IMPLEMENTATION OF AN ALGEBRAIC BYPASS TRANSITION MODEL INTO TWO-EQUATION TURBULENCE MODEL FOR A FINITE VOLUME METHOD SOLVER

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1 Introduction

The mathematical model of turbulent flow, based on the finite volume method and two-equation turbulence model, was extended by an algebraic model of the bypass transition taking into account the effect of free-stream turbulence and pressure gradient on the laminar/turbulent transition. The intermittent feature of flow in the transition region is described by the algebraic relation for the intermittency parameter and empirical relations for the onset and length of the transition region.

2 Formulation of the problem

We solve a system of averaged Navier-Stokes equations in two spatial dimensions

$$\frac{\partial \mathbf{w}}{\partial t}+\frac{\partial \mathbf{f}}{\partial x}+\frac{\partial \mathbf{g}}{\partial y}=\frac{\partial \mathbf{r}}{\partial x}+\frac{\partial \mathbf{s}}{\partial y} \tag{1}$$

with **w** vector of conserved variables, **f** and **g** are inviscid flux vectors, **r** and **s** are viscous flux vectors. We consider perfect gas ($\kappa = 1.4$) only. The system of governing equation was closed by the turbulence model with the turbulent viscosity. The two-equation k-ω SST model proposed by Menter [1] was considered. Appropriate initial and boundary conditions are prescribed.

3 Transition model

Transition models are based on the algebraic and/or transport equation for the intermittency coefficient γ. Nevertheless, all the transition models are dependent in some extent on empirical relation for the transition onset. Therefore, existing models of bypass transition have a limited range of applicability. We deal with the transition model based on the empirical relation for the intermittency coefficient

$$\gamma = 1-\exp\left[-\hat{n}\sigma\left(\mathrm{Re}_x-\mathrm{Re}_{xt}\right)^2\right] \tag{2}$$

proposed by Narasimha [2]. For simplicity, it is supposed that the intermittency coefficient is dependent on x coordinate only. The position of the transition onset is described by the Reynolds number Re_{xt} determined by means of the momentum Reynolds number $\mathrm{Re}_{\vartheta t} = f\,(Tu, \lambda_t)$ where Tu (%) is the free-stream turbulence level and λ_t is the pressure gradient parameter. Both parameters are considered at the location of the transition onset. Příhoda et al. [3] proposed the relation

$$\mathrm{Re}_{\vartheta t}=\mathrm{Re}_{\vartheta to}\left[1+0.25\exp(-\mathrm{Tu})\frac{1-\exp(-40\lambda_t)}{1+0.4\exp(-40\lambda_t)}\right] \tag{3}$$

based on experiments of Fasihfar and Johnson [4] and the relation for the flat plate flow according to Mayle [5].

The length of the transition region is given by parameters describing the spot generation rate $\hat{n}$ and spot propagation parameter σ. The simplest correlation uses the empirical parameter

$$N=\hat{n}\sigma\,\mathrm{Re}_{\vartheta t}^{3} \tag{4}$$

proposed by Narasimha [2] . For flat plate flow, it can be expressed by the relation

$$N_o=0.86\,\mathrm{x}\,10^{-3}\,\mathrm{Tu}^{-0.564} \tag{5}$$

given by Gostelow, Blunden and Walker [6]). The effect of the pressure gradient is correlated by empirical relations

$$N=0.86\,\mathrm{x}\,10^{-3}\exp(2.134\lambda_t\ln\mathrm{Tu}-59.23\lambda_t-0.564\ln\mathrm{Tu}) \qquad \text{for} \quad \lambda_t<0$$

$$N=0.86\,\mathrm{x}\,10^{-3}\,\mathrm{Tu}^{-0.564}\exp\left(-10\sqrt{\lambda_t}\right) \qquad \text{for} \quad \lambda_t>0 \tag{6}$$

proposed by Solomon, Walker, Gostelow [7].

To avoid the calculation of the momentum Reynolds number in cases with complicated geometry and unstructured meshes, the vorticity Reynolds number Re_ν is used. According to Menter et al. [8], these two parameters can be correlated for the Blasius boundary layer by the relation

$$\mathrm{Re}_\vartheta=\frac{\mathrm{Re}_{\nu\max}}{2.193} \tag{7}$$

where $\mathrm{Re}_{\nu\max}$ is the maximum of the vorticity Reynolds number

$$\mathrm{Re}_{\nu\max}=\max_y\left(\frac{y^2|\Omega|}{\nu}\right) \tag{8}$$

where y is the distance to the nearest wall and Ω is the absolute value of vorticity

$$\Omega=\sqrt{2\Omega_{ij}\Omega_{ij}} \tag{9}$$

The relation (7) does not change too much for flows with pressure gradient.

4 Numerical solution

The finite volume method of the cell centered type with the modification of the approximative Roe's Riemann solver, the linear least square reconstruction and the Barth's limiter was developed. The viscous fluxes are discretized in the central manner on a mesh dual to the cell faces. Time integration is performed with linearized Euler backward formula and local time stepping is used. The system of equations is solved with GMRES method and ILU(0) preconditioning. The method works on general unstructured meshes.

