UNIVERZITA PALACKÉHO V OLOMOUCI

AUTOREFERÁT DISERTAČNÍ PRÁCE

Vědecká rada AV ČR, Praha

prof. RNDr. dr hab. Jan Andres, CSc.

TOPOLOGICAL PRINCIPLES FOR ORDINARY DDIFFERENTIAL EQUATIONS

Vědní obor 11–04–09 matematická analýza a příbuzné obory

Autoreferát disertace k získání vědecké hodnosti DSc.

Olomouc, 2010

Disertace byla vypracována v rámci výzkumného záměru *Matematické modely a struktury* (MSM 6198959214).

Disertant: prof. RNDr. dr hab. Jan Andres, CSc.
katedra matematické analýzy a aplikací matematiky
přírodovědecké fakulty Univerzity Palackého
Tř. 17. listopadu 12, 771 46 Olomouc

Oponenti:

Autoreferát byl odeslán dne:

Obhajoba disertace se koná dne: v hod.
před komisí pro obhajoby doktorských disertačních prací v oboru 11–04–09 *matematická analýza a příbuzné obory* na

S disertací je možno se seznámit na

prof. RNDr. Michal Křížek, DrSc.
předseda komise pro obhajoby doktorských
disertačních prací v oboru 11–04–09
matematická analýza a příbuzné obory
Matematický ústav AV ČR
Žitná 25, 115 67 Praha 1

a) Recent state of problems under investigation and the related literature

The present dissertation is identical with the monographic chapter B[2] in the handbook [CDF3]. Unlike variational methods, the *topological methods* for differential equations and inclusions with given constraints (e.g. boundary conditions) are based on the fixed point theorems. The *fixed point theory* is nowadays an autonomous advanced discipline which roughly consists of two main parts: (i) metric fixed point theory (Banach–type theorems) and (ii) topological fixed point theory (Schauder–type theorems). The recent state of fixed point theory is reflected by e.g. the handbooks [KS] and [BFGJ]. The handbook [BFGJ] includes author's further monographic chapter B[3] titled "*Applicable fixed point principles*". Further author's papers related to the development of fixed point theory and its applications are: C1[12], C1[20], C1[23], C1[33], C1[37], C1[40], C1[41], C1[43], C1[49], C1[52], C1[54], C1[56], C1[58], C1[60], C1[62]. In the frame of fractal theory, besides the standard application of the Banach theorem, we also extended the fixed point theory in hyperspaces in: [BFGJ, Chapter 6], C1[14], C1[29], C1[32], C1[34], C1[49]. There are four journals exclusively devoted to the fixed point theory and its applications: Fixed Point Theory and Applications, Journal of Fixed Point Theory and Applications, Fixed Point Theory and JP Journal of Fixed Point Theory and Applications.

Continuation principles for the solvability of various types of problems for differential equations and inclusions are based on the homotopical properties of topological invariants like degrees, fixed point indices, Lefschetz and Nielsen numbers. Their survey can be partly found e.g. in the monographs of M. A. Krasnosel'skii and P. P. Zabreiko [KZ], R. E. Gaines and J. Mawhin [GM], [Ma], S. Fučík [Fu], P. Fitzpatrick et al. [FZ], K. Deimling [De], M. I. Kamenskii et al. [KOZ], J. Andres and L. Górniewicz A[1], etc. Some contributions in the Handbook of Differential Equations (Ordinary Differential Equations) [CDF1], [CDF2], [CDF3], [BF] are relied on these principles, too. Further continuation principles were formulated in our papers C1[5], C1[7], C1[8], C1[13], C1[21], C1[36], C1[41], C1[56], C1[57], C1[58], C1[74]. The most closely related journal is Topological Methods in Nonlinear Analysis.

Our continuation principles can also be associated with multivalued maps. That is why they are applicable for differential inclusions. There is a big progress in *multivalued analysis*, especially in the last two decades. This can be seen e.g. from the Handbook of Multivalued Analysis in two volumes [HP1] and [HP2]. The journal Set–Valued Analysis is exclusively devoted to multivalued problems.

Our *existence results* concern boundary value problems on compact as well as noncompact (e.g. infinite) intervals, differential equations and inclusions in finite–dimensional (Euclidean) as well as infinite–dimensional (Banach) spaces. We are also interested in *multiplicity results* (by means of the Nielsen number, estimating from below the number of fixed points and the associated solutions) and the *topological structure of solution sets*. Let us note that, for multiplicity results, the additivity of degrees is usually applied. For instance, if the zero degree in a given domain becomes nontrivial in its subdomain, then the sign of the degree must be opposite, on the remaining set, which implies the existence of at least two solutions. On the other hand, the nontraditional application of the Nielsen number is rather difficult, but effective. The rare results of the other authors have been collected e.g. in [Fe], [Br], C1[26]. Our results concerning the application of the Nielsen theory can be found in C1[19], C1[21], C1[23], C1[26], C1[37], C1[48], C1[52], C1[55], C1[56], C1[60], C2[7], C2[9]. Unlike in the results of R. F. Brown and M. Fečkan (cf. [Br], [Fe]), where the parameters had to be implemented to simplify the calculations, our results apply also without any parameter. Moreover, since we have to our disposal the multivalued Nielsen theory, our techniques also apply to differential inclusions.

The investigation of the topological structure of solution sets mostly concerns the initial value problems. For boundary value problems, the related results are rather rare. In the past, the Hukuhara–Kneser type results (i.e. continua of solutions) were mainly achieved. Recently, the R_δ-structure of solutions (i.e. special continua, more general than convex compact sets) are appreciated. The related results of the other authors are partly described in the monograph [DMNZ]. Our further results were published in papers: C1[1], C1[43], C1[50], C1[54], C1[58]. In the monograph A[1], the whole chapters III-2 and III-3 are devoted to this type of problems.

The multiplicity results, in the frame of newly extended Nielsen theory, represent probably our most original contribution in this field. Further rather original results are those concerning asymptotic boundary value problems, because they are not treated sequentially.

Because of the pages limit, we study only differential systems of the first order. Furthermore, the insufficient attention is also paid to the verification of the transversality condition, guaranteeing the fixed point free boundary of given domains, which are naturally required in our methods. This requirement can be satisfied e.g. by means of bounding (Liapunov–like) functions which was systematically elaborated in our papers: C1[4], C1[7], C1[8], C1[9], C1[13], C1[39], C1[47], C2[2], C2[5]. In the papers C1[1], C1[4], C1[5], C1[8], C1[9], differential systems of the second order were examined.

In the meantime, many results in the presented work B[2] have been naturally improved or extended (e.g. by means of the effectively expressed transversality condition just mentioned, in C1[7], C1[13], C1[36], C1[47]). As an illustrative example, we can mention in these lines the stimulating Theorem 1.1 in Introduction of B[2]. More concretely, unlike the standard Sharkovskii cycle coexistence theorem which cannot be applied to differential equations, Theorem 1.1 represents a version of the celebrated Sharkovskii theorem for differential equations without uniqueness. For this goal, we published a series of papers C1[25], C1[26], C1[31], C1[35], C1[39], C1[42], C2[4], D[3], D[5] which lead to formulating Theorem 1.1 in B[2]. Nevertheless, the authors of [OO] have rather surprisingly shown, by means of an upper and lower solutions technique, that the mentioned Theorem 1.1 does not at all depend of the new (Sharkovskii's) ordering of positive integers. The explanation and generalization of these results was published in our papers C1[6], C1[16]. Further extensions of the Sharkovskii theorem were formulated by ourselves in C1[10], C1[11], C1[15], C1[22], C2[1].

The above results can serve the stimulating arguments for the application of multivalued analysis and, in particular, the investigation of differential inclusions.

There is also another motivation for the investigation of *multivalued ODEs*, i.e. differential inclusions, because of a strict connection with

(i) optimal control problems for ODEs,

(ii) Filippov solutions of discontinuous ODEs,

(iii) implicit ODEs,

etc.

ad (i): Consider a *control problem* for

$$\dot{x} = f(t, x, u), \quad u \in U, \tag{1}$$

where $f : [0, \tau] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ and $u \in U$ are control parameters such that $u(t) \in \mathbb{R}^n$, for all $t \in [0, \tau]$. In order to solve a control problem for (1), we can define a multivalued map $F(t, x) := \{f(t, x, u)\}_{u \in U}$. The solutions of (1) are those of

$$\dot{x} \in F(t, x), \tag{2}$$

and the same is true for a given control problem.

ad (ii): If function f is discontinuous in x, then Carathéodory theory can not be applied for solving e.g. the Cauchy (initial value) problems

$$\begin{cases} \dot{x} = f(t, x), \\ x(0) = x_0, \end{cases} \tag{3}$$

where $f : [0, \tau] \times \mathbb{R}^n \to \mathbb{R}^n$.

