SIMULATION OF THE FORCED AIRFOIL OSCILLATION WITH k-ω TURBULENT MODEL[1]

Jaroslav Pelant / Martin Kyncl
Výzkumný a zkušební letecký ústav, Praha

Abstract
This paper deals with numerical solution of the Navier-Stokes equations for turbulent flow. It describes numerical method, based on finite volume method, and special handeling of numerical boundary conditions for moving grids. The explicit time marching procedure is used. We show the analysis concerning boundary condition for the wall on move. It is based on the one-side modification of the Riemann problem and its solution. Suggested method was programmed and tested for the simulation of an airfoil oscillating with given amplitude and frequency around chosen point.

1 Formulation of the Navier-Stokes equations for turbulent flow

We consider the Navier-Stokes equations in the conservation form with dimensions. We apply the conservation law of mass-continuity, momentum, and energy for a volume element through which fluid is flowing. For the three-dimensional case this system of the Navier-Stokes equations has the form

$$\frac{\partial}{\partial t}q + \frac{\partial}{\partial x}f(q) + \frac{\partial}{\partial y}g(q) + \frac{\partial}{\partial z}h(q) - \left(\frac{\partial}{\partial x}r(q) + \frac{\partial}{\partial y}s(q) + \frac{\partial}{\partial z}d(q)\right) = 0 \tag{1}$$

where

$$q = (\varrho, \varrho u, \varrho v, \varrho w, e)$$

$$f(q) = (\varrho u, \varrho u^2 + p, \varrho uv, \varrho uw, (e+p)u)$$

$$g(q) = (\varrho v, \varrho vu, \varrho v^2 + p, \varrho vw, (e+p)v)$$

$$h(q) = (\varrho w, \varrho wu, \varrho wv, \varrho w^2 + p, (e+p)w)$$

$$r(q) = \left(0, \tau_{xx}, \tau_{xy}, \tau_{xz}, u\tau_{xx} + v\tau_{xy} + w\tau_{xz} + \left(\frac{\mu}{P_r} + \frac{\mu_T}{P_{r_T}}\right)\frac{\kappa\,\partial\varepsilon}{\partial x}\right)$$

$$s(q) = \left(0, \tau_{xy}, \tau_{yy}, \tau_{zy}, u\tau_{xy} + v\tau_{yy} + w\tau_{zy} + \left(\frac{\mu}{P_r} + \frac{\mu_T}{P_{r_T}}\right)\frac{\kappa\,\partial\varepsilon}{\partial y}\right)$$

$$d(q) = \left(0, \tau_{xz}, \tau_{yz}, \tau_{zz}, u\tau_{xz} + v\tau_{yz} + w\tau_{zz} + \left(\frac{\mu}{P_r} + \frac{\mu_T}{P_{r_T}}\right)\frac{\kappa\,\partial\varepsilon}{\partial z}\right)$$

$$\tau_{xx} = (\mu + \mu_T)\left(+\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\right) - \frac{2\varrho k}{3}$$

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$$\tau_{yy} = (\mu + \mu_T)\left(-\frac{2}{3}\frac{\partial u}{\partial x} + \frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\right) - \frac{2\varrho k}{3}$$

$$\tau_{zz} = (\mu + \mu_T)\left(-\frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} + \frac{4}{3}\frac{\partial w}{\partial z}\right) - \frac{2\varrho k}{3},$$

and

$$\tau_{xy} = \tau_{yx} = (\mu + \mu_T)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\tau_{yz} = \tau_{zy} = (\mu + \mu_T)\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$$

$$\tau_{zx} = \tau_{xz} = (\mu + \mu_T)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),$$

with p the pressure, ϱ the density, (u, v, w) vector of velocity, x, y, z the space coordinates, and t the time. Further, k denotes turbulent kinetic energy of flux components of the velocity, ω is the specific turbulent dissipation, P_r is laminar and P_{r_T} is turbulent Prandtl constant number, μ is the dynamic viscosity coefficient dependent on temperature, $\mu_T = \varrho k/\omega$ is the eddy-viscosity coefficient. In the energy equation, e denotes the total energy

