Recent Developments in Digital Mathematics Libraries

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Institute of Mathematics AS CR, Prague

DiPP 2014

Looking back to the recent history

1980s TeX

1981 P. Ginsparg: small HTTP server for authors to upload preprints written in TeX

1990s Electronic platforms for publishing, the first digitization projects (Gallica, JSTOR, GDZ); Google

http://www.nsf.gov/awardsearch/showAward?AWD_ID=9411306

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Award Abstract #9411306

The Stanford Integrated Digital Library Project

NSF Org:	IIS Division of Information & Intelligent Systems
Initial Amendment Date:	September 16, 1994
Latest Amendment Date:	October 5, 1998
Award Number:	9411306
Award Instrument:	Cooperative Agreement
Program Manager:	Stephen Griffin IIS Division of Information & Intelligent Systems CSE Directorate for Computer & Information Science & Engineering
Start Date:	September 1, 1994
Expires:	August 31, 1999 (Estimated)
Awarded Amount to Date:	$4,516,573.00
Investigator(s):	Hector Garcia-Molina (Former Principal Investigator)
Sponsor:	Stanford University 3160 Porter Drive Palo Alto, CA 94304-1212 (650)723-2300
NSF Program(s):	DIGITAL SOCIETY&TECHNOLOGIES, ARTIFICIAL INTELL & COGNIT SCI, INFORMATION & KNOWLEDGE MANAGE, ROBOTICS, HUMAN COMPUTER INTER PROGRAM, ADVANCED NET INFRA & RSCH
Program Reference Code(s):	9139, HPCC, 6850, 9216
Program Element Code(s):	6850, 6856, 6855, 6398, 6840, 6845, Z093, 4090, Z026, Z410, Z596, Z564, Z971, Y494

ABSTRACT

This project - the Stanford Integrated Digital Library Project (SIDLP) - is to develop the enabling technologies for a single, integrated and "universal" library, proving uniform access to the large number of emerging networked information sources and collections. These include both on-line versions of pre-existing works and new works and media of all kinds that will be available on the globally interlinked computer networks of the future. The Integrated Digital Library is broadly defined to include everything from personal information collections, to the collections that one finds today in conventional libraries, to the large data collections shared by scientists. The technology developed in this project will provide the "glue" that will make this worldwide collection usable as a unified entity, in a scalable and economically viable fashion.

BOOKS/ONE TIME PROCEEDING

Please see http://www-diglib.stanford.edu for a list of publications.. "Please see http://www-diglib.stanford.edu for a list of publications.", 09/01/1994-08/31/1999, 1998, "World-Wide Web list of publications.".

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The National Science Foundation, 4201 Wilson Boulevard, Arlington, Virginia 22230, USA
Tel: (703) 292-5111, FIRS: (800) 877-8339 | TDD: (800) 281-8749

Text Only Version

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2001	IMU: *Call to all mathematicians to make their publications electronically available* http://www.nap.edu/catalog.php?record_id=18619
2002	World Digital Mathematics Library, with new impulses in 2006, 2012, 2014

