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Various Aspects of Hořava-Lifshitz Gravity

Komise pro obhajoby doktorských prací v oboru jaderné, subjaderné a matematické fyziky

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Accompanying papers

I J. Kluson, *"Branes at Quantum Criticality,"* JHEP **0907** (2009) 079 [arXiv:0904.1343 [hep-th]].

II J. Kluson, *"New p+1 dimensional nonrelativistic theories from Euclidean stable and unstable Dp-branes,"* Phys. Rev. D **80** (2009) 046004 [arXiv:0905.1483 [hep-th]].

III J. Kluson, *"Horava-Lifshitz f(R) Gravity,"* JHEP **0911** (2009) 078 [arXiv:0907.3566 [hep-th]].

IV J. Kluson, *"New Models of f(R) Theories of Gravity,"* Phys. Rev. D **81** (2010) 064028 [arXiv:0910.5852 [hep-th]].

V J. Kluson, *"String in Horava-Lifshitz Gravity,"* Phys. Rev. D **82** (2010) 086007 [arXiv:1002.2849 [hep-th]].

VI J. Kluson, *"Note About Hamiltonian Formalism of Modified $F(R)$ Hořava-Lifshitz Gravities and Their Healthy Extension,"* Phys. Rev. D **82** (2010) 044004 [arXiv:1002.4859 [hep-th]].

VII J. Kluson, *"Note About Hamiltonian Formalism of Healthy Extended Horava-Lifshitz Gravity,"* JHEP **1007** (2010) 038 [arXiv:1004.3428 [hep-th]].

VIII J. Kluson and K. L. Panigrahi, *"T-Duality For String in Horava-Lifshitz Gravity,"* Eur. Phys. J. C **71** (2011) 1595 [arXiv:1006.4530 [hep-th]].

IX J. Kluson, *"Horava-Lifshitz Gravity And Ghost Condensation,"* Phys. Rev. D **82** (2010) 124011 [arXiv:1008.5297 [hep-th]].

X J. Kluson, *"Hamiltonian Analysis of Non-Relativistic Covariant RF-Diff Horava-Lifshitz Gravity,"* Phys. Rev. D **83** (2011) 044049 [arXiv:1011.1857 [hep-th]].

XI J. Kluson, S. Nojiri, S. D. Odintsov and D. Saez-Gomez, *"$U(1) Invariant F(R)$ Horava-Lifshitz Gravity,"* Eur. Phys. J. C **71** (2011) 1690 [arXiv:1012.0473 [hep-th]].

XII J. Kluson, *"Lagrange Multiplier Modified Horava-Lifshitz Gravity,"* Eur. Phys. J. C **71** (2011) 1820 [arXiv:1101.5880 [hep-th]].

XIII J. Kluson, S. 'i. Nojiri and S. D. Odintsov, *"Covariant Lagrange multiplier constrained higher derivative gravity with scalar projectors,"* Phys. Lett. B **701** (2011) 117 [arXiv:1104.4286 [hep-th]].

XIV J. Kluson, *"Note About Weyl Invariant Horava-Lifshitz Gravity,"* Phys. Rev. D **84** (2011) 044025 [arXiv:1104.4200 [hep-th]].

XV J. Kluson, *"Note About Equivalence of $F(R)$ and Scalar Tensor Horava-Lifshitz Gravities,"* Phys. Rev. D **84** (2011) 104014 [arXiv:1107.5660 [hep-th]].

XVI M. Chaichian, J. Kluson, M. Oksanen and A. Tureanu, *"On higher derivative gravity with spontaneous symmetry breaking: Hamiltonian analysis of new covariant renormalizable gravity,"* Phys. Rev. D **87** (2013) 064032 [arXiv:1208.3990 [gr-qc]].

Abstract

Cílem této doktorské práce je podat přehled mého příspěvku k problematice studia Hořavovy-Lifšicovy teorie gravitace. V její první části stručně popíši problémy tykající se kvantování gravitační teorie. V další části shrnu základní principy Hořavovy-Lifšicovy gravitace. Poté podám přehled různých verzí této teorie a budu diskutovat jejich konsistenci. Dále popíši hamiltonovskou formulaci těchto teorií, která má fundamentální význam pro určení fyzikálních stupňů volnosti. V závěru stručně shrnu základní fakta týkající se Hořavovy-Lifšicovy teorie a nastíním směry, kterými by se měl následující výzkum této teorie ubírat. Závěrečná část této práce obsahuje přehled mých prací týkající se této problematiky.

1 Introduction

1.1 Gravity and Renormalizability

One of the most striking problem of the current theoretical physics is an inconsistency between quantum mechanics and general relativity. In more details, when we consider the perturbative general relativity as an ordinary quantum field theory we find that this theory is not renormalizable which is in strict difference with the success of the Standard Model description of the particle physics. This fact means that even if the general relativity is very succesful for the description of the classical gravitational phenomena it should be viewed as an effective theory that breaks down at some scale. Beyond that scale general relativity is not able to describe the gravitational interaction on space time and it is not possible to consruct its quantum version using conventional quantization techniques.

If we accept the point of view that the general relativity is an effective theory then we can say that the Einstein-Hilbert action contains only the lowest order terms in curvature expansion. The natural question is whether inclusion of the higher order curvature terms could make the general theory renormalizable theory. This could work when we recognize that such terms could modify the propagator of the graviton at high energies [1]. However the price what we pay for such a renormalizable theory of gravity is hight: This theory contains ghost degrees of freedom and are therefore not unitary. This is a general property of all higher time derivatives theories.

On the other hand there is an interesting possibility to modify the propagator by adding higher order spatial derivatives without adding the higher order time derivatives. Intuitively we should expect to find theory with improved high energy behavior in ultraviolet regime (UV) without having problems with the higher order time derivatives. Clearly such a presumption implies that the time and spatial coordinates should be treated on different footing and consequently the resulting theory is not Lorentz invariant. On the other hand we can still hope that following picture emerges: We have theory that is not Lorentz invariant at high energies but where the theory is renormalizable due to the Lorentz non-invariant propagator but at the low energy regime (IR) the Lorentz invariance could be recovered or at least the Lorentz violations in the IR could stay below current experimental constraints.

In December 2008 and January 2009 Hořava formulated his proposal [2, 3] which is now known as Hořava-Lifshitz gravity. This proposal was extensively studied either from theoretical or phenomonological points of view. The main idea of given proposal is simple: try to modify gravity at UV scale so that the theory is renormalizable. In the next we review this framework and its major development.