$$e = \varrho\varepsilon + \frac{1}{2}\varrho(u^2 + v^2 + w^2),$$

where $\varepsilon = p/\varrho(\kappa - 1)$ is the internal energy of a unit mass of the fluid with the constant $\kappa > 1$. The system of equations (1) is an open system for turbulent flow. If the turbulent kinetic energy $k = 0$, then the system of equations (1) is a closed system of the Navier-Stokes equations for a laminar flow. If $k = 0$ and $\mu = 0$, then (1) are the Euler equations. The system studied (1) can be rewritten into the differential symbolic form

$$\frac{\partial \alpha_i}{\partial t} + \frac{\partial \beta_i}{\partial x} + \frac{\partial \gamma_i}{\partial y} + \frac{\partial \delta_i}{\partial z} = 0$$

and in the integral form it reads

$$\int_{\Delta t} \mathrm{d}t \int_{\partial\Omega} ((\beta_i, \gamma_i, \delta_i), \mathbf{n})\mathrm{d}s = -\int_{\Delta t} \mathrm{d}t \int_{\Omega} \frac{\partial \alpha_i}{\partial t}\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z,$$

where $i = 1, 2, 3, 4, 5$, Ω is from the space $\mathrm{R}^3(x, y, z)$. (,) denotes the scalar product. $\mathbf{n}$ is a normal vector to the surface $\partial\Omega$. The positive orientation is given by the outward direction. Here s is the integral measure in the surface $\partial\Omega$. Using the integral form we can study a flow with shock waves, too. For example we can apply the one-dimensional system of the same equations for the predictor of the numerical method at the special points of a chosen grid in the region Ω.

2 The k - ω turbulence model

Turbulent model is described by following equations

$$\frac{\partial \varrho k}{\partial t}+\frac{\partial \varrho k u}{\partial x}+\frac{\partial \varrho k v}{\partial y}+\frac{\partial \varrho k w}{\partial z}=P_k-\beta^* \varrho \omega k+\frac{\partial}{\partial x}\left(\left(\mu+\sigma_k \mu_{T}\right)\frac{\partial k}{\partial x}\right)+$$
$$+\frac{\partial}{\partial y}\left(\left(\mu+\sigma_k \mu_{T}\right)\frac{\partial k}{\partial y}\right)+\frac{\partial}{\partial z}\left(\left(\mu+\sigma_k \mu_{T}\right)\frac{\partial k}{\partial z}\right) \tag{2}$$
$$\frac{\partial \varrho \omega}{\partial t}+\frac{\partial \varrho \omega u}{\partial x}+\frac{\partial \varrho \omega v}{\partial y}+\frac{\partial \varrho \omega w}{\partial z}=P_\omega-\beta^* \varrho \omega^2+\frac{\partial}{\partial x}\left(\left(\mu+\sigma_\omega \mu_{T}\right)\frac{\partial \omega}{\partial x}\right)+$$
$$+\frac{\partial}{\partial y}\left(\left(\mu+\sigma_\omega \mu_{T}\right)\frac{\partial \omega}{\partial y}\right)+\frac{\partial}{\partial z}\left(\left(\mu+\sigma_\omega \mu_{T}\right)\frac{\partial \omega}{\partial z}\right)+C_D, \tag{3}$$

where k the turbulent kinetic energy and ω the turbulent dissipation are functions of time t and space coordinates x, y, z. The production terms P_k and P_ω are given by formulas

$$P_k=\tau_{xx}\frac{\partial u}{\partial x}+\tau_{xy}\frac{\partial u}{\partial y}+\tau_{yx}\frac{\partial v}{\partial x}+\tau_{yy}\frac{\partial v}{\partial y}+\tau_{xz}\frac{\partial u}{\partial z}+\tau_{zx}\frac{\partial w}{\partial x}+\tau_{yz}\frac{\partial v}{\partial z}+\tau_{zy}\frac{\partial w}{\partial y}+\tau_{zz}\frac{\partial w}{\partial z},$$

where functions τ are defined in Chapter 1 for $\mu = 0$.