Making however the *Filippov regularization* of f, namely

$$F(t, x) := \bigcap_{\delta > 0} \bigcap_{\substack{r \subset [0,\tau] \times \mathbb{R}^n \\ \mu(r) = 0}} \overline{\mathrm{conv}} f(O_\delta((t, x) \setminus r)), \tag{4}$$

where $\mu(r)$ denotes the Lebesque measure of the set $r \subset \mathbb{R}^n$ and

$$O_\delta(y) := \{z \in [0, \tau] \times \mathbb{R}^n \mid |y - z| < \delta\},$$

multivalued F is well-known to be again upper Carathéodory (shortly u-Carathéodory) with nonempty, convex and compact values, provided only f is measurable and satisfies $|f(t,x)| \leq \alpha + \beta|x|$, for all $(t,x) \in [0,\tau] \times \mathbb{R}^n$, with some nonnegative constants α, β. Thus, by a *Filippov solution* of $\dot{x} = f(t,x)$, it is so understood a Carathéodory solution of (2), where F is defined in (4). As an example from physics, dry friction problems can be solved in this way.

ad (iii): Let us consider the *implicit differential equation*

$$\dot{x} = f(t,x,\dot{x}), \tag{5}$$

where $f : [0,\tau] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a compact (continuous) map and the solutions are understood in the sense of Carathéodory. We can associate with (5) the following two differential inclusions:

$$\dot{x} \in F_1(t,x) \tag{6}$$

and

$$\dot{x} \in F_2(t,x), \tag{7}$$

where $F_1(t,x) := \mathrm{Fix}(f(t,x,\,\cdot\,))$, i.e. the fixed point set of $f(t,x,\,\cdot\,)$ w.r.t. the last variable, and $F_2 \subset F_1$ is a (multivalued) lower semicontinuous selection of F_1. The sufficient condition for the existence of such a selection F_2 reads (see e.g. [AG, Chapter III.11, pp. 558–559]):

$$\dim \mathrm{Fix}(f(t,x,\,\cdot\,)) = 0, \quad \text{for all } (t,x) \in [0,\tau] \times \mathbb{R}^n, \tag{8}$$

where dim denotes the topological (covering) dimension.

Denoting by $S(f), S(F_1), S(F_2)$ the sets of all solutions of initial value problems to (5), (6), (7), respectively, one can prove (see [AG, p. 560] that, under (8), $S(f) = S(F_1) \subset S(F_2) \neq \emptyset$.

Although there are several monographs devoted to multivalued ODEs (see e.g. [AG], [AC], [KOZ], [De], [FG], [HP2], [Ki], [MM], [Sm], [To]), topological principles were presented mainly for single-valued ODEs (besides [AG], [De], [KOZ] and [FG] for differential inclusions, see e.g. [Fu], [FZ], [GM], [KZ], [KW], [Ma]). Hence, we consider without special distinguishing differential equations as well as inclusions; both in Euclidean and Banach spaces. All solutions of problems under our consideration (even in Banach spaces) will be understood at least in the sense of Carathéodory. Thus, in view of the indicated relationship with problems (i)–(iii), many obtained results can be also employed for solving optimal control problems, problems for systems with variable structure, implicit boundary value problems, etc.

The reader exclusively interested in single-valued ODEs can simply read "continuous", instead of "upper semicontinuous" or "lower semicontinuous", and replace the inclusion symbol $\in$ by the equality $=$, in the given differential inclusions.

b) Main aims

The main aims of our dissertation are the following:

1. To formulate sufficiently general *continuation principles based on the correctly defined indices of fixed points* (resp. topological degrees) and to develop the methods for the solvability of a large class of boundary value problems on compact as well as noncompact intervals, for differential equations and inclusions.

2. To formulate general *continuation principle based on the correctly defined Nielsen number* and to develop the method for obtaining the multiplicity results concerning especially periodic solutions of differential equations and inclusions.

3. On the basis of methods from the point 1., to *establish effective criteria of solvability* of a large class of boundary value problems on compact as well as noncompact intervals, for differential equations and inclusions.

4. On the basis of a method from the point 2., to *establish effective criteria for a lower estimate of the number of* especially periodic solutions to differential equations and inclusions.

c) Applied methods

All the applied methods can be characterized as topological. In the larger context, problems under investigation belong to nonlinear analysis.

Our methods applied to nonlinear boundary value problems employ Schauder's idea of linearization (parametrization) and subsequent transformation of a given problem to an existence or multiplicity fixed point problem. Fixed points of the associated Hammerstein operators represent solutions of given problems, or — in case of Poincaré translation operators along the trajectories of differential systems — determine solutions.

Various versions of the Lefschetz fixed point theorem, which is a far–reaching generalization of the well–known Schauder fixed point theorem, is used rather as a normalization property for a very general fixed point index or Nielsen number. The fixed point index can be understood as a relative degree which can be applied (unlike the absolute degree) in nonnormable (e.g. Fréchet) spaces.

As it was already pointed out, our methods can be also applied to multivalued operators which can be technically convenient. On the other hand, one should then know the topological structure of solution sets of the fully or partly (Schauder–like) linearized problems.

Besides the existence and multiplicity results, our methods allow us to detect the localization of solutions, i.e. we can look for solutions in given sets. Since we use the degree arguments, some solutions can escape from these sets or further solutions can exist outside these sets.

Homotopical properties of all the applied topological invariants guarantee the validity of obtained results, under slight homotopical deformations. In other words, the obtained results are in this sense stable under homotopical deformations.

d) Sample of dissertation results

In the entire text, all spaces are metric and by a (multivalued) map $\varphi : X \multimap Y$, i.e. $\varphi : X \to 2^Y \setminus \{\emptyset\}$, we mean the one with nonempty, closed values.

The existence results (in Section 5) and the multiplicity results (in Section 6) for differential equations and inclusions are based on the application of topological invariants (Lefschetz number, Nielsen number, fixed point index, degree theory, etc.) developed for multivalued maps in mostly Fréchet spaces (see e.g. our monograph [AG]). It will be, therefore, convenient to recall at least the basic related notions.

A *Fréchet space* is a complete, metrizable, locally convex space. Its topology can be generated by a family of seminorms. If it is normable, then it becomes Banach.

By AR (or ANR) we denote, as usual, the class of *absolute retracts* (or *absolute neighbourhood retracts*), namely X is an AR (or ANR) if each embedding $h : X \to Y$, i.e. $h : X \to h(X)$ is a homomorphism, into a metrizable space Y, such that $h(X) \subset Y$ is closed, is a retract (or a neighbourhood retract of Y). It is well-known that every ANR is a retract of some open subset of a normed space and that every retract of an open subset of a convex set in a Fréchet space is an ANR. Furthermore, every AR is *contractible*, i.e. homotopically equivalent to a one point space, and every ANR X is *locally contractible*, namely locally contractible in each of its points $x \in X$ which means that, for every $\varepsilon > 0$, there exists $\delta > 0$ ($\delta < \varepsilon$) such that the ball $B(x, \delta)$ is contractible in $B(x, \varepsilon)$. If there exists a decreasing sequence $\{X_n\}$ of compact, contractible sets X_n such that $X = \cap\{X_n \mid n = 1, 2, \ldots\}$, then X is called an *R_δ-set*. Let us note that any R_δ-set is *acyclic* w.r.t. any continuous theory of homology (e.g. the Čech homology), i.e. homologically equivalent to a one point space, and so any R_δ-set is nonempty, compact and connected. The following hierarchies hold for metric spaces:

$$\begin{array}{c} \text{contractible} \subset \text{acyclic} \\ \cup \\ \text{convex} \subset \text{AR} \subset \text{ANR}, \end{array}$$

$$\text{compact, convex} \subset \text{compact AR} \subset \text{compact, contractible} \subset R_\delta \subset \text{compact, acyclic},$$

and all the above inclusions are proper.

A map $\varphi : X \multimap Y$ is said to be *upper semicontinuous* (u.s.c.) if, for every open $U \subset Y$, the set $\{x \in X \mid \varphi(x) \subset U\}$ is open in X. It is said to be *lower semicontinuous* (l.s.c.) if, for every open $U \subset Y$, the set $\{x \in X \mid \varphi(x) \cap U \neq \emptyset\}$ is open in X. If it is both u.s.c. and l.s.c., then it is called *continuous*. A compact-valued map $\varphi : X \multimap Y$, or equivalently $\varphi : X \to \mathcal{K}(Y) := \{K \subset Y \mid K \text{ is compact}\}$, is continuous if and only if it is *Hausdorff-continuous*, i.e. continuous w.r.t. the metric d in X and the Hausdorff-metric d_H in $\mathcal{B}(Y) := \{D \subset Y \mid D \text{ is nonempty and bounded}\}$, where $d_H(A, B) := \inf\{\varepsilon > 0 \mid A \subset O_\varepsilon(B) \text{ and } B \subset O_\varepsilon(A)\}$ and $O_\varepsilon(D) := \{x \in X \mid \exists y \in D : d(x, y) < \varepsilon\}$. Observe that every single-valued u.s.c. or l.s.c. map is continuous in the usual sense.

A u.s.c. map with R_δ-(acyclic) values will be called an R_δ-(acyclic) map. R_δ-maps $\varphi : X \multimap Y$ can be identified here with J-maps, written $\varphi \in J(X,Y)$. A typical example of J-maps are the Hammerstein operators representing boundary value problems for ordinary differential inclusions. A map which is a finite composition of compact-valued acyclic maps is called *admissible* (see e.g. [AG]).

The class of admissible maps contains u.s.c. maps with convex and compact values, u.s.c. maps with contractible and compact values, R_δ-maps, acyclic maps with compact values and their compositions. Moreover, the class of admissible maps is, unlike the mentioned subclasses, closed under composition, i.e. composition of admissible maps remains admissible. A typical example of admissible maps are Poincaré's translation operators along the trajectories of systems of ordinary differential inclusions. Compact and condensing admissible maps are, therefore, extremely important in the whole work.

For the sake of simplicity, we shall present here only the results for differential equations and inclusions in finite–dimensional (Euclidean) spaces.

As a sample of results having the character of a method, let us introduce the following theorem (denoted in our dissertation as Corollary 4.1). Those who are not familiar with multivalued analysis can simply read u-Carathéodory as Carathéodory, i.e. consider particular single-valued case.

