ON A CLASS OF m-POINT BOUNDARY VALUE PROBLEMS

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Abstract. We investigate the existence of positive solutions for a nonlinear second-order differential system subject to some m-point boundary conditions. The nonexistence of positive solutions is also studied.

Keywords: differential system, boundary condition, positive solution, fixed point theorem

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1. Introduction

We consider the nonlinear second-order differential system

$$\text{(S)}\qquad \begin{cases} u''(t) + b(t)f(v(t)) = 0 \\ v''(t) + c(t)g(u(t)) = 0, \ t \in (0,T), \end{cases}$$

with the m-point boundary conditions

$$\text{(BC)}\qquad \begin{cases} \beta u(0) - \gamma u'(0) = 0, \ u(T) = \sum\limits_{i=1}^{m-2} a_i u(\xi_i) + b_0 \\ \beta v(0) - \gamma v'(0) = 0, \ v(T) = \sum\limits_{i=1}^{m-2} a_i v(\xi_i) + b_0, \ m \in \mathbb{N}, \ m \geqslant 3. \end{cases}$$

In this paper we study the existence and nonexistence of positive solutions of (S), (BC). In the case $b_0 = 0$ and $b(t) = \lambda\tilde{b}(t)$, $c(t) = \mu\tilde{c}(t)$, the existence of positive solutions with respect to a cone has been investigated in [13]. In [12] the authors studied the existence and nonexistence of positive solutions for the m-point boundary value problem on time scales

$$\begin{cases} u^{\Delta\nabla}(t) + a(t)f(u(t)) = 0, \ t \in (0,T) \\ \beta u(0) - \gamma u^{\Delta}(0) = 0, \ u(T) - \sum\limits_{i=1}^{m-2} a_i u(\xi_i) = b, \ m \geqslant 3, \ b > 0. \end{cases}$$

In recent years the existence of positive solutions of multi-point boundary value problems for second-order or higher-order differential or difference equations has been the subject of investigation by many authors (see [1]–[11], [14]–[17]).

We shall suppose that the following conditions are verified:

(H1) $\beta,\ \gamma \geqslant 0$, $\beta+\gamma > 0$, $a_i > 0$ for $i = \overline{1, m-2}$, $a_{m-2} \geqslant 1$, $0 < \xi_1 < \xi_2 < \ldots < \xi_{m-2} < T$, $b_0 > 0$, $T > \sum\limits_{i=1}^{m-2} a_i\xi_i$, $d = \beta\Big(T - \sum\limits_{i=1}^{m-2} a_i\xi_i\Big) + \gamma\Big(1 - \sum\limits_{i=1}^{m-2} a_i\Big) > 0$.

(H2) $b, c\colon [0,T] \to [0,\infty)$ are continuous functions and there exist $t_0,\ \tilde{t}_0 \in [\xi_{m-2}, T)$ such that $b(t_0) > 0$, $c(\tilde{t}_0) > 0$.

(H3) $f, g\colon [0,\infty) \to [0,\infty)$ are continuous functions that satisfy the conditions

a) there exists $c_0 > 0$ such that $f(u) < c_0/L$, $g(u) < c_0/L$ for all $u \in [0, c_0]$,

b) $\lim\limits_{u\to\infty} f(u)/u = \infty$, $\lim\limits_{u\to\infty} g(u)/u = \infty$,

where

$$L = \max\Big\{\frac{\beta T+\gamma}{d}\int_0^T (T-s)b(s)\,\mathrm{d}s,\quad \frac{\beta T+\gamma}{d}\int_0^T (T-s)c(s)\,\mathrm{d}s\Big\}.$$

2. Preliminaries

In this section we present some auxiliary results from [12] and [13] related to the second-order differential system with boundary conditions

$$u''(t) + y(t) = 0,\ \ t \in (0,T), \tag{2.1}$$

$$\beta u(0) - \gamma u'(0) = 0,\ \ u(T) - \sum_{i=1}^{m-2} a_i u(\xi_i) = 0. \tag{2.2}$$

Lemma 2.1 ([12], [13]). *If $\beta \neq 0$, $d \neq 0$, $0 < \xi_1 < \ldots < \xi_{m-2} < T$, then the solution of* (2.1), (2.2) *is given by*

$$u(t) = \frac{\beta t+\gamma}{d}\int_0^T (T-s)y(s)\,\mathrm{d}s - \frac{\beta t+\gamma}{d}\sum_{i=1}^{m-2} a_i\int_0^{\xi_i} (\xi_i - s)y(s)\,\mathrm{d}s$$
$$- \int_0^t (t-s)y(s)\,\mathrm{d}s,\ \ 0 \leqslant t \leqslant T.$$