$$P_\omega=\frac{\alpha_\omega \omega P_k}{k}, \text{ where } \alpha_\omega=\frac{\beta}{\beta^*}-\frac{\sigma_\omega \kappa^2}{\sqrt{\beta^*}} \text{ and } \sigma_k,\ \beta^*,\ \beta,\ \sigma_\omega \text{ a } \kappa \text{ are constants.}$$

C_D is defined as

$$C_D=\sigma_d\,\frac{\varrho}{\omega}\,\max\left\{\frac{\partial k}{\partial x}\frac{\partial \omega}{\partial x}+\frac{\partial k}{\partial y}\frac{\partial \omega}{\partial y}+\frac{\partial k}{\partial z}\frac{\partial \omega}{\partial z},0\right\},$$

where σ_d is constant. This turbulent model k-ω (2), (3) with equations (1) presents closed system of equations.

3 Modification of the Riemann problem for turbulent flow for one-dimensional space

In this Chapter we reduce the system of equations (1) to the form

$$\frac{\partial \varrho}{\partial t}+\frac{\partial u \varrho}{\partial x}=0$$
$$\frac{\partial \varrho u}{\partial t}+\frac{\partial (p+\varrho u^2)}{\partial x}+\frac{\partial}{\partial x}\left(\frac{2\varrho k}{3}\right)=0$$
$$\frac{\partial \varrho(\varepsilon+\frac{1}{2}u^2)}{\partial t}+\frac{\partial(\varrho u(\varepsilon+\frac{1}{2}u^2)+pu)}{\partial x}+\frac{\partial}{\partial x}\left(\frac{2\varrho k u}{3}\right)=0.$$

It is convenient to define abstract term of pressure $\tilde{p}=p+2k\varrho/3$. Using p instead of letter $\tilde{p}$, then the mentioned equations have the following form

$$\frac{\partial \varrho}{\partial t}+\frac{\partial u \varrho}{\partial x}=0 \tag{4}$$

$$\frac{\partial \varrho u}{\partial t} + \frac{\partial (p + \varrho u^2)}{\partial x} = 0 \tag{5}$$

$$\frac{\partial \varrho(\varepsilon + \frac{1}{2}u^2)}{\partial t} + \frac{\partial (\varrho u(\varepsilon + \frac{1}{2}u^2) + pu)}{\partial x} = 0. \tag{6}$$

We are interested in the solution of this system in an arbitrary small neighbourhood round a chosen point with given initial condition formed by two different states. This is so-called Riemann problem for 1D Euler equations. There exists entropy weak solution to this problem, see [1]. Solution of this problem leads to nonlinear system of equations, one cannot express the solution in closed form, though it is possible to evaluate the solution with chosen accuracy. Mentioned equations can be seen in [1], [2],[3].

4 Boundary conditions on a wall on the move

Let's suppose boundary on move $\partial\Omega$, and let us choose point X belonging to this boundary. Let the move of the chosen point X be defined by vector of velocity U, V, W at time t. Here we can suppose, that axis x and U are in the direction of outer normal to $\partial\Omega$, as the system studied is invariant to rotation. We neglect tangential derivatives and analogously to thoughts in Chapter 3 we deal with one-dimensional problem (4), (5), (6). The turbulent kinetic energy is $k = 0$ on the fix wall on the move. The limit state values $p_1, \varrho_1, u_1, v_1, w_1$ from the inside of the region Ω are known. This represents one-side initial condition for our problem. This condition is not enough to get unique (weak) solution of the local problem. Further, we consider the vector of velocity U, V, W as the boundary condition at the point X if it is possible. Our aim is the reconstruction of the state values at the boundary point X in time. State values in an arbitrary small neighbourhood of the point (X, t) fulfill a non–linear system of algebraic equations mentioned in Chapter 3, which are rewritten equations in the limit form at (X, t), equations are derived in cf. [1], [2]. Solving this non–linear system for given p_1, ϱ_1, u_1 and U it is possible to compute state values P – pressure and R – density at the point (X, t). For $U \leq u_1$ we obtain a shock wave going in the direction into the region Ω. The pressure P behind the shock wave is a solution of the equation

$$-U + u_1 = \frac{P - p_1}{\sqrt{\varrho_1 \left(\frac{\kappa+1}{2}P + \frac{\kappa-1}{2}p_1\right)}}.$$

