INDIRECT MODELLING OF A DIFFUSION PROCESS IN THE FLOW SYSTEM WITH RETENTION

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Abstract
The contribution deals with a certain mathematic model of pelt washing by water in the flow system with retention which is an important part of a leather process. In the process the raw hide is changed into a leather. The process is extremely demanding of sources, especially the water, electric power and chemical agents. The requirements for environmental protection confirm the necessity of making use of possibilities of mathematical modelling by the help of computer algebraic system for optimization and controlling of the process.

Keywords
Mathematic model, flow washing of pelt, calcium hydroxide concentration field, deliming of pelt, computer algebraic system.

INTRODUCTION AND FORMULATION OF PROBLEM

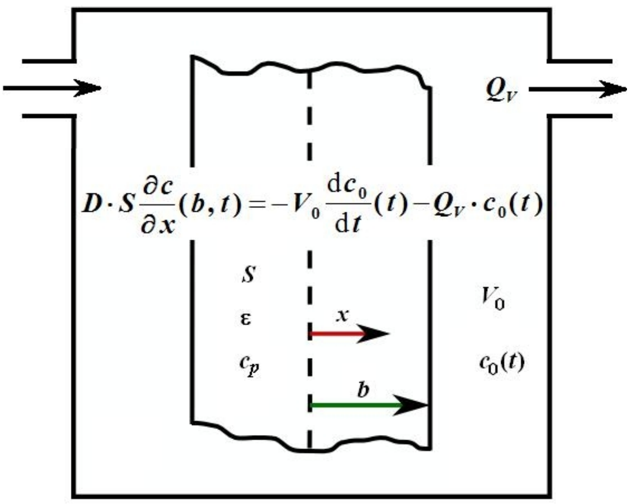

Fig. 1 Model of flow washing with retention

During the process the washing liquid with the volume flow rate $Q_V = \mathrm{d}V_0 / \mathrm{d}t$ is fed continually into and out of the system (see Fig. 1) whereas a certain constant volume V_0 of washing liquid is held in the washing equipment. The pelt with volume V stays during the washing process in the washing equipment. In our case the pure water represents the washing liquid and the wash-out component (calcium hydroxide) is made up by the solid elements which are not soluble in the washing water.

V	volume of solid phase (i.e. of pelt sample)	m^3
V_0	volume of washing water as a deliming reagent in washing equipment	m^3
Q_V	volume flow rate (the time derivative $\mathrm{d}V_0/\mathrm{d}t$)	$m^3 \cdot s^{-1}$
t	time	s
c	$Ca(OH)_2$ volume concentration in pelt	$kg \cdot m^{-3}$
c_0	$Ca(OH)_2$ volume concentration in washing equipment	$kg \cdot m^{-3}$
c_p	$Ca(OH)_2$ initial concentration in pelt	$kg \cdot m^{-3}$
D	effective diffusion coefficients of washing component from solid phase	$m^2 \cdot s^{-1}$
D^*	modified diffusion coefficient (of washing)	$m^2 \cdot s^{-1}$
x	position coordinate	m
b	half thickness of pelt	m
ε	porosity (ratio of poroses volume to pelt sample volume)	1
Na	soak number (the ratio V_0/V)	1
q_n	n-th root of a certain transcendent equation	1
A	sorption balance constant (from Langmuir's sorption isotherm)	1
B	sorption balance constant (from Langmuir's sorption isotherm)	$m^3 \cdot kg^{-1}$
S	area of pelt	m^2
F_o	Fourier criterium (dimensionless time)	1
C	$Ca(OH)_2$ dimensionless volume concentration in pelt	1
C_0	$Ca(OH)_2$ dimensionless volume concentration in washing equipment	1
X	dimensionless space coordinate	1
L	dimensionless flow of washing liquid	1

The solution of model is given by the field of concentration $c(x, t)$ of calcium on the half-stripe domain $G = \left\{ (x,t) \in \mathbf{R} \times \mathbf{R}^+ \;\middle|\; 0 \le x \le b,\ t > 0 \right\}$. We want to find the function $c(x, t)$ that satisfies the initial boundary problem

$$D \frac{\partial^2 c}{\partial x^2} = \frac{\partial c}{\partial t} + \frac{\partial c_A}{\partial t} \tag{1}$$

$$c_A = \frac{Ac}{Bc+1} \tag{2}$$

$$c(x,0) = c_p \tag{3}$$

$$c_0(0) = 0 \tag{4}$$

$$c(b,t) = \varepsilon \cdot c_0(t) \tag{5}$$

$$\frac{\partial c}{\partial x}(0,t) = 0 \tag{6}$$

$$D \cdot S\ \frac{\partial c}{\partial x}(b,t) \ = -V_0 \cdot \frac{\mathrm{d} c_0}{\mathrm{d} t}(t) - Q_V \cdot c_0(t)\,. \tag{7}$$

Equation (1) represents calcium ions diffusion from pelt sample in the direction of water bath. The expression of the right hand side last term of equation depends on desorption mechanism of washing component from solid phase. If we suppose that diffusion is determining for change rate of concentration then it is possible to express the dependence of c_A (of bound component) on the unbound component c by the relation of Langmuir's sorption isotherm (2). Condition

(3) shows the initial distribution of calcium ions concentration in solid phase-pelt. Relation (4) describes that we use pure water for pelt bath washing. Relation (5) holds under condition of a perfectly mixed liquid phase. Boundary condition (6) denotes that field of concentration in solid phase is symmetric. Boundary balance condition (7) denotes the equality of the diffusion flux at the boundary between the solid and the liquid phases with the speed of accumulation of the diffusing element in the surrounding.