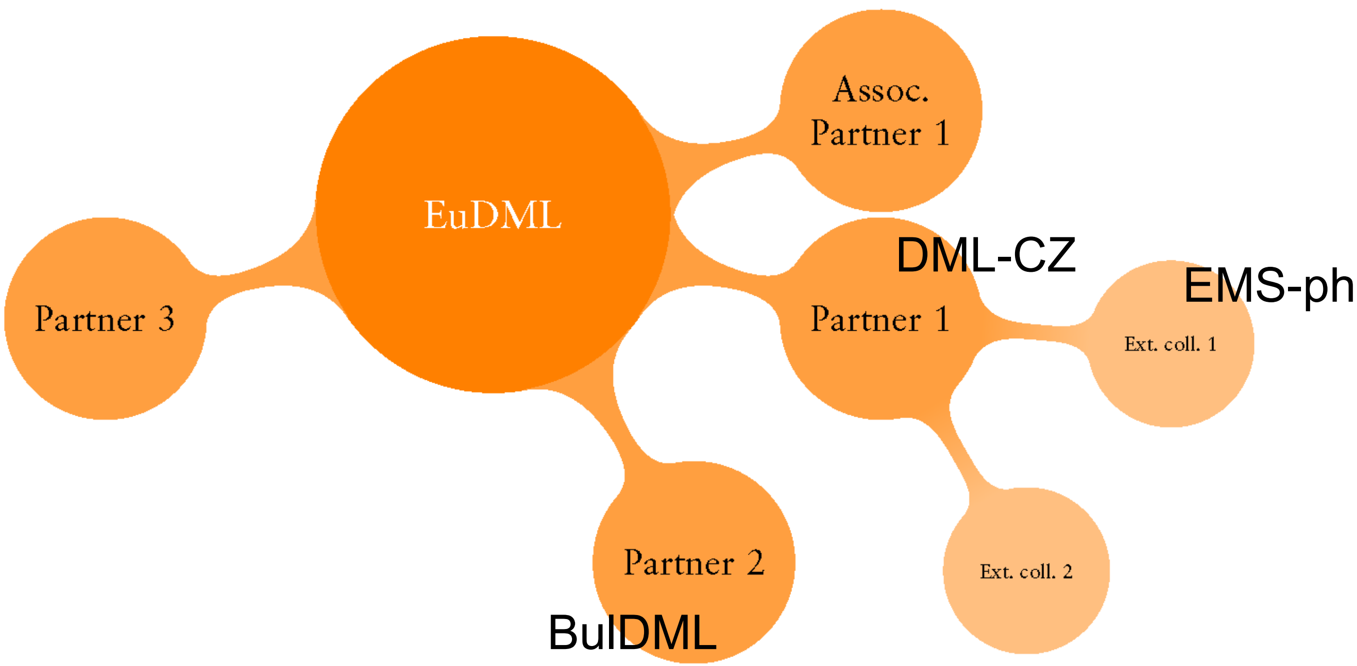
“Classical” digital libraries

- Archiving and access to texts with permanent URL
- Sufficiently reach metadata for
  - searching authors, titles, key words, MSC codes, text
  - references lookup
  - interlinking documents
- Further services
  - search for semantically similar documents
  - formula search
  - ...

NUMDAM GDZ Göttinger Digitalisierungszentrum icm bdim: Biblioteca Digitale Italiana di Matematica Un progetto SIMAI e UMI ∂ML e

eLibrary DML-CZ BulDML ...

EuDML: distributed digital library



EuDML http://eudml.org

EuDML – basic principles

- The texts in EuDML must have been scientifically validated and formally published.
- EuDML items must be open access after a finite embargo period. Once documents contributed to the library are made open access due to this policy, they cannot revert to close access later on.
- The digital full text of each item contributed to EuDML must be archived physically at one of the EuDML member institutions.

EuDML – sustainable development

- 2010–2013: project of 14 partners partly funded by the EC
- Since 2014: international association of 12 partners
  - The European Mathematical Society
  - FIZ Karlsruhe – Leibniz Institut für Informationsinfrastruktur GmbH
  - Interdisciplinary Centre for Mathematical and Computational Modelling, University of Warsaw
  - Université Joseph Fourier, Grenoble 1
  - University of Birmingham
  - Institute of Mathematics and Informatics BAS, Sofia
  - Institute of Mathematics AS CR, Praha
  - Ionian University, Corfu
  - Società Italiana per la Matematica Applicata e Industriale
  - Unione Matematica Italiana
  - Niedersächsische Staats- und Universitätsbibliothek Göttingen
  - Masaryk University, Brno

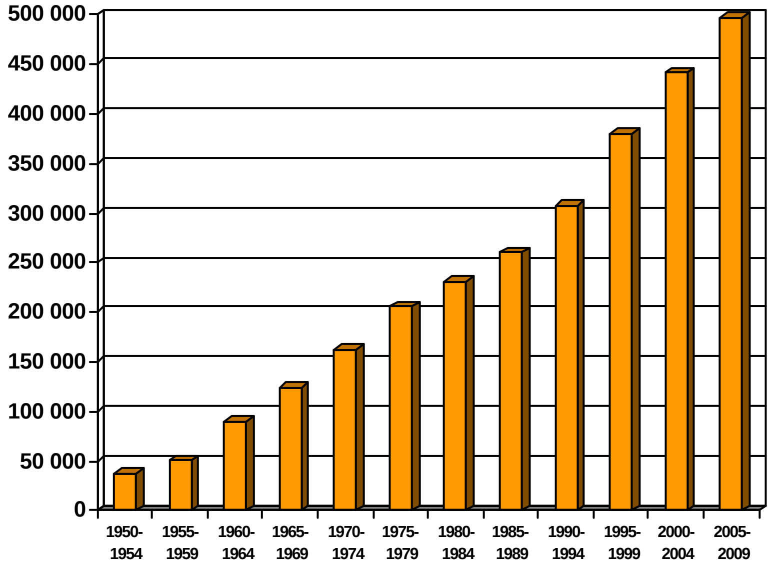
Mathematical corpus

- zbMATH (including Jahrbuch since 1868):
  - > 3 million items
  - 3000 journals and series
  - 170 000 books
  - the eldest item from 1826
- MathSciNet (since 1940)
  - 2,9 million items
  - 2000 journals and series
  - 100 000 books since 1960

Growth of mathematics literature

	arXiv	MathSciNet	WoS
2008	14 373	86 533	20 908
2009	16 319	87 279	22 390
2010	18 765	87 162	22 079
2011	21 287	89 638	22 716
2012	24 176	92 191	23 760

5years increments in zbMATH



New challenges

- NRC Report (NSF, IMU, Sloan Foundation) *Developing a 21st century global library for mathematics research*

- Stefan Banach (quoted by S. Ulam, 1957):

  Good mathematicians see analogies between theorems; great mathematicians see analogies between analogies.

III. *Generalized linear differential equations*

1. The generalized linear differential equation and its basic properties

We assume that $\mathbf{A}\colon [0,1] \to L(R_n)$ is an $n \times n$-matrix valued function such that $\operatorname{var}_0^1 \mathbf{A} < \infty$ and $\mathbf{g} \in BV_n[0,1] = BV_n$.

The generalized linear differential equation will be denoted by the symbol

$$\mathrm{d}\mathbf{x} = \mathrm{d}[\mathbf{A}]\,\mathbf{x} + \mathrm{d}\mathbf{g} \tag{1,1}$$

which is interpreted by the following definition of a solution.

