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Performance and implementation aspects of an isogeometric mortat-based contact algorithm

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Introduction

The main difficulty in contact analysis is non-smoothness. It arises from inequality constraints as well as geometric discontinuities inducted by the spatial discretization. Contact analysis based on traditional finite elements utilizes element facets to describe contact surfaces. The facets are C^0 continuous so that the surface normals can experience jump across facet boundaries leading to artificial oscillations in contact force and pressure. A remedy to this geometric discontinuity could provide isogeometric analysis (IGA). The fundamental idea is to accurately describe a physical domain by proper representation (e.g. NURBS) and then to utilize the same basis for analysis. This is in contrast with the classical finite element method where the basis is given in advance by the element type. Consequently the physical domain could be approximated inaccurately. A more detailed description can be found in [1].

Isogeometric NURBS-based contact analysis has some additional advantages:

- preserving geometric continuity,
- facilitating patch-wise contact search,
- supporting a variationally consistent formulation,
- and having a uniform data structure for the contact surface and the underlying volumes.

Closest point projection

Given a slave point $\mathbf{x}_s \in \mathbb{R}^3$, a master point $\mathbf{x}_m \in \Gamma$ satisfying

$$\mathbf{x}_m = \arg\min_{\mathbf{x}\in\Gamma} \|\mathbf{x} - \mathbf{x}_s\|$$

is defined as the closest projection of $\mathbf{x}_s$ onto the surface Γ. The parametric coordinates of such a point may be obtained via the minimization of the squared distance function defined as

$$d(\boldsymbol{\xi}) := \frac{1}{2}(\mathbf{x}(\boldsymbol{\xi}) - \mathbf{x}_s)\cdot(\mathbf{x}(\boldsymbol{\xi}) - \mathbf{x}_s)$$

At the stationary point, the gradient of the squared distance function must vanish, thus

$$\nabla d = \nabla\mathbf{x}(\mathbf{x} - \mathbf{x}_s) = \mathbf{0} \quad \text{at } \mathbf{x}_m$$

To attack this problem, a bunch of numerical method were tested [2].

Method	AVRG NITER	DNF NITER	AVRG CPU [ms]
Newton-Raphson	14	19	34
Least square projection	52	5427	76
Sphere approx. method	26	0	75
Torus approx. method	7	0	20
Steepest descent	52	183	137 (60.8 %)
Broyden	166	922	442 (55.4 %)
BFGS	12	8	48 (45.2 %)
Simplex	199	0	303

Bibliography

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Explicit dynamic contact algorithm

An algorithm, originally proposed in [3], was adapted to the isogeometric analysis and expanded to explicit dynamics. The main idea is as follow. The contact boundary value problem is formulated in the weak sense

$$\delta\boldsymbol{\Pi}_{\mathrm{int,ext}}(\mathbf{u},\delta\mathbf{u}) + \delta\boldsymbol{\Pi}_{\mathrm{c}}(\mathbf{u},\delta\mathbf{u}) = 0$$
$$g_{\mathrm{N}}(\mathbf{u}) \geq 0$$

where $\delta\boldsymbol{\Pi}_{\mathrm{c}}$ was proposed in the form

$$\delta\boldsymbol{\Pi}_{\mathrm{c}}(\mathbf{u},\delta\mathbf{u}) = -\int_{\Gamma_{c1}} \varepsilon_{\mathrm{N}} g_{\mathrm{N}} \delta\mathbf{u}\,\mathrm{d}\Gamma - \int_{\Gamma_{c2}} \varepsilon_{\mathrm{N}} g_{\mathrm{N}} \delta\mathbf{u}\,\mathrm{d}\Gamma$$

Note that the contact virtual work is integrated over both contact boundaries Γ_{c1} and Γ_{c2} so that the algorithm preserves symmetry. Consequently, after FE discretization the action-reaction principle is not explicitly fulfilled. However, it should be shown that the equilibrium is recovered during the mesh refinement.