The solution is

$$P = \frac{1}{2}\left(2p_1 + \frac{\kappa+1}{2}\varrho_1(u_1 - U)^2 \pm \sqrt{D_{is}}\right), \tag{7}$$

where

$$D_{is} = \left(2p_1 + \varrho_1\frac{\kappa+1}{2}(u_1 - U)^2\right)^2 - 4p_1^2 + 4(u_1 - U)^2\varrho_1\frac{\kappa-1}{2}p_1.$$

The inequality $p_1 \leq P$ must be fulfilled. The solution with the sign minus implies the inequality

$$8p_1\varrho_1\frac{\kappa-1}{2}(u_1 - U)^2 \leq 0,$$

which is impossible. So we got a unique solution (7) with the sign plus. The density behind the wave is

$$R = \frac{a_1\varrho_1}{a_1 - \varrho_1(u_1 - U)},$$

where

$$a_1 = \sqrt{\varrho_1\left(\frac{\kappa+1}{2}P + \frac{\kappa-1}{2}p_1\right)}.$$

The velocity of this wave is

$$D_1 = u_1 - \frac{a_1}{\varrho_1}.$$

Now, let $U > u_1$. In this case an expansion wave moves in the direction into the region Ω. The pressure P behind or inside of this wave is given by the equation

$$U = u_1 + \frac{2}{\kappa-1}c_1\left(1 - \left(\frac{P}{p_1}\right)^{\frac{\kappa-1}{2\kappa}}\right), \text{ where } c_1 = \sqrt{\frac{\kappa p_1}{\varrho_1}}.$$

The solution

$$P = p_1\left(\frac{-U + u_1 + \frac{2}{\kappa-1}c_1}{\frac{2}{\kappa-1}c_1}\right)^{\frac{2\kappa}{\kappa-1}}.$$

Further, it is necessary to determine the sound speed c_1^* for given velocity U from the equation

$$U = u_1 + \frac{2}{\kappa-1}c_1 - \frac{2}{\kappa-1}c_1^* \quad \text{or} \quad c_1^* = c_1\left(\frac{P}{p_1}\right)^{\frac{\kappa-1}{2\kappa}}.$$

The speeds of the front and the back of the expansion wave are $D_1 = u_1 - c_1$, and $D_1^* = U - c_1^*$. Sought density behind rarefaction wave is $R = \varrho_1(\frac{P}{p_1})^{\frac{1}{\kappa}}$.

5 Numerical time-step method on moving grid for 2D

Let a quadrangle grid be given by points $(x_{j,k}, y_{j,k}) j = 1,..,J, k = 1,..,K$ in a plane at time t. This grid is dependent on time. Points $(xx_{j,k}, yy_{j,k})$ define the other grid with the same indexes at time $t+\tau$. All the points define 3D space cells $\Omega_{j,k}$ from space $\mathrm{R}^3(\mathrm{t,x,y})$. To show the principle of this method we choose an arbitrary cell of the time-space grid. For the sake of simplicity this cell is denoted

$$\Omega = ((x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), (xx_1, yy_1), (xx_2, yy_2), (xx_2, yy_3), (xx_4, yy_4)).$$

Now we want to use equations (1), (2), (3) in the symbolic form

$$\int\!\!\int_{\partial\Omega}((\alpha_i, \beta_i, \gamma_i), \mathbf{n})\mathrm{d}s = \int\!\!\int\!\!\int_{\Omega} f_i(x,y,t)\mathrm{d}x\mathrm{d}y\mathrm{d}t, \tag{8}$$