1.1. Definition. Let $[a,b] \subset [0,1]$, $a < b$; a function $\mathbf{x}\colon [a,b] \to R_n$ is said to be a solution of the *generalized linear differential equation* (1,1) on the interval $[a,b]$ if for any $t, t_0 \in [a,b]$ the equality

$$\mathbf{x}(t) = \mathbf{x}(t_0) + \int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s) + \mathbf{g}(t) - \mathbf{g}(t_0) \tag{1,2}$$

is satisfied.

In the original papers of J. Kurzweil (cf. [1], [2]) on generalized differential equations and in other papers in this field the notation

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\tau} = \mathrm{D}[\mathbf{A}(t)\,\mathbf{x} + \mathbf{g}(t)]$$

was used for the generalized linear differential equation.

It is evident that the generalized linear differential equation can be given on an arbitrary interval $[a,b] \subset R$ instead of $[0,1]$.

If $\mathbf{x}_0 \in R_n$ and $t_0 \in [a,b] \subset [0,1]$ are fixed and $\mathbf{x}\colon [a,b] \to R_n$ is a solution of (1,1) on $[a,b]$ such that $\mathbf{x}(t_0) = \mathbf{x}_0$, then $\mathbf{x}$ is called the solution of the initial value (Cauchy) problem

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on $[a,b]$.

1.2. Remark. If $\mathbf{B}\colon [0,1] \to L(R_n)$ is an $n \times n$-matrix valued function, continuous on $[0,1]$ with respect to the norm of a matrix given in I.1.1 and $\mathbf{h}\colon [0,1] \to R_n$ is continuous on $[0,1]$, then the initial value problem for the linear ordinary differential equation

$$\mathbf{x}' = \mathbf{B}(t)\,\mathbf{x} + \mathbf{h}(t)\,, \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \tag{1,4}$$

is equivalent to the integral equation

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{B}(s)\,\mathbf{x}(s)\,\mathrm{d}s + \int_{t_0}^{t} \mathbf{h}(s)\,\mathrm{d}s\,, \qquad t \in [0,1]\,.$$

If we denote $\mathbf{A}(t) = \int_0^t \mathbf{B}(r)\,\mathrm{d}r$, $\mathbf{g}(t) = \int_0^t \mathbf{h}(r)\,\mathrm{d}r$ for $t \in [0,1]$, then this equation can be rewritten into the equivalent Stieltjes form

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s) + \mathbf{g}(t) - \mathbf{g}(t_0)\,, \qquad t \in [0,1]\,.$$

The functions $\mathbf{A}\colon [0,1] \to L(R_n)$, $\mathbf{g}\colon [0,1] \to R_n$ are absolutely continuous and therefore also of bounded variation. In this way the initial value problem (1,4) has become the initial value problem of the form (1,3) with $\mathbf{A}, \mathbf{g}$ defined above and both problems are equivalent. Essentially the same reasoning yields the equivalence of the problem (1,4) to an equivalent Stieltjes integral equation when $\mathbf{B}\colon [0,1] \to L(R_n)$, $\mathbf{h}\colon [0,1] \to R_n$ are assumed to be Lebesgue integrable and if we look for Carathéodory solutions of (1,4).

1.3. Theorem. *Assume that* $\mathbf{A}\colon [0,1] \to L(R_n)$ *is of bounded variation on* $[0,1]$, $\mathbf{g} \in BV_n$. *Let* $\mathbf{x}\colon [a,b] \to R_n$ *be a solution of the generalized linear differential equation* (1,1) *on the interval* $[a,b] \subset [0,1]$. *Then* $\mathbf{x}$ *is of bounded variation on* $[a,b]$.

Proof. By the definition 1.1 of a solution of (1,1) the integral $\int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for every $t, t_0 \in [a,b]$. Hence by I.4.12 the limit $\lim_{t \to t_0+} \int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for $t_0 \in [a,b)$ and $\lim_{t \to t_0-} \int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for $t_0 \in (a,b]$. Hence by (1,2) the solution $\mathbf{x}(t)$ of (1,1) possesses onesided limits at every point $t_0 \in [a,b]$ and for every point $t_0 \in [a,b]$ there exists $\delta > 0$ and a constant M such that $|\mathbf{x}(t)| \le M$ for $t \in (t_0 - \delta, t_0 + \delta) \cap [a,b]$. By the Heine-Borel Covering Theorem there exists a finite system of intervals of the type $(t_0 - \delta, t_0 + \delta)$ covering the compact interval $[a,b]$. Hence there exists a constant K such that $|\mathbf{x}(t)| \le K$ for every $t \in [a,b]$. If now $a = t_0 < t_1 < \ldots < t_k = b$ is an arbitrary subdivision of $[a,b]$, we have by I.4.27

$$|\mathbf{x}(t_i) - \mathbf{x}(t_{i-1})| \le \left| \int_{t_{i-1}}^{t_i} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s) \right| + |\mathbf{g}(t_i) - \mathbf{g}(t_{i-1})|$$
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is satisfied.

In the original papers of J. Kurzweil (cf. [1], [2]) on generalized differential equations and in other papers in this field the notation

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\tau} = \mathrm{D}[\mathbf{A}(t)\,\mathbf{x} + \mathbf{g}(t)]$$

was used for the generalized linear differential equation.