The application of the FEM for spatial discretization and the central difference method (CDM) for temporal integration yields

$$\mathbf{M}\mathbf{U}_{n+1} = \Delta t^2\left[\mathbf{R} + \mathbf{R}_{c12}(\mathbf{U}_n) + \mathbf{R}_{c21}(\mathbf{U}_n) - \mathbf{F}(\mathbf{U}_n)\right] + \mathbf{M}(2\mathbf{U}_n - \mathbf{U}_{n-1})$$

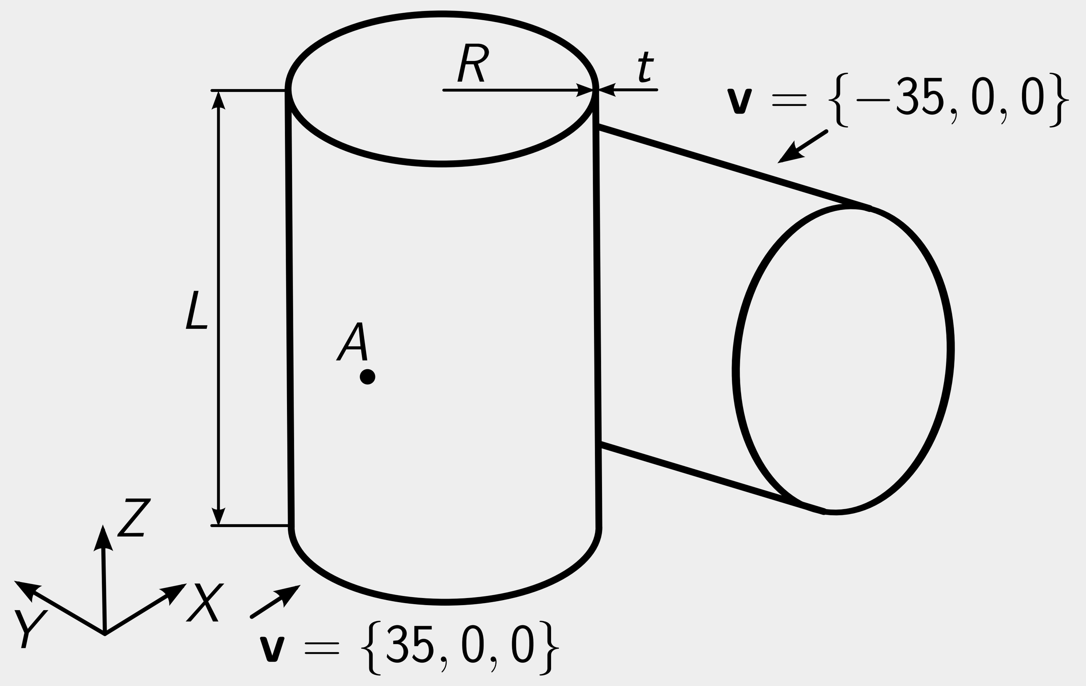
The stability of the integration process requires time step to be smaller or equal to $2/\omega_{\max}$. The consistent element mass matrix rising from the variational formulation has the form