The intermittency coefficient resulting from the transition model gives its value on the boundary. Inside the computational domain we take the value γ at the point on the wall boundary, which lies closest to the considered place. Using the Boussinesq´s hypothesis, the intermittency coefficient influences in the transition region the effective viscosity given by the relation

$$\mu_{ef}=\mu+\gamma\mu_t \tag{10}$$

where μ_t is the turbulent viscosity. Following corresponding modifications of the SST turbulence model were made in transport equations for the turbulent energy k and for the specific dissipation rate ω.

The transition model is not valid on the leading edge. This region has to be excluded from the consideration.

5 Numerical results

5.1 ERCOFTAC test cases

We consider three test cases related to flat-plate transitional 2D boundary layers flows with different free-stream turbulence level. Test cases T3A, T3B, and T3Aminus from the ERCOFTAC database [9, 10] were chosen. The domain of solution consists of rectangle [-0.2, 1.58] $\times$ [0, 0.3]. The wall boundary is located at $y = 0$, $x \geq 0$. The symmetry is imposed at $y = 0$, $x < 0$. Output is at $x = 1.58$. At all the other boundaries the free stream is prescribed.

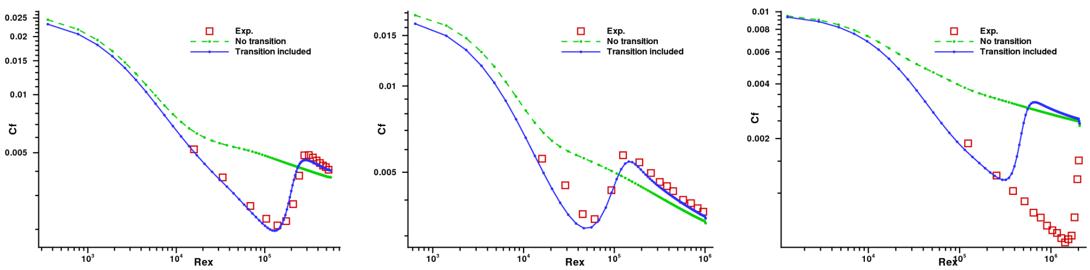
Constant value of free stream static pressure p and density ρ is considered. As the free stream turbulence is isotropic, the decay of turbulence level can be approximated as

$$Tu(\%) = 100\frac{\sqrt{\overline{u^2}}}{\overline{U}} = C(1000x + 610)^{-5/7} \tag{11}$$

with values of C given in Tab.1. The equation (11) was used to determine inlet boundary conditions for k and ω. The other conditions are given in Tab.1 as well.

Case	U_e (m/s)	Tu_o (%)	C
T3A	5.4	3.0	274
T3B	9.4	6.0	560
T3Aminus	19.8	0.9	120

Table 1: Free stream conditions for ERCOFTAC test cases



T3A T3B T3Aminus

Figure 1: Distribution of skin friction coefficient for ERCOFTAC test cases

The computations for test cases T3A and T3B give a good agreement with theoretical investigation and experimental data. But for test case T3Aminus with turbulence level Tu about 1%, the predicted transition onset starts sooner in comparison with experimental data. It can be caused by many factors. Besides free-stream turbulence and pressure gradient, the transition is influenced by surface roughness, noise and vibrations of the experimental facility. Further experimental data can be influenced by the used experimental technique and by the method of determining of the transition onset. Therefore a considerable scatter exits in experimental data used for empirical correlations.

5.2 SE1050 turbine cascade

The further test case represents the transonic flow through SE1050 blade cascade. The chosen blade cascade SE 1050 was designed for the last stage of an axial steam turbine of large output. The transonic flow has a relatively complex structure of the flow field especially in the exit part of the cascade [11]. Experiments on the SE 1050 blade cascade were carried out in the High-Speed Wind Tunnel of the Institute of Thermomechanics [12].

Stagnation values of the pressure, density and inlet angle are prescribed at the inlet. Mean value of the pressure is prescribed at the outlet. We also prescribe zero normal derivatives of the conserved variables at the inlet and outlet. No-slip boundary condition was used on the adiabatic wall. Suitable boundary conditions are chosen for the SST turbulence model.

The selected test case is characterized by the outlet isentropic Mach number $Ma_2 = 1.198$, the inlet angle $\alpha_1 = 19.3^o$ and the Reynolds number $Re = 1.5 \times 10^6$. We have chosen two different turbulence levels Tu = 1% and Tu = 3%.

The hybrid mesh consists of 27249 nodes and 42345 elements. Layers of quadrilateral elements are present along the blade, in the wake and along the outlet boundary (to prevent reflection of the out-running shock waves).

The computation was carried out for three modes: “laminar” ($\gamma \equiv 0$), with the transition model included, and “turbulent” ($\gamma \equiv 1$). The flow-fields depicted by the Mach number isolines are compared in Fig.2. The skin-friction distribution is shown in Fig.3. The difference between the laminar and turbulent shock wave/boundary layer interaction is clearly visible, both on Mach number isolines and the skin-friction distribution. The survey of loss coefficients is given in Tab.2. The problem with turbulence level of Tu = 3% didn't converge properly, due to limit-cycle type feedback between turbulence and transition model.