It is evident that the generalized linear differential equation can be given on an arbitrary interval $[a,b] \subset R$ instead of $[0,1]$.

If $\mathbf{x}_0 \in R_n$ and $t_0 \in [a,b] \subset [0,1]$ are fixed and $\mathbf{x}\colon [a,b] \to R_n$ is a solution of (1,1) on $[a,b]$ such that $\mathbf{x}(t_0) = \mathbf{x}_0$, then $\mathbf{x}$ is called the solution of the initial value (Cauchy) problem

$$\mathrm{d}\mathbf{x} = \mathrm{d}[\mathbf{A}]\,\mathbf{x} + \mathrm{d}\mathbf{g}\,, \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \tag{1,3}$$

on $[a,b]$.

1.2. Remark. If $\mathbf{B}\colon [0,1] \to L(R_n)$ is an $n \times n$-matrix valued function, continuous on $[0,1]$ with respect to the norm of a matrix given in I.1.1 and $\mathbf{h}\colon [0,1] \to R_n$ is continuous on $[0,1]$, then the initial value problem for the linear ordinary differential equation

$$\mathbf{x}' = \mathbf{B}(t)\,\mathbf{x} + \mathbf{h}(t)\,, \qquad \mathbf{x}(t_0) = \mathbf{x}_0 \tag{1,4}$$

is equivalent to the integral equation

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{B}(s)\,\mathbf{x}(s)\,\mathrm{d}s + \int_{t_0}^{t} \mathbf{h}(s)\,\mathrm{d}s\,, \qquad t \in [0,1]\,.$$

If we denote $\mathbf{A}(t) = \int_0^t \mathbf{B}(r)\,\mathrm{d}r$, $\mathbf{g}(t) = \int_0^t \mathbf{h}(r)\,\mathrm{d}r$ for $t \in [0,1]$, then this equation can be rewritten into the equivalent Stieltjes form

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s) + \mathbf{g}(t) - \mathbf{g}(t_0)\,, \qquad t \in [0,1]\,.$$

The functions $\mathbf{A}\colon [0,1] \to L(R_n)$, $\mathbf{g}\colon [0,1] \to R_n$ are absolutely continuous and therefore also of bounded variation. In this way the initial value problem (1,4) has become the initial value problem of the form (1,3) with $\mathbf{A}, \mathbf{g}$ defined above and both problems are equivalent. Essentially the same reasoning yields the equivalence of the problem (1,4) to an equivalent Stieltjes integral equation when $\mathbf{B}\colon [0,1] \to L(R_n)$, $\mathbf{h}\colon [0,1] \to R_n$ are assumed to be Lebesgue integrable and if we look for Carathéodory solutions of (1,4).

1.3. Theorem. *Assume that* $\mathbf{A}\colon [0,1] \to L(R_n)$ *is of bounded variation on* $[0,1]$, $\mathbf{g} \in BV_n$. *Let* $\mathbf{x}\colon [a,b] \to R_n$ *be a solution of the generalized linear differential equation* (1,1) *on the interval* $[a,b] \subset [0,1]$. *Then* $\mathbf{x}$ *is of bounded variation on* $[a,b]$.

Proof. By the definition 1.1 of a solution of (1,1) the integral $\int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for every $t, t_0 \in [a,b]$. Hence by I.4.12 the limit $\lim_{t \to t_0+} \int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for $t_0 \in [a,b)$ and $\lim_{t \to t_0-} \int_{t_0}^{t} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for $t_0 \in (a,b]$. Hence by (1,2) the solution $\mathbf{x}(t)$ of (1,1) possesses onesided limits at every point $t_0 \in [a,b]$ and for every point $t_0 \in [a,b]$ there exists $\delta > 0$ and a constant M such that $|\mathbf{x}(t)| \le M$ for $t \in (t_0 - \delta, t_0 + \delta) \cap [a,b]$. By the Heine-Borel Covering Theorem there exists a finite system of intervals of the type $(t_0 - \delta, t_0 + \delta)$ covering the compact interval $[a,b]$. Hence there exists a constant K such that $|\mathbf{x}(t)| \le K$ for every $t \in [a,b]$. If now $a = t_0 < t_1 < \ldots < t_k = b$ is an arbitrary subdivision of $[a,b]$, we have by I.4.27

$$|\mathbf{x}(t_i) - \mathbf{x}(t_{i-1})| \le \left|\int_{t_{i-1}}^{t_i} \mathrm{d}[\mathbf{A}(s)]\,\mathbf{x}(s)\right| + |\mathbf{g}(t_i) - \mathbf{g}(t_{i-1})|$$
$$\le K \operatorname{var}_{t_{i-1}}^{t_i} \mathbf{A} + |\mathbf{g}(t_i) - \mathbf{g}(t_{i-1})|$$

III. *Generalized linear differential equations*

1. The generalized linear differential equation and its basic properties

We assume that $\mathbf{A}: [0,1] \to L(R_n)$ is an $n \times n$-matrix valued function such that $\operatorname{var}_0^1 \mathbf{A} < \infty$ and $\mathbf{g} \in BV_n[0,1] = BV_n$.