Lemma 2.2 ([13]). *Under the assumptions of Lemma 1, the Green function for the boundary value problem (2.1), (2.2) is given by*

$$G(t,s)=\begin{cases}\dfrac{\beta t+\gamma}{d}\left[(T-s)-\sum\limits_{j=i}^{m-2}a_j(\xi_j-s)\right]-(t-s) & \text{if } \xi_{i-1}\leqslant s<\xi_i,\ s\leqslant t,\\ & i=\overline{1,m-2}\ (\xi_0=0),\\ \dfrac{\beta t+\gamma}{d}\left[(T-s)-\sum\limits_{j=i}^{m-2}a_j(\xi_j-s)\right] & \text{if } \xi_{i-1}\leqslant s<\xi_i,\ s\geqslant t,\\ & i=\overline{1,m-2},\\ \dfrac{\beta t+\gamma}{d}(T-s)-(t-s) & \text{if } \xi_{m-2}\leqslant s\leqslant T,\ s\leqslant t,\\ \dfrac{\beta t+\gamma}{d}(T-s) & \text{if } \xi_{m-2}\leqslant s\leqslant T,\ s\geqslant t.\end{cases}$$

Lemma 2.3 ([12]). *If $\beta>0$, $\gamma\geqslant 0$, $d>0$, $a_i>0$ for all $i=\overline{1,m-2}$, $0<\xi_1<\xi_2<\ldots<\xi_{m-2}<T$, $\sum\limits_{i=1}^{m-2}a_i\xi_i\leqslant T$ and $y\in C([0,T])$, $y(t)\geqslant 0$ for all $t\in[0,T]$, then the unique solution u of the problem (2.1), (2.2) satisfies $u(t)\geqslant 0$ for all $t\in[0,T]$.*

Lemma 2.4 ([13]). *If $\beta>0$, $\gamma\geqslant 0$, $d>0$, $0<\xi_1<\xi_2<\ldots<\xi_{m-2}<T$, $a_i>0$ for $i=\overline{1,m-2}$, $a_{m-2}\geqslant 1$, $T\geqslant\sum\limits_{i=1}^{m-2}a_i\xi_i$, $y\in C([0,T])$, $y(t)\geqslant 0$ for all $t\in[0,T]$, then the solution of the problem (2.1), (2.2) satisfies*

$$u(t)\leqslant\frac{\beta T+\gamma}{d}\int_0^T(T-s)y(s)\,\mathrm{d}s,\ 0\leqslant t\leqslant T,$$
$$u(\xi_j)\geqslant\frac{\beta\xi_j+\gamma}{d}\int_{\xi_{m-2}}^T(T-s)y(s)\,\mathrm{d}s,\ \forall\, j=\overline{1,m-2}.$$

Lemma 2.5 ([12]). *We assume that $\beta>0$, $\gamma\geqslant 0$, $d>0$, $0<\xi_1<\xi_2<\ldots<\xi_{m-2}<T$, $a_i>0$ for all $i=\overline{1,m-2}$, $T>\sum\limits_{i=1}^{m-2}a_i\xi_i$ and $y\in C([0,T])$, $y(t)\geqslant 0$ for all $t\in[0,T]$. Then the solution of the problem (2.1), (2.2) verifies $\inf\limits_{t\in[\xi_1,T]}u(t)\geqslant r\|u\|$, where*

$$r=\min_{2\leqslant s\leqslant m-2}\left\{\frac{\xi_1}{T},\frac{\sum_{i=1}^{m-2}a_i(T-\xi_i)}{T-\sum_{i=1}^{m-2}a_i\xi_i},\frac{\sum_{i=1}^{m-2}a_i\xi_i}{T},\frac{\sum_{i=1}^{s-1}a_i\xi_i+\sum_{i=s}^{m-2}a_i(T-\xi_i)}{T-\sum_{i=s}^{m-2}a_i\xi_i}\right\}$$

and $\|u\|=\sup\limits_{t\in[0,T]}|u(t)|$.

3. Main results

First we present an existence result for the positive solutions of (S), (BC).