THEOREM 1 *Consider the boundary value problem*

$$\begin{cases} \dot{x}(t) \in F(t, x(t)), & \text{for a.a. } t \in J, \\ x \in S, \end{cases} \tag{9}$$

where J is a given real interval, $F : J \times \mathbb{R}^n \multimap \mathbb{R}^n$ is a u-Carathéodory map and S is a subset of $\mathrm{AC}_{\mathrm{loc}}(J, \mathbb{R}^n)$.

Let $G : J \times \mathbb{R}^n \times \mathbb{R}^n \multimap \mathbb{R}^n$ be a u-Carathéodory map such that

$$G(t, c, c) \subset F(t, c), \quad \text{for all } (t, c) \in J \times \mathbb{R}^n.$$

Assume that

(i) there exists a retract Q (e.g. a convex closed subset or its homeomorphic image) of $C(J, \mathbb{R}^n)$ such that the associated problem

$$\begin{cases} \dot{x}(t) \in G(t, x(t), q(t)), & \text{for a.a. } t \in J, \\ x \in S \cap Q \end{cases} \tag{10}$$

has an R_δ-set $T(q)$ (e.g. convex and compact or a singleton) of solutions, for each $q \in Q$, *(ii) there exists a locally integrable function* $\alpha : J \to \mathbb{R}$ *such that*

$$|G(t, x(t), q(t))| \leq \alpha(t), \quad \textit{a.e. in } J,$$

for any $(q, x) \in \Gamma_T$*, where* Γ_T *denotes the graph of the solution operator* $T : Q \multimap C(J, \mathbb{R}^n)$.

(iii) $T(Q)$ *is bounded in* $C(J, \mathbb{R}^n)$ *and* $\overline{T(Q)} \subset S$.

Then problem (9) has a solution $x(\,.\,)$ *such that* $x(t) \in \overline{Q}$*, for all* $t \in J$.

If, in particular, $J = [a, b]$ (i.e. compact), then we can give the following theorem (denoted in our dissertation as Corollary 4.3).

THEOREM 2 *Consider the boundary value problem (9), where* $J = [a, b]$ *is this time a compact interval,* $F : J \times \mathbb{R}^n \multimap \mathbb{R}^n$ *is a u-Carathéodory map and* $S \subset \mathrm{AC}(J, \mathbb{R}^n)$.

Let $G : J \times \mathbb{R}^n \times \mathbb{R}^n \times [0, 1] \multimap \mathbb{R}^n$ *be a u-Carathéodory map such that* $G(t, c, c, 1) \subset F(t, c)$*, for all* $(t, c) \in J \times \mathbb{R}^n$*. Assume that*

(i) there exist a (bounded) retract Q *(e.g. a convex closed subset of its homeomorphic image) of* $C(J, \mathbb{R}^n)$ *such that its interior* $Q \backslash \partial Q$ *is nonempty (open) and a closed bounded subset* S_1 *of* S *such that the associated problem*

$$\begin{cases} \dot{x}(t) \in G(t, x(t), q(t), \lambda), & \textit{for a.a. } t \in J, \\ x \in S_1 \end{cases} \tag{11}$$

is solvable with an R_δ*-set* $T(q)$ *(e.g. convex and compact or a singleton) of solutions, for each* $(q, \lambda) \in Q \times [0, 1]$*, and conditions*

(ii) there exists a locally integrable function $\alpha : J \to \mathbb{R}$ *such that*

$$|G(t, x(t), q(t), \lambda)| \leq \alpha(t), \quad \textit{a.e. in } J,$$

for any $(q, \lambda, x) \in \Gamma_T$*, where* T *denotes the set-valued map which assigns to any* $(q, \lambda) \in Q \times [0, 1]$ *the set of solutions of (11)*

and

(iii) $T(Q \times \{0\}) \subset Q$,

hold true,

(iv) the solution map T *has no fixed points on the boundary* ∂Q *of* Q*, for every* $(q, \lambda) \in Q \times [0, 1]$.

Then problem (9) has a solution.

For multiplicity results, we can introduce the following theorem (denoted in our dissertation as Theorem 4.3). It will be convenient to start with one definition whose idea is due to R. F. Brown.

DEFINITION 1 We say that the mapping $T : Q \multimap U$ is *retractible onto* Q, where U is an open subset of $C(J, \mathbb{R}^n)$ containing Q, if there is a (continuous) retraction $r : U \to Q$ and $p \in U \setminus Q$ with $r(p) = q$ implies that $p \notin T(q)$.

Its advantage consists in the fact that, for a retractible mapping $T : Q \multimap U$ onto Q with a retraction r in the sense of Definition 1, its composition with r, $r|_{T(Q)} \circ T : Q \multimap Q$, has a fixed point $\widehat{q} \in Q$ if and only if $\widehat{q}$ is a fixed point of T.

THEOREM 3 *Let $G : J \times \mathbb{R}^n \times \mathbb{R}^n \multimap \mathbb{R}^n$ be u-Carathéodory map and assume that*

(i) there exists a closed, connected subset Q of $C(J, \mathbb{R}^n)$ with a finitely generated abelian fundamental group such that, for any $q \in Q$, the set $T(q)$ of all solutions of the linearized problem

$$\begin{cases} \dot{x}(t) \in G(t, x(t), q(t)), & \text{for a.a. } t \in J, \\ x \in S \end{cases} \tag{12}$$

is R_δ (e.g. convex and compact or a singleton),

(ii) $T(Q)$ is bounded in $C(J, \mathbb{R}^n)$ and $\overline{T(Q)} \subset S$,

(iii) there exists a locally integrable function $\alpha : J \to \mathbb{R}$ such that

$$|G(t, x(t), q(t))| := \sup\{|y| \mid y \in G(t, x(t), q(t))\} \le \alpha(t), \quad \text{a.e. in } J,$$

for any pair $(q, x) \in \Gamma_T$, where Γ_T denotes the graph of T.

Assume, furthermore, that

(iv) the solution operator $T : Q \multimap U$, related to (12), is retractible onto Q with a retraction r in the sense of Definition 1.

At last, let

$$G(t, c, c) \subset F(t, c) \tag{13}$$

for a.a. $t \in J$ and any $c \in \mathbb{R}^n$. Then the original problem (9) admits at least $N(r|_{T(Q)} \circ T)$ solutions belonging to Q, where N stands for the Nielsen number.

REMARK 1 The definition of the Nielsen number is rather sophisticated and its calculation is usually a difficult task.

REMARK 2 In the (single-valued) case of Carathéodory ODEs, we can only assume in Theorem 3(i) that the linearized problem (12) is uniquely solvable. Moreover, the requirement that the fundamental group $\pi(Q)$ of Q to be finitely generated and abelian can be then omitted.

Application of Theorem 2 leads, for instance, to the following existence result (denoted in our dissertation as Corollary 5.2)

THEOREM 4 *Consider problem*

$$\begin{cases} \dot{x}(t) + A(t)x(t) \in F(t, x(t)), & \textit{for a.a. } t \in [0, \tau], \\ x(0) = x(\tau), \end{cases}$$

where $F(t, x) \equiv F(t + \tau, x)$ satisfies the conditions:

(i) $F : [0, \tau] \times \mathbb{R}^n \multimap \mathbb{R}^n$ is a u-Carathéodory mapping with nonempty, compact and convex values,

(ii) there are two nonnegative Lebesgue-integrable functions $\delta_1, \delta_2 : [0, \tau] \to [0, \infty)$ such that

$$|F(t, x)| \le \delta_1(t) + \delta_2(t)|x|, \quad \textit{for a.a. } t \in [0, \tau] \textit{ and all } x \in \mathbb{R}^n,$$

where $|F(t, x)| = \sup\{|y| \mid y \in F(t, x)\}$.

Let $G : [0, \tau] \times \mathbb{R}^n \times \mathbb{R}^n \times [0, 1] \multimap \mathbb{R}^n$ be a product-measurable u-Carathéodory map such that $G(t, c, c, 1) \subset F(t, c)$, for all $(t, c) \in [0, \tau] \times \mathbb{R}^n$.

Assume that A is a piece-wise continuous (single-valued) bounded τ-periodic $(n \times n)$-matrix whose Floquet multipliers lie off the unit cycle, jointly with

(iv) there exists a (bounded) retract Q of $C([0, \tau], \mathbb{R}^n)$ such that $Q \setminus \partial Q$ is nonempty (open) and such that $G(t, x, q(t), \lambda)$ is Lipschitzian in x with a sufficiently small Lipschitz constant, for a.a. $t \in [0, \tau]$ and each $(q, \lambda) \in Q \times [0, 1]$,

(v) there exists a Lebesgue integrable function $\alpha : [0, \tau] \to [0, \infty)$ such that

$$|G(t, x(t), q(t), \lambda)| \le \alpha(t), \quad \textit{a.e. in } [0, \tau]$$

for any $(x, q, \lambda) \in \Gamma_T$ (i.e. from the graph of T), where T denotes the set-valued map which assigns, to any $(q, \lambda) \in Q \times [0, 1]$, the set of solutions of

$$\begin{cases} \dot{x}(t) + A(t)x(t) \in G(t, x(t), q(t), \lambda), & \textit{for a.a. } t \in [0, 1], \\ x(0) = x(\tau), \end{cases}$$

(vi) $T(Q \times \{0\}) \subset Q$ holds and ∂Q is fixed point free w.r.t. T, for every $(q, \lambda) \in Q \times [0, 1]$.

Then the inclusion $\dot{x} + A(t)x \in F(t, x)$ admits a τ-periodic solution.

Remark 3 Since in Theorem 4 the associated homogeneous problem has obviously only the trivial solution, the requirement $T(Q \times \{0\}) \subset Q$ reduces to $\{0\} \subset Q$, provided $G(t, x, q, \lambda) = \lambda G(t, x, \lambda)$, $\lambda \in [0, 1]$.

