitional boundary layer. An attempt to verify the effect of turbulence length scales upon the prediction of the transition onset is also interpreted. To support the results validation, a numerical investigation utilized Menter el. al [7] model in ANSYS-CFX tool has been represented beside the ERCOFTAC Test Cases T3A, T3B and T3AM for skin friction coefficient.*

Keywords: Transition onset; Integral scheme; Transport equation model; Boundary layer; Turbulence intensity; Turbulence length scale

1. INTRODUCTION

The subject of laminar-turbulent transition is of considerable practical interest and has a wide range of engineering applications due to the fact that transition controls the evolution of important aerodynamic quantities such as drag or heat transfer. Transition in boundary layer flows in turbomachines and aerospace devices is known to be affected by various parameters, such as freestream turbulence, pressure gradient and separation, Reynolds number, Mach number, turbulent length scale, wall roughness, streamline curvature and heat transfer. Due to this variety of parameters, there is no mathematical model exist that can predict the onset and length of the transition region. In addition to the influence of these parameters upon transition origination, the poor understanding of the fundamental mechanisms which lead initially small disturbances to transition may also caused this lack. At present, there are three main concepts used to model transition in industry. The first approach is based on the stability theory where the successful technique is so-called e^N method. This method is based on the local linear stability theory and the parallel flow assumption in order to calculate the growth of the disturbance amplitude from the boundary layer neutral point to the transition location. A shortcoming of this technique indicates that it is not compatible with the current CFD methods because the typical industrial Navier-Stokes solutions are not accurate enough to evaluate the stability equation. Moreover since it is based on the linear stability theory, it cannot predict the transition due to non-linear effects such as high freestream turbulence or surface roughness. The second approach uses the conventional turbulence models such as the two-equation turbulence model of Launder and Sharma [1]. The disadvantages of this solution that first, ignores the transition physics and the importance of the transition zone completely and secondly, it is fabricated especially to deals with flows where the transitional region covers a large portion of the flow field. The main concept of the construction of these models that, the calibration of the damping functions in these models is based on reproducing the viscous sublayer behavior, not on predicting transition from laminar to turbulent flow. The last approach is usage of the concept of intermittency to blend the flow from laminar to turbulent regions. The development of intermittency in this technique is based on the observations from the experimental work. Due to these observations, empirical relationship can be established to correlate the onset location and growth rate of the transition. To achieve this task, most correlations usually relate the important affected parameters in the physical domain of study, such as free-stream turbulence level, Tu, and pressure gradient to the transition momentum thickness Reynolds number. Well-known correlations in literature are that of Mayle [2] and Abu-Ghannam and Shaw [3]. This technique is quite often used for the steady boundary layer on a flat plate. The empirical correlation of the transition region can be used within differential methods such as Forest [4], McDonald and Fish [5], Arnal et al. [6], Menter et al. [7] and Cebeci and

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Cousteix [8] and solved numerically to predict the development of the transitional boundary layer or can be included with existed approximate integral methods of laminar and turbulent boundary layers such as in Abu-Ghannam and Shaw [3], Fraser, C. J. and Milne, J. S. [9], Davenport, Schetz and Wang [10], Chris Kirney [11] and Martin Hepperle [12] and thus the entire development of boundary layer can be predicted.

The main objective of the present paper is to evaluate the capability of approximate integral calculations in predicting the transition onset and development for three different boundary layers characteristics. The proposed scheme is constructed of well-structured integral methods of laminar and turbulent flows with empirical correlation of transition region. In addition, the attempt to verify the effect of turbulence length scales upon the prediction of the transition onset is interpreted.