Theorem 3.1. *Assume that the assumptions* (H1), (H2), (H3)a) *hold. Then the problem* (S), (BC) *has at least one positive solution for* $b_0 > 0$ *sufficiently small.*

P r o o f. We consider the problem

$$h''(t) = 0, \ t \in (0, T), \tag{3.1}$$
$$\beta h(0) - \gamma h'(0) = 0, \ h(T) - \sum_{i=1}^{m-2} a_i u(\xi_i) = 1.$$

The solution $h(t)$, $t \in (0, T)$ of $(3.1)_1$ is $h(t) = C_1 t + C_2$. Because $\beta h(0) - \gamma h'(0) = 0$ we have $\beta C_2 - \gamma C_1 = 0$, and so $C_2 = \gamma\beta^{-1} C_1$. Therefore $h(t) = C_1 t + \gamma\beta^{-1} C_1$. By the condition $h(T) = \sum\limits_{i=1}^{m-2} a_i h(\xi_i) + 1$ we obtain $C_1 T + \gamma\beta^{-1} C_1 = \sum\limits_{i=1}^{m-2} a_i (C_1 \xi_i + \gamma\beta^{-1} C_1) + 1$, hence $C_1 = \beta/d$.

So

$$h(t) = \frac{\beta t + \gamma}{d}, \quad t \in [0, T]. \tag{3.2}$$

We now define $x(t)$, $y(t)$, $t \in (0, T)$ by

$$u(t) = x(t) + b_0 h(t), \ v(t) = y(t) + b_0 h(t), \quad t \in (0, T).$$

Then (S), (BC) can be equivalently written as

$$\begin{cases} x''(t) + b(t) f(y(t) + b_0 h(t)) = 0 \\ y''(t) + c(t) g(x(t) + b_0 h(t)) = 0, \quad t \in (0, T), \end{cases} \tag{3.3}$$

with the boundary conditions

$$\begin{cases} \beta x(0) - \gamma x'(0) = 0, \quad \beta y(0) - \gamma y'(0) = 0, \\ x(T) = \sum\limits_{i=1}^{m-2} a_i x(\xi_i), \quad y(T) = \sum\limits_{i=1}^{m-2} a_i y(\xi_i). \end{cases} \tag{3.4}$$

Using the Green function given in Lemma 2.2, a pair $(x(t), y(t))$ is a solution of problem (3.3), (3.4) if and only if

$$\begin{cases} x(t) = \int_0^T G(t,s) b(s) f(\int_0^T G(s,\tau) c(\tau) g(x(\tau) + b_0 h(\tau))\, \mathrm{d}\tau + b_0 h(s))\, \mathrm{d}s, \\ y(t) = \int_0^T G(t,s) c(s) g(x(s) + b_0 h(s))\, \mathrm{d}s, \ 0 \leqslant t \leqslant T, \end{cases} \tag{3.5}$$

where $h(t)$, $t \in [0, T]$ is given by (3.2).

We consider the Banach space $X = C([0,T])$ with the supremum norm $\|\cdot\|$ and define the set

$$K = \{x \in C([0,T]),\ 0 \leqslant x(t) \leqslant c_0,\ \forall\, t \in [0,T]\} \subset X.$$

We also define the operator $\Psi\colon\ K \to X$ by

$$\Psi(x)(t) = \int_0^T G(t,s)b(s)f\biggl(\int_0^T G(s,\tau)c(\tau)g(x(\tau)+b_0h(\tau))\,\mathrm{d}\tau + b_0h(s)\biggr)\,\mathrm{d}s, \qquad 0 \leqslant t \leqslant T.$$

For sufficiently small $b_0 > 0$, (H3) a) yields

$$f(y(t)+b_0h(t)) \leqslant \frac{c_0}{L}, \quad g(x(t)+b_0h(t)) \leqslant \frac{c_0}{L}, \quad \forall\, x,y \in K,\ \forall\, t \in [0,T].$$

Then for any $x \in K$ we obtain, by using Lemma 2.3, that $\Psi(x)(t) \geqslant 0,\ \forall\, t \in [0,T]$. By Lemma 2.4 we also have

$$\begin{aligned} y(s) &\leqslant \frac{\beta T+\gamma}{d}\int_0^T (T-\tau)c(\tau)g(x(\tau)+b_0h(\tau))\,\mathrm{d}\tau \\ &\leqslant \frac{c_0}{L}\frac{\beta T+\gamma}{d}\int_0^T (T-\tau)c(\tau)\,\mathrm{d}\tau \leqslant \frac{c_0}{L}L = c_0,\ \forall\, s \in [0,T] \end{aligned}$$

and

$$\begin{aligned} \Psi(x)(t) &\leqslant \frac{\beta T+\gamma}{d}\int_0^T (T-s)b(s)f(y(s)+b_0h(s))\,\mathrm{d}s \\ &\leqslant \frac{c_0}{L}\frac{\beta T+\gamma}{d}\int_0^T (T-s)b(s)\,\mathrm{d}s \leqslant \frac{c_0}{L}L = c_0,\ \forall\, t \in [0,T]. \end{aligned}$$

Therefore $\Psi(K) \subset K$.

