Reaction-diffusion systems with unilateral terms and spatial patterns

Milan Kučera
Institute of Mathematics, Academy of Sciences, Prague, Czech Republic
kucera@math.cas.cz

Let us consider a system

$$\frac{\partial u}{\partial t} = d_1 \Delta u + f(u,v), \quad \frac{\partial v}{\partial t} = d_2 \Delta u + g(u,v) + s_- v^- - s_+ v^+$$

in a bounded domain in R^N with Neumann or mixed boundary conditions. Here f, g are real functions, $f(0,0) = g(0,0) = 0$, d_1, d_2 are diffusion parameters, v^- and v^+ denote the negative and positive part of v, and s_-, s_+ are non-negative functions of the space variable with supports satisfying supp $s_- \cup$ supp $s_+ \neq \emptyset$, supp $s_- \cap$ supp $s_+ = \emptyset$. The unilateral term $s_- v^-$ or $s_+ v^+$ can describe a source or sink active only in those places of supp s_- or supp s_+ where the value of v is negative or positive, respectively. The system can describe a biochemical reaction, u and v denote deviations of concentrations of reactants from a certain positive spatially homogeneous steady state, so that also negative values of u, v have a good sense. Assumptions guaranteeing Turing's diffusion-driven instability for the case $s_- = s_+ \equiv 0$ are considered, i.e. for $s_- = s_+ \equiv 0$ the trivial solution of the system without any diffusion ($d_1 = d_2 = 0$) is asymptotically stable, but as a solution of the system with diffusion terms it is stable only for some diffusion parameters (domain of stability D_S) and unstable for the others (domain of instability). An influence of the unilateral terms to a location of bifurcations of spatially non-homogeneous stationary solutions (spatial patterns) will be discussed. For our system with unilateral terms, there are bifurcations of spatial patterns also in the domain D_S, where a bifurcation for the system without unilateral terms is excluded. In spite of that the system is non-potential, in some cases a variational approach can be used in a certain non-direct way for finding critical points (i.e. d_1, d_2 suspected from bifurcation) in D_S. If s_-, s_+ are sufficiently small, then combining it with a topological approach, we can show that in some cases there is really a bifurcation of spatial patterns in D_S. In particular, spatial patterns exist for a larger domain of d_1, d_2 than in standard models without any unilateral term. Some biological aspects and numerical experiments showing also an influence of unilateral terms to the form of spatial patterns will be mentioned.