2. TRANSITION PROCESS

Transition process from laminar to turbulent flow is demonstrated as a result of a sequence of complicated phenomena which are influenced by many factors. Part of these factors is due to the environmental flow conditions, whereas the remaining factors are emanating due to generated excitation respect to flow exhibition around artificial constructions. The level of influenced factors upon transition process is mainly appearing in the way of how the transition to turbulence exists. The structural development in the natural transition region of boundary layer follows a certain sequence process. According to the theory of stability, the first step in the transition process is the presence of self-excited disturbances in the laminar boundary layer. The growth of these small disturbances so called TS-waves follows the exponential law in the first propagation, thus can be describes by linear stability theory. Further downstream, when the perturbations reach certain amplitude, their propagation starts to deviate from that predicted by linear growth. The initially two-dimensional Tollmien-Schlichting waves with respect to some experiments investigations are distorted into a series of "Peaks" and "valleys", known as $\Lambda-\text{structures}$. The formation of $\Lambda-\text{structures}$ downstream is due to superimposed of the three-dimensional disturbances caused by secondary instabilities. Further downstream, three-dimensional and nonlinear effects are increased. Due to the nonlinear development of the disturbances the peak-valley structures are stretched and form horseshoe vortices. These $\Lambda-\text{vortices}$ decay downstream into small and small vortices which finally replaced by turbulent "spots". The onset of transition can be defined in the exact location of streamwise where the first spots are presented. At this location the velocity profile is reshaped from that profile of the laminar plate boundary layer solved by Blasius to the profile of the fully turbulent plate boundary layer. This is revealed in strong decrease in the shape factor H, while a great increase in the friction drag is observed. Moreover, a great increase in the boundary-layer thickness occurs [13]. Continuous developing of the turbulent spots initiates the transition to fully turbulent boundary-layer flow as the last stage of transition process. In transition region, the isolated spots within which a fully developed turbulent flow exists appear successively in a random fashion in time and space and grow as they are washed downstream. Thus the flow at any point in the region becomes turbulent during those periods of time during which a spot moves over it and is laminar for the remainder. Although it is well-known that the transition is random phenomenon, it may be possible to determine the fraction of the total time in which the flow is turbulent as an average taken over an appropriate interval of time. In fact this fraction, which is called intermittency factor (γ), has been determined by Schubauer and Klebanoff [14] and Dhawan and Narasimha [15] from their experimental data record. In the same manner it can be reasonable to assume the existence of a sort of probability function specifying the rate of spot formation per unit area.

To evaluate the parameters behavior within the laminar-turbulent transition region, some authors [16] admitted a linear combination of turbulent and laminar results weighted by an exponential probability function (intermittency) which was developed based on the experiments performed by Schubauer and Klebanoff [14] whereas in the program which is used for the present results, only the proposed relations of [3] have been used to evaluate the development of the integral parameters. A description of the construction details and calculation techniques of the code can be found in reference [17].

3. PHYSICAL DOMAIN OF TRANSITION

The mathematical model implemented for the prediction of transitional boundary layer requires:

1) The solution of velocity field over the flat plate with sharp-leading edge.
2) The calculation of the momentum and energy boundary layers equations in integral form.

The freestream velocity and the turbulence level in the flow were specified at the leading edge of the plate as input parameters. Since the scheme is limited to flows with constant thermophysical properties and isothermal surface temperature, the rest of input parameters are standard and defined inside the program. With this standard input data, the program calculated the integral parameters along the flat plate for three different types of boundary layer characteristics. These parameters are skin-friction coefficient, momentum thickness and shape factor. The physical domain of the mathematical model is presented in Fig. 1

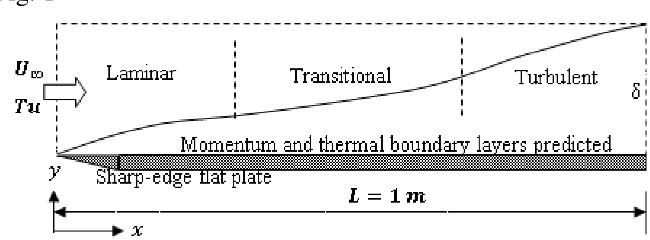

Figure 1: Physical domain of the mathematical model

4. Calculation procedure

Three well-known test cases: T3AM, T3A and T3B, which are widely examined for the transitional boundary layer are considered in the present evaluation where the transition is induced by the external freestream turbulence (bypass transition) rather than by the development of T-S waves (natural transition). These experimental works are classified as the low, moderate and high freestream turbulence intensity cases respectively. All test cases are boundary-layer flows on a flat plate with a sharp leading edge under zero-pressure-gradient condition.