1.2 Lorentz violation as a field theory regulator

As we wrote briefly above the main idea how to find the renormalizable theory is to give up Lorentz invariance at high energies. Note that this idea goes beyond the gravity and it has been considered in the past for other fields and further branches of theoretical physics, as for example condensed matter physics. We can demonstrate this idea on the simple example of the scalar field, following [4].

It is important to stress that there is nothing wrong when we presume that Lorentz invariance is broken at hight energies. On the other hand the Lorentz violations are severely constrained in a wide range of energies and especially in the IR. In other words the main question is whether we can construct a field theory that exhibits Lorentz violations which in the far UV lead to renormalizability but remains consistent Lorentz invariant theory at low energies. Let us consider following scalar field action

$$S_\phi = \int dt d^d x \left(\dot{\phi}^2 - \sum_{m=1}^{z} a_m \phi(-\triangle)^m \phi + \sum_{n=1}^{M} g_n \phi^n \right) , \tag{1}$$

where

$$\dot{\phi} \equiv \frac{\partial \phi}{\partial t} , \tag{2}$$

and where

$$\triangle = \frac{1}{\sqrt{\gamma}} \partial_i [\sqrt{\gamma} \gamma^{ij} \partial_j] \tag{3}$$

is the spatial Laplacian with γ_{ij} is the spatial metric that in the flat space time is diagonal $\gamma_{ij} = \mathrm{diag}(1, \ldots, 1)$. Further, z and M are positive integers that will be specified below. We call theory as "power counting renormalizable"when all of its interaction terms scale like energy to some non-positive power so that Feynmann diagrams are expected to be convergent. To see this explicitly we firstly choose the engineering dimensions of space and time as

$$[dt] = [\kappa]^{-z} , \quad [dx] = [\kappa]^{-1} , \tag{4}$$

where κ is symbol with dimension of momentum. The main requirement is that the action is dimensionless so that from the kinetic term we determine the dimension of the scalar field

$$[\phi] = [\kappa]^{(d-z)/2} . \tag{5}$$

Then checking that the potential contribution in the action is also dimensionless implies the scaling dimensions of a_m and g_n

$$[g_n] = [\kappa]^{d+z+\frac{n}{2}(z-d)} , \quad [a_m] = [\kappa]^{2(z-m)} . \tag{6}$$

We clearly see that a_m has non-negative momentum dimension for all m. Further, g_n has non-negative momentum dimension for all n when $z \geq d$. When $z < d$ we see that g_n has non-negative momentum dimension only when $n \leq \frac{2(d+z)}{d-z}$. Let us now consider concrete values of d and z. For example, in case $d = 3$ we find that the theory is renormalizable for $z = 1$ and for $M = 4$ which is well known fact that the usual relativistic ϕ^4 theory is power counting renormalizable in $3+1$ dimensions. However there is an interesting class of the scalar field theories where $z = d$ that is renormalizable for $z = 3$ in $3 + 1$ dimensions.

In case of the graviton the situation is slightly different due to the fact that the graviton self-interaction vertices are more complicated. Explicitly, in case of the scalar field we have interaction vertices where the momenta do not enter these interactions. On the other hand in case of the graviton we have self-interactions vertices that contain spatial derivatives. Clearly this fact slightly complicates the situation however does not spoil the power-counting renormalizability as long as $z \geq d$ and the action contains operators with at least $2d$ spatial derivatives.

In summary, all these arguments provide a strong support that the field theories that contain at least $2d$ spatial derivatives in $d + 1$ dimensions are power counting renormalizable.

2 Non-Relativistic Gravity

In this section we give a brief description of the exact structure of a gravity that has the characteristics mentioned in the previous sections. For simplicity we will consider an explicit case of 3 dimensions. The basic requirement is that the theory should have only two time derivatives but at least 6 spatial derivatives. However the fact that we have more spatial derivative than time derivatives implies that these two derivatives should be treated differently. This fact can be naturally incorporated into the theory when we work in Arnowitt-Deser-Misner (ADM) decomposition of the spacetime

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) . \tag{7}$$

Due to the fact that time is treated differently than space we have to pick a preferred foliation of spacetime. Clearly such an action cannot be invariant under the full diffeomorphism as in case of general relativity. However it turns out that given action can be invariant under restricted set: foliation preserving diffeomorphism which is the space-independent time reparameterization together with time-dependent spatial diffeomorphism

$$t' = f(t) \ , \ x' = x^i(t, x^j) \ . \tag{8}$$

The basic requirement for the construction of the theory is that the action has to respect given symmetries.

It is easy to see that the only covariant quantity under (8) that contains the time derivative of the spatial metric is the extrinsic curvature

$$K_{ij} = \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \ , \tag{9}$$

where $\dot{g}_{ij}$ denotes differentiation with respect to time coordinate and ∇_i is the covariant derivative associated with the spatial metric g_{ij}. Note also that K_{ij} transforms as a scalar under time reparameterization $t' = f(t)$.

Further, the requirement that the theory should be second order in time derivatives in order to avoid the presence of the ghosts implies that the action should contain terms quadratic in the extrinsic curvature. It is also important to stress that there are no invariants under symmetry (8) that contain time derivatives of the lapse N or the shift N_i without including higher order time derivatives of g_{ij} as well. Therefore the most general action that we can consider is

$$S = \frac{M_{pl}^2}{2} \int d^3\mathbf{x} dt N \sqrt{g} \left[K^{ij} \mathcal{G}_{ijkl} K^{kl} - \mathcal{V}(g_{ij}, N) \right] \ , \tag{10}$$

where M_{pl} is a constant which can be identified with the Planck mass, g is the determinant of the spatial metric g_{ij} and where $\mathcal{G}^{ijkl}$ is the generalized de Witt metric

$$\mathcal{G}^{ijkl} = \frac{1}{2} \left(g^{ik} g^{jl} + g^{il} g^{jk} \right) - \lambda g^{ij} g^{kl} \ , \tag{11}$$

where λ is dimensionless coupling constant. $\mathcal{V}$ generally depends on g_{ij} and N and their spatial derivatives. It does not contain time derivatives and also cannot depend on the shift N_i since there are no suitable invariants that are invariant under (8). Finally the power counting renormalizability requires that $\mathcal{V}$ contains terms that are sixth order in spatial derivatives. Of course, we can consider the theory that contain spatial derivatives of arbitrary order but for simplicity we restrict ourselves to the theories that contain sixth order spatial derivatives but not higher.

3 Potential and various versions of the theory

We see that there are many possibilities how to construct the potential $\mathcal{V}$. In fact, different choices lead to the different version of the theory. We would like to give list of some of them.