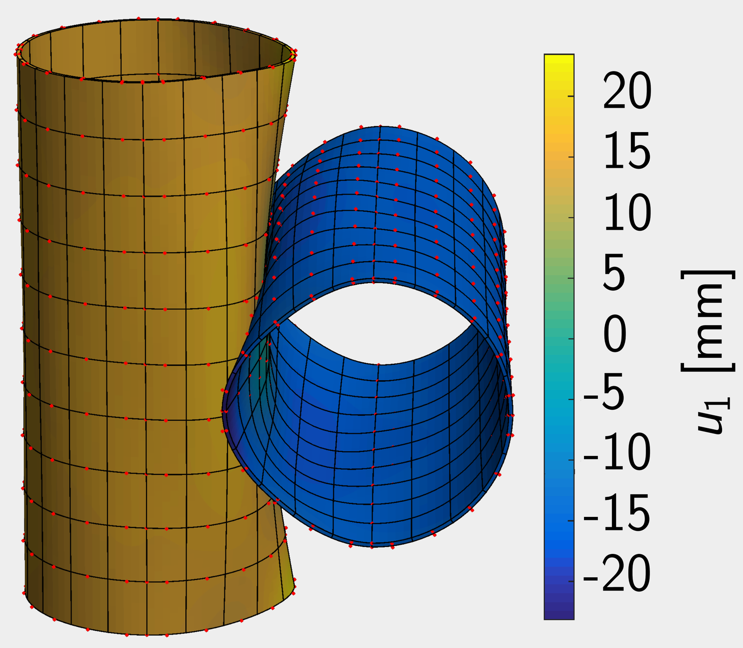
$$\mathbf{M}_e = \int_{\Omega_e} \rho\mathbf{H}^{\mathrm{T}}\mathbf{H}\,\mathrm{d}\Omega$$

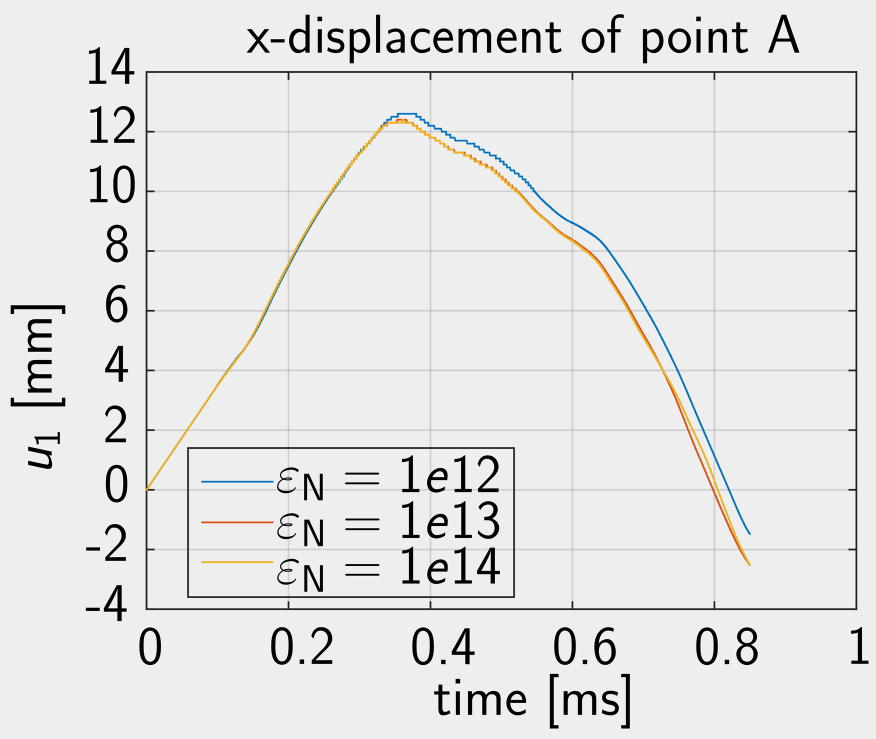
The efficient solution of the resulting system of equations requires diagonalization of the mass matrix. The common techniques are the row sum method and HRZ method [4].

Impact between two tubes

Given parameters: $R = 0.1$ m, $L = 0.46$ m, $t = 0.003$ m, Young's modulus $E = 200$ GPa, Poisson's ratio $\nu = 0.3$ and mass density $\rho = 7840$ kg $\cdot$ m^{-3}.







Conclusions

This paper addressed the utilization of the NURBS based isogeometric analysis in an explicit contact-impact algorithm. More attention were payed to the problem of the closest point projection. It was found that the most efficient method for the local contact search is the torus approximation method. The robustness of the method were examined by means of numerical example which involves impact between two tubes.

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