Incompressible flow is considered in this exercise, so that the fluid properties such as density and kinematic viscosity are constant and equal to $1.2\ kg/m^3$ and $1.51 \cdot 10^{-5}\ m^2/s$ respectively. In the calculation, the freestream turbulence intensity and freestream velocity are specified before the run of the program as boundary conditions. According to the specific case under study where only the zero pressure gradient condition is of interest, so that the zero normal gradient of pressure has specified in the program.

Since it seems to be benefit to compare the integral model with some existed and well-known numerical models, two transition models of different intermittency transport equations namely; models of Suzen and Huang [18], and Menter et al. [7], which were used to predict the experimental test cases of Coupland [19], are included in the results. These experimental data are specially selected to test the capability of transition models to predict the effect of freestream turbulence on the development of transition in the laminar boundary layer under the zero pressure gradient condition. Comparisons are also made for all test cases between the recent model and one turbulence model, that is, the k-ε model of Launder and Sharma [1], which has been known as the best model among all two-equation models for transitional flow. The results data of these computational models are obtained from the ref. [20].

$Case$	$U_\infty\ (m/s)$	Tu (%)	$Re_{\theta S}$	Re_L
T3AM	19.8	0.98	810	$2.24 \cdot 10^6$
T3A	5.4	3.35	272	$6.12 \cdot 10^5$
T3B	9.4	6.14	180	$1.07 \cdot 10^6$

Table 1: Flow characteristics

$Model$	$Empirical\ Correlation$
Abu-Ghannam & Shaw[3]	$Re_{\theta S} = 163 + \exp(6.91 - Tu)$
Menter et al. [7]	$Re_{\theta S} = 803.73(Tu + 0.6067)^{-1.027}$
Suzen et al. [18]	$Re_{\theta S} = [120 + 150(Tu)^{-2/3}]coth[4(0.3 \cdot 10^5 K_t)]$

Table 2: Empirical correlation used in models [20]

Model	$Re_{\theta S}$
Abu-Darag and Horak [17]	539
Suzen and Huang [18]	326
Menter et al. [7]	500
Launder and Sharma [1]	612

Table 3: Transition onset defines by momentum thickness Reynolds number corresponds to T3AM

Model	$Re_{\theta S}$
Abu-Darag and Horak [17]	198
Suzen and Huang [18]	224
Menter et al. [7]	196
Launder and Sharma [1]	240

Table 4: Transition onset defines by momentum thickness Reynolds number corresponds to T3A

Model	$Re_{\theta S}$
Abu-Darag and Horak [17]	165
Suzen and Huang [18]	198
Menter et al. [7]	113
Launder and Sharma [1]	200

Table 5: Transition onset defines by momentum thickness Reynolds number corresponds to T3B

5. RESULTS AND DISCUSSION

The predicted results of the considered model Abu-Darag is compared with the experimental data of ERCOFTAC Test cases T3AM, T3A and T3B in addition to three numerical models; Launder and Sharma [1], Suzen and Huang [18] and the model of Menter et al. [7]. Details about these models can be found in ref. [20] and the transition onset defines by momentum thickness Reynolds number corresponds to T3AM, T3A and T3B are presented in Tables 3, 4 and 5 respectively. The comparison has been carried out for skin friction coefficient at three different boundary layer flow conditions. In the first case the flow is subjected to low freestream turbulence intensity while in the second and third cases the flow is subjected to moderate and high level of freestream turbulence intensities respectively.