Remark 4 The requirement concerning a fixed point free boundary ∂Q of Q in Theorem 4 can be verified by means of bounding (Liapunov-like) functions (see C1[13], C1[36], C1[47] and cf. [AG, Chapter III.8]).

Application of Theorem 3 leads to the following multiplicity result (denoted in our dissertation as Theorem 6.2).

Theorem 5 *Let suitable positive constants δ_1, δ_2 exist such that the inequalities*

$$\begin{cases} \dfrac{1}{|a|}|e_0\delta_2^{1/m} - G| \geq \delta_1 > \left(\dfrac{H}{f_0}\right)^n, \\ \dfrac{1}{|b|}|f_0\delta_2^{1/n} - H| \geq \delta_2 > \left(\dfrac{G}{e_0}\right)^m \end{cases} \tag{14}$$

are satisfied for constants e_0, f_0, G, H estimating the product-measurable u-Carathéodory or l-Carathéodory multivalued functions (with nonempty, convex and compact values) e, f, g, h as above, for constants a, b with $ab > 0$ and for odd integers m, n with $\min(m, n) \geq 3$. Then system

$$\begin{cases} \dot{x} + ax \in e(t, x, y)y^{(1/m)} + g(t, x, y), \\ \dot{y} + by \in f(t, x, y)x^{(1/n)} + h(t, x, y), \end{cases} \tag{15}$$

admits at least two entirely bounded solutions. In particular, if multivalued functions e, f, g, h are still ω-periodic in t, then system (15) admits at least tree ω-periodic solutions, provided the sharp inequalities appear in (14).

Remark 5 Unfortunately, because of the invariance (w.r.t. the solution operator T_1) of the subdomains

$$\left\{ q(t) \in C\Big(\Big[-\frac{\omega}{2}, \frac{\omega}{2}, \mathbb{R}^2\Big)\Big| 0 < \delta_1 \leq q_1(t) \leq R \wedge 0 < \delta_2 \leq q_2(t) \leq R \right\}$$

and

$$\left\{ q(t) \in C\Big(\Big[-\frac{\omega}{2}, \frac{\omega}{2}, \mathbb{R}^2\Big)\Big| -R \leq q_1(t) \leq -\delta_1 < 0 \wedge -R \leq q_2(t) \leq -\delta_2 < 0 \right\},$$

for each $\omega \in (-\infty, \infty)$, the same result can also be obtained, for example, by means of the fixed point index.

Remark 6 In order to avoid the handicap in Remark 5, we were able to modify the result in Theorem 5 for a functional planar system. The related result is denoted in our dissertation as Theorem 6.3.

e) Presentations and applications

Many results in the present dissertation were generalized and extended in further papers. They also became a basis for a further elaboration of our PhD students in their thesis (7 finished doctors and 2 PhD students under our supervision). The author presented these results at many international conferences and seminars at the universities. The results also allowed us an intensive international collaboration (especially with Italian and recently with French colleagues). The author had many invited talks at 2 American and many European universities. In the last three years he was a visiting professor at Université Paris 1–Panthéon–Sorbonne; before he was for many years a visiting professor at Universita di Roma 1–LaSapienza.

The majority of the results in our dissertation has a character of new methods. These methods can be applied e.g. in the optimal control problems or in the problems, where the generating vector fields are discontinuous at space variables (e.g. dry friction problems).

The publications were supported by the Council of Czech Government (MSM 619 895 92 14).

References

[AC] J.-P. Aubin and A. Cellina, *Differential Inclusions*, Springer, Berlin, 1984.

[AG] J. Andres and L. Górniewicz, *Topological Fixed Point Principles for Boundary Value Problems*, Kluwer, Dordrecht, 2003.

[BF] F. Battelli, M. Fečkan (eds.): *Handbook of Differential Equations. Ordinary differential equations*, Vol. IV. Elsevier, Amsterdam, 2008.

[BFGJ] R. F. Brown, M. Furi, L. Górniewicz, B. Jiang (eds.): *Handbook of Topological Fixed Point Theory.* Springer, Berlin, 2005.

[Br] R. F. Brown: *More about Nielsen theories and their applications*, In: [BFGJ], pp. 433–462.

[CDF1] A. Cañada, P. Drábek, A. Fonda (eds.): *Handbook of Differential Equations. Ordinary differential equations*, Vol. I. Elsevier, Amsterdam, 2004.

[CDF2] A. Cañada, P. Drábek, A. Fonda (eds.): *Handbook of Differential Equations. Ordinary differential equations*, Vol. II. Elsevier, Amsterdam, 2005.

[CDF3] A. Cañada, P. Drábek, A. Fonda (eds.): *Handbook of differential equations. Ordinary differential equations*, Vol. III. Elsevier, Amsterdam, 2006.

[De] K. Deimling: *Multivalued Differential Equations.* De Gruyter, Berlin, 1992.

[DMNZ] R. Dragoni, J. W. Macki, P. Zecca: *Solution Sets of Differential Equations in Abstract Spaces.* Longman, Harlow, 1996.

[Fe] M. Fečkan: *Multiple solutions of nonlinear equations via Nielsen fixed–point theory: a survey.* In: T. M. Rassias (ed.): Nonlinear Analysis in Geometry and Topology, Palm Harbor, 2000, pp. 77–97.

[FG] M. Frigon and A. Granas (eds.), *Topological Methods in Differential Equations and Inclusions*, Kluwer, Dordrecht, 1995.

[Fu] S. Fučík: *Solvability of Nonlinear Equations and Boundary Value Problems.* Reidel, Dordrecht, 1980.

[FZ] M. Furi, P. Zecca (eds.): *Topological Methods for Ordinary Differential Equations.* LNM 1537, Springer, Berlin, 1993.

[GM] R. E. Gaines and J. Mawhin: *Coincidence Degree and Nonlinear Differential Equations.* LNM 568, Springer, Berlin, 1977.

[HP1] S. Hu, N. S. Papageorgiou: *Handbook of Multivalued Analysis*, Vol. I: Theory. Kluwer, Dordrecht, 1997.

[HP2] S. Hu, N. S. Papageorgiou: *Handbook of Multivalued Analysis*, Vol. II: Applications. Kluwer, Dordrecht, 2000.

[KOZ] M. I. Kamenskii, V. V. Obukhovskii and P. Zecca, *Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Space*, W. deGruyter, Berlin, 2001.

[Ki] M. Kisielewicz, *Differential Inclusions and Optimal Control*, Kluwer, Dordrecht, 1991.

[KS] W. Kirk, B. Sims (eds.): *Handbook of Metric Fixed Point Theory*. Kluwer, Dordrecht, 2001.

[KZ] M. A. Krasnoselśkii, P. P. Zabreiko: *Geometrical Methods of Nonlinear Analysis*. Springer, Berlin, 1984.

[KW] W. Krawcewicz and J. Wu, *Theory of Degrees with Applications to Bifurcations and Differential Equations*, J. Willey & Sons., New York, 1997.

[Ma] J. Mawhin: *Topological Degree Methods in Nonlinear Boundary Value Problems.* AMS, Providence, R. I., 1979.

[MM] M. D. P. Monteiro Marques, *Differential Inclusions in Nonsmooth Mechanical Problems*, Birkhäuser, Basel, 1993.

[OO] F. Obersnel, P. Omari: *Period two implies chaos for a class of ODEs*, Proc. Amer. Math. Soc. **135**, 7 (2007), 2055–2058.

[Sm] G. V. Smirnov, *Introduction to the Theory of Differential Inclusions*, Grad. Stud. Math., Vol. 41, Amer. Math. Soc., Providence, RI, 2002.

[To] A. A. Tolstonogov, *Differential Inclusions in a Banach Space*, Kluwer, Dordrecht, 2000.

List of author's publications

The present dissertation is identical with the monographic chapter B[2]. In March 2010, the author found 302 citations (without self–citations) of his publications; 113 citations were found on WOS and 27 publications were cited in monographs.

A. Monographs

A[**1**] Andres, J., Górniewicz, L.: *Topological Fixed Point Principles for Boundary Value Problems.* Kluwer, Dordrecht, 2003.

B. Chapters in monographs

B[**1**] Andres, J.: *Periodic-type solutions of differential inclusions.* In: Advances in Mathematical Research, Vol. 8 (A. R. Baswell, ed.), Nova Sciences Publishers, New York, 2009, pp. 295–353.

B[**2**] Andres, J.: *Topological principles for ordinary differential equations.* In: Handbook of Differential Equations, Ordinary Differential Equations, vol. 3 (A. Cañada, P. Drábek, A. Fonda, eds.), Elsevier, Amsterdam, 2006, pp. 1–101.

B[**3**] Andres, J.: *Applicable fixed point principles.* In: Handbook of Topological Fixed Point Theory (R. F. Brown, M. Furi, L. Górniewicz, B. Jiang, eds.), Springer, Berlin, 2005, pp. 687–739.

C. Scientific papers

C1 Scientific papers in journals

C1[**1**] Andres, J., Pavlačková, M.: *Topological structure of solution sets to asymptotic boundary value problems.* J. Diff. Eqns **248** (2010)), 127–150. **IF:** 1.349

C1[**2**] Andres, J.: *On de Saussure's principle of linearity and visualization of language structures.* Glottotheory 2, 2 (2010), 1–14.

C1[**3**] Andres, J.: *On a conjecture about the fractal structure of language.* J. Quant. Linguistics **17**, 2 (2010), 101–122.