3.1 Detailed balance

In the first formulation of the HL gravity P. Hořava proposed that the potential $\mathcal{V}$ should be defined using so named *detalied balance* which is inspired by condensed matter systems. This principle says that the potential should be derived from a superpotential W

$$\mathcal{V} = E^{ij}\mathcal{G}_{ijkl}E^{kl} \ , \tag{12}$$

where

$$E^{ij} = \frac{1}{\sqrt{g}}\frac{\delta W}{\delta g_{ij}} \ . \tag{13}$$

Then the most general action that we can write with $\mathcal{V}$ satisfying the conditions above is

$$\begin{aligned} S_{db} &= \frac{M_{pl}^2}{2}\int d^3\mathbf{x}dtN\sqrt{g}\left[K_{ij}\mathcal{G}^{ijkl}K_{kl} - \frac{\alpha^4}{M_{pl}^4}C_{ij}C^{ij} + \frac{2\alpha^2\beta}{M_{pl}^3}\frac{\epsilon^{ijk}}{\sqrt{g}}R_{il}\nabla_j R^l{}_k - \right. \\ &- \left. \frac{\beta^2}{M_{pl}^2}R^{ij}R_{ij} + \frac{\beta^2}{4}\frac{1-4\lambda}{1-3\lambda}R^2 + \frac{\beta^2\zeta}{1-3\lambda}R - \frac{3\beta^2\zeta^2}{1-3\lambda}M_{pl}^2\right] \ , \end{aligned} \tag{14}$$

where ϵ^{ijk} is the Levi-Civita symbol and where

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}}\nabla^k\left(R^j{}_l - \frac{1}{4}\delta^j{}_l R\right) \tag{15}$$

is the Cotton tensor and α, β and ζ are dimensionless couplings. It is important to stress that there are only 3 new couplings for a total of 6 terms in $\mathcal{V}$.

The advantages of the detailed balance formulation of the HL gravity is that it reduces the number of terms that we can consider in the potential which could simplify the action. On the other hand there is nothing fundamental about detailed balance so it can be considered as an assumption that simplify the calculation.

3.2 Projectable HL gravity

There is another restriction that simplifies given theory which is called as *projectability.* This is a presumption that the lapse is just a function of time $N = N(t)$. It is important to stress that there is no fundamental principle behind this assumption. One reason for such a choice is that only in this case we can impose the gauge fixing $N = 1$ as in general relativity which is not possible in case without projectability since the foliation preserving diffeomorphism only allows time independent reparameterizations.

The assumption of the projectability implies that all terms with spatial derivative of N vanish. In fact, the potential $\mathcal{V}$ depends on the metric and its spatial derivatives which means that the action includes all of the curvature invariants that can be constructed from g_{ij} up to six spatial derivatives. An important simplification occurs in three dimensions where the Weyl tensor vanishes identically and the Riemann tensor can be expressed in terms of Ricci tensor. Using Bianchi identities and ignoring the boundary terms we find the most general action

$$\begin{aligned} S_p = \frac{M_{pl}^2}{2}\int d^3\mathbf{x}dtN\sqrt{g}\Big(&K_{ij}\mathcal{G}^{ijkl}K_{kl} - g_0 M_{pl}^{-1} - g_1 R - g_2 M_{pl}^{-2}R^2 - \\ &- g_3 M_{pl}^{-2}R_{ij}R^{ij} - g_4 M_{pl}^{-4}R^3 - g_5 M_{pl}^{-4}R(R_{ij}R^{ij}) - \\ &- g_6 M_{pl}^{-4}R^i{}_jR^j{}_kR^k{}_i - g_7 M_{pl}^{-4}R\nabla^2 R - g_8 M_{pl}^{-4}\nabla_i R_{jk}\nabla^i R^{jk}\Big)\ , \end{aligned} \tag{16}$$

where g_i are dimensionless couplings. We should mention following remarks considering this action:

- When we impose projectability together with detailed balance we find that the action S_{db} is the same apart from the fact that N is the function of time only.
- There are just more 3 operators in the most general projectable action than in the one with detailed balance which means that the detailed balance does not simplify the action enough. For that reason the projectable version is more studied than the detailed balance one.
- g_0 controls the value of the cosmological constant which is not restricted.
- There are two types of Lorentz violating terms in the action. The ones that are contained in $\mathcal{V}$. These terms are suppressed by some scale

that can be determined by tuning the coupling g_2 to g_8. The second one is contained in the kinetic term and is related to the fact that λ is not generally equal to 1.

- All couplings are running. We generally hope that λ runs to 1 in the IR or sufficiently close to it in order to satisfy experimental constraints. As a result we find that the theory could effectively reduce to general relativity and diffeomorpism invariance would emerge as IR (approximate) symmetries. However given analysis requires to apply the renormalization group machinery to given problem which as far as we know has not been done yet.

3.3 Non-projectable Hořava-Lifshitz gravity

The most general formulation of the HL gravity corresponds to the case when neither detailed balance nor projectability are enforced. This version is known as *non-projectable HL gravity.* It is important to stress that when we abandon detailed balance then adding just some specific forms of the extra terms is not right way how to construct the most general form of the non-projectable HL gravity. In fact, radiative corrections will generate all possible terms compatible with the symmetries of the theory and hence all such terms should be taken into account. The crucial point of the non-projectable HL gravity is that the potential term should contain also terms constructed using the vector quantity

$$a_i = \frac{1}{N}\partial_i N \ , \tag{17}$$

as contractions of it with itself or curvature terms that lead to invariants [14]. The lowest order invariant that can be constructed from a_i is $a_i a^i$ so that the action will be of the form

$$S_{np} = \frac{M_{pl}^2}{2}\int d^3\mathbf{x} dt N\sqrt{g}\left[K_{ij}\mathcal{G}^{ijkl}K_{kl} + \zeta R + \eta a_i a^i + \frac{1}{M_A^2}L_4 + \frac{1}{M_B^4}L_6\right] \ , \tag{18}$$

where L_4 and L_6 include all possible 4th and 6th order operators respectively that one can construct using a_i and g_{ij}. This is the version of the theory that was firstly analyzed in [14] in order to resolve some inconsistencies in the non-projectable version of the theory which we will mention below. We would like to make following comments considering non-projectable version of HL gravity:

- The action above can be extended by inclusion of the cosmological constant.
- The scales M_A and M_B that suppress the higher order operators are arbitrary. This is an analog of the situation in the projectable case where we have arbitrary dimensionless couplings g_i.
- The number of operators in the non-projectable case is larger than in the projectable case.
- If we believe that the general relativity should be recovered in the IR we should demand that λ goes to 1 and η has to run to 0.