The results of the skin friction coefficient for the three flow conditions were represented in Figures 2, 3 and 4. The importance of determination of this parameter revealed from the fact that it is playing a vital role in indicating the starting and ending points of transition. In addition, the determination of the growth rate characteristics of transition enables to determine the exact length of transition. The analytic skin friction coefficient for laminar and turbulent flows in case of the flat plate boundary layer flow with zero pressure gradient condition can be determined from $C_f = 0.664/Re^{1/2}$ and $C_f = 0.027/Re^{1/7}$ respectively and displayed by the dash lines in Figure 2, 3 and 4. The variation of the friction coefficient within the transition region indicates the growth rate and length of transition.

For boundary layer flow subjected to low level of turbulence intensity illustrated in Figure 2, the present model predicts very early the transition onset compared with the experimental work of T3AM. The flow specifications for the three flow conditions were tabulated in Table 1. According to K. Suluksna and E. Juntasaro [20] the experimental data indicate that the transition starts at Reynolds numbers of $1.40 \cdot 10^6$ corresponding to the momentum thickness Reynolds numbers of 810 while for the present model the transition starts at Reynolds numbers of $0.65 \cdot 10^6$ corresponding to the momentum thickness Reynolds numbers of 539. In both Abu Darag model and T3AM, the development of the boundary layer from laminar to turbulent flow is not yet complete within the length of the flat plate which is $100\,cm$ for Abu Darag model and $170\,cm$ for T3AM, and hence no ending point of transition appears. The difference in transition onset between these two cases is $50\,cm$ as clearly indicated in Figure 2. In comparison of the present model with Suzen and Huang model [18], the later one shows an early onset of transition at momentum thickness Reynolds numbers of 326. The k-ε model of Launder and Sharma [1] specifically from Figure 2 verified that the model has the capability to predict the transition at low turbulence intensity condition but with rapid rate of transition which results in short length of transition region. Menter et al. model [7] according to [20] starts the transition at $Re_{\theta S} = 500$ which means before the Abu Darag model, but with very slow rate of transition and very close to the laminar flow.

The case of the boundary layer flow subjected to moderate level of turbulence intensity is depicted in Figure 3. The model of Abu Darag represents an earlier deviation of the friction coefficient from the laminar line compared with the experimental work of T3A. The transition starts at Reynolds numbers of $0.93 \cdot 10^5$, corresponding to the momentum thickness Reynolds numbers of 198 and end at $Re_x = 2.46 \cdot 10^5$ corresponds to 549.4 while the transition in the experimental data T3A case starts at Reynolds number of $1.35 \cdot 10^5$ corresponding to the momentum thickness Reynolds number of 272 and ends at the Reynolds number of $3.09 \cdot 10^5$ or the momentum thickness Reynolds number of 628 [35]. This difference in transition onset is equal to approximately $11.74\,cm$. The model of Menter et al. [7] agrees fairly well with the case T3A. When compare the computed skin friction coefficient with the experimental data, the model is able to handle satisfactory transition onset and transition length. In case of the model of Suzen and Huang [18], the deviation of the skin friction coefficient from the laminar line shows a slight delayed onset of transition in the case of moderate freestream turbulent intensity compared with T3A and therefore more delayed with Abu Darag model is existed. The variation of the skin friction coefficient during transition region indicates an earlier origination of transition of Abu Darag model which it seems to be close to the result of k-ε model of Launder and Sharma [1] but with slow growth rate of the transition. Since the later model was considered to be suitable for use in simulating the transition at high level of turbulence [20], thereby this earlier-prediction between the present integral model result and the results of T3A, Suzen and Huang model [18], and Menter et al. [7] it may be returns to either the difference in length scale of turbulence, L_x, in case of the experimental works of Abu-Ghannam and Shaw [3], and T3A test case or due to the difference in empirical correlations used by the models, Abu-Darag and Horak [17], Suzen and Huang [18], and Menter et al. [7] in case of the computational exercise. These empirical correlations are presented in Table 2 and used to predict the onset and length of transition region in the models. The last case presented in this paper in Figure 4 is the boundary layer flow subjected to high freestream turbulence intensity. Although the empirical correlation of the transition onset prediction of Abu-Ghannam and Shaw model [3] is based on the experimental investigation for turbulence intensity ranging from 0.3 to 5%, the present model is able to predict satisfactory the transition in this case which is about 6%. In the present model the transition starts at Reynolds numbers of $6.7 \cdot 10^4$, corresponding to the momentum thickness Reynolds numbers of 165 and end at $Re_x = 1.88 \cdot 10^5$ corresponds to 487.6 whereas the transition in the experimental data T3B case starts at Reynolds number of $5.90 \cdot 10^4$ corresponding to the momentum thickness Reynolds number of 180 and ends at the Reynolds number of $1.25 \cdot 10^5$ or the momentum thickness Reynolds number of 337 [20]. Two numerical models are able to predict the transition before the present model, namely Menter et al. [7] and the two-equation turbulence model of Launder and Sharma [1]. Menter et al. model [7] predicts the transition at $Re_{\theta S} = 113$ [20] while the model of Suzen and Huang [18], and the experimental work T3B are start the transition at 198 and 180 respectively [20].

