Homework assignment

L5: Regression models for item description

Assignment date: **30.10.2018**
Deadline: **5.11.2018 23:59**
Slides: http://www.cs.cas.cz/martinkova/NMST570
Note: Send answers to drabinova@cs.cas.cz
Name:

1 Interpretation of logistic regression models

Ex. 1.1 Logistic regression model for probability of correct answer on total scores X to given item is given by:

$$\mathsf{P}(Y = 1|X, b_0, b_1) = \frac{\exp(b_0 + b_1 X)}{1 + \exp(b_0 + b_1 X)}. \tag{1}$$

1. How do you interpret parameters b_0 and b_1? [0.4]

2. Fill in the table below with probabilities of correct answer to item with parameters $b_0 = -5$ and $b_1 = 0.5$ [1]

Total score X	0	5	10	15	20
$\mathsf{P}(Y = 1\|X, b_0, b_1)$					

Ex. 1.2 Consider logistic regression model on standardized total scores Z, that is

$$\mathsf{P}(Y = 1|Z, \tilde{b_0}, \tilde{b_1}) = \frac{\exp(\tilde{b_0} + \tilde{b_1} Z)}{1 + \exp(\tilde{b_0} + \tilde{b_1} Z)}. \tag{2}$$

1. What is the interpretation of parameters $\tilde{b_0}$ and $\tilde{b_1}$? [0.4]

2. What is the value of parameter $\tilde{b_0}$ and $\tilde{b_1}$ from model (2) when parameters of model (1) are $b_0 = -5$ and $b_1 = 0.5$ and mean of total scores X is $M = 10$ and their standard deviation is $SD = 3$? Provide whole calculation. [1]
HINT: Use $Z = (X - M)/SD$ for arbitrary X and solve $\mathsf{P}(Y = 1|X, b_0, b_1) = \mathsf{P}(Y = 1|Z, \tilde{b_0}, \tilde{b_1})$

Ex. 1.3 Consider logistic regression model on standardized total scores Z with IRT parameterization, that is

$$\mathsf{P}(Y = 1|Z, a, b) = \frac{\exp(a(Z - b))}{1 + \exp(a(Z - b))}. \tag{3}$$

1. How do you interpret parameters a and b? [0.4]

2. Sketch curve for probability of correct answer and show how it is related to parameters a and b. [0.5]

3. What is the relation between parameters a and b in model (3) and parameters $\tilde{b_0}$ and $\tilde{b_1}$ in model (2)? [0.75]
HINT: Solve $\mathsf{P}(Y = 1|Z, \tilde{b_0}, \tilde{b_1}) = \mathsf{P}(Y = 1|Z, a, b)$

4. What are the values of parameters a and b when values of parameters $\tilde{b_0}$ and $\tilde{b_1}$ are given by second part of Exercise 1.2.2? [0.35]

2 Extensions of logistic regression models

Ex. 2.1 Consider non-linear extension of logistic regression model on Z-scores with IRT parameterization, that is

$$\mathsf{P}(Y = 1|Z, a, b, c, d) = c + (d - c)\frac{\exp(a(Z - b))}{1 + \exp(a(Z - b))}. \tag{4}$$

1. How do you interpret parameters c and d? [0.4]

2. Sketch curve for probability of correct answer and show how it is related to parameters c and d. Describe how is now defined parameter b. [0.75]

3. Fill table below with probabilities of correct answer for parameters $a = 1$, $b = 0$ and Z-score $Z = b$ with various values of parameters c and d. [0.8]

c	0	0.2	0	0.2
d	1	1	0.9	0.9
$\mathsf{P}(Y = 1\|Z, a, b, c, d)$				

Ex. 2.2 Consider multinomial model, that is

$$\mathsf{P}(Y = k|Z, a_k, b_k) = \frac{\exp(a_k(Z - b_k))}{\sum_{j=1}^{K} \exp(a_j(Z - b_j))},\ k = 0, 1, 2, \tag{5}$$

where $a_0 = 0, b_0 = 0, a_1 = 1.5, b_1 = -0.6, a_2 = 2$ and $b_2 = 0$.

1. Fill table below with probabilities of $k = 0$, $k = 1$ and $k = 2$ for different levels of Z. [1.25]

Z	-1.8	-0.6	0	0.6	1.8
$\mathsf{P}(Y = 0)$					
$\mathsf{P}(Y = 1)$					
$\mathsf{P}(Y = 2)$					

2. Sketch curves for probabilities of $k = 0$, $k = 1$ and $k = 2$ in one plot. For what levels of Z do the curves cross? [0.5]

3 ShinyItemAnalysis

Run `ShinyItemAnalysis` (online or locally) and change data to HCI.

1. For item 4 interpret estimates of parameters obtained by various models (Logistic IRT Z, Logistic 3P IRT Z and Logistic 4P IRT Z). Which model does fit the best using likelihood ratio test? (Regression/Model Comparison) [0.5]

2. For item 19 interpret estimates of parameters obtained by various models (Logistic IRT Z, Logistic 3P IRT Z and Logistic 4P IRT Z). Which model does fit the best using likelihood ratio test? (Regression/Model Comparison) [0.5]

4 Provide feedback

Here you can provide feedback on lecture, lab session and/or materials (slides, HW assignment, ShinyItemAnalysis manual) [1pt bonus] :)