Suppose that considered concentration $c(x, t)$ of calcium hydroxide is very low, i.e. $B \cdot c << 1$, we can define domain, in which sorption isotherm comes into the linear form and thus the relation (2) will be in the more simple form

$$c_A = A \cdot c \tag{8}$$

and system of equations (1 – 7) will be in the linear form

$$\frac{\partial c}{\partial t} - D^* \frac{\partial^2 c}{\partial x^2} = 0 \quad \left(\text{where constant } D^* = \frac{D}{1+A} \right). \tag{9}$$

MAIN RESULTS

We introduce dimensionless variables for the solution of equation (9) with additional conditions (3 – 7)

$$C = \frac{c}{c_p}, \; C_0 = \frac{c_0}{c_p}, \; F_o = \frac{D \cdot t}{b^2 \cdot (1+A)}, \; X = \frac{x}{b}, \; Na = \frac{V_0}{V}, \; L = \frac{Q_V \cdot b^2}{D^* \cdot V_0}. \tag{10}$$

By means of Laplace transformation we obtain analytic solution. Final solution given by dimensionless concentration field $C(X, F_o)$ in pelt holds

$$C(X, F_o) = 2\sum_{n=1}^{\infty} \frac{[DV_0 q_n^2 - (1+A)Q_V b^2] e^{-q_n^2 \cdot F_o} \cos(q_n \cdot X)}{q_n [DV_0 q_n (q_n \sin q_n - 2\cos q_n) - (1+A)(DV\varepsilon + Q_V b^2)\sin q_n - (1+A)DV\varepsilon q_n \cos q_n]} \tag{11}$$

where q_n is the n-th positive root of the following transcendent equation

$$\tan q = \frac{(1+A)Q_V b^2 - q^2 V_0 D}{(1+A)DV\varepsilon q}. \tag{12}$$

Now it follows the graphics processing of calcium hydroxide concentration field solution $C(X, F_o)$ from (11) in pelt in case of flow washing with retention by the computer algebraic system Maple.

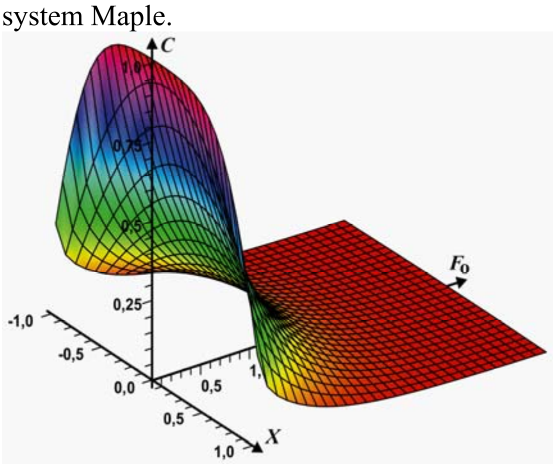

Fig. 2 Model of calcium hydroxide concentration field *C* in pelt in case of flow washing with retention

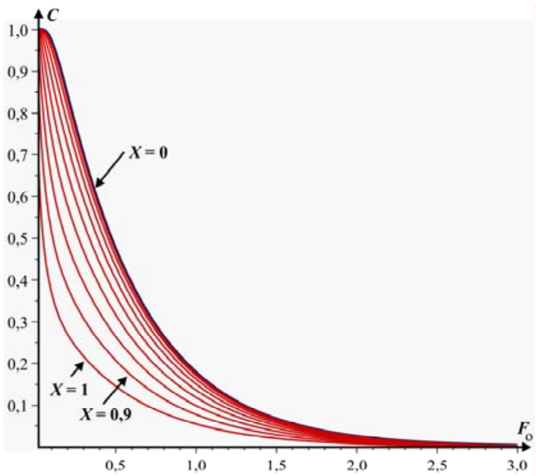

Fig. 3 Auxiliary view from the previous figure for uniform distribution of dimensionless distances *X*

CONCLUSION

The above mentioned indirect model is space-one-dimensional but it is sufficiently usable.This model generalizes the model of bath washing of the pelt because it includes the brand-new volume flow rate term ($Q_V = \mathrm{d}V_0/\mathrm{d}t$) as in the boundary balance condition (7) as well as in the solution (11).

Calcium hydroxide concentration field in the pelt doesn't show so much marked changes in the case of flow washing with respect of bath washing in the time-dependence near the beginning of flow washing.

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