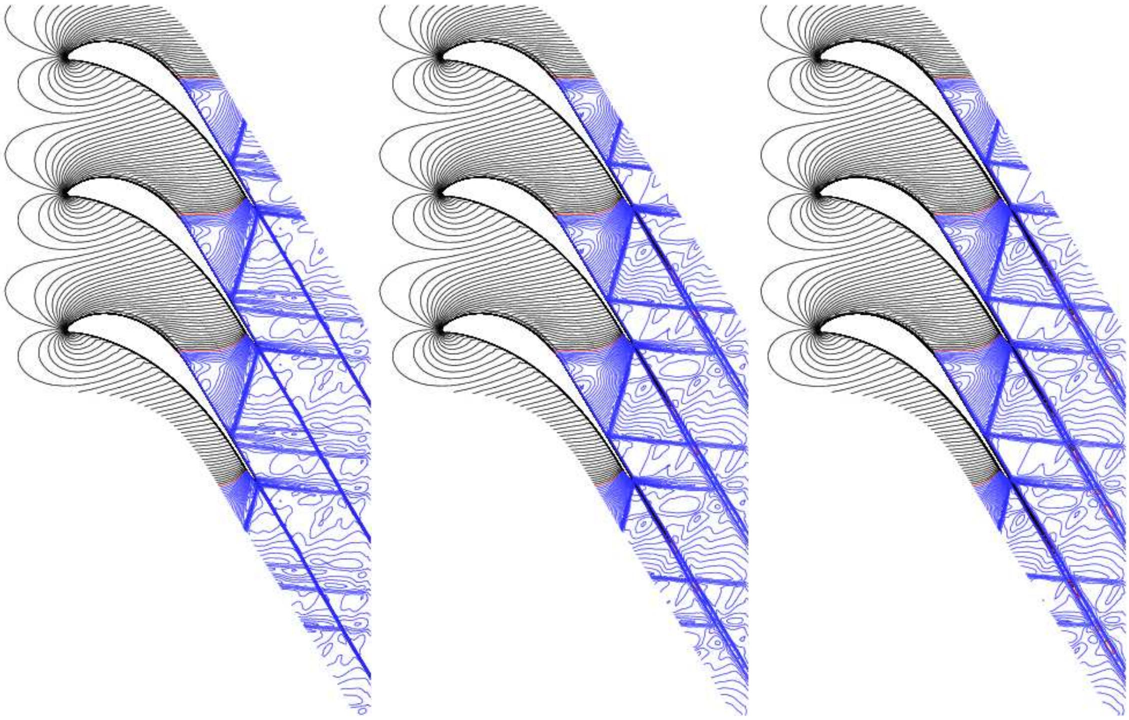
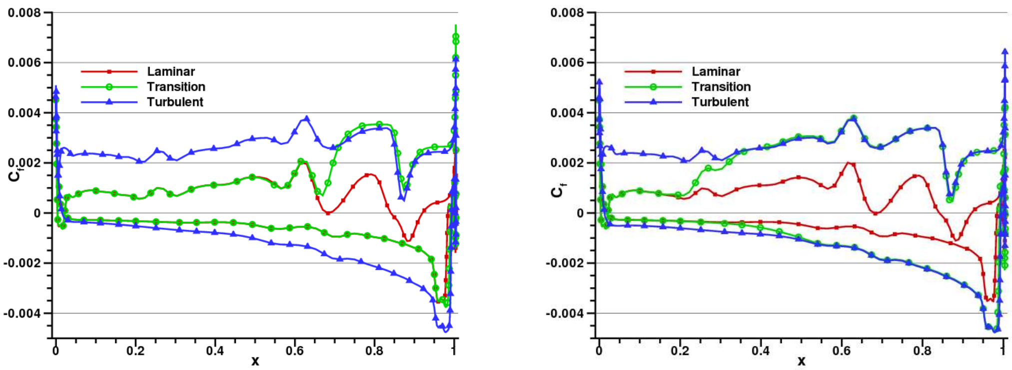


Figure 2: Mach number isolines for the SE1050 blade cascade, left to right: “laminar” computation ($\gamma \equiv 0$), transition model included, “turbulent” computation ($\gamma \equiv 1$)



a) Tu = 1% b) Tu = 3%

Figure 3: Distribution of the skin friction coefficient for the SE1050 blade cascade

	Loss coefficient ξ(%)	
	Tu = 1%	Tu = 3%
“Laminar” computation	1.3	1.3
Computation with transition model included	2.3	2.5
“Turbulent” computation	2.6	2.8
Experiment	4.5	

Table 2: Loss coefficient for the SE1050 blade cascade

6 Conclusions

The proposed algebraic bypass transition model was implemented into the unstructured finite volume method solver. The calculation procedure was validated for transitional flat-plate boundary layers with different free-stream turbulence level and for transonic flow through the SE1050 blade cascade. The transition in internal flows is influenced by many various factors and so the prediction of the bypass transition is the most problematic part of the whole calculation. Further progress can be achieved by the application of the transition model based on a transport equation for the intermittency coefficient.

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