The generalized linear differential equation will be denoted by the symbol

$$(1,1) \qquad d\mathbf{x} = d[\mathbf{A}]\,\mathbf{x} + d\mathbf{g}$$

which is interpreted by the following definition of a solution.

1.1. Definition. Let $[a,b] \subset [0,1]$, $a < b$; a function $\mathbf{x}: [a,b] \to R_n$ is said to be a solution of the *generalized linear differential equation* (1,1) on the interval $[a,b]$ if for any $t, t_0 \in [a,b]$ the equality

$$(1,2) \qquad \mathbf{x}(t) = \mathbf{x}(t_0) + \int_{t_0}^{t} d[\mathbf{A}(s)]\,\mathbf{x}(s) + \mathbf{g}(t) - \mathbf{g}(t_0)$$

is satisfied.

In the original papers of J. Kurzweil (cf. [1], [2]) on generalized differential equations and in other papers in this field the notation

$$\frac{d\mathbf{x}}{d\tau} = D[\mathbf{A}(t)\,\mathbf{x} + \mathbf{g}(t)]$$

was used for the generalized linear differential equation.

It is evident that the generalized linear differential equation can be given on an arbitrary interval $[a,b] \subset R$ instead of $[0,1]$.

If $\mathbf{x}_0 \in R_n$ and $t_0 \in [a,b] \subset [0,1]$ are fixed and $\mathbf{x}: [a,b] \to R_n$ is a solution of (1,1) on $[a,b]$ such that $\mathbf{x}(t_0) = \mathbf{x}_0$, then $\mathbf{x}$ is called the solution of the initial value (Cauchy) problem

$$(1,3) \qquad d\mathbf{x} = d[\mathbf{A}]\,\mathbf{x} + d\mathbf{g}, \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$

on $[a,b]$.

1.2. Remark. If $\mathbf{B}: [0,1] \to L(R_n)$ is an $n \times n$-matrix valued function, continuous on $[0,1]$ with respect to the norm of a matrix given in I.1.1 and $\mathbf{h}: [0,1] \to R_n$ is continuous on $[0,1]$, then the initial value problem for the linear ordinary differential equation

$$(1,4) \qquad \mathbf{x}' = \mathbf{B}(t)\,\mathbf{x} + \mathbf{h}(t), \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$

is equivalent to the integral equation

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} \mathbf{B}(s)\,\mathbf{x}(s)\,ds + \int_{t_0}^{t} \mathbf{h}(s)\,ds, \qquad t \in [0,1].$$

If we denote $\mathbf{A}(t) = \int_0^t \mathbf{B}(r)\,dr$, $\mathbf{g}(t) = \int_0^t \mathbf{h}(r)\,dr$ for $t \in [0,1]$, then this equation can be rewritten into the equivalent Stieltjes form

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^{t} d[\mathbf{A}(s)]\,\mathbf{x}(s) + \mathbf{g}(t) - \mathbf{g}(t_0), \qquad t \in [0,1].$$

The functions $\mathbf{A}: [0,1] \to L(R_n)$, $\mathbf{g}: [0,1] \to R_n$ are absolutely continuous and therefore also of bounded variation. In this way the initial value problem (1,4) has become the initial value problem of the form (1,3) with $\mathbf{A}, \mathbf{g}$ defined above and both problems are equivalent. Essentially the same reasoning yields the equivalence of the problem (1,4) to an equivalent Stieltjes integral equation when $\mathbf{B}: [0,1] \to L(R_n)$, $\mathbf{h}: [0,1] \to R_n$ are assumed to be Lebesgue integrable and if we look for Carathéodory solutions of (1,4).

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Proof. By the definition 1.1 of a solution of (1,1) the integral $\int_{t_0}^{t} d[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for every $t, t_0 \in [a,b]$. Hence by I.4.12 the limit $\lim_{t \to t_0+} \int_{t_0}^{t} d[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for $t_0 \in [a,b)$ and $\lim_{t \to t_0-} \int_{t_0}^{t} d[\mathbf{A}(s)]\,\mathbf{x}(s)$ exists for $t_0 \in (a,b]$. Hence by (1,2) the solution $\mathbf{x}(t)$ of (1,1) possesses onesided limits at every point $t_0 \in [a,b]$ and for every point $t_0 \in [a,b]$ there exists $\delta > 0$ and a constant M such that $|\mathbf{x}(t)| \le M$ for $t \in (t_0 - \delta, t_0 + \delta) \cap [a,b]$. By the Heine-Borel Covering Theorem there exists a finite system of intervals of the type $(t_0 - \delta, t_0 + \delta)$ covering the compact interval $[a,b]$. Hence there exists a constant K such that $|\mathbf{x}(t)| \le K$ for every $t \in [a,b]$. If now $a = t_0 < t_1 < \ldots < t_k = b$ is an arbitrary subdivision of $[a,b]$, we have by I.4.27

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$$\le K \operatorname{var}_{t_{i-1}}^{t_i} \mathbf{A} + |\mathbf{g}(t_i) - \mathbf{g}(t_{i-1})|$$

Specialized sources of information

- Formulae: Springer, EuDML, zbMATH

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SAMPLE SEARCHES

- \sqrt{{{1}\over {N-3}}
- \frac{{\alpha_{\gamma}}}
- \tilde{\beta}
- {\hbox{PE}}
- "\epsilon" AND "2\pi"
- "\bar \delta _q" OR "\frac{dw}{dz}"