where $i = 1,2,3,4,5,6$. Ω is from the space $\mathrm{R}^3(t,x,y)$. (,) denotes the scalar product. $\mathbf{n}$ is a normal vector to $\partial\Omega$. The positive orientation is given by the outward direction. Here s is the integral measure in the surface $\partial\Omega$. Using the integral form we can study a flow with shock waves, too. For example we can apply one dimensional system of the same equations for the predictor of the numerical method at the special points of a chosen grid in the region Ω for special two-dimensional case. Now we want to use equation (8) for this special cell Ω. So, let Ω_d be the lower side of Ω at time t, Ω_u be the upper side at time $t+\tau$ and Ω_f, Ω_r, Ω_l, Ω_h are other sides called walls. Integral equation (8) has the special form

$$\alpha_i^u\|\Omega_u\| - \alpha_i^d\|\Omega_d\| + Q_f + Q_r + Q_l + Q_h = \int\!\!\int\!\!\int_{\Omega} f_i(x,y,t)\mathrm{d}x\mathrm{d}y\mathrm{d}t, \tag{9}$$

where

$$\|\Omega_d\| = \frac{1}{2}((x_3 - x_1)(y_4 - y_2) - (x_4 - x_2)(y_3 - y_1)) = \Omega(x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4),$$

and analogously

$$\|\Omega_u\| = \Omega(xx_1, yy_1, xx_2, yy_2, xx_3, yy_3, xx_4, yy_4)$$

$$\|\Omega_r\| = \Omega(x_2, y_2, xx_2, yy_2, xx_1, yy_1, x_1, y_1)$$

$$\|\Omega_f\| = \Omega(x_1, y_1, xx_1, yy_1, xx_4, yy_4, x_4, y_4)$$

$$\|\Omega_h\| = \Omega(x_3, y_3, xx_3, yy_3, xx_2, yy_2, x_2, y_2)$$

$$\|\Omega_l\| = \Omega(x_4, y_4, xx_4, yy_4, xx_3, yy_3, x_3, y_3)$$

$$Q_r = \alpha_i^r \|\Omega_r\| + \beta_i^r \tau(\bar{y}_2 - \bar{y}_1) - \gamma_i^r \tau(\bar{x}_2 - \bar{x}_1)$$

$$Q_f = \alpha_i^f \|\Omega_f\| + \beta_i^f \tau(\bar{y}_1 - \bar{y}_4) - \gamma_i^f \tau(\bar{x}_1 - \bar{x}_4)$$

$$Q_h = \alpha_i^h \|\Omega_h\| + \beta_i^h \tau(\bar{y}_3 - \bar{y}_2) - \gamma_i^h \tau(\bar{x}_3 - \bar{x}_2)$$

$$Q_l = \alpha_i^l \|\Omega_l\| + \beta_i^l \tau(\bar{y}_4 - \bar{y}_3) - \gamma_i^l \tau(\bar{x}_4 - \bar{x}_3),$$

$$\text{where } \bar{x}_i = \frac{x_i + xx_i}{2}, \ \bar{y}_i = \frac{y_i + yy_i}{2}.$$

and the upper index at α_i, β_i, γ_i means the value on a side of the same notation. Using equation (9) for α_i^u, it is possible to obtain state values p, ϱ, u, v, k, ω at the centre of Ω_u if other values are known at other centres of five sides. The state values for Ω_d are known from last time step t The main problem is to obtain state values on the sides Ω_f, Ω_h, Ω_l, Ω_r called walls. Any definition cannot keep the laws of conservation. Let us choose one wall. If this wall is inside the grid, then we can define state values from either side of the wall in the middle of the segment at time t. For this case it is possible to apply the result of Chapter 3. If the mentioned wall is on the boundary, we can apply solution of the boundary-initial Riemann problem. Chapter 4 shows analysis of this problem for the wall on the body surface. Boundary problems for inlet and outlet walls were analyzed in [2]. Chapter $3, 4$ can be used because of properties of system (1) in the space $\mathrm{R}^3(x, y, z)$, which is invariant with respect to arbitrary rotation matrix transformation. Time step τ for each wall is bounded by elementary shocks or expansion waves coming from opposite walls. We know velocities of those waves using Chapter 3, 4 or [2], . The last problem which is necessary to explain is how to define state values on either side at the centre of a common boundary of two cells when state values are known at the centres of all the cells at the same time t. For this case it is possible to apply some different schemes. But those schemes must lead to the highest-accuracy yielding a third-order truncation error. We used scheme with so-called Van-Albada limiter.