3.4 Horava-Lifshitz gravity with other symmetries

The actions that were presented above are invariant under foliation preserving diffeomorphism (8) and not under the full set of diffeomorphism. However the fact that the theory possesses less symmetries than general relativity implies that there is a possibility of the existence of a additional scalar mode that could be dangerous for the consistency of given theory.

Way how to eliminate this unwanted mode is l non-trivially extend the gauge symmetry of the theory so that it will have as many generators per space-time point as general relativity. This was firstly performed in [7] when the action was suitable modified in order to have an extra $U(1)$ symmetry. Explicitly, the action presented there has the form

$$\begin{aligned} S_{extraU(1)} = \frac{M_{pl}^2}{2} \int d^3\mathbf{x} dt N\sqrt{g} \Big[& K_{ij}\mathcal{G}^{ijkl}K_{kl} - \mathcal{V}(g,N) + \\ & + \nu\Theta^{ij}(2K_{ij} - \nabla_i\nabla_j\nu) - A(R-2\Omega)/N \Big] \end{aligned} \tag{19}$$

where Ω is a constant, A acts as a Lagrange multiplier, ν is an auxiliary scalar field and

$$\Theta^{ij} \equiv R^{ij} - \frac{1}{2}g^{ij}R + \Omega g^{ij} , \tag{20}$$

and where $\mathcal{V}$ contains 6th order operators. It was argued in [7] that at IR this theory reduces to general relativity and the extra gauge symmetry leads to the spin 2 graviton excitation. It was then shown in [8] that this form of the HL gravity is in fact equivalent to the Lagrange multiplier extension of the HL gravity which means that we add the additional term to the action. This term is constructed from the Lagrange multiplier that multiplies some function of canonical variables. Clearly such a term imposes an additional constraint in the theory which eliminates one degree of freedom. On the other hand such construction seems to be rather artifical and deserves further investigation.

4 Consistency of the projectable Version

In this section we perform more detailed analysis of the consistency of the projectable version of HL gravity. The significant property of the projectable version is that the lapse function $N(t)$ depends on time and hence the variation of the action with respect to N does not lead to the usual local Hamiltonian constraint but to a global constraint. In other words this is the constraint in the form of an integral over space. This is crucial difference with respect to the non-projectable case.

An important point in each physical theory is the number of physical degrees of freedom and their nature. The general answer on this question can be found in the Hamiltonian formulation of given theory when we identify all constraints, determine whether they are the first class or the second class constraints and then we perform the counting of the physical degrees of freedom [20]. However this procedure could be very troublesome due to the fact that we firstly have to find Hamiltonian of given theory, then determine all constraints and calculate the Poisson brackets between them. An alternative way how to determine physical content of given theory at least in some situation is to analyze the linearized equations of motion around some background. The simplest case is the flat background and then after suitable gauge fixing it is possible to find following equation of motion for propagating a spin-2 mode

$$\ddot{\tilde{H}}_{ij} = -[g_1\partial^2 + g_3 M_{pl}^{-2}\partial^4 + g_0 M_{pl}^{-4}\partial^6]\tilde{H}_{ij} \ , \tag{21}$$

where $\tilde{H}_{ij}$ is transverse and traceless

$$\partial_i \tilde{H}_{ij} = 0 \ , \delta^{ij}\tilde{H}_{ji} = 0 \ . \tag{22}$$

The crucial point is that the less symmetry we have the more degrees of freedom will be unconstrained so that we expect to find more excitations than in case of GR. It turns out that this is really the case and there is an extra scalar degree of freedom whose linerized dynamics is governed by the action

$$S_2^p = -M_{pl}^2 \int d^3\mathbf{x}dt \left[\frac{1}{c_h^2}\dot{h}^2 + h\partial^2 h + \frac{8g_2 + 3g_3}{M_{pl}^2}(\partial^2 h)^2 - \frac{8g_7 - 3g_8}{M_{pl}^2}(\partial^2 h)^3 \right] , \quad (23)$$

where

$$c_h^2 = \frac{1-\lambda}{3\lambda - 1} , \partial^2 = \partial^i \partial_i \ . \quad (24)$$

From the action (23) we see that the scalar mode is ghost (mode with negative kinetic term) on condition when $1 > \lambda > 1/3$. In fact, it turns out that in order the resulting instability not to be visible we have to demand that $|1 - \lambda| < 10^{-61}$. This value is clearly very low and it is difficult to believe that renormalization group flow could drive λ to this value.

It was shown in [10, 11] that when we perform the linearized analysis around de Sitter background we find that the scalar mode exhibit better behavior. On the other hand the qualitatively the result is the same: λ needs to be sufficiently close to 1 for the instability not to have visible consequences.

It is important to stress that the previous analysis was based on the linearized dynamics when the perturbative action is quadratic in h. However it was shown in many papers that such perturbative treatment breaks down when λ approaches 1 when the scalar mode is strongly coupled. In fact, it turns out that the strong coupling scale is phenomenologically unacceptably low since it is well known that we can tread gravity perturbatively at low energies. On the other hand we can ask the question whether when we include non-linear effect we could resolve the strong coupling problem by an analogue of the Vainstein mechanism in massive gravity [1]. In fact, it was shown in [13] that this is really possible in case of spherically symmetric, static configurations. Unfortunately definitive conclusion has not been reached yet and this proposal deserves further study.

5 Non-projectable version: Its dynamics and consistency

We consider non-projectable version of the theory with no-detailed balance imposed. The dynamics of the spin 2 graviton is the same as in the projectable version so that we will not discuss it here. Further, as in the projectable case there is an extra scalar degree of freedom whose dynamics is

[1] For recent review, see [12].

governed by the action

$$S_2^{np} = -M_{pl}^2 \int d^3\mathbf{x}dt \left[\frac{1}{c_h^2}\dot{h}^2 + \frac{\eta-2}{\eta} h\partial^2 h\right] \ . \qquad (25)$$