SAMPLE RESULT

Self-intersections of random walks on lattices

Acta Mathematica Hungarica *(2002) 96:187-220, August 01, 2002*

$$P\left({E_n^{\left(d \right)} ,{\text{i}}{\text{.o}}{\text{.}}} \right) = 0\quad {\text{or}}\quad {\text{1}}$$

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```
P\left( {E_n^{\left( d \right)} ,{\text{i}}{\text{.o}}{\text{.}}}
\right) = 0\quad {\text{or}}\quad {\text{1}}
```

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– Eric Hellman, http://go-to-hellman.blogspot.com

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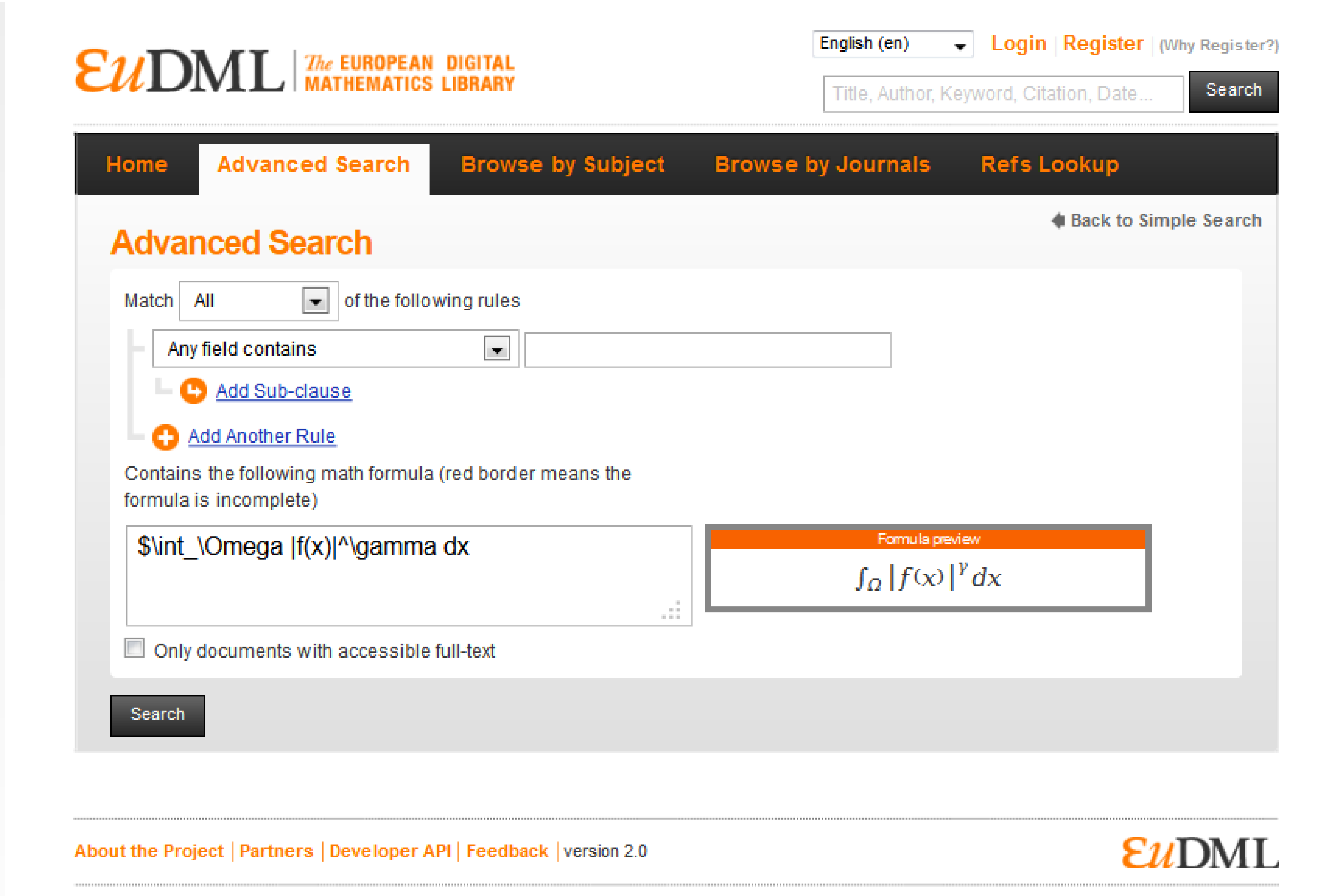
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Examples

- m < \infty — Explanation: Query queries are formulated in **LaTeX**.
- \sin(x) — Explanation: Standard **math commands** are supported but mathematical variables are not instantiated.
- ?a^2 + ?b^2 — Explanation: **Search variables** should be marked by a preceding **question mark** and ended by **whitespace characters**.
- ?a+?b = ?b+?a — Explanation: Variables can occur multiple times and receive **identical instantiations**; this query matches commutativity.
- ?f(?a+?b) — Explanation: Query variables are also allowed in **functional position**.

MathSearch is a new service that allows to search for mathematical formulae in documents indexed in zbMATH. It offers open access to formulae retrieval using the MathWebSearch system, which is a content-based search engine for MathML formulae based on substitution tree indexing. The first prototype is a result of a research project with the Jacobs University Bremen, funded by the Leibniz association, which aims at developing concepts and

Specialized sources of information

- Formulae: Springer, EuDML, zbMATH
- Theorems: Wikipedia

List of theorems

From Wikipedia, the free encyclopedia

This is a **list of theorems**, by Wikipedia page. See also

- Classification of finite simple groups
- List of fundamental theorems
- List of lemmas
- List of conjectures
- List of inequalities
- List of mathematical proofs
- List of misnamed theorems

Most of the results below come from pure mathematics, but some are from theoretical physics, economics, and other applied fields.

0–9 [edit]

- 15 and 290 theorems (*number theory*)
- 2π theorem (*Riemannian geometry*)

A [edit]

- AF+BG theorem (*algebraic geometry*)
- ATS theorem (*number theory*)
- Abel's binomial theorem (*combinatorics*)
- Abel's curve theorem (*mathematical analysis*)

Specialized sources of information

- Formulae: Springer, EuDML, zbMATH
- Theorems: Wikipedia
- Proofs: Mizar, The Coq Proof Assistant, Wikipedia

Mizar Home Page

Current Mizar Version: 8.1.02 (Download)
MML Version: 5.22.1191 (10 Dec 2013)
Unpacked distribution can be browsed here (HTML-linked articles, plain-text articles, abstracts).

Contents:

- Mizar Project,
- Mizar Language,
- Mizar System,
- Bibliography of Mizar Project,
- Mizar People,
- Mizar Mathematical Library,
- Formalized Mathematics,
- Association of Mizar Users.

Mizar mirror sites at:

- University of Bialystok, Bialystok, Poland (main site),
- SunSite ICM, Warsaw, Poland,
- Auburn University, Auburn, USA,
- Shinshu University, Nagano, Japan.

Google™ [Search with Google]
Search: mizar.org

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What is Coq ?

Coq is a formal proof management system. It provides a formal language to write mathematical definitions, executable algorithms and theorems together with an environment for semi-interactive development of machine-checked proofs. Typical applications include the formalization of programming languages semantics (e.g. the CompCert compiler certification project or Java Card EAL7 certification in industrial context), the formalization of mathematics (e.g. the full formalization of the 4 color theorem or constructive mathematics at Nijmegen) and teaching.