6 Modification of the production terms P_k, and P_ω, and time steps for two-equation models of turbulence

A shock may cause problems in computations using standard eddy-viscosity two-equation models. To avoid that effect in eddy-viscosity models the production may be limited. So the production term P_k can be limited with the inequality

$$|P_k| \le P_{lim} = \varrho\, k \sqrt{\sum_{i,j=1}^{3} S_{ij} S_{ji}} \quad ,$$

where

$$S_{11} = \frac{1}{2}\left(\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial w}{\partial z}\right)$$

$$S_{22} = \frac{1}{2}\left(\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial w}{\partial z}\right)$$

$$S_{33} = \frac{1}{2}\left(\frac{4}{3}\frac{\partial w}{\partial z} - \frac{2}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right)$$

$$S_{12} = S_{21} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$S_{13} = S_{31} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$S_{23} = S_{32} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right).$$

The new productions

$$\tilde{P}_k = min(P_k, P_{lim})$$

and

$$\tilde{P}_\omega = \frac{\alpha_\omega \omega \tilde{P}_k}{k}.$$

Modificated time step $\Delta t_{k\omega}$ of a time step Δt received during a computation of the Navier-Stokes equations with respect to the transported quantity of k or ω in the turbulent model is given by the equation

$$\Delta t_{k\omega} = \frac{\Delta t}{1 - \Delta t\ min(\frac{R(\varrho k)}{\varrho k}, \frac{R(\varrho \omega)}{\varrho \omega}, 0)},$$

where symbol R means $R(q) = \frac{\partial q}{\partial t}$.

Initial conditions for k and ω are defined in the following formulae. The kinetic turbulent energy

$$k_\infty = 10^{-6}(u_\infty^2 + v_\infty^2 + w_\infty^2) = 10^{-6}\lambda_\infty^2 c_*^2,$$

where λ_∞ is Laval number of the free flow. The turbulent dissipation

$$\omega_\infty = \frac{\varrho_\infty k_\infty}{10^{-2}\mu_\infty},$$

where ϱ_∞ is density and μ_∞ is dynamic viscosity for the free flow.

$$\mu_\infty = 1.716 * 10^{-5} \left(\frac{T_\infty}{T_Z}\right)^{\frac{2}{3}} \frac{T_Z + T_S}{T_\infty + T_S},$$

where T_∞ is the temperature

$$T_\infty = T_O \left(1 - \frac{\kappa - 1}{\kappa + 1}\lambda_\infty^2\right),$$

$T_Z = 273.15$, $T_S = 110.6$, and T_O is total temperature.

The physical wall boundary conditions on k and ω are

$$k = 0, \quad \frac{\partial k}{\partial y} = 0,$$

and

$$\omega = const\ \frac{6\mu}{\beta \varrho y_c^2},$$

where y is the wall distance in the wall-normal direction, and y_c is the wall normal distance to the first cell centre, and $const = 120$.

7 Numerical results

Suggested method was programmed and paralelization was used. Figure 1 demonstrates 2D turbulent flow computation around oscillating airfoil. We chose NACA0012 airfoil oscillating round point [0,03;0] with 30Hz frequency and amplitude $\pm 2^o$. Geometry shown at Figure 1, fluid flows from left side towards right. Figures 1,2 illustrates solution after 18.900.000 iterations. Computational mesh consisted of 256x60 quadrilaterals. The solution after 78.000.000 iterations is shown at 3,4. Average statical pressure $p_r = 66471.3904802231$ was chosen at the outlet. Boundary condition at the inlet part was composed of total pressure $p_o = 101325$, temperature $T_o = 273.15$ and component of velocity $v_{tan} = 0$ tangential to boundary.