This is the action that is derived by linearizating the action (18) around the flat space and considering only the lowest order operators. Note that c_h^2 is defined in (24). On the other hand the low momentum phase velocity of the scalar is equal to

$$c_h'^2 = c_h^2 \frac{\eta - 2}{\eta} \ . \qquad (26)$$

h is the ghost mode when $c^2 > 0$ that implies that it is the ghost for

$$1 > \lambda > 1/3 \ . \qquad (27)$$

On the other hand the dispersion relation for the plane wave $e^{i\omega t - ik_i x^i}$ takes the form

$$\omega^2 = c_h'^2 k^i k_i \qquad (28)$$

so that there will be no tachyon instability for $c_h'^2 > 0$. These conditions imply following allowed ranges of λ and η [9]

$$\lambda > 1 \ , \quad 0 < \eta < 2 \ , \qquad (29)$$

or

$$\lambda < 1/3 \ , 0 < \eta < 2 \ . \qquad (30)$$

In the second region in the parameter space λ is very far from 1 where $\lambda = 1$ is the value required of general relativity so that the second region is not considered at all. It is important to stress that the presence of the operator $a_i a^i$ in the action (18) that is the lowest order operator which contributes to the quadratic action has a crucial impact on the consistency of the theory since now there is a region in parameter space (29) where h is not ghost and is also classically stable. Note that conclusion is not affected by inclusion of the higher order operators [14]. It is also important to stress that when we impose the detailed balance condition the term $\eta a_i a^i$ is not allowed and hence these nice properties are lost. Further, as we show below, the non-projectable version without the terms $a_i a^i$ is non-perturbativelly inconsistent from the Hamiltonian point of view.

5.1 Strong coupling

We argued above that the non-projectable version exhibits improved scalar dynamics with respect to the projectable version on condition when the detailed balance is not imposed. However it is also important to check whether the scalar mode is strongly coupled at low energies. To do this we have to consider the cubic action for the scalar [15]

$$S_3^{np} = M_{pl}^2 \int d^3\mathbf{x}dt \left(\left(1 - \frac{4(1-\eta)}{\eta^2} \right) h(\partial h)^2 - \frac{2}{c_h^4} \dot{h} \partial_i h \frac{\partial^i}{\partial^2} \dot{h} + \right.$$
$$\left. + \left(\frac{3}{2} + \frac{1}{\eta} \right) \left[\frac{1}{c_h^4} h \left(\frac{\partial_i \partial_j \dot{h}}{\partial^2} \right) - \frac{(2c_h^2+1)}{c_h^4} h \dot{h} \right] \right) , \tag{31}$$

where $\frac{1}{\partial^2}$ means the operator inverse to the operator ∂^2. As the next step we have to canonically normalize the low energy quadratic action which needs following redefinition

$$t = \sqrt{\frac{\eta}{2-\eta}} \frac{\hat{t}}{|c_h|} , \quad h = \left(\frac{\eta}{2-\eta} \right)^{1/4} \sqrt{|c_h|} \frac{\hat{h}}{M_{pl}} . \tag{32}$$

Then the cubic action takes the form

$$S_3^{np} = \frac{(2-\eta)^2}{\eta^{1/2} c_h'^{3/2} M_{pl}} \int d\hat{t} d^3x \left(c_h^2 \left(1 - \frac{8(1-\eta)}{(2-\eta)^2} \right) \hat{h}(\partial \hat{h})^2 - 2\hat{h}' \partial_i \hat{h} \frac{\partial^i}{\partial^2} \hat{h}' + \right.$$
$$\left. + \left(\frac{3}{2} + \frac{1}{\eta} \right) \left[\hat{h} \left(\frac{\partial_i \partial_j}{\partial^2} \hat{h}' \right)^2 - \left(\frac{2\eta c_h'^2}{2-\eta} + 1 \right) \hat{h} (\hat{h}')^2 \right] \right) , \tag{33}$$

where now $h' = \frac{dh}{d\hat{t}}$. We see that the cubic interactions are suppressed with respect to the quadratic ones by various scales $f(|\lambda - 1|, \eta) M_{pl}$ where f is an algebraic function whose form depends on which term we consider.

We know that $|\lambda - 1|$ and η measure deviations from Lorentz invariance which occur at arbitrary low scales. Of course, they have to be small for the theory to avoid experimental constraints on Lorentz violations. Let us consider the most interesting case where $c_h' \sim 1$ which is the value prefered some experimental constraints. Then we have $\eta \sim |\lambda - 1|$ and it turns out that the strong coupling scale occurs at $M_{sc} \sim 10^{15} GeV$. In other words the strong coupling scale is too high to be phenomenologically accessible. On the other hand the fact that there is a strong coupling scale implies the serious question considering the renormalizability of the theory since the arguments of the renormalizability were based on presumption that the perturbative treatment can be used to arbitrary high energies [15].

However the situation is more complicated since the strong coupling arguments given here do not consider the role of the higher order operators in the action. The way how to deal with these operators was proposed in [16]. It was argued there that in order to avoid the strong coupling we have to lower the scale that supresses the higher order operators in the action (18) below M_{sc}. In other words we have to impose $M_A \sim M_B \sim M_*$ where $M_* < M_{sc}$. However this procedure clearly requires an introduction of a second scale that is different from M_{pl} and also a hierarchy of scales. We also need a large dimensionless coupling since $M_{pl} \gg M_*$. On the other hand it was argued in [16] that this is technically natural. Explicitly, the careful analysis shows that M_* has to constrained as [15, 16]

$$10^{15}\mathrm{GeV} \gtrsim M_* \gtrsim 10^{11}\mathrm{GeV} \ . \tag{34}$$

This result suggests that there is a comfortable invterval for M_* within which non-projectable HL gravity avoids the strong coupling and also detectable Lorentz violations at least with current experimental accuracy.

6 Hamiltonian Formalism

It is very instructive to perform the Hamiltonian formulation of the HL gravity. It turns out that it is crucial whether we consider either projectable or non-projectable version. On the other hand the detailed balance does not play any significant role in the Hamiltonian formulation. The reason why we perform the Hamiltonian formulation is that it is very powerful in the analysis of the constraint structure and determining the number of the physical degrees of freedom even in the case of the full non-linear theory.

6.1 Projectable HL gravity

In this section we review the Hamiltonian formulation of the projecable HL gravity, following mainly the original formulation [2, 3] [2].