Using standard arguments we deduce that Ψ is completely continuous (Ψ is compact: for any bounded set $B \subset K$, $\Psi(B) \subset K$ is relatively compact, by Arzèla-Ascoli theorem, and Ψ is continuous). By the Schauder fixed point theorem, we conclude that Ψ has a fixed point $x \in K$. This element together with y given by (3.5) represents a solution for (3.3), (3.4). This shows that our problem (S), (BC) has a positive solution $u = x + b_0h$, $v = y + b_0h$ for sufficiently small b_0. □

In what follows we present sufficient conditions for nonexistence of positive solutions of (S), (BC).

Theorem 3.2. *Assume that the assumptions* (H1), (H2), (H3)b) *hold. Then the problem* (S), (BC) *has no positive solution for* b_0 *sufficiently large.*

Proof. We suppose that (u,v) is a positive solution of (S), (BC). Then $x = u - b_0 h$, $y = v - b_0 h$ is a solution for (3.3), (3.4), where h is the solution of problem (3.1). By Lemma 2.3 we have $x(t) \geqslant 0$, $y(t) \geqslant 0$, $\forall\, t \in [0,T]$, and by (H2) we deduce that $\|x\| > 0$, $\|y\| > 0$. Using Lemma 2.5 we also have $\inf_{t\in[\xi_1,T]} x(t) \geqslant r\|x\|$ and $\inf_{t\in[\xi_1,T]} y(t) \geqslant r\|y\|$, where r is defined in Lemma 2.5.

Using now (3.2)—the expression for h, we deduce that

$$\inf_{t\in[\xi_1,T]} h(t) = \frac{\beta\xi_1+\gamma}{d} \geqslant \frac{\xi_1 h(T)}{T} = \frac{\xi_1}{T}\frac{\beta T+\gamma}{d}.$$

Therefore $\inf_{t\in[\xi_1,T]} h(t) \geqslant \frac{\xi_1}{T}\|h\|$ ($\|h\| = h(T)$). We denote $\delta = \min\{\xi_1/T, r\}$. Then

$$\inf_{t\in[\xi_1,T]} (x(t)+b_0h(t)) \geqslant \delta(\|x\| + b_0\|h\|) \geqslant \delta\|x+b_0h\|$$

and

$$\inf_{t\in[\xi_1,T]} (y(t)+b_0h(t)) \geqslant \delta(\|y\| + b_0\|h\|) \geqslant \delta\|y+b_0h\|.$$

We now consider

$$R = \frac{d}{\delta(\beta\xi_{m-2}+\gamma)}\left(\min\left\{\int_{\xi_{m-2}}^{T}(T-s)c(s)\,\mathrm{d}s, \int_{\xi_{m-2}}^{T}(T-s)b(s)\,\mathrm{d}s\right\}\right)^{-1} > 0.$$

By (H3)b), for R defined above we deduce that there exists $M > 0$ such that $f(u) > 2Ru$, $g(u) > 2Ru$ for all $u \geqslant M$.

We consider $b_0 > 0$ sufficiently large such that

$$\inf_{t\in[\xi_1,T]} (x(t)+b_0h(t)) \geqslant M \text{ and } \inf_{t\in[\xi_1,T]} (y(t)+b_0h(t)) \geqslant M.$$

By using Lemma 2.4 and the above considerations, we have

$$\begin{aligned}
y(\xi_{m-2}) &\geqslant \frac{\beta\xi_{m-2}+\gamma}{d}\int_{\xi_{m-2}}^{T}(T-s)c(s)g(x(s)+b_0h(s))\,\mathrm{d}s\\
&\geqslant \frac{\beta\xi_{m-2}+\gamma}{d}\int_{\xi_{m-2}}^{T}(T-s)c(s)\cdot 2R(x(s)+b_0h(s))\,\mathrm{d}s\\
&\geqslant \frac{\beta\xi_{m-2}+\gamma}{d}\int_{\xi_{m-2}}^{T}(T-s)c(s)\cdot 2R\inf_{\tau\in[\xi_{m-2},T]}(x(\tau)+b_0h(\tau))\,\mathrm{d}s\\
&\geqslant \frac{\beta\xi_{m-2}+\gamma}{d}\int_{\xi_{m-2}}^{T}(T-s)c(s)\cdot 2R\inf_{\tau\in[\xi_1,T]}(x(\tau)+b_0h(\tau))\,\mathrm{d}s\\
&\geqslant \frac{\beta\xi_{m-2}+\gamma}{d}\int_{\xi_{m-2}}^{T}(T-s)c(s)\cdot 2R\delta\|x+b_0h\| \geqslant 2\|x+b_0h\| \geqslant 2\|x\|.
\end{aligned}$$