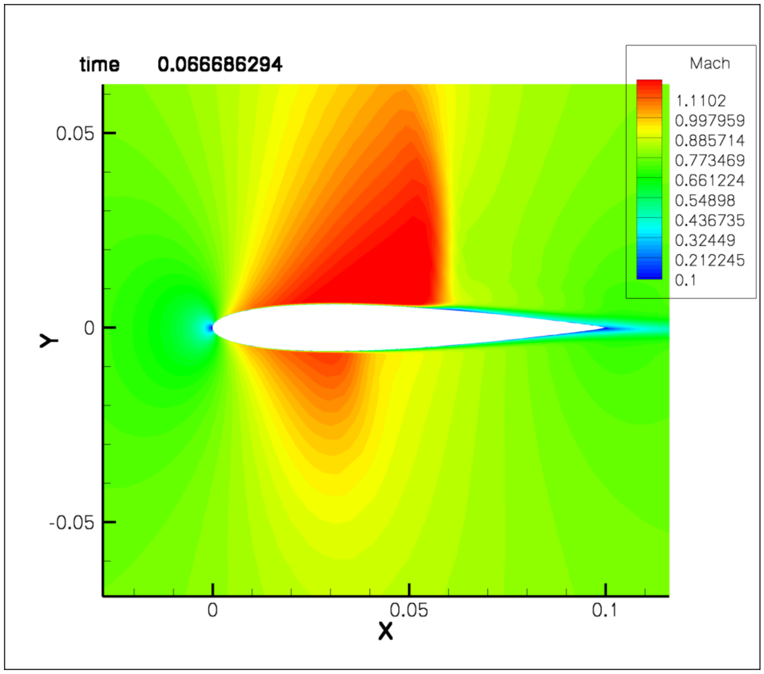

Figure 1: Nonstationary turbulent flow, Mach number isolines, solution after 18.900.000 iterations

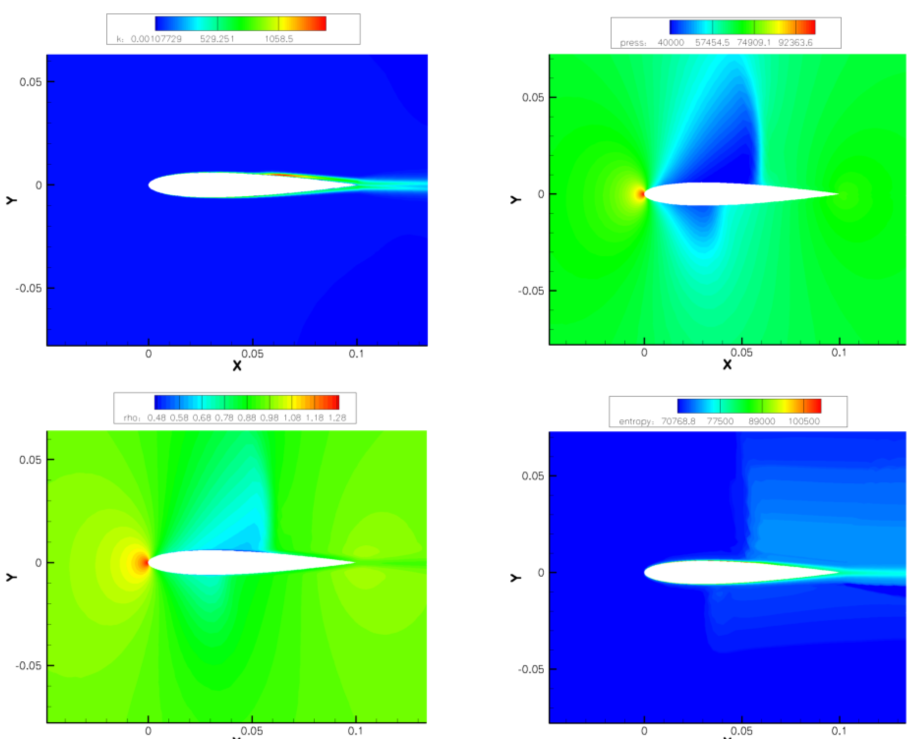

Figure 2: Nonstationary turbulent flow, k, pressure, density and entropy isolines, solution after 18.900.000 iterations