In case of the projectable version we have $N = N(t)$ and the Hamiltonian formulation is rather straightforward. More explicitly, let us consider following general action

$$S = \int dt d^3\mathbf{x} N\sqrt{g}(K_{ij}\mathcal{G}^{ijkl}K_{kl} - \mathcal{V}(h)) \ , \tag{35}$$

[2] More detailed treatment can be found in accompanied papers.

where K_{ij} is the extrinsic derivative

$$K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i) \ , \quad (36)$$

and where the generalized metric $\mathcal{G}^{ijkl}$ is defined as

$$\mathcal{G}^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl} \ , \quad (37)$$

where λ is real constant. Finally $\mathcal{V}(g)$ is general function of g_{ij} and its covariant derivative. Now we are ready to perform the Hamiltonian analysis of theory defined by the action (35). We firstly determine the momenta conjugate to N, N^i, g_{ij} from (35)

$$\begin{aligned} p_N(\mathbf{x}) &= \frac{\delta S}{\delta \partial_t N(\mathbf{x})} \approx 0 \ , \quad p^i(\mathbf{x}) = \frac{\delta S}{\delta \partial_t N_i(\mathbf{x})} \approx 0 \ , \\ p^{ij}(\mathbf{x}) &= \frac{\delta S}{\delta \partial_t g_{ij}(\mathbf{x})} = \frac{1}{\kappa^2}\sqrt{g}\mathcal{G}^{ijkl}K_{kl} \ . \end{aligned} \quad (38)$$

Then it is easy to find corresponding Hamiltonian

$$H = \int d^3\mathbf{x}(N\mathcal{H}_T + N^i\mathcal{H}_i) \ , \quad (39)$$

where

$$\begin{aligned} \mathcal{H}_T &= \frac{\kappa^2}{\sqrt{g}}\pi^{ij}\mathcal{G}_{ijkl}\pi^{kl} + \frac{1}{\kappa^2}\sqrt{g}\mathcal{V} \ , \\ \mathcal{H}_i &= -2g_{ik}\nabla_j\pi^{jk} \ . \end{aligned} \quad (40)$$

Now the requirement of the stability of the primary constraints $p_N \approx 0 \ , p_i(\mathbf{x}) \approx 0$ implies following secondary constraints:

$$\begin{aligned} \partial_t p_N &= \{p_N, H\} = -\int d^3\mathbf{x}\mathcal{H}_T \equiv -\Phi_N \approx 0 \ , \\ \partial_t p_i &= \{p_i, H\} = -\mathcal{H}_i \approx 0 \ . \end{aligned} \quad (41)$$

It is important to stress that Φ_N is the integrated constraint unlike the local Hamiltonian constraint in general relativity which is a direct consequence of the projectability condition. Clearly by definition we find

$$\{\Phi_N, \Phi_N\} = 0 \ , \left\{\Phi_N, \mathbf{T}_S(N^i)\right\} = 0 \tag{42}$$

where $\mathbf{T}_S(N^i) = \int d^3\mathbf{x} N^i \mathcal{H}_i$. This result shows that Φ_N is the first class constraint. However since Φ_N is global constraint it cannot affect the local dynamics which means that cannot be gauge fixed in order to eliminate some degree of freedom. In fact, the local constraints are $p^i \approx 0, \mathcal{H}^i \approx 0$. As a result we find that the projectable version of the HL gravity contains one additional propagating scalar mode with respect to the general relativity with all crucial consequences that were reviewed above.

6.2 Non-projectable version of HL gravity

Let us now consider non-projectable case when the requirement of the preservation of the constraint $p_N(t, \mathbf{x}) \approx 0$ implies an existence of the secondary constraint

$$\mathcal{H}_T(\mathbf{x}) \approx 0 \ . \tag{43}$$

Now however the crucial point is that for general form of the potential $\mathcal{V}$ given theory is non-consistent [17, 18, 19]. This inconsistency follows from the calculation of the Poisson brackets between the smeared forms of the constraints $\mathcal{H}_T$

$$\{\mathbf{T}_T(N), \mathbf{T}_T(M)\} = \int d^3\mathbf{x}(N(E_3^{ijk} D_{ijk} M - E_2^{ij} D_{ij} M + E_1^i D_i M + E_0 M) \ , \tag{44}$$

where $E_n^{i_1 \dots i_n}, n = 0, 1, 2, 3$ are tensor densities that depend on the canonical variables h_{ij}, π^{ij} and their spatial derivatives and where also $D_{ij} = D_{(i} D_{j)}$ etc.

Now in order to have the Hamiltonian constraint (43) to be preserved under time evolution we should calculate the Poisson bracket between (43) and $H = \int d^3\mathbf{x} N(\mathbf{x}) \mathcal{H}(\mathbf{x})$. Using (44) we find that in order the constraint $\mathcal{H}_T$ to be preserved we have to demand

$$E_3^{ijk} D_{ijk} N - E_2^{ij} D_{ij} N + E_1^i D_i N + E_0 N \approx 0 \ . \tag{45}$$

There are now two possible ways how to interpret the condition (45). The first one is to regard it as a new secondary constraint that imposes further constraints on the variables h_{ij} and π^{ij}. Considering N to be arbitrary we find following additional constraints

$$E_3^{ijk} \approx 0 \ , \quad E_2^{ij} \approx 0 \ , \quad E_1^i \approx 0 \ , \quad E_0 \approx 0 \ . \tag{46}$$

For generic potential we find a over constrained theory with no gravitational dynamics since all degrees of freedom are fixed. Since this is rather dissapointing result we mean that we should consider the second interpretation which seems to be more natural. Explicitly, we claim that (45) is the condition for the lapse N that has the role of a Lagrange multiplier in the action. We also mean that this is more natural interpretation from the point of view of the general analysis of the constrained systems [20]. The detailed analysis of the equation (45) performed in [18] showed that the only solution of (45) is $N = 0$. This is very unsatisfactory result that implies that the Hamiltonian is zero and hence there is no time evolution.

In summary, we either have too few gravitational degrees of freedom or there is no time evolution at all. Then we claim that given theory is dynamically inconsistent.

6.3 Healthy extended Hořava-Lifshitz gravity

It turns out that there is a way how to extend non-projectable HL gravity which could be dynamically consistent [14]. As we reviewed above the extension consists with the introducing the variable $a_i = \frac{\partial_i N}{N}$ to the potential so that it now has the form $\mathcal{V} = \mathcal{V}(g, a_i)$. The Hamiltonian analysis of this theory was firstly performed in [21] that was further elaborated in [22]. Let us briefly review the analysis presented in [21]. We consider the action in the form

$$S = M_p^2 \int dt d^3\mathbf{x} N\sqrt{h}(K_{ij}\mathcal{G}^{ijkl}K_{kl} - \mathcal{V}(h_{ij}, a_i)) \ . \tag{47}$$

The momenta conjugate to N, N^i are the primary constraints of the theory

$$\pi_N \approx 0 \ , \quad \pi_i \approx 0 \ . \tag{48}$$

Now the total Hamiltonian has the same form as above when the Hamiltonian constraint is

$$\mathcal{H}_0 = \frac{1}{M_p^2\sqrt{h}}\pi^{ij}\mathcal{G}_{ijkl}\pi^{kl} + M_p^2\sqrt{h}\mathcal{V}(h_{ij}, a_i) \ . \tag{49}$$