More about Coq

How to get it ?

The stable version of Coq is version 8.4. Released in August 2012, it features a uniform modular evolution of the arithmetical libraries, a new proof engine providing bullets, and various improvements of existing features, especially regarding CoqIDE, the tactics, the tactic language, the specification language.

Get Coq

Documentation

The reference documentation for Coq are the Reference Manual and the

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Recent news

- Coq is hiring a specialized engineer for 2 years
  07-18-14
- Coq 8.4pl4 is out
  05-12-14
- Coq received ACM Software System 2013 award
  04-19-14

All news

WIKIPEDIA
The Free Encyclopedia

Category:Proof assistants

From Wikipedia, the free encyclopedia

*The main article for this category is **Proof assistant**.*

Pages in category "Proof assistants"

The following 24 pages are in this category, out of 24 total. This list may not reflect recent changes (learn more).

- Automated proof checking
- Proof assistant

A

- Agda (programming language)
- ALF (proof assistant)
- Automath

C

- Coq

E

- Epigram (programming language)

H

- HOL (proof assistant)

H cont.

- HOL Light

I

- Isabelle (proof assistant)

J

- Jape (software)

K

- KeY

L

- LEGO (proof assistant)
- Logic for Computable Functions
- Logical framework

M

- Matita

M cont.

- Metamath
- MINLOG
- Mizar system

N

- NuPRL

P

- PhoX
- Prototype Verification System

Q

- QED manifesto

T

- Total functional programming

Specialized sources of information

- Formulae: Springer, EuDML, zbMATH
- Theorems: Wikipedia
- Proofs: Mizar, The Coq Proof Assistant, Wikipedia
- Functions: MathWorld, Wolfram Functions Site

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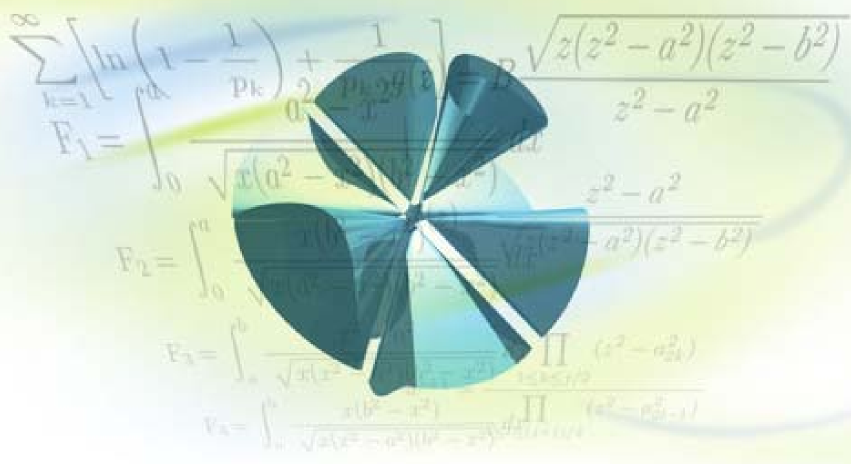

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- Mathieu & Spheroidal Functions
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- **ALPHABETICAL INDEX**

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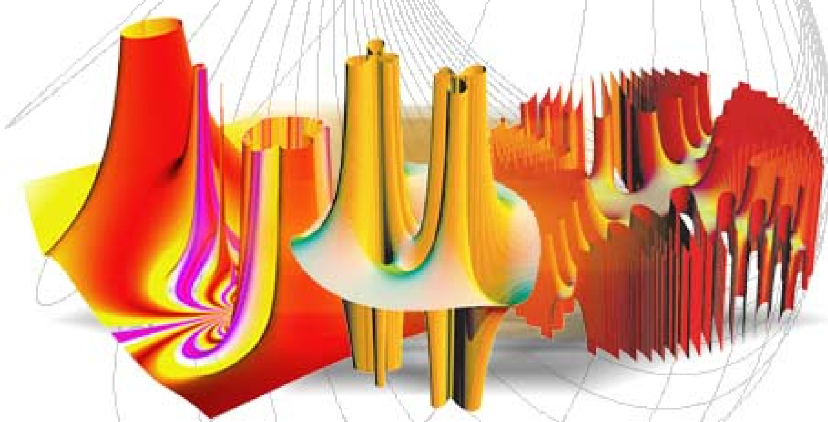

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- Functions: MathWorld, Wolfram Functions Site
- Sequences: On-line Encyclopedia of Integer Sequences
- Group, rings, fields: Wikipedia (Lie groups), Wikipedia (finite simple groups), ...
- Identities: Piezas
- Problem solving sites: Polymath, Mathoverflow

The polymath blog

January 20, 2014

Two polymath (of a sort) proposed projects

Filed under: discussion,polymath proposals — Gil Kalai @ 5:20 pm
Tags: Convexity, polymath-proposals, Riemann Hypothesis

This post is meant to propose and discuss a polymath project and a sort of polymath project.

I. A polymath proposal: Convex hulls of real algebraic varieties.

One of the interesting questions regarding the polymath endeavor was:

> Can polymath be used to develop a theory/new area?

My idea is to have a project devoted to develop a theory of "convex hulls of real algebraic varieties". The case where the varieties are simply a finite set of points is a well-developed area of mathematics – the theory of convex polytopes, but the general case was not studied much. I suppose that for such a project the first discussions will be devoted to raise questions/research directions. (And mention some works already done.)