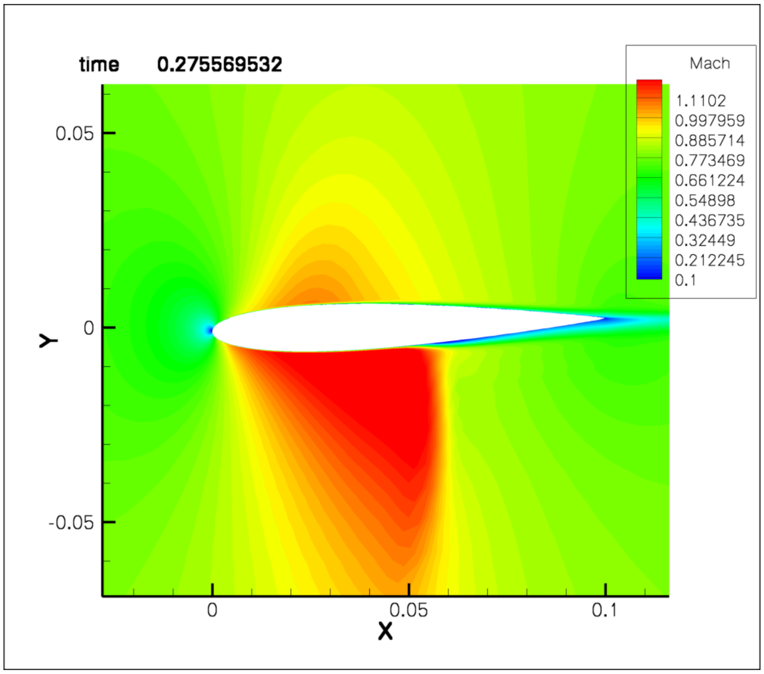

Figure 3: Nonstationary turbulent flow, Mach number isolines, solution after 78.000.000 iterations

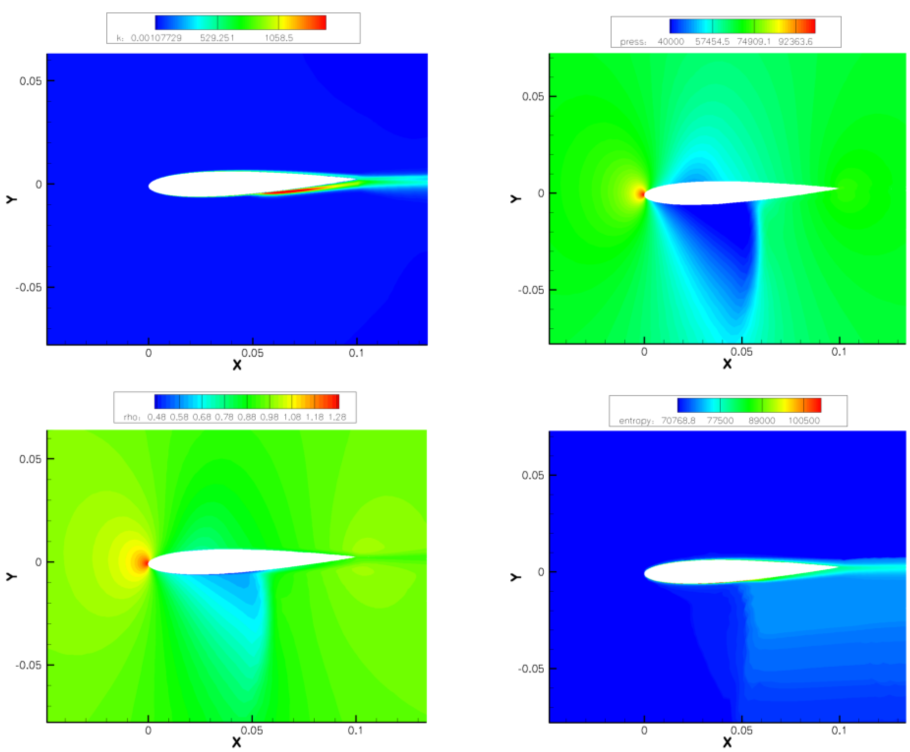

Figure 4: Nonstationary turbulent flow, k, pressure, density and entropy isolines, solution after 78.000.000 iterations

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