C1[**4**] Andres, J., Kožušníková, M., Malaguti, L.: *On the Floquet problem for second-order Marchaud differential systems.* J. Math. Anal. Appl. **351**, 1 (2009), 360–372. $\boxed{\textbf{IF: } 1.046}$

C1[**5**] Andres, J., Pavlačková, M.: *Asymptotic boundary value problems for second-order differential systems.* Nonlin. Anal. **71**, 5-6 (2009), 1462–1473. $\boxed{\textbf{IF: } 1.295}$

C1[**6**] Andres, J., Fürst, T., Pastor, K.: *Sharkovskii's theorem, differential inclusions, and beyond.* Topol. Meth. Nonlin. Anal. **33**, 1 (2009), 149–168. $\boxed{\textbf{IF: } 0.594}$

C1[**7**] Andres, J., Malaguti, L., Taddei, V.: *On boundary value problems in Banach spaces.* Dynam. Syst. Appl. **18** (2009), 275–302. $\boxed{\textbf{IF: } 0.718}$

C1[**8**] Andres, J., Kožušníková, M., Malaguti, L.: *Bound sets approach to boundary value problems for vector second-order differential inclusions.* Nonlin. Anal. **71**, 1-2 (2009), 28–44. $\boxed{\textbf{IF: } 1.295}$

C1[**9**] Andres, J., Malaguti, L, Pavlačková, M.: *Strictly localized bounding functions for vector second-order boundary value problems.* Nonlin. Anal., T.M.A. **71**, 12 (2009), 6019–6028. $\boxed{\textbf{IF: } 1.295}$

C1[**10**] Andres, J.: *Randomization of Sharkovskii-type theorems.* Proc. Amer. Math. Soc. **136**, 4 (2008), 1385–1395; Erratum: **136**, 10 (2008), 3733–3734. $\boxed{\textbf{IF: } 0.584}$

C1[**11**] Andres, J., Fürst, T., Pastor, K.: *Full analogy of Sharkovskii's theorem for a lower semicontinuous maps.* J. Math. Anal. Appl. **340**, 2 (2008), 1132–1144. $\boxed{\textbf{IF: } 1.046}$

C1[**12**] Andres, J., Pastor, K., Šnyrychová, P.: *Simple fixed point theorems on linear continua.* Cubo. A Math. J. **10**, 4 (2008), 27–43.

C1[**13**] Andres, J., Malaguti, L., Taddei, V.: *A bounding functions approach to multivalued boundary value problems.* Dynamic Syst. Appl. **16** (2007), 37–48. $\boxed{\textbf{IF: } 0.718}$

C1[**14**] Andres, J., Väth, M.: *Calculation of Lefschetz and Nielsen numbers in hyperspaces for fractals and dynamical systems.* Proc. Amer. Math. Soc. **135** (2007), 479–487. $\boxed{\textbf{IF: } 0.584}$

C1[**15**] Andres, J., Pastor, K., Šnyrychová, P.: *A multivalued version of Sharkovskii's theorem holds with at most two exceptions.* J. Fixed Point Th. Appl. **2** (2007), 153–170.

C1[**16**] Andres, J., Fürst, T., Pastor, K.: *Period two implies all periods for a class of ODEs: a multivalued map approach.* Proc. Amer. Math. Soc. **135**, 10 (2007), 3187–3191. $\boxed{\textbf{IF: } 0.584}$

C1[**17**] Andres, J., Bersani, A.-M., Grande, R. F.: *Hierarchy of almost-periodic function spaces.* Rendiconti Mat. Appl. **26** (2006), 121–188.

C1[**18**] Andres, J., Górniewicz, L.: *Periodic solutions of dissipative systems revisited.* Fixed Point Theory Appl. ID 65195 (2006), 1–12. $\boxed{\textbf{IF: } 0.728}$

C1[**19**] Andres, J., Fürst, T.: *An example of application of the Nielsen theory to integro-differential equations.* Proceed. Amer. Math. Soc. **134** (2006), 1985–1993. $\boxed{\textbf{IF: } 0.584}$

C1[**20**] Andres, J., Górniewicz, L.: *Note on topological degree for monotone-type multivalued maps.* Fixed Point Th. **7**, 2 (2006), 191–199.

C1[**21**] Andres, J., Fürst, T.: *Nontrivial application of Nielsen theory to differential systems.* J. Diff. Eqns **231** (2006), 313–330. $\boxed{\textbf{IF: } 1.349}$

C1[**22**] Andres, J., Šnyrychová, P., Szuca, P.: *Sharkovskii's theorem for connectivity G_δ-relations.* Int. J. Bifurc. Chaos **16**, 8 (2006), 2377–2393. $\boxed{\textbf{IF: } 0.870}$

C1[**23**] Andres, J., Wong, P.: *Relative Nielsen theory for noncompact spaces and maps.* Topology Appl. **153** (2006), 1961–1974. $\boxed{\textbf{IF: } 0.362}$

C1[**24**] Andres, J., Bersani, A. M., Radová, L.: *Almost-periodic solutions in various metrics of higher-order differential equations with a nonlinear restoring term.* Acta Univ. Palacki. Olomuc., Fac. rer. nat., Math. **45** (2006), 7–29.

C1[**25**] Andres, J., Pastor, K.: *A version of Sharkovskii's theorem for differential equations.* Proceed. Amer. Math. Soc. **135** (2005), 449–453. $\boxed{\textbf{IF: } 0.584}$

C1[**26**] Andres, J.: *Nielsen number and differential equations.* Fixed point Theory Appl. **2** (2005), 137–167. $\boxed{\textbf{IF: } 0.728}$

C1[**27**] Andres, J., Jüttner, L., Pastor, K.: *On a multivalued version of the Sharkovskii theorem and its application to differential inclusions, II.* Set-Valued Anal. **13** (2005), 47–68. $\boxed{\textbf{IF: } 0.714}$

C1[**28**] Andres, J., Bednařík, D., Pastor, K.: *On the notion of derivo-periodicity.* J. Math. Anal. Appl. **303**, 2 (2005), 405–417. $\boxed{\textbf{IF: } 1.046}$

C1[**29**] Andres, J., Fišer, J., Gabor, G., Lesniak, K.: *Multivalued fractals.* Chaos, Solitons, Fractals **24**, 3 (2005), 665–700. $\boxed{\textbf{IF: } 2.980}$

C1[**30**] Andres, J., Górniewicz, L.: *A note to the paper of D. Richeson and J. Wiseman.* Anal. Stiint. Univ. "Al. I. Cuza", Iai, **51**, 1, Matem. 2, (2005), 259–264.

C1[**31**] Andres, J.: *Period three implications for expansive maps in $\Re^n$.* J. Difference Eqns Appl. **10**, 1 (2004), 17–28. $\boxed{\textbf{IF: } 0.867}$

C1[**32**] Andres, J., Fišer, J.: *Metric and topological multivalued fractals.* Int. J. Bifurc. Chaos **14**, 4 (2004), 1277–1289. $\boxed{\textbf{IF: } 0.870}$

C1[**33**] Andres, J., Väth, M.: *Two topological definitions of a Nielsen number for coincidences of noncompact maps.* Fixed Point Th. Appl. **1**, 1 (2004), 49–69. $\boxed{\textbf{IF: } 0.728}$

C1[**34**] Andres, J.: *Continuation principles for fractals.* Fixed Point Theor. **5**, 2 (2004), 165–180.

C1[**35**] Andres, J., Pastor, K.: *On a multivalued version of the Sharkovskii theorem and its application to differential inclusions, III.* Topol. Meth. Nonlin. Anal. **22** (2003), 369–386. **IF:** 0.594

C1[**36**] Andres, J., Malaguti, L., Taddei, V.: *Bounded solutions of Carathéodory differential inclusions: a bound sets approach.*, Abstr. Appl. Anal. **2003**, 9 (2003), 547–571. **IF:** 0.644

C1[**37**] Andres, J., Górniewicz, L., Jezierski, J.: *Periodic points of multivalued mappings with applications to differential inclusions on tori.*, Topol. Appl. **127**, 3 (2003), 447–472. **IF:** 0.362

C1[**38**] Andres, J., Fišer, J.: *Fractals generated by differential equations.* Dynam. Syst. Appl. **11**, 4 (2002), 471–480. **IF:** 0.718

C1[**39**] Andres, J., Jüttner, L.: *Period three plays a negative role in a multivalued version of Sharkovskii's theorem.* Nonlin. Anal. **51**, 6 (2002), 1101–1104. **IF:** 1.295

C1[**40**] Andres, J., Väth, M.: *Coincidence index for noncompact mappings on nonconvex sets.*, Nonlinear Funct. Anal. Appl. **7**, 4 (2002), 619–658.

C1[**41**] Andres, J., Bader, R.: *Asymptotic boundary value problems in Banach spaces.*, J. Math. Anal. Appl. **274**, 1 (2002), 437–457. **IF:** 1.046

C1[**42**] Andres, J., Fišer, J., Jüttner, L.: *On a multivalued version of the Sharkovskii theorem and its application to differential inclusions.* Set-Valued Anal. **10**, 1 (2002), 12–14. **IF:** 0.714

C1[**43**] Andres, J., Gabor, G., Górniewicz, L.: *Acyclicity of solution sets to differential inclusions.* Nonlin. Anal. **49**, 5 (2002), 671–688. **IF:** 1.295

C1[**44**] Andres, J., Jüttner, L.: *Periodic solutions of discontinuous differential systems.*, Nonlinear Anal. Forum **6**, 2 (2001), 391–407.

C1[**45**] Andres, J., Bersani, A. M.: *Almost-periodicity problem as a fixed-point problem for evolution inclusions.*, Topol. Methods Nonlinear Anal. **18**, 2 (2001), 337–349. **IF:** 0.594

C1[**46**] Andres, J., Krajc, B.: *Periodic solutions in a given set of differential systems.*, J. Math. Anal. Appl. **264**, 2 (2001), 495–509. **IF:** 1.046

C1[**47**] Andres, J., Malaguti, L., Taddei, V.: *Floquet boundary value problems for differential inclusions: a bound sets approach.*, Z. Anal. Anwend. **20**, 3 (2001), 709–725.

