NUMERICAL SOLUTION OF STEADY AND UNSTEADY FLOW OVER A PROFILE IN A CHANNEL

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Abstract

The work deals with steady and unsteady solution of subsonic flow over a profile DCA 18% in a channel. For the computation the predictor-corrector MacCormack scheme with modified TVD Causon's artificial dissipation is used. Firstly, the steady state solution compared to the experimental results is presented. Than a simple unsteady model based on pressure change at the outlet area of the computational domain and finaly an unsteady model obtained with the use of ALE method (moving mesh) are presented.

Mathematical Model The behaviour of flow in both cases (steady and unsteady) is described by the system of compressible Euler equations in conservation form:

$$W_t + F_x + G_y = 0. \tag{1}$$

where the vector of conservative variables W and inviscid fluxes F, G are

$$W = \left\|\rho, \rho u, \rho v, e\right\|^T,\ F = \left\|\rho u, \rho u^2 + p, \rho uv, (e+p)u\right\|^T,\ G = \left\|\rho v, \rho uv, \rho v^2 + p, (e+p)v\right\|^T.$$

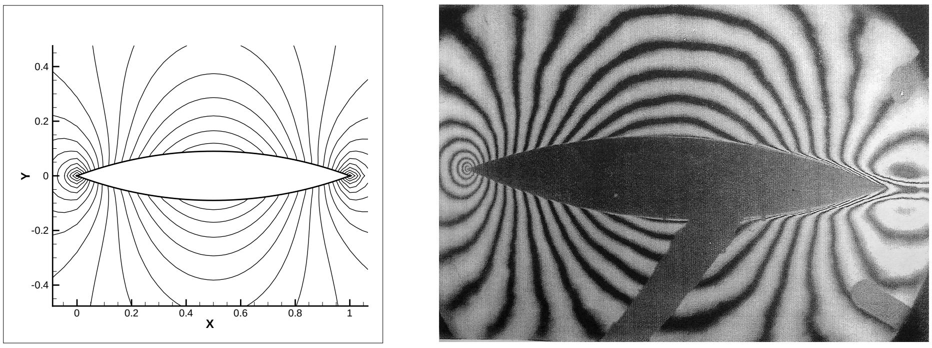
To solve this system following relation (equation of state for ideal gas) is added $p = (\kappa - 1)\left[e - \frac{1}{2}\rho(u^2 + v^2)\right], \quad \kappa = \frac{c_p}{c_v}$. We consider ρ - density, (u, v) - velocity vector, p - pressure and e - total energy per unit volume. Boundary conditions in the inlet area are given by three prescribed values, fourth one is extrapolated, whereas in the outlet area the conditions are given by prescribed pressure.

Nonstationary effects Two models of nonstationary flow have been implemented:

1. The first one was caused by pressure change in the outlet area of computational domain given by the condition $p_{outlet} = p_\infty(1 + 0.2 sin(ft))$, where $f\,[s^{-1}]$ is frequency and $t\,[s]$ is time.

2. Considering the second model, prescribed oscillations of the profile fixed in the point of an elastic exis were given by the formula $\varphi = \varphi_0\, sin(2\pi f t)$, where $\varphi\,[rad]$ is the angle of rotation of the profile from equilibrium position and $\varphi_0\,[rad]$ is amplitude of oscillations. To treat the vibrating profile, the ALE method is used (the mesh is deformed with respect to profile rotation).

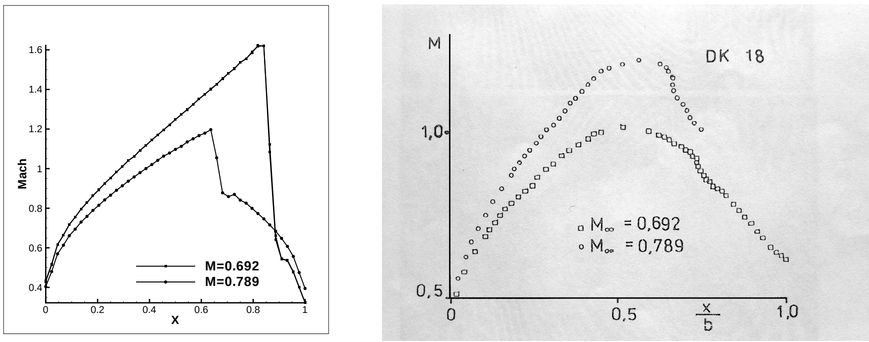
Numerical Scheme The system (1) was numericaly solved by finite volume method with the use of predictor-corrector MacCormack scheme (cell-centered form) with Jameson's and modified Causon's TVD artificial dissipation. Computational area was discretized by structured H-type mesh containing 156×112 cells (i.e. 17472). Development of nonstationary flow in the case of ALE computation was observed on behaviour of lift coefficient given as $c_n = \frac{\oint P dx}{\frac{1}{2} u_{ref}^2 \rho_{ref}}$.