In general (but perhaps more so for an open-ended project), I would like to see also polymath projects which are on longer time scale than existing ones but perhaps less intensive, and that people can "get in" or "spin-off" at will in various times.

II. A polymath-of-a-sort proposal: Statements about the Riemann Hypothesis

The Riemann hypothesis is arguably the most famous open question in mathematics. My view is that it is premature to try to attack the RH by a polymath project (but I am not an expert and, in any case, a mathoverflow

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- Identities: Piezas
- Problem solving sites: Polymath, Mathoverflow
- Annotations: comments, discussion groups and threads
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New Babel → need of documentation and aggregation

Towards 21st Century Global Library for Mathematics Research – recommendations

- A DML organization should be created to manage and encourage the creation of a knowledge-based library of mathematical concepts such as theorems and proofs… It should be an advocate for the mathematics community and help develop plans for development and funding of open information systems of use to mathematicians.
- A primary role of the DML should be to provide a platform that engages the mathematical community in enriching the library's knowledge base and identifies connections in the data.
- The DML should rely on citation indexing, community sourcing, and a combination of other computationally based methods for linking among articles, concepts, authors, and so on.
- Community engagement and the success of community-sourced efforts need to be continuously evaluated throughout DML development and operation to ensure that DML missions continue to align with community needs and that community engagement efforts are effective.

Towards 21st Century Global Library for Mathematics Research – recommendations

- The DML should be open and built to cooperate with both researchers and existing services. In particular, the content (knowledge structures) of the library, at least for vocabularies, tags, and links, should also be open, although the library will link to both open and copyright-restricted literature.
- The DML should serve as a nexus for the coordination of research and research outcomes, including community endorsements, and encourage best practices to facilitate knowledge management in research mathematics.
- The initial DML planning group should set up a task force of suitable experts to produce a realistic plan, timeline, and prioritization of components, using this report as a high-level blueprint, to present to potential funding agencies (both public and private).
- The DML needs to build an ongoing relationship with the research communities spanning mathematics, computer science, information science, and related areas concerned with knowledge extraction and structuring in the context of mathematics and to help translation of developments in these areas from research to large scale application.

Towards 21st Century Global Library for Mathematics Research – next steps

- Panel discussion at the ICM 2014 in Seoul (IMU CEIC): working group formed to
  - write concrete road map and incremental budget
  - derive proposals to funders from this document
- Collaboration on the EuDML level, developments in local repositories including the BulDML and DML-CZ
  - extending the content
  - enhancing metadata
  - developing specialized tools (formula search, ...)
  - including new types of files (images, videos, simulations, ...)
  - securing long term preservation
  - following the EuDML policies, namely the eventual open access
  - getting users involved (annotations, ...)