C1[**48**] Andres, J.: *Nielsen number, Artin braids, Poincaré operators and multiple nonlinear oscillations.* Nonlin. Anal. **47**, 2 (2001), 1017–1028. **IF:** 1.295

C1[**49**] Andres, J.: *Some standard fixed-point theorems revisited.* Atti Sem. Mat. Fis. Univ. Modena **49** (2001), 455–471.

C1[**50**] Andres, J.: *Ordinary differential equations in the lack of uniqueness.* Atti Sem. Mat. Fis. Univ. Modena **49** (2001), 247–267.

C1[**51**] Andres, J., Bersani, A. M., Leśniak, K.: *On some almost-periodicity problems in various metrics.* Acta Appl. Math. **65**, 1–3 (2001), 35–57. $\boxed{\textbf{IF:}\ 0.430}$

C1[**52**] Andres, J., Górniewicz, L., Jezierski, J.: *Relative versions of the multivalued Lefschetz and Nielsen theorems and their application to admissible semi–flows.* Topol. Meth. Nonlin. Anal. **16**, 1 (2000), 73–92. $\boxed{\textbf{IF:}\ 0.594}$

C1[**53**] Andres, J., Fišer, J.: *Basins of attraction for triple logistic maps.* Annal. Math. Silesianae **14** (2000), 23–32.

C1[**54**] Andres, J., Gabor, G., Górniewicz, L.: *Topological structure of solution sets to multivalued asymptotic problems.* Zeitschrift für Anal. und ihre Anw. **19**, 1 (2000), 35–60.

C1[**55**] Andres, J.: *A nontrivial example of application of the Nielsen fixed–point theory to differential systems: problem of Jean Leray.* Proceed. Amer. Math. Soc. **128** (2000), 2921–2931. $\boxed{\textbf{IF:}\ 0.584}$

C1[**56**] Andres, J.: *Multiple bounded solutions of differential inclusions: the Nielsen theory approach.* J. Diff. Eqns **155** (1999), 285–310. $\boxed{\textbf{IF:}\ 1.349}$

C1[**57**] Andres, J.: *Almost-periodic and bounded solutions of Carathéodory differential inclusions.* Diff. Int. Eqns **112**, 6 (1999), 887–912.

C1[**58**] Andres, J., Gabor, G., Górniewicz, L.: *Boundary value problems on infinite intervals.* Trans. Amer. Math. Soc. **351**, 12 (1999), 4861–4903. $\boxed{\textbf{IF:}\ 1.014}$

C1[**59**] Andres, J.: *Existence of two almost-periodic solutions of pendulum-type equations.* Nonlin. Anal. **37** (1999), 797–804. $\boxed{\textbf{IF:}\ 1.295}$

C1[**60**] Andres, J., Górniewicz, L., Jezierski, J.: *A generalized Nielsen number and multiplicity results for differential inclusions.* Topol. Appl. **100**, 2–3 (1999), 193–209. $\boxed{\textbf{IF:}\ 0.362}$

C1[**61**] Andres, J., Krajc, B.: *Bounded solutions in a given set of differential systems.* J. Comp. Appl. Math. **113**, 1–2 (1999), 73–82. $\boxed{\textbf{IF:}\ 1.048}$

C1[**62**] Andres, J., Górniewicz, L.: *From the Schauder fixed-point theorem to the applied multivalued Nielsen theory.* Topol. Meth. Nonlin. Anal. **14**, 2 (1999), 229–238. $\boxed{\textbf{IF:}\ 0.594}$

C1[**63**] Andres, J., Fišer, J., Jüttner, L., Velecká, I.: *Population dynamics of Bithynia tentaculata.* Acta UPO, Fac. rer. nat., Math. **37** (1998), 7–34.

C1[**64**] Andres, J., Jüttner, L., Velecká, I.: *Mathematical modelling of population dynamics of freshwater mollusc Bithynia tentaculata (Linné, 1758) (Gastropoda: Prosobranchia).* Acta UPO, Fac. rer. nat., Biol. **36** (1998), 83–100.

C1[**65**] Andres, J.: *Periodic solutions of quasi–linear functional differential inclusions.* Functional Diff. Eqns **5**, 3–4 (1998), 287–296.

C1[**66**] Andres, J.: *Nonlinear rotations.* Nonlin. Anal., T.M.A. **30**, 1 (1997), 495–503. **IF:** 1.295

C1[**67**] Andres, J.: *A target problem for differential inclusions with state-space constraints.* Demonstratio Math. **30**, 4 (1997), 783–790.

C1[**68**] Andres, J., Krajc, B.: *Unified approach to bounded, periodic and almost periodic solutions of differential systems.* Annal. Math. Silesianae **11** (1997), 39–53.

C1[**69**] Andres, J.: *On the multivalued Poincaré operators.* Topol. Meth. Nonlin. Anal. **10**, 1 (1997), 171–182. **IF:** 0.594

C1[**70**] Andres, J.: *Existence of periodic and bounded solutions of the generalized Liénard equation under forcing.* Rep. Math. Phys. **39**, 1 (1997), 91–98. **IF:** 0.6

C1[**71**] Andres, J., Sanchez, L.: *A note on vector boundary value problems.* Nonlin. Diff. Eqns, T.M.A. **3**, 1–2 (1997), 49–58.

C1[**72**] Andres, J., Fišer, J.: *Bifurcations of a triple logistic map.* Acta UPO, Phys. **36** (1997), 217–224.

C1[**73**] Andres, J.: *Existence, uniqueness and instability of large-period harmonics to the third-order nonlinear ordinary differential equations.* J. Math. Anal. Appl. **199** (1996), 445–457. **IF:** 1.046

C1[**74**] Andres, J., Górniewicz, L., Lewicka, M.: *Partially dissipative periodic processes.* Banach Center Publ. **35** (1996), 109–118.

C1[**75**] Andres, J.: *Splay-phase and anti-splay-phase orbits for equivariant set-valued flows on tori.* Diff. Eqns and Dynam. Systems **4**, 1 (1996), 89–98.

C1[**76**] Andres, J.: *Concluding remarks on problem of Moser and conjecture of Mawhin.* Annal. Math. Silesianae **10** (1996), 57–65.

C1[**77**] Andres, J., Turský, T.: *Asymptotic estimates of solutions and their derivatives of nth-order nonhomogenous ordinary differential equations with constant coefficients.* Discussiones Math. **16**, 1 (1996), 75–89.

C1[**78**] Andres, J., David, Z.: *Basin of attraction metamorphoses for planar endomorphisms.* Acta UPO, Fac. rer. nat., Phys. 35 (1996), 221–251.

C1[**79**] Andres, J., Turský, T.: *On the method of Esclangon.* Acta UPO, Math. **35** (1996), 7–20.

C1[**80**] Andres, J.: *Large-period forced oscillations to higher-order pendulum-type equations.* Diff. Eqns and Dynam. Systems **3**, 4 (1995), 407–421.

C1[**81**] Andres, J.: *Recent results on third-order nonlinear ODEs.* J. Nigerian Math. Soc. **14** (1995), 41–66.

C1[**82**] Andres, J.: *Further remarks on problem of Moser and conjecture of Mawhin.* Topol. Meth. Nonlin. Anal. **6**, 1 (1995), 163–174. **IF:** 0.594

C1[**83**] Andres, J., Kučera, R.: *Note on the observation of comets in 1664 and 1665 by the Olomouc scholar P. Valentin Stansel, S.J.* Acta UPO **120**, Phys. 34 (1995), 207–218.

C1[**84**] Andres, J.: *Note on the problem of de la Vallée Poussin.* Publ. Math. Debrecen **45**, 1–2 (1994), 145–152. **IF:** 0.346

C1[**85**] Andres, J.: *Periodic derivatives and the Wirtinger-type inequalities.* Acta UPO, Fac. rer. nat. **116**, Phys. 33 (1994), 177–192.

C1[**86**] Andres, J., Koutná, M., Vlček, V.: *Asymptotic estimates of solutions to the fifth-order nonhomogenous linear ordinary differential equations with almost constant coefficients.* Acta UPO, Fac. rer. nat. **116**, Phys. 33 (1994), 193–233.

C1[**87**] Andres, J.: *Derivo-periodic boundary value problems for nonautonomous ordinary differential equations.* Riv. Mat. Pura Appl. **13** (1993), 63–90.

C1[**88**] Andres, J.: *Several remarks to problem of Moser and conjecture of Mawhin.* Boll. U. M. I. **7**, 7-A (1993), 377–386. **IF:** 0.059

C1[**89**] Andres, J., Vlček, V.: *Green's functions for periodic and anti-periodic BVPs to second-order ODEs.* Acta UPO, Fac. rer. nat. **110**, Math. 32 (1993), 7–16.

C1[**90**] Andres, J., Staněk, S.: *Note to the Lagrange stability of excited pendulum-type equations.* Math. Slovaca **43**, 5 (1993), 617–630. **IF:** 0.051

C1[**91**] Andres, J.: *Transformation theory for nondissipative systems: Some remarks and simple application in examples.* Acta UPO, Fac. rer. nat. **111**, Phys. 32 (1993), 125–132.