Note the crucial difference with the previous case when now the potential depends on a_i. In fact, the requirement of the preservation of the constraints

$\pi_N \approx 0, \pi_i \approx 0$ leads to the secondary constraint

$$\mathcal{C} = \mathcal{H}_0 - \frac{1}{N} D_i V^i \approx 0 \ , \quad \mathcal{H}_i \approx 0 \ , \tag{50}$$

where we defined the vector density

$$V^i(\mathbf{x}) = M_p^2 \frac{\delta}{\delta a_i(\mathbf{x})} \int N\sqrt{h}\mathcal{V}(h_{ij}, a_i) \ . \tag{51}$$

The momentum constraints $\mathcal{H}_i$ are the same as in projectable case and their smeared forms are generators of spatial diffeomorphism. It is important that $\mathcal{C} \approx 0$ depends on N and its spatial derivatives. This fact implies

$$\{\mathcal{C}, p_N\} \neq 0 \tag{52}$$

which shows that $\mathcal{C}$ and π_N are the second class constraints that may be used for eliminating N and π_N from the set of canonical variables. More precisely, for general potential the constraint $\mathcal{C}$ is quite complicated partial differential equation for the lapse and we presume that there exists the solution of this constraint. The scalar constraint determines N up to a constant rescaling a time-dependent prefactor. This freedom left in N is associated with a time reparameterization symmetry.

It turns out that there is another first class constraint. Let us consider the combination

$$\Pi_N = \int d^3\mathbf{x} N p_N \approx 0 \tag{53}$$

that has following non-zero Poisson brackets

$$\{N, \Pi_N\} = N \ , \{\pi_N, \Pi_N\} = -\pi_N \ . \tag{54}$$

It turns out that the vector a_i is invariant

$$\{a_i, \Pi_N\} = 0 \ . \tag{55}$$

It is important to check the stability of the constraint Π_N

$$\partial_t \Pi_N = \{\Pi_N, H_T\} = -\int d^3\mathbf{x} N\mathcal{H}_0 \equiv -\Phi_0 \approx 0 \ . \tag{56}$$

In other words there is the second global constraint

$$\Phi_0 = \int d^3\mathbf{x} N\mathcal{H}_0 \ . \tag{57}$$

It turns out that Π_N has weakly vanishing Poisson brackets with all constraints and hence it is the first class constraint.

As the final point we have to check the stability of the secondary constraints. We are not going to discuss the stability of the spatial diffeomorphism constraints since it is trivial and we focus on the constraint $\mathcal{C}$. Let us consider the total Hamiltonian that contains all constraints in the form

$$H_T = \int d^3\mathbf{x}(N\mathcal{H}_0 + \Gamma\mathcal{C} + w_N N p_N) \ , \tag{58}$$

where we ignored constraints related to the spatial diffeomorphism $\pi_i \approx 0$, $\mathcal{H}_i \approx 0$. Now the requirement of the stability of the constraint $\pi_N \approx 0$ implies

$$\partial_t \pi_N = \{\pi_N, H_T\} \approx \int d^3\mathbf{x}\Gamma(\mathbf{x}) \{\pi_N, \mathcal{C}(\mathbf{x})\} = 0 \ . \tag{59}$$

The equation above becomes the partial differential equation for Γ that has solution $\Gamma = 0$. Then the requirement of the stability of the constraint $\mathcal{C}$ implies

$$\{\mathcal{C}, H\} = \int d^3\mathbf{x} N \left(\{\mathcal{C}, \mathcal{H}_0(\mathbf{x})\} + w_N(\mathbf{x}) \{\mathcal{C}, p_N(\mathbf{x})\}\right) = 0 \ . \tag{60}$$

This equation can be solved for w_N. It is important to stress that w_N is not completely solved by (60) that determines w up to a solution of the homogeneous equation

$$\int d^3\mathbf{x} N(\mathbf{x}) w_N(\mathbf{x}) \{\mathcal{C}, p_N(\mathbf{x})\} = 0 \ . \tag{61}$$

This has the solution when $w_N = \kappa$ since in this case this Poisson bracket is zero due to the fact that it is equal to $\{\Pi_N, \mathcal{C}\}$ and as we argued above this is zero. In other words the general solution of the equation (60) is

$$w_N = \bar{w}_N[h_{ij}, \pi^{ij}, N] + \kappa \ , \tag{62}$$

where $\bar{w}_N[h_{ij}, \pi^{ij}, N]$ is particular solution of (60). This solution can be substituted back into the Hamiltonian and we obtain the final form of the Hamiltonian

$$H = \int d^3\mathbf{x} N(\mathcal{H}_0 + \bar{w}_N p_N) + \kappa \int N p_N \ , \tag{63}$$

where we see that the Hamiltonian is linear combination of the first class constraints Φ_0 and Π_N where the first one is the generator of the global time reparameterization and the second one corresponds to the constant rescaling of N. Note that the constraints $\mathcal{C}, \pi_N$ are the second class constraints that vanish strongly and can be used to express N as the function of the phase space variables. For detailed treatment see [22]. It is important to stress the now we have consistent theory from the Hamiltonian point of view that however still contains an additional scalar mode. However as we argued above it is possible to choose parameters in the potential in such a way that given model is phenomonologically acceptable.

7 Conclusion

Hořava-Lifshitz gravity is very promising attempt how to define renormalizable theory of gravity. We shown that there are two versions: projectable or non-projectable Hořava-Lifshitz gravity. We argued that the projectable version suffers from serious problems due to the presence of the scalar mode which is either clasically or quantum mechanically unstable and also exhibits a strong coupling at low energies. On the other hand non-projectable version, when all operators allowed by symmetries are included, seems to evercome all these problems.

Despite the intensive study of HL gravity there are still open issues with it. There are following two most important ones: It was argued that the theory is power-counting renormalizable. Even if this is a strong indication of UV completeness the renormalizability beyond the power counting has not be proved yet. Further, the renormalization group flow of the various couplings has not been studied and hence we do not really know whether the theory approaches general relativity in IR ($\lambda \rightarrow 1$, $\eta \rightarrow 0$) or not. The second important problem is to analyze the role of matter and its coupling to the gravity. The matter action will have to include higher order spatial derivatives that implies that there will be modifications in the dispersion relations of matter fields. Clearly such a modifications have to be restricted by experimental bounds. Further, the coupling between matter and the scalar graviton could lead to violations of the equivalence principle. We see that more work is needed in order to see whether HL gravity is the right way in the construction of the renormalizable theory of gravity.