The model of Suzen and Huang [18] in the cases of moderate and high freestream turbulent intensities shows a delayed onset of transition compared with the model of Abu Darag while this result is vice versa in case of low turbulence intensity. In all cases, a rapid variation of the skin friction coefficient with a slight overshoot at the end of the transition region is obtained which implies a rapid growth rate of transition produced by the model leading to a shorter transition length. When compare the result of the model of Menter et al. [7] for the computed skin friction coefficient with the experimental data, the model gives a too early onset of transition and an unsatisfactory transition length for the case of high freestream turbulence intensity.

6 CONCLUSIONS

The evaluation of Abu-Darag and Horak scheme verified that the model is capable of reproducing the three test cases of the transitional boundary layers: low, moderate and high freestream turbulence intensities.

The evaluation of the code is also performed for two transition model with intermittency transport equations and Two-equation turbulence model and their results are analyzed. In case of low turbulence level, the model Abu-Darag and Horak [17] is approximately agree with the model of Menter et al. [7] in terms of momentum thickness Reynolds number and both of them are predict transition earlier than T3AM whereas Suzen and Huang model [18] starts the transition earlier than all of them. The same conclusion is obtained for the case of moderate turbulence intensity with one difference that, Suzen and Huang model [18] predicts the transition later than both Abu-Darag and Horak [17], and Menter et al. [7], and earlier than T3A. In case of high level of freestream turbulence intensity, Menter et al. [7] predicts transition onset too earlier than all models while Abu-Darag and Horak [17] predicts transition onset slight earlier than T3B, and Suzen and Huang model [18].

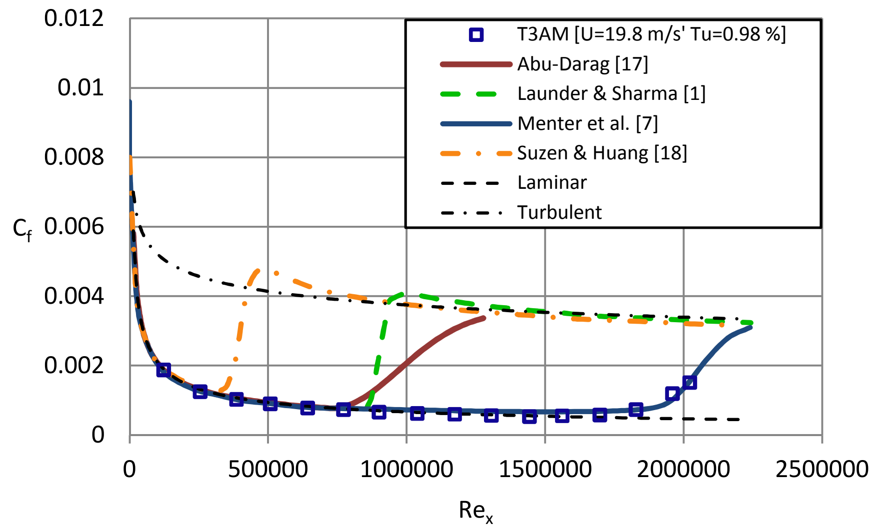

Figure 2: Predicted skin friction coefficient in transitional boundary layer subjected to low turbulence intensity