C1[**92**] Andres, J.: *Existence of periodic solutions for an n-th order differential equation with nonlinear restoring term and time-variable coefficients.* J. Math. Anal. Appl. **167**, 1 (1992), 1–11. **IF:** 1.046

C1[**93**] Andres, J.: *Note to the asymptotic behaviour of solutions of damped pendulum equations under forcing.* Nonlin. Anal., T. M. A. **18**, 8 (1992), 705–712. **IF:** 1.295

C1[**94**] Andres, J.: *Asymptotic properties of solutions to quasi-linear differential systems.* J. Comput. Appl. Math. **41** (1992), 57–64. **IF:** 1.048

C1[**95**] Andres, J., Vlček, V.: *On four-point regular BVPs for second-order quasi-linear ODEs.* Acta UPO, Fac. rer. nat. **105**, Math. 31 (1992), 37–44.

C1[**96**] Andres, J., Vlček, V.: *Square integrable processes in nonlinear oscillators—II.* Acta UPO, Fac. rer. nat. **106**, Phys. 31 (1992), 153–158.

C1[**97**] Andres, J.: *Lagrange stability of higher-order analogy of damped pendulum equations.* Acta UPO, Fac. rer. nat. **106**, Phys. 31 (1992), 159–164.

C1[**98**] Andres, J.: *O Hurwitzově problému pro posunuté mnohočleny* [*On the problem of Hurwitz for the shifted polynomials*]. Acta UPO, Fac. rer. nat. **106**, Phys. 31 (1992), 165–169.

C1[**99**] Andres, J., Vlček, V.: *Periodic solutions of the third order parametric differential equations involving large nonlinearities.* Math. Slovaca **41**, 4 (1991), 337–349. **IF:** 0.051

C1[**100**] Andres, J.: *Note to a certain third-order nonlinear differential equation related to the problem of Littlewood.* Fasc. Math. **23** (1991), 5–8.

C1[**101**] Andres, J.: *Periodic solutions of the third-order differential equation with right-hand side in the form of nonlinear restoring term plus general gradient-like part.* Acta UPO, Fac. rer. nat. **100**, Math. 30 (1991), 143–148.

C1[**102**] Andres, J., Vlček, V.: *Note to the existence of periodic solutions for higher-order differential equations with nonlinear restoring term and time-variable coefficients.* Acta UPO, Fac. rer. nat. **100**, Math. 30 (1991), 149–157.

C1[**103**] Andres, J., Vlček, V.: *Square integrable processes in nonlinear oscillators.* Acta UPO, Fac. rer. nat. **101**, Phys. 30 (1991), 91–104.

C1[**104**] Andres, J., Gaudenzi, M., Zanolin, F.: *A transformation theorem for periodic solutions of nondissipative systems.* Rend. Sem. Mat. Univers. Politecn. Torino **48**, 2 (1990), 171–186.

C1[**105**] Andres, J.: *Four-point and asymptotic boundary value problems via a possible modification of Poincaré's mapping.* Math. Nachr. **149** (1990), 155–162. **IF:** 0.537

C1[**106**] Andres, J.: *Periodic derivative of solutions to nonlinear differential equations.* Czech. Math. J. **40**, 3 (1990), 353–360. **IF:** 0.210

C1[**107**] Andres, J.: *Note to the paper of Fučík and Mawhin.* CMUC **31**, 2 (1990), 223–226.

C1[**108**] Andres, J.: *On some modification of the Levinson operator and its application to a three-point boundary value problem.* Acta UPO, Fac. rer. nat. **97**, Math. 29 (1990), 35–43.

C1[**109**] Andres, J.: *A four-point boundary value problem for the second-order ordinary differential equations.* Arch. Math. (Basel) **53** (1989), 384–389. **IF:** 0.5

C1[**110**] Andres, J.: *On a certain nonlinear differential equation of the third order involving the sinusoidal restoring term.* Fasc. Math. **20** (1989), 97–102.

C1[**111**] Andres, J., Vlček, V.: *Asymptotic behaviour of solutions to the N-th order nonlinear differential equation under forcing.* Rend. Ist. Matem. Univ. Trieste **21**, 1 (1989), 128–143.

C1[**112**] Andres, J.: *Instability of a real classical solution of the second harmonic generation equations with respect to a boson operator correction.* Acta UPO, Fac. rer. nat. **94**, Phys. 28 (1989), 49–55.

C1[**113**] Andres, J., Vlček, V.: *On the existence of square integrable solutions and derivatives to fourth and fifth order equations.* Acta UPO, Fac. rer. nat. **94**, Math. 28 (1989), 65–86.

C1[**114**] Andres, J.: *Structure of the phase-space for the third-order forced nonlinear differential equations belonging to the generalized class D' in the sense of Levinson.* Fasc. Math. **18** (1988), 63–67.

C1[**115**] Andres, J.: *Asymptotic properties of solutions of a certain third order differential equation with the oscillatory restoring term.* Acta UPO, Fac. rer. nat. **91**, Math. 27 (1988), 201–210.

C1[**116**] Andres, J., Mikolajski, J., Palát, J.: *Über die Trichotomie von Lösun*gen einer nichtlinearen Vektordifferentialgleichung zweiter Ordnung. Acta UPO, Fac. rer. nat. **91**, Math. 27 (1988), 211–224.

C1[**117**] Andres, J.: *On a boundary value problem for $x''' = f(t, x, x', x'')$.* Acta UPO, Fac. rer. nat. **91**, Math. 27 (1988), 289–298.

C1[**118**] Andres, J.: *Solution with periodic second derivative of a certain third order differential equation.* Math. Slovaca **37**, 3 (1987), 239–245. **IF:** 0.051

C1[**119**] Andres, J., Štrunc, M.: *Lagrange-like stability of local cycles to a certain forced phase-locked loop described by the third–order differential equation.* Rev. Roum. Sci. Techn. **32**, 2 (1987), 219–223.

C1[**120**] Andres, J.: *On local ω-cycles to certain third order nonlinear differential equations.* Fasc. Math. **17** (1987), 49–54.

C1[**121**] Andres, J.: *Dichotomies for solutions of a certain third order nonlinear differential equation which is not from the class D.* Fasc. Math. **17** (1987), 55–62.

C1[**122**] Andres, J.: *On dissipativity of quantum optical systems.* J. Math. Phys. **28**, 2 (1987), 489–491. **IF:** 1.085

C1[**123**] Andres, J.: *On dissipativity of parametric generation process.* Acta UPO, Fac. rer. nat. **88**, Phys. 26 (1987), 71–78.

C1[**124**] Andres, J.: *Time development of coherent states involving A_3-catastrophe.* Acta UPO, Fac. rer. nat. **88**, Phys. 26 (1987), 79–85.

C1[**125**] Andres, J.: *Higher kind periodic orbits.* Acta UPO, Fac. rer. nat. **88**, Phys. 26 (1987), 87–92.

C1[**126**] Andres, J., Palát, J.: *Über die Existenz einer begrenzten und periodischen Lösung der nichtlinearisierten Jacobischen Gleichung mit negativ definitivem Träger.* Acta UPO, Fac. rer. nat. **88**, Math. 26 (1987), 85–94.

C1[**127**] Andres, J.: *Symmetry in a certain simply perturbed Hamiltonian systems.* Acta UPO, Fac. rer. nat. **88**, Math. 26 (1987), 131–138.

C1[**128**] Andres, J.: *Dynamical systems modelled by third order differential equations with special respect to an influence of the restoring terms on the properties of solutions.* Acta UPO, Fac. rer. nat. **88**, Math. 26 (1987), 139–148.

C1[**129**] Andres, J.: *Boundedness results of solutions to the equation* $x''' + ax'' + g(x)x' + h(x) = p(t)$ *without the hypothesis* $h(x)\operatorname{sgn} x \leq 0$ *for* $|x| > R$. Atti Accad. Naz. Lincei **80**, 7–12 (1986/87), 533–539.

C1[**130**] Andres, J.: *Boundedness of solutions of the third order differential equation with the oscillatory restoring and forcing terms.* Czech. Math. J. **36**, 1 (1986), 1–6. $\boxed{\textbf{IF: } 0.210}$

C1[**131**] Andres, J.: *On stability and instability of the roots of the oscillatory function in a certain nonlinear differential equation of the third order.* Čas. pěst. mat. 3 (1986), 225–229.

C1[**132**] Andres, J., Voráček, J.: *Existence periodických řešení parametrické nelineární diferenciální rovnice třetího řádu* [*Existence of periodic solutions of the third-order parametric nonlinear differential equation*]. Kniž. odb. věd. sp. VUT v Brně, B-106 (1986), 7–15.

C1[**133**] Andres, J.: *A useful proposition to nonlinear differential systems with a solution of the prescribed asymptotic properties.* Acta UPO, Fac. rer. nat. **85**, Math. 25 (1986), 157–164.

C1[**134**] Andres, J.: *On the "Poincaré–Lyapunov"—like systems of three differential equations.* Acta UPO, Fac. rer. nat. **85**, Math. 25 (1986) 165–180. Erratum: Acta UPO, Fac. rer. nat. **94**, Math. 28 (1989), 61–63.

C1[**135**] Andres, J., Štrunc, M.: *On stability in second harmonic generation with a time-dependent classical pumping.* Kniž. odb. věd. sp. VUT v Brně, B-112 (1986), 9–14.

C1[**136**] Andres, J.: *Periodic boundary value problem for certain nonlinear differential equations of the third order.* Math. Slovaca **35**, 3 (1985), 305–309. $\boxed{\textbf{IF: } 0.051}$

C1[**137**] Andres, J., Voráček, J.: *Periodic solutions to a nonlinear parametric differential equation of the third order.* Atti Accad. Naz. Lincei **77**, 3–4 (1984/85), 81–86.

C1[**138**] Andres, J., Voráček, J.: *Periodic solutions of a certain parametric third order differential equation.* Kniž. odb. věd. sp. VUT v Brně, B-94 (1983), 7–11.