The author of this thesis also believes that the results described in the thesis are very important and bring new information about various topics in Hořava-Lifshitz gravity, especially in its Hamiltonian formulation and $F(R)$ generalization.

8 The papers

Paper I

In this paper we propose new non-relativistic $p+1$ dimensional theory. This theory is defined in such a way that the potential term obeys the principle of detailed balance where the generating action corresponds to p-brane action. This condition ensures that the norm of the vacuum wave functional of p+1 dimensional theory is equal to the partition function of p-brane theory.

Paper II

The program initiated in the previous paper was continued in the second paper. We extend the analysis presented there to the case of stable and unstable Dp-branes.

Paper III

This paper is devoted to the construction of new type of $f(R)$ theories of gravity that are based on the principle of detailed balance. It is a generalization of the original Petr Hořava's idea of construction of the general relativity based on the principle of detailed balance. We discuss two versions of these theories with and without the projectability condition.

Paper IV

In this paper we continue our analysis of generalized $f(R)$ theories of gravity that we began in previous paper. We introduce new models of f(R) theories of gravity that are generalization of Horava-Lifshitz gravity.

Paper V

Here we discuss the Hořava-Lifshitz gravity from different point of view. We generalize the analysis of the dynamics of point particle in Horava-Lifshitz background to the case of string probe when we replace the Hamiltonian constraint of the Polyakov string with the constraint that breaks Lorentz invariance of target space-time. Then we find corresponding Lagrangian and argue that the world-sheet theory is invariant under foliation preserving diffeomorphism. Finally we discuss the Hamiltonian dynamics and show that this is well defined on condition that the world-sheet lapse function obeys the projectability condition.

Paper VI

This note is devoted to the study of Hamiltonian formalism of modified F(R) Horava-Lifshitz theories of gravity that were proposed in [23]. We also study Hamiltonian formulation of the healthy extended Horava-Lifshitz gravities and show that these theories have many unusual properties that imply their possible inconsistency.

Paper VII

In this paper we continue the study of the Hamiltonian formalism of the healthy extended Hořava-Lifshitz gravity. We find the constraint structure of given theory and argue that this is the theory with the second class constraints. Then we discuss physical consequence of this result. We also apply the Batalin-Tyutin formalism of the conversion of the system with the second class constraints to the system with the first class constraints to the case of the healthy extended Hořava-Lifshitz theory.

Paper VIII

We continue our study of the Lorentz breaking string theories. These theories are defined as string theory with modified Hamiltonian constraint which breaks the Lorentz symmetry of target space-time. We analyze properties of this theory in the target space-time that possesses isometry along one direction. We also derive the T-duality rules for Lorentz breaking string theories and show that they are the same as that of Buscher's T-duality for the relativistic strings.

Paper IX

In this paper we formulate RFDiff invariant $f(R)$ Horava-Lifshitz gravity that are theories which are invariant under restricted diffeomorphism

$$x'^i = x^i + \zeta^i(\mathbf{x}, t) \ , \quad t' = t + \delta t \ , \delta t = \text{const} \ . \tag{64}$$

We show that these theories are related to the ghost condensation in the projectable version of Horava-Lifshitz gravity.

Paper X

We perform the Hamiltonian analysis of non-relativistic covariant Horava-Lifshitz gravity in the formulation presented recently in [24]. We argue that the resulting Hamiltonian structure is in agreement with the original construction of non-relativistic covariant Hořava-Lifshitz gravity presented in [7]. Then we extend this construction to the case of RFDiff invariant Hořava-Lifshitz theory. We find well behaved Hamiltonian system with the number of the first and the second class constraints that ensure the correct number of physical degrees of freedom of gravity.

Paper XI

This paper is devoted to the study of various aspects of projectable $F(R)$ Hořava-Lifshitz gravity. We show that some versions of $F(R)$ Hořava-Lifshitz gravity may have stable de Sitter solution and unstable flat space solution. In this case, the problem of scalar graviton does not appear because flat space is not vacuum state. Generalizing the $U(1)$ Hořava-Lifshitz theory proposed in [7], we formulate $U(1)$ extension of scalar theory and of $F(R)$ Hořava-Lifshitz gravity. The Hamiltonian approach for such the theory is developed in full detail. It is demonstrated that its Hamiltonian structure is the same as for the non-relativistic covariant Hořava-Lifshitz gravity. The spectrum analysis performed around flat background indicates towards the consistency of the theory because it contains graviton with only transverse polarization.

Paper XII

We consider RFDiff invariant Hořava-Lifshitz gravity action with additional Lagrange multiplier term that is a function of scalar curvature. We find its Hamiltonian formulation and we show that the constraint structure implies the same number of physical degrees of freedom as in general relativity.

Paper XIII

We formulate higher derivative gravity with Lagrange multiplier constraint and scalar projectors. Its gauge-fixed formulation as well as vector fields formulation is developed and corresponding spontaneous Lorentz symmetry breaking is investigated. We show that the only propagating mode is higher derivative graviton while scalar and vector modes do not propagate.

Paper XIV

We construct Hořava-Lifshitz gravities that are invariant under anisotropic Weyl scaling. This construction is based on an extension of the group of symmetries of healthy extended Hořava-Lifshitz gravity and RFDiff invariant Hořava-Lifshitz gravity. We find their Hamiltonian formulation and determine their constraint structure.

Paper XV

In this note we study the relation between F(R) and scalar tensor Horava-Lifshitz gravity. We find that due to the broken diffeomorphism invariance corresponding scalar tensor theory has more complicated form than in case of the full diffeomorphism invariant F(R) theory of gravity. We also show that in the low energy limit this theory flows to the relativistic scalar tensor theory of gravity.

Paper XVI

In order to explore some general features of modified theories of gravity which involve higher derivatives and spontaneous Lorentz and/or diffeomorphism symmetry breaking, we study the recently proposed new version of covariant renormalizable gravity (CRG). CRG attains power-counting renormalizability via higher derivatives and introduction of a constrained scalar field and spontaneous symmetry breaking. We obtain an Arnowitt-Deser-Misner representation of the CRG action in four-dimensional spacetime with respect to a foliation of spacetime adapted to the constrained scalar field. The resulting action is analyzed by using Hamiltonian formalism. We discover that CRG contains two extra degrees of freedom. One of them carries negative energy (a ghost) and it will destabilize the theory due to its interactions. This result is in contrast with the original paper [25], where it was concluded that the theory is free of ghosts and renormalizable when we analyze fluctuations on the flat background.

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