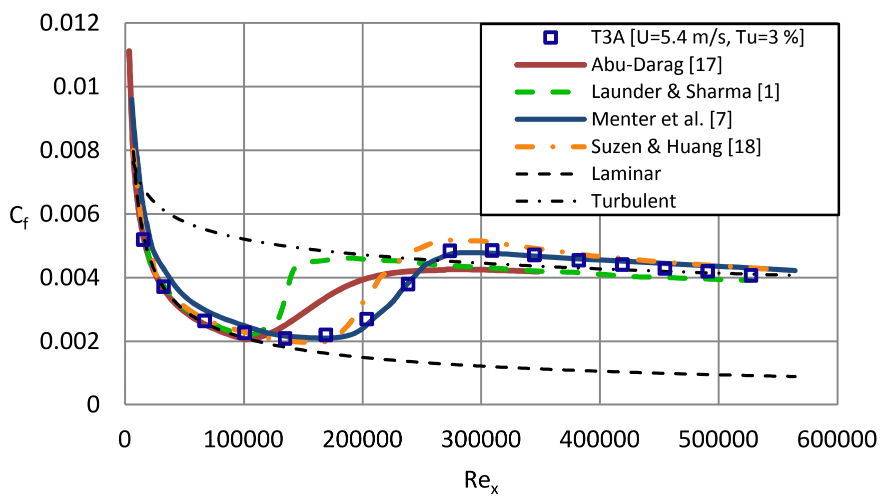

Figure 3: Predicted skin friction coefficient in transitional boundary layer subjected to moderate turbulence intensity

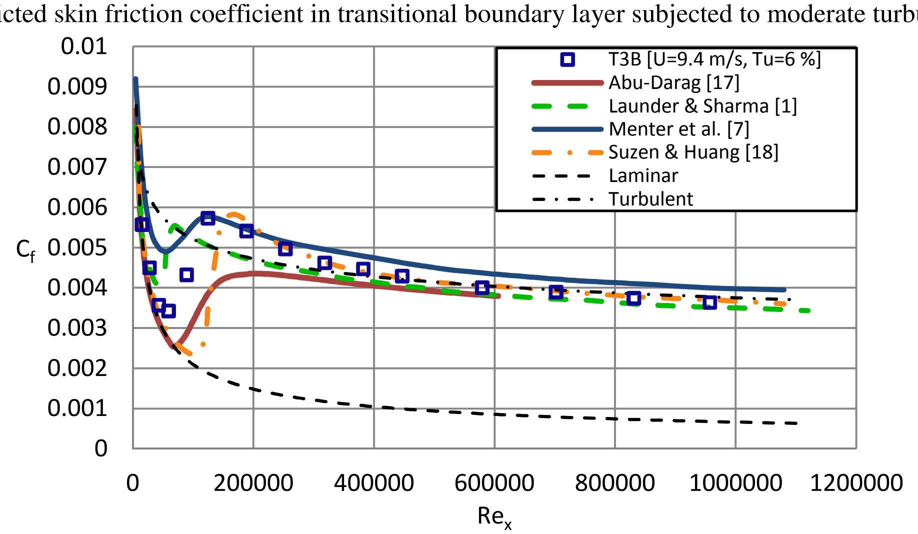

Figure 4: Predicted skin friction coefficient in transitional boundary layer subjected to high turbulence intensity

ACKNOWLEDGEMENT
This work, which has been carried out at the Department of Mechanical Engineering at the University of Defence, is supported by the Czech Science Foundation (project No. P101/10/0257) and by the research scheme of the Faculty of Military Technologies SV-K216.

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