C2 Papers in proceedings

C2[**1**] Andres, J.: *Coexistence of random subharmonic ocillations.* In: Proc. EQUADIFF, Vienna, (P. Szmolyan, ed.), to appear.

C2[**2**] Andres, J.: *Bound sets approach to BVPs: retrospective a perspectives.* In: Proc. Conf. on BVPs, Santiago de Compostela, Spain (ed. by A. Cabada and J. Nieto), American Institute of Physics, New York, 2009, 29–38.

C2[**3**] Andres, J., Fišer, J.: *Iterated multifunction systems.* In: Proceed. of the Eight Int. Conf. on Difference Eqns and Applications (S. Elaydi et al.), Chapman & Hall/CRC, London, 2005, 15–22.

C2[**4**] Andres, J.: *Sharkovskii's theorem and differential equations.* In : Proceed. of the EQUADIFF 2003 (Conference held in Hasselt, Belgium) (F. Dumortier et al., eds.), World Scientific, Singapore, 2005, 821–826.

C2[**5**] Andres, J.: *Using the integral manifolds to solvability of boundary value problems.* In: "Set Valued Mappings with Applications in Nonlinear Analysis" (ed. by R. P. Agarwal and D. O'Regan), Ser. Math. Anal. Appl. **3**, Taylor and Francis, London, 2002, 27–38.

C2[**6**] Andres, J.: *Poincaré's translation multioperator revisited.* In: "Proceed. of the 3^{rd} Symp. on Nonlin. Anal." (ed. by W. Kryszewski and A. Nowakowski), LN in Nonlin. Anal. **3**, 2002, 7–22.

C2[**7**] Andres, J.: *Nielsen number and multiplicity results for multivalued boundary value problems.*, Grossinho, M. R. (ed.) et al., Nonlinear analysis and its applications to differential equations. Papers from the autumn school on nonlinear analysis and differential equations, Lisbon, Portugal, September 14-October 23, 1998. Boston, MA: Birkhäuser. Prog. Nonlinear Differ. Equ. Appl. 43, 175-187 , 2001.

C2[**8**] Andres, J., Górniewicz, L.: *On the Banach contraction principle for multivalued mappings.* In: "Approximation, Optimization and Mathematical Economics" (M. Lassonde, ed.). Physica-Verlag, Springer, Berlin, 2001, 14–23.

C2[**9**] Andres, J., Górniewicz, L., Jezierski, J.: *Noncompact version of the multivalued Nielsen theory and its application to differential inclusions.* Diff. Inclusions and Optimal Control, LN in Nonlin. Anal. **2** (1998), 33–50.

C2[**10**] Andres, J.: *Ważewski-type results without transversality.* Proceed. of Equadiff 95 (Lisbon, July 24–29) World Scientific, Singapore, 1998, 233–238.

C2[**11**] Andres, J.: Bounded, almost-periodic and periodic solutions of quasi-linear differential inclusions. Differential Inclusions and Optimal Control. Lecture Notes in Nonlinear Analysis 2 (1998), 19–32.

C2[**12**] Andres, J.: *Problem of Barbashin in the case of forcing.* In: Qualit. Theory of Diff. Eqns. (Szeged 1988), Colloq. Math. Soc. J. Bolyai **53**, North Holland, Amsterdam–New York, 1989, 9–16.

C2[**13**] Andres, J.: *On a possible modification of Levinson's operator.* In: Proceed. of ICNO XI, Budapest, 1987, 345–348.

C2[**14**] Andres, J.: *Андрес, J.: Асимптотические свойства решений одного дифференциалного уравнения третьего порядка с колебательной функцией* [*Asymptotic properties of solutions of a certain differential equation of the third order with the oscillatory function*]. In: Линейные операторы и их приложения [Linear Operators and Their Applications], Baku, 1986, 109–111.

D. Further papers

D[**1**] Andres, J.: *Meze lidského poznání a meze poznání člověka očima matematika.* In: Současné integrující přístupy k pojetí člověka, Vyd. UP Olomouc, 2009, 52–56.

D[**2**] Andres, J.: *La scienza postmoderna e il cristianesimo.* Quadermi di Simmetria **4**, Roma, (2007), 1–14.

D[**3**] Andres, J.: *Šarkovského věta pro diferenciální rovnice (Role čísel 1, 2, 3, 4, 5 pro diferenciální rovnice).* In: Seminář z teorie diferenciálních rovnic 2001–2005 (Zápisky přednášek), (J. Bouchala, B. Krajc, editoři), Katedra apl. mat. FEI VŠB-TU Ostrava, Ostrava, 2005, 35–39.

D[**4**] Andres, J.: *Tři zdroje a tři součásti geometrizujících vizí Ladislava Daka.* Mat. fyz. inf. **7** (2005), 423–432; **8** (2005), 493–498.

D[**5**] Andres, J: *Šarkovského věta a diferenciální rovnice.* Pokroky mat. fyz. ast. **49**, 2 (2004), 151–159.

D[**6**] Andres, J., Fišer, J., Chajda, I.: *Talmudická triangulace kruhu a její možné dusledky [Talmudic triangulation of the circle and its possible consequences].* Českosl. čas. fyz. **49**, 4 (1999), 234–244.

D[**7**] Andres, J., Górniewicz, L., Nistri, P. (eds.): Differential Inclusions and Optimal Control. LN in Nonlin. Anal. 2, Schauder Center for Nonlinear Studies, Toru, 1998.

D[**8**] Andres, J.: *Matematika v díle Pavla A. Florenského [Mathematics in the work of P. A. Florenskii].* Universum **29** (1998), 29–36.

D[**9**] Andres, J.: *Matematicko-fyzikální analýza kardiálních modelů [Mathematical and physical analysis of cardiac models].* Českoslov. čas. fyz. **47**, 4–5 (1997), 310–312.

D[**10**] Andres, J., Fišer, J.: *Řez zlatý, stříbrný a bronzový [Golden, silver and bronze means].* Pokroky mat. fyz. astr. **6** (1995), 307–317.

D[**11**] Andres, J., Křesálek, V.: *Katastrofy, chaos a krize v optice [Catastrophs, chaos and crises in optics].* Jemná mech. opt. **10** (1987), 307–310.

E. Some invited lectures

- Mini–Workshop on Multivalued Analysis
  Ancona, Italy (May 28–29, 2009)
  Multivalued chaos (main speaker)

- Conference on Boundary value Problems
  Santiago de Compostela , Spain (September 16–19, 2008)
  Bound sets approach to BVPs: Retrospective and perspectives

- Topological Theory of Fixed and Periodic Points
  Bedlewo, Poland (July 22–28, 2007)
  Randomized Sharkovskii–type theorems

- EQUADIFF 2007
  Vienna, Austria (August 5–11, 2007)
  Coexistence of random subharmonic solutions

- Trends in Differential Equations and Dynamical Systems
  Modena, Italy (November 29–30, 2007)
  Bounding functions and multivalued boundary value problems
  (main speaker)

- Progress on Difference Equations
  Homburg, Saar, Germany (March 6–10, 2006)
  Hyperchaos

- 21st European Conference on Operational Research
  Reykjavik, Island (July 2–5, 2006)
  Multivalued boundary value problems

- Fractals and Markov Operators
  L'Aquila, Italy (September 12–14, 2005)
  Fixed point theory in hyperspaces with application to fractals
  (main speaker)

- Conference on Nielsen Theory and Related Topics
  St John's, Newfoundland, Canada (June 28–July 2, 2004)
  Nielsen number and differential equations (plenary talk)

- International Conference on Nonlinear Operators,
  Differential Equations and Applications
  Cluj, Romania (August 24–27, 2004)
  Continuation principles for fractals

- Functional Analysis and Approximation
  Bologna, Italy (December 15–17, 2004)
  On multifractals

- EQUADIFF 2003
  Hasselt, Belgium (July 22–26, 2003)
  Sharkovskii's theorem and differential equations

- 3rd Polish Symposium on Nonlinear Analysis
  Lódź, Poland (January 29–31, 2001)
  Poincaré's translation multioperator revisited (plenary talk)

- 3rd World Congress of Nonlinear Analysis
  Catania, Italy (July 19–26, 2000)
  Multiple bounded solutions of differential inclusions

- Ninth Meeting on Real Analysis and Measure Theory
  Grado, Italy (September 15–19, 2000)
  Some standard fixed–point theorems revisited (plenary talk)

- 2nd Symposium on Nonlinear Analysis (in honour
  of centinary of Juliusz Schauder's birth)
  Toruń, Poland (September 13–17, 1999)
  From the Schauder theorem to the applied multivalued Nielsen theory
  (plenary talk)

- International Conference on Functional Differential Equations
  Ariel, Israel (June 29–July 2, 1998)
  Periodic solutions of functional differential inclusions

- Nonlinear Analysis and Differential Equations
  Lisbon, Portugal (September–October, 1998)
  Nielsen number and multiplicity results
  for multivalued boundary value problems

- Differential Inclusions and Optimal Control
  Int. S. Banach Math. Center, Warsaw, Poland
  (September 22–October 3, 1997)
  Bounded, almost–periodic and periodic solutions
  of quasi–linear differential inclusions (plenary talk)

- 2nd World Congress of Nonlinear Analysis
  Athens, Greece (July 10–17, 1996)
  Nonlinear rotations

- Topological Methods in Differential Inclusions
  Int. S. Banach Math. Center, Warsaw, Poland (October 10–15, 1994)
  Ważewski's principle without transversality (plenary talk)

- Ordinary Differential Equations and Their Applications
  Firenze, Italy (September 20–24, 1993)
  On partially dissipative systems