And then we obtain

$$\|x\| \leqslant \frac{1}{2} y(\xi_{m-2}) \leqslant \frac{1}{2}\|y\|. \tag{3.6}$$

In a similar manner we deduce $x(\xi_{m-2}) \geqslant 2\|y + b_0 h\| \geqslant 2\|y\|$ and so

$$\|y\| \leqslant \frac{1}{2} x(\xi_{m-2}) \leqslant \frac{1}{2}\|x\|. \tag{3.7}$$

By (3.6) and (3.7) we obtain $\|x\| \leqslant \frac{1}{2}\|y\| \leqslant \frac{1}{4}\|x\|$, which is a contradiction, because $\|x\| > 0$. Then, when b_0 is sufficiently large, our problem (S), (BC) has no positive solution. $\square$

4. An example

We consider $T = 1$, $b(t) = bt$, $c(t) = ct$, $t \in [0,1]$, $b, c > 0$; $\beta = 3$, $\gamma = \frac{1}{12}$, $m = 5$, $\xi_1 = \frac{1}{4}$, $\xi_2 = \frac{1}{2}$, $\xi_3 = \frac{3}{4}$, $a_1 = \frac{1}{4}$, $a_2 = \frac{1}{3}$, $a_3 = 1$. Then $d = \frac{1}{72} > 0$ and the condition $T > \sum\limits_{i=1}^{m-2} a_i \xi_i$ is verified ($1 > \frac{47}{48}$).

We also consider the functions f, $g\colon [0,\infty) \to [0,\infty)$, $f(x) = \tilde{a}x^3/(x+1)$, $g(x) = \tilde{b}x^3/(x+1)$ with $\tilde{a}$, $\tilde{b} > 0$. We have $\lim\limits_{x\to\infty} f(x)/x = \lim\limits_{x\to\infty} g(x)/x = \infty$. The constant L from (H3) is in this case

$$L = \max\left\{\frac{\beta+\gamma}{d}\int_0^1 (1-s)bs\,\mathrm{d}s, \frac{\beta+\gamma}{d}\int_0^1 (1-s)cs\,\mathrm{d}s\right\} = 37\max\{b,c\}.$$

We choose $c_0 = 1$ and if we select $\tilde{a}$ and $\tilde{b}$ satisfying the conditions

$$\tilde{a} < \frac{2}{L} = \frac{2}{37\max\{b,c\}} = \frac{2}{37}\min\left\{\frac{1}{b}, \frac{1}{c}\right\},\ \ \tilde{b} < \frac{2}{L} = \frac{2}{37\max\{b,c\}} = \frac{2}{37}\min\left\{\frac{1}{b}, \frac{1}{c}\right\},$$

then we obtain $f(x) \leqslant \tilde{a}/2 < 1/L$, $g(x) \leqslant \tilde{b}/2 < 1/L$ for all $x \in [0,1]$.

Thus all the assumptions (H1)–(H3) are verified. By Theorem 3.1 and Theorem 3.2 we deduce that the nonlinear second-order differential system

$$\begin{cases} u''(t) + bt\dfrac{\tilde{a}v^3(t)}{v(t)+1} = 0 \\[2ex] v''(t) + ct\dfrac{\tilde{b}u^3(t)}{u(t)+1} = 0, \quad t \in (0,1) \end{cases}$$

with the boundary conditions

$$\begin{cases} u'(0) = 36u(0), \quad u(1) = \frac{1}{4}u(\frac{1}{4}) + \frac{1}{3}u(\frac{1}{2}) + u(\frac{3}{4}) + b_0 \\ v'(0) = 36v(0), \quad v(1) = \frac{1}{4}v(\frac{1}{4}) + \frac{1}{3}v(\frac{1}{2}) + v(\frac{3}{4}) + b_0, \end{cases}$$

has at least one positive solution for sufficiently small $b_0 > 0$ and no positive solution for sufficiently large b_0. $\square$

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