(a) Numerical result.

(b) Experimental result.

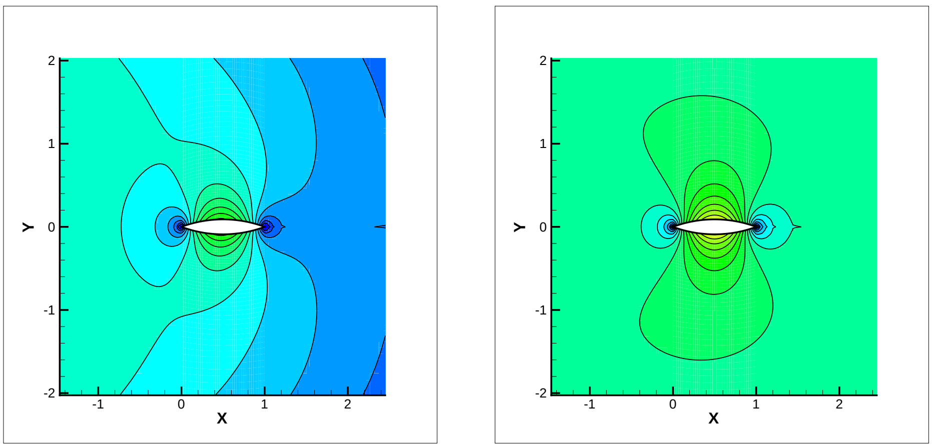
Figure 1: DCA 18%, $M_\infty = 0.526$, $\alpha = 0°$, isolines of Mach number, stationary computation.



(a) Numerical result.

(b) Experimental result.

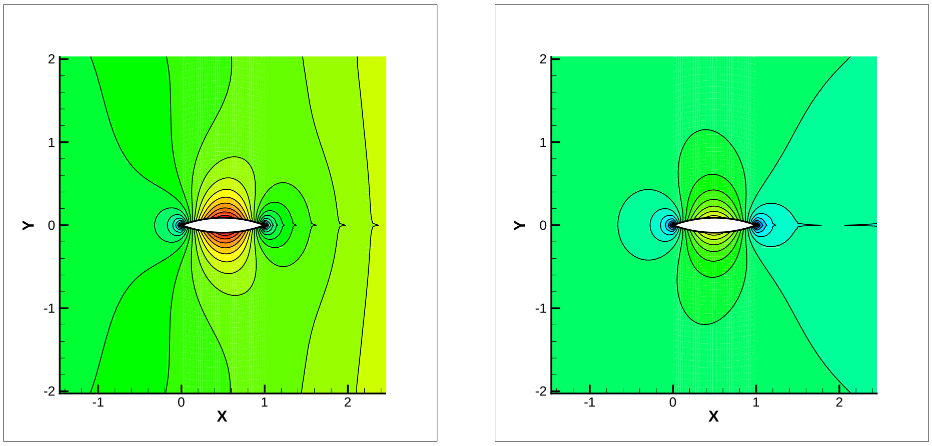
Figure 2: DCA 18%, $M_\infty = 0.526, \alpha = 0°$, behaviour c_p for various inlet velocities, stationary computation.



(a) $t = \frac{1}{2}\pi$.

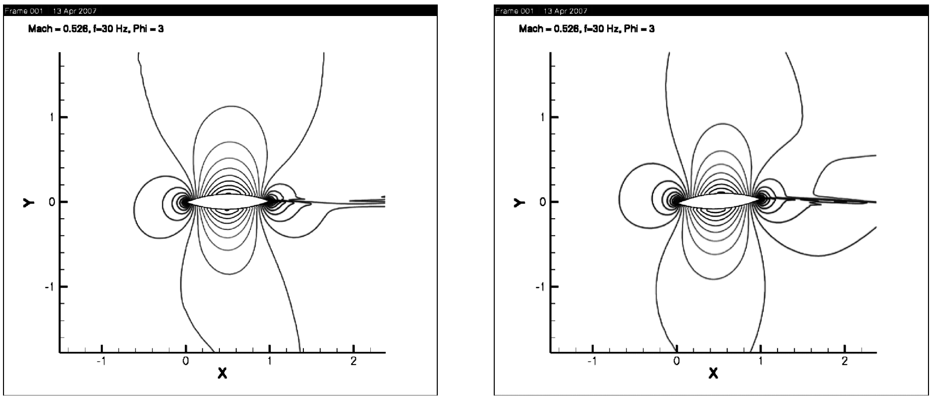
(b) $t = \pi$.

Numerical Results At first, we present numerical results for stationary flow compared with experimental results of the Institute of Thermomechanics CAS followed by the results of mentioned nonstationary models.

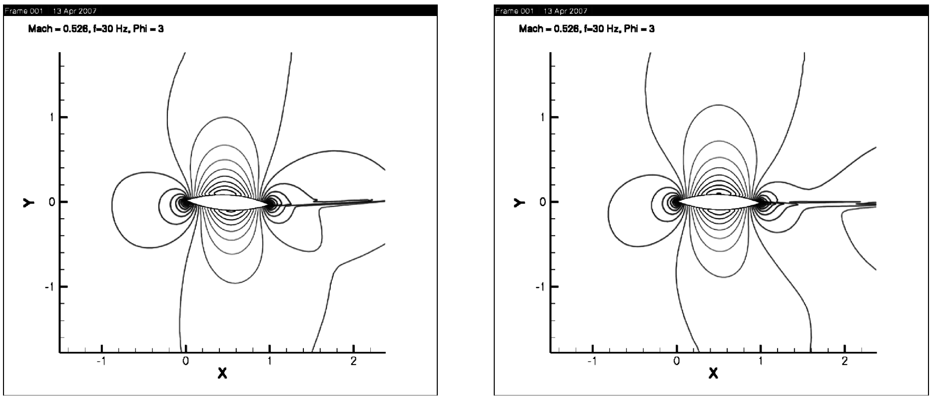


(c) $t = \frac{3}{2}\pi$.. (d) $t = 2\pi$.

Figure 3: DCA 18%, $M_\infty = 0.526, \alpha = 0°$, Mach number isolines, unsteady computation - pressure change in the outlet.



(a) $t = \frac{1}{2}\pi$. (b) $t = \frac{1}{2}\pi$.



(c) $t = \frac{3}{2}\pi$. (d) $t = 2\pi$.

Figure 4: DCA 18%, $M_\infty = 0.526$, Mach number isolines, unsteady computation, prescribed oscillations, ALE.

Conclusion The numerical method solving steady and unsteady inviscid compressible flow around a profile with one degree of freedom in a channel has been developed and preliminary results (showing

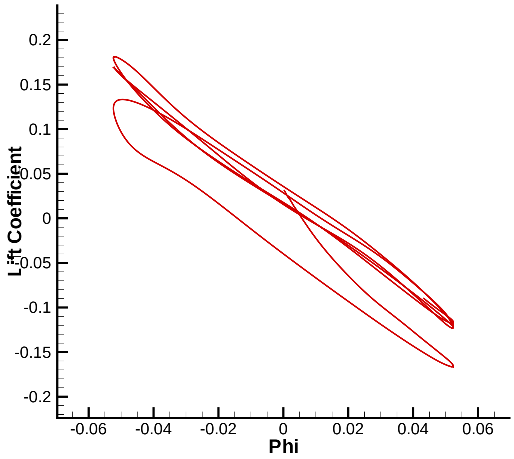


Figure 5: DCA 18%, $M_\infty = 0.526$, lift coefficient behaviour, unsteady computation, prescribed oscillations, ALE.

all the flow characteristic as expected) have been presented. Future steps intended are implementing a model able to handle one or two number of freedom and also flow induced